GAUGE BOSON PAIRS at the LHC:

ELECTROWEAK CORRECTIONS and MONTE CARLO IMPLEMENTATION

J.H. Kühn

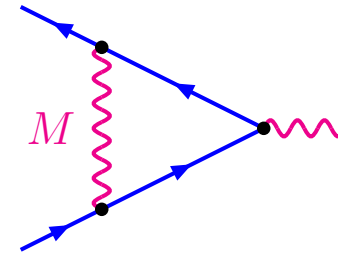**(with A. Bierweiler, S. Gieseke, T.Kasprzik, A. Penin, S. Uccirati)**

CERN, 6.12.2013

I. Introduction

"Typical" size of electroweak corrections: $\frac{\alpha_{\text{weak}}}{\pi} \approx 10^{-2}$

new aspects at LHC: $\sqrt{\hat{s}} \approx$ 1-2TeV $\gg M_{W,Z}$

strong enhancement of negative corrections

one-loop example: massive U(1) M

$$\Rightarrow \text{ Born} * \left[1 + \frac{\alpha}{4\pi} \left(-\ln^2 \frac{s}{M^2} + 3\ln \frac{s}{M^2} - \frac{7}{2} + \frac{\pi^2}{3} \right) \right]$$

$\frac{s}{M^2}$	$-\ln^2 \frac{s}{M^2}$	$+3\ln \frac{s}{M^2}$	$-\frac{7}{2} + \frac{\pi^2}{3}$	Σ	$* 4 \frac{\alpha_{\text{weak}}}{4\pi}$
$\left(\frac{1000}{80}\right)^2$	-25.52	$+15.15$	-0.21	-10.6	-13%
$\left(\frac{2000}{80}\right)^2$	-41.44	$+19.31$	-0.21	-22.3	-27%

(four-fermion cross section $\Rightarrow$ factor 4)

- leading $\log^2$ multiplied by $\text{(charge)}^2 = I(I+1) = \begin{cases} 3/4 & I = 1/2 \\ 2 & I = 1 \end{cases}$

  $\Rightarrow$ further enhancement for W-pairs by nearly factor 2.

- important subleading logarithms (NLL+...)

- One-loop up to $\mathcal{O}(40\%) \rightarrow$ two-loop terms may be relevant

- interplay between electroweak and QCD corrections

- important differences between fermions and electroweak gauge bosons

- important differences between long. and transverse gauge bosons ($I = 1/2$ vs. $I = 1$)

- Small transverse momenta, close to WW threshold!

  $\rightarrow$ complete calculation with masses required

Gauge Boson Pair Production: W^+W^-, $W^{\pm}Z$, ZZ, $\gamma\gamma$

Two Approaches:

- dominant, logarithmically enhanced terms via evolution equation & separation of QED
  $\Rightarrow$ one- and two-loop terms in NNLL
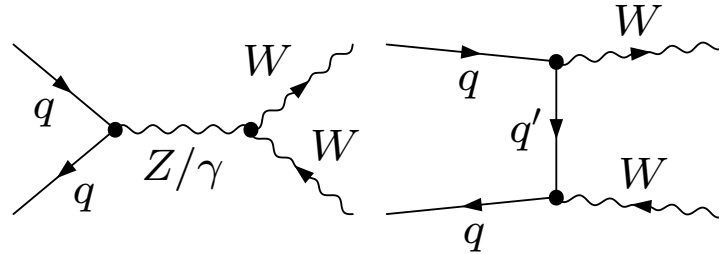 J.H.K., Metzler, Penin, Uccirati: JHEP 1106 (2011) 143
  related work based on SCET: Manohar,...

- one-loop calculation, including $M_W^2/\hat{s}$ terms and real radiation: full NLO
  Bierweiler, Kasprzik, J.H.K., Uccirati
  (JHEP 1211 (2012)093[arXiv:1208.3147], arXiv:1208.3404, arXiv:1305.5402)
  earlier work: logarithmically enhanced terms only, including W decays and off-shell effects: Accomando, Denner, Kaiser
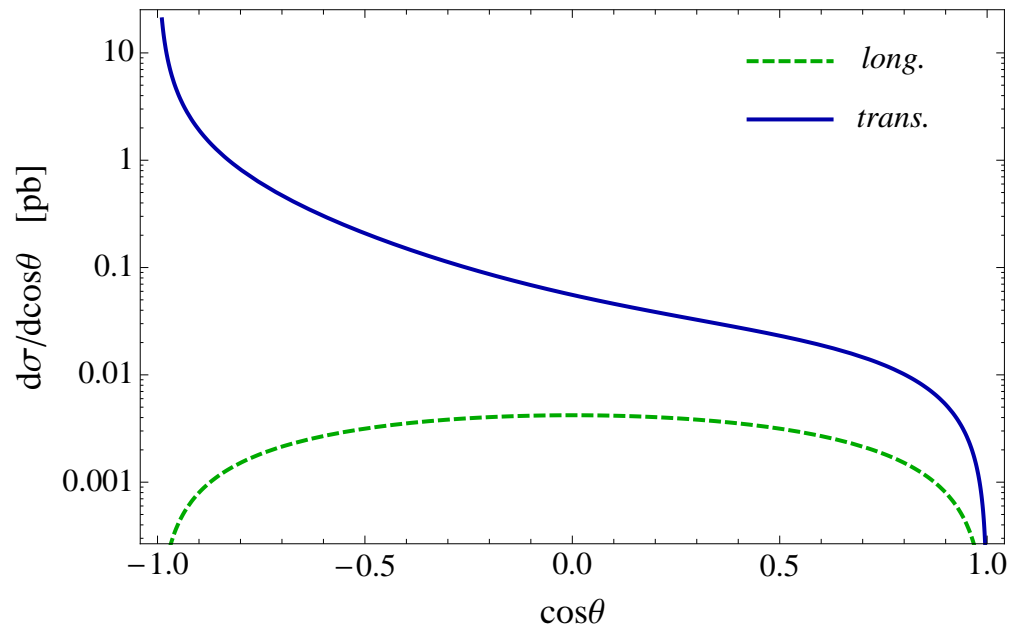
- Aim: Combine the two approaches
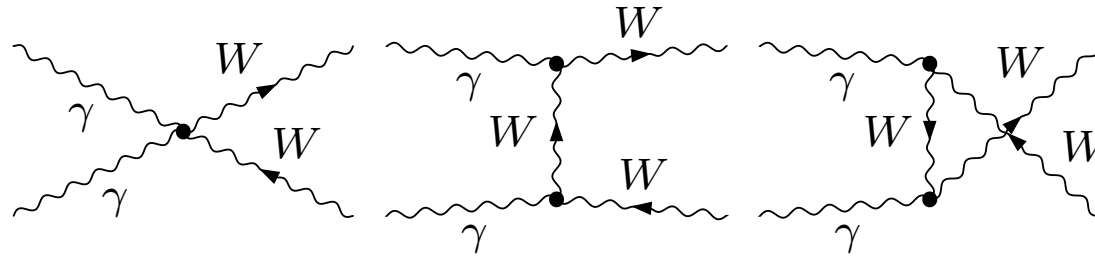 $\rightarrow$ Implementation in Monte Carlo

II. One-Loop Results for $q\bar{q}$-process

$$u\bar{u} \to W^+W^-, \ \sqrt{s} = 1 \text{ TeV}$$
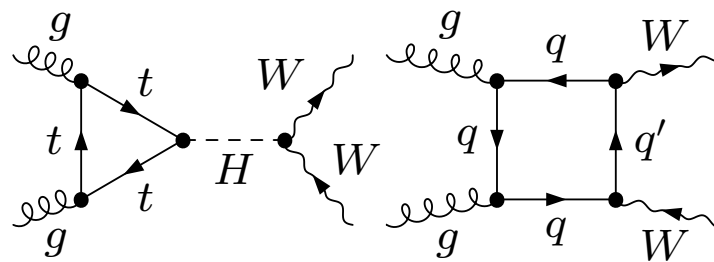

- Strong enhancement for $\Theta \to 180°$
- dominance of transverse W

Also included: $\gamma\gamma \to WW$

also included: $gg \to WW$

One Loop Corrections

- On-shell, G_μ-scheme (M_W, M_Z, G_μ)

- virtual and real photon radiation included
  remaining collinear singularities "absorbed" in PDFs
  ($q\bar{q}$: MSTW2008LO; $\gamma\gamma$: MRST2004QED/NNPDF2.3QED)

- two independent calculations

- NLO QCD recalculated
  $\rightarrow$ giant corrections from events with soft W and hard jet: jet or p_T-veto
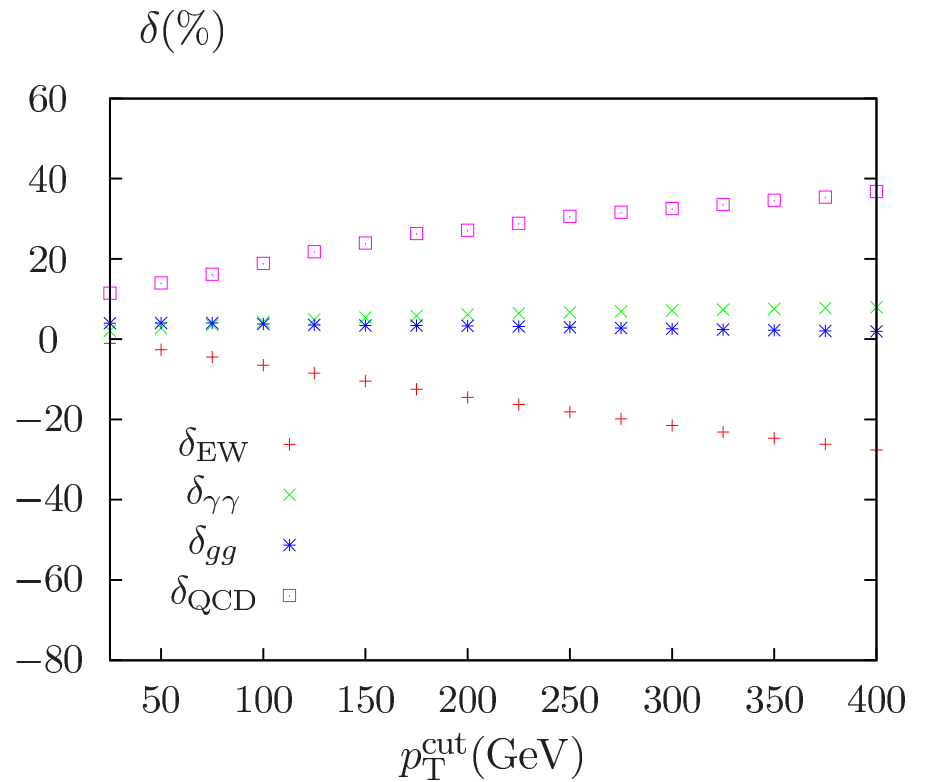 to select "balanced" pairs
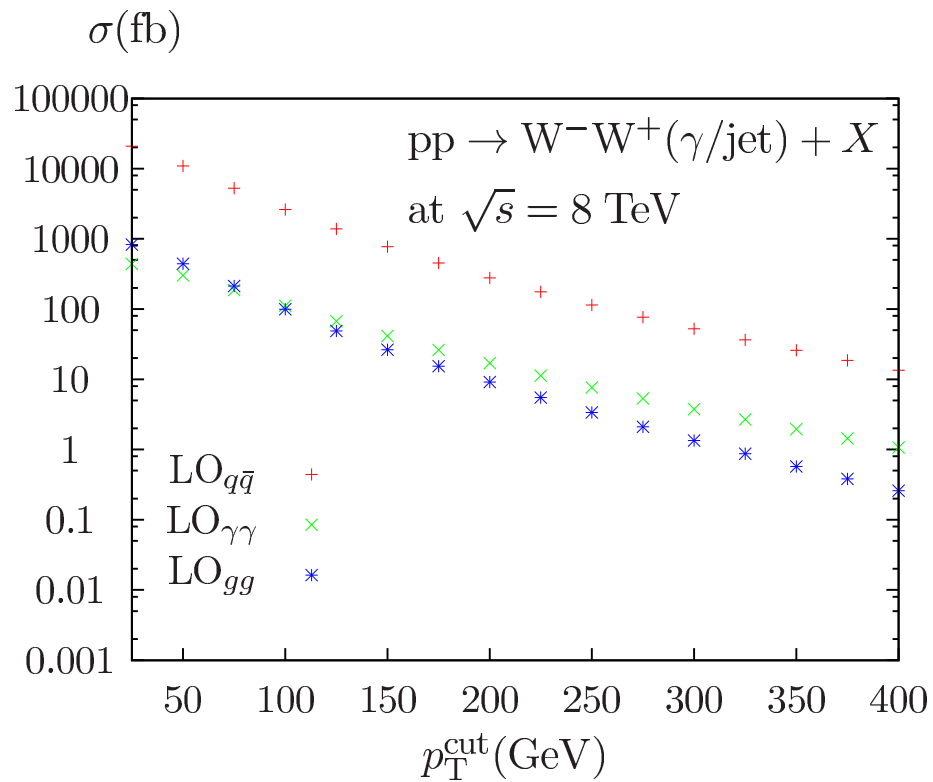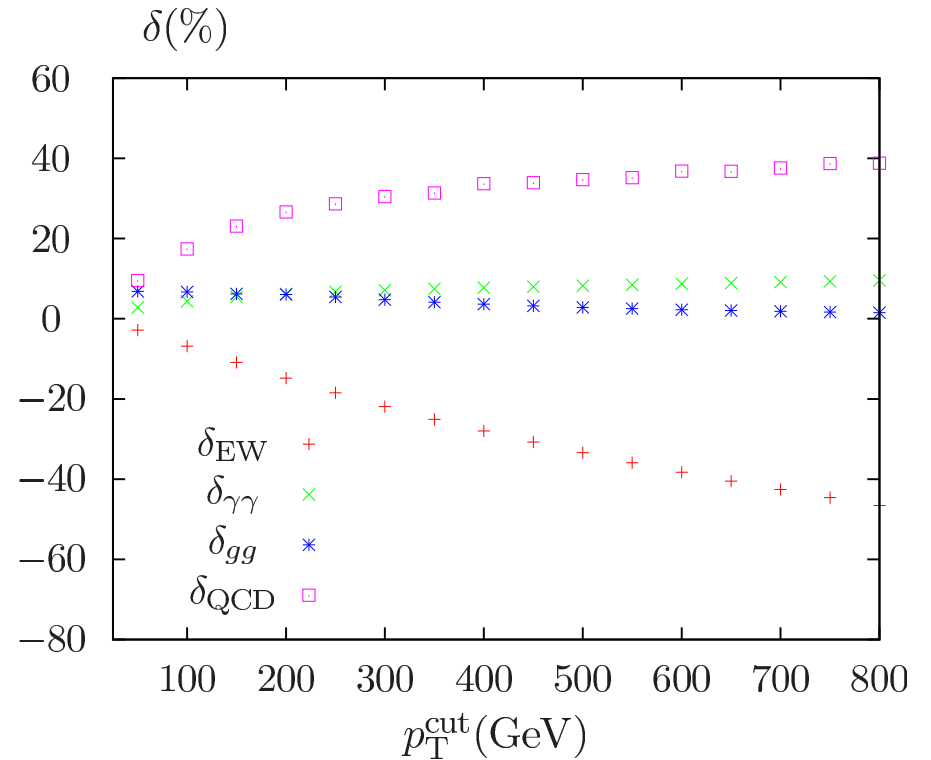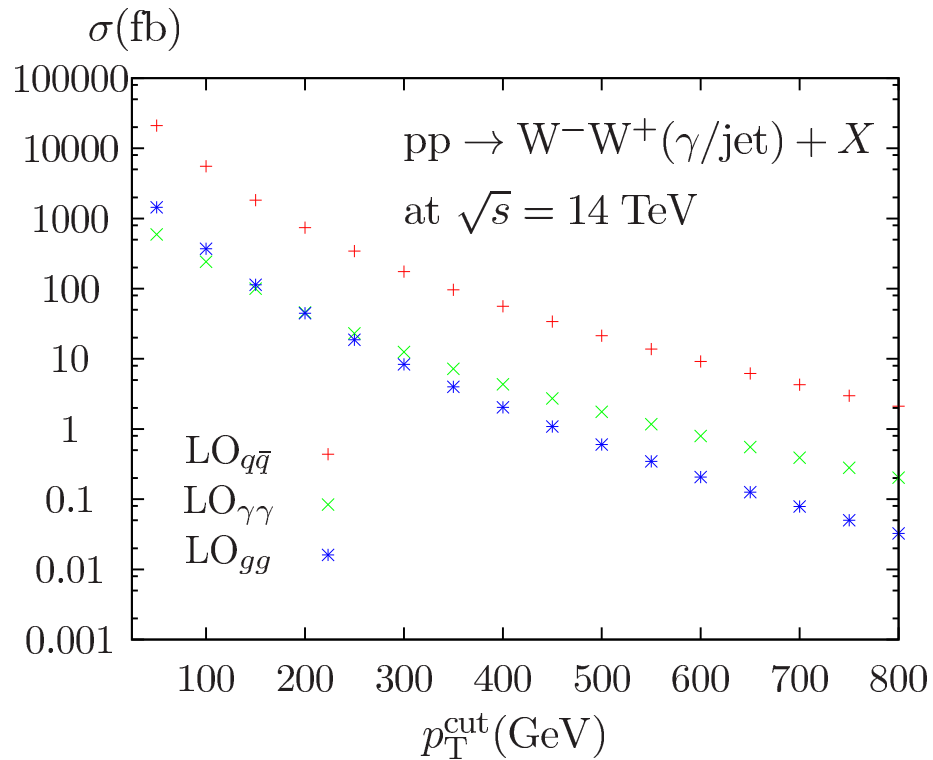
(related results: Baglio, Ninh, Weber)

- surprisingly large contribution from $\gamma\gamma \to WW$ at large $\hat{s}$ and **large** rapidity (small $\hat{t}$).

- $\sigma(\gamma\gamma \to W^+W^-) \to \frac{8\pi\alpha^2}{M_W^2}$

- large negative corrections at large $\hat{s}$ and small rapidity

- pronounced modification of angular distribution ($\to$ mimics anomalous couplings) (compare Accomando, Denner, Kaiser)

- sizable QCD corrections
  (disregard events with $p_{T,\text{jet}} > p_{T,W}/2$ for visible jets)

- combination of QCD and EW corrections? additive vs. multiplicative!

One-Loop Results

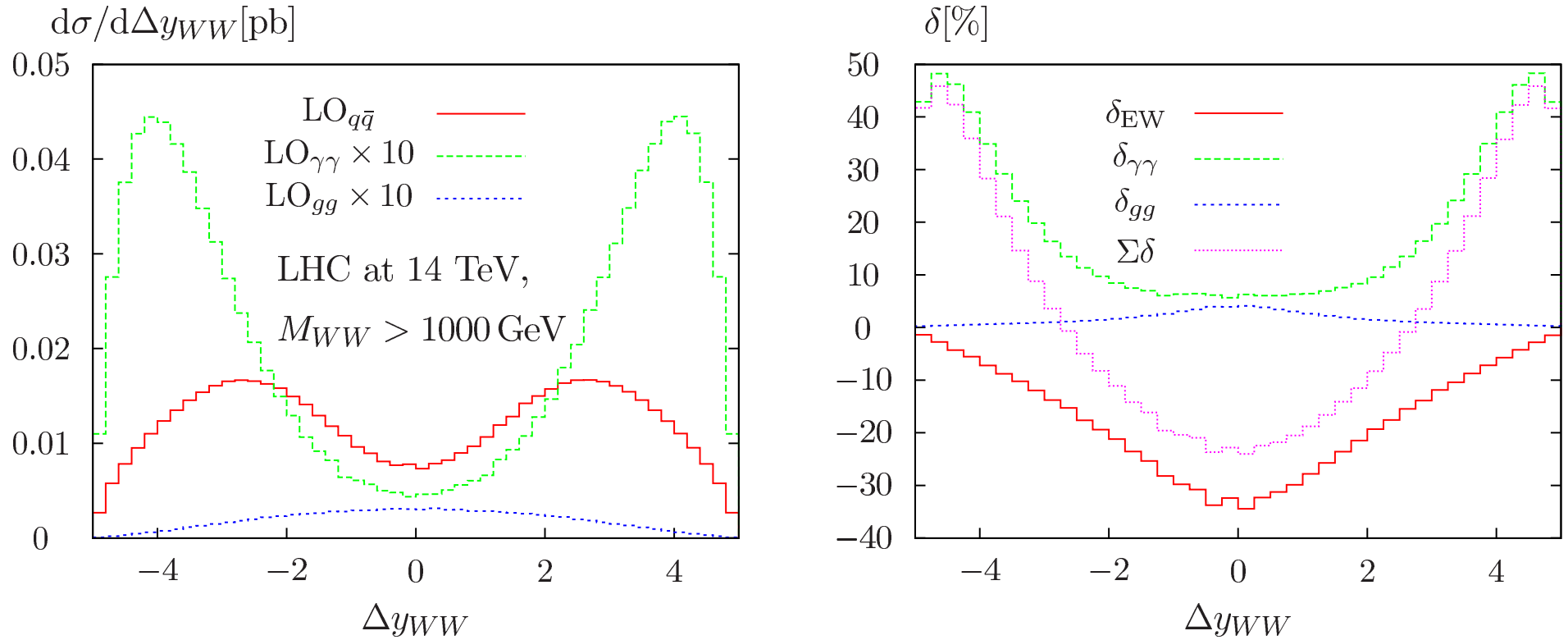**Total cross sections LHC8, $p_T > p_{T,\text{cut}}$ ($\int \mathcal{L}\mathrm{d}t = 25$ fb^{-1} per experiment)**

$\sigma(\text{fb})$

$\delta(\%)$

$\mathrm{pp} \to \mathrm{W}^-\mathrm{W}^+(\gamma/\mathrm{jet}) + X$

at $\sqrt{s} = 8$ TeV

$\mathrm{LO}_{q\bar{q}}$

$\mathrm{LO}_{\gamma\gamma}$

LO_{gg}

δ_{EW}

$\delta_{\gamma\gamma}$

δ_{gg}

δ_{QCD}

$p_{\mathrm{T}}^{\mathrm{cut}}(\text{GeV})$

$p_{\mathrm{T}}^{\mathrm{cut}}(\text{GeV})$

Total cross sections LHC14, $p_T > p_{T,\text{cut}}$
(Hope for $\int \mathcal{L}\mathrm{d}t = 300$ fb^{-1} per experiment)

assume 10% efficiency

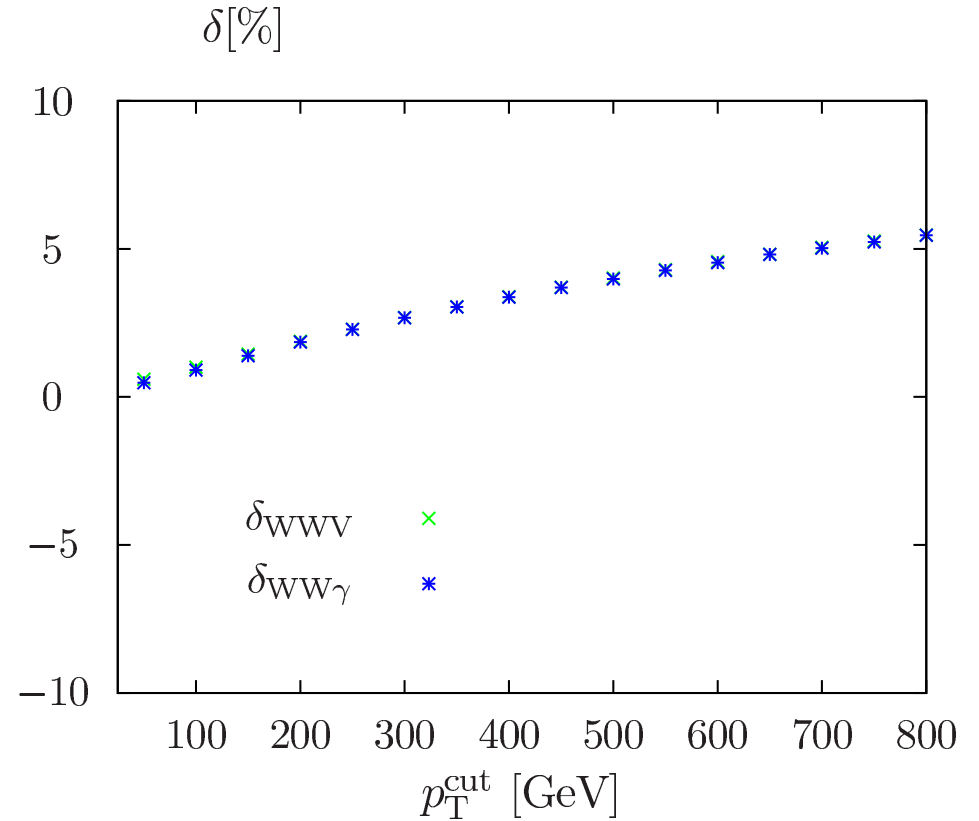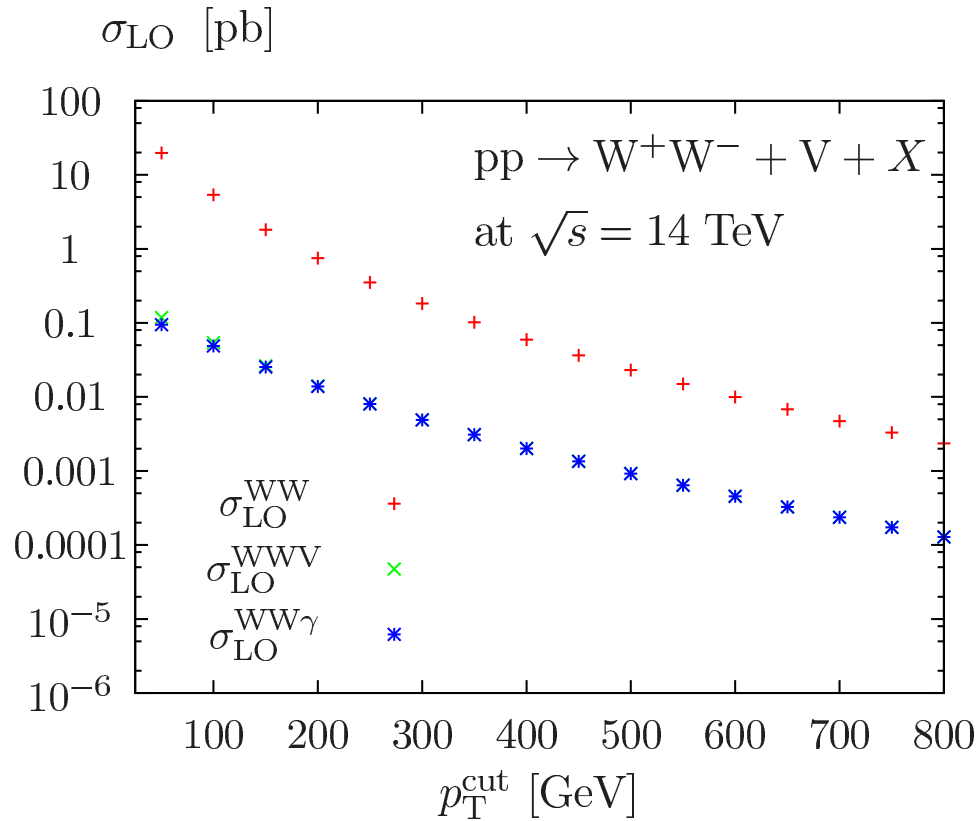$\rightarrow$ up to $p_T^{\text{cut}} \sim 800$ GeV

Differential LO cross sections for the W-boson rapidity gap with a minimal invariant mass of 1000 GeV at the LHC14. On the right-hand-side, the corresponding relative rates due to photon- and gluon-induced channels w.r.t. the $q\bar{q}$-contributions are shown, as well as the EW corrections.

Real W, Z Radiation: Compensation?

- soft and/or collinear radiation may (partly) compensate or overcompensate virtual corrections:

- model study (Bell, J.K., Rittinger arXiv:1004.4117; EPJC)
  strong dependence on cuts!
  asymptotic energies (multi-TeV)

- semi-realistic evaluation (on-shell W, Z) (Bierweiler, Kasprzik, J.K.)
  $q\bar{q} \longrightarrow W^+W^-(\gamma)$ (Born + one-loop)
  vs. $q\bar{q} \longrightarrow W^+W^-Z$
       $q\bar{q} \longrightarrow W^+W^-W^+$ + c.c.

- request large invariant mass of V-pair
  or large p_T of both Vs

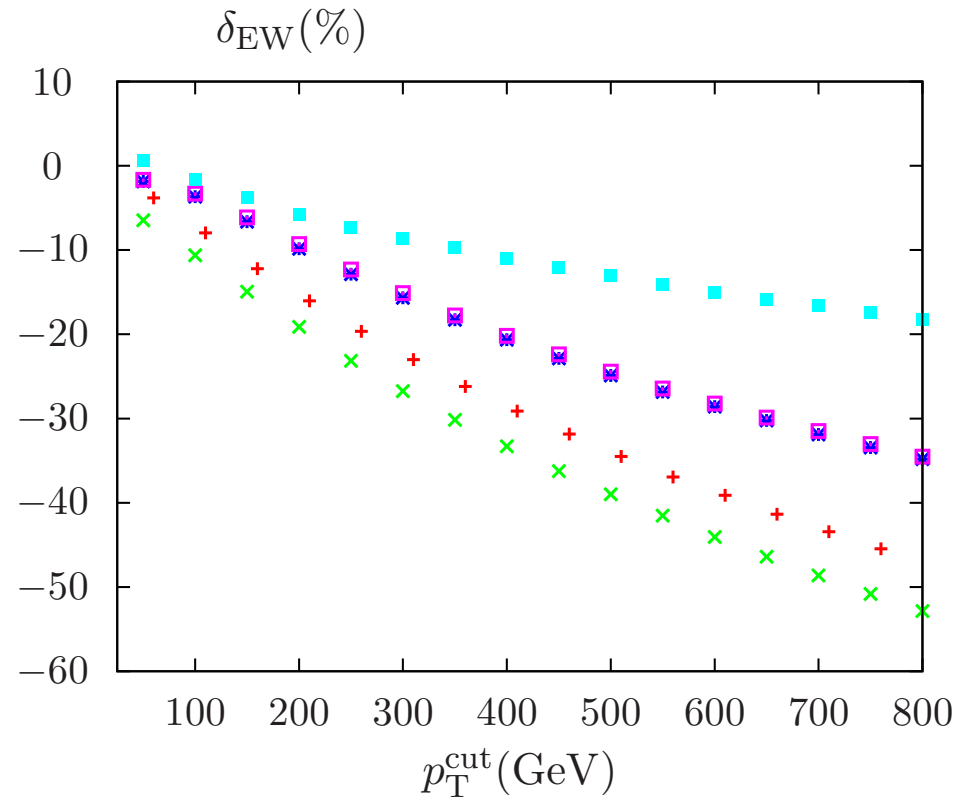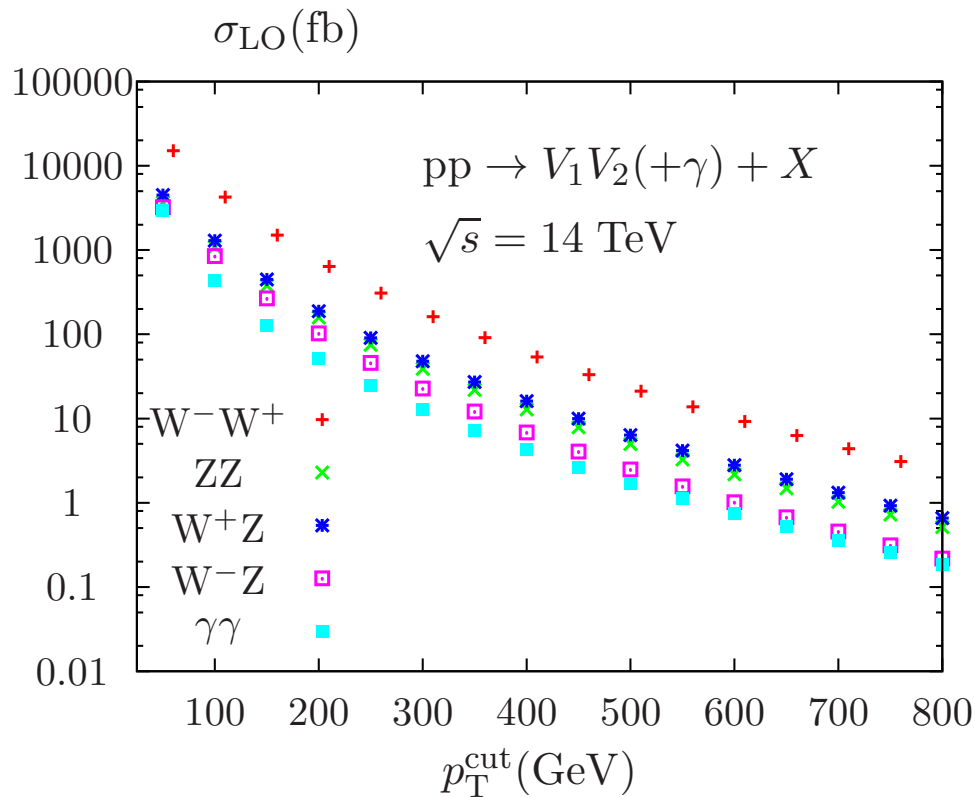Aim: real radiation taken care of by MC $\Rightarrow$ different final states:

$\rightarrow$ different angular distributions
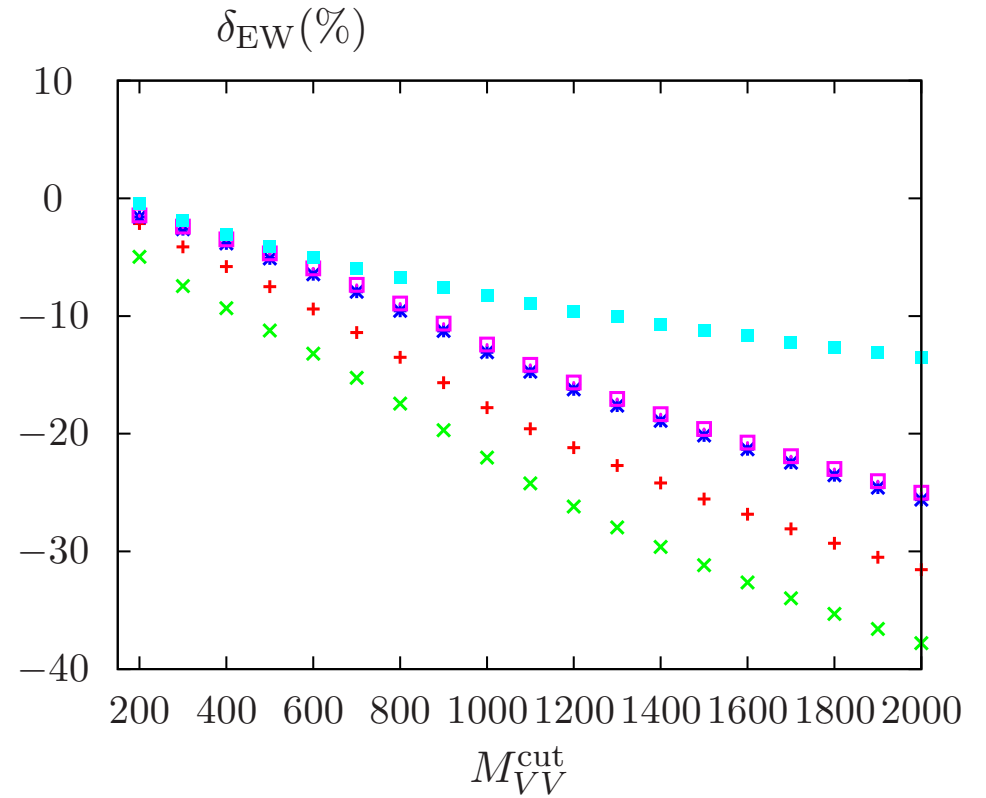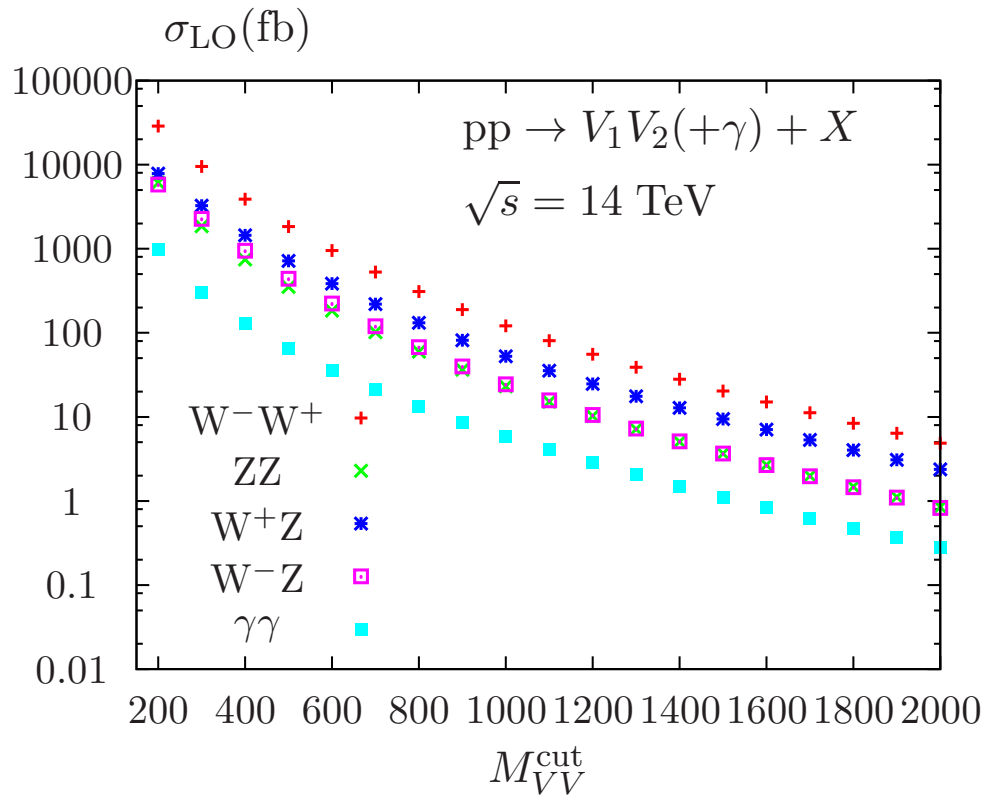$\quad W^+W^-$: $q\bar{q} \rightarrow W^+W^-Z$, $W^+W^-W^\pm$

($\delta_{VV\gamma}$ strongly depends on cuts; here: $p_{T,\gamma} > 15$ GeV, $|y_\gamma| < 2.5$)
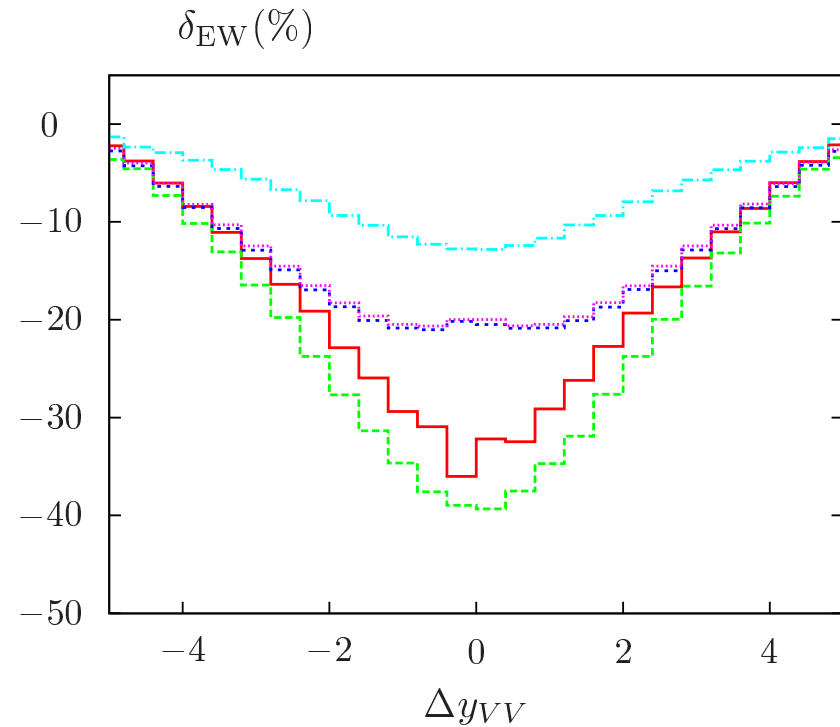
Similar setup, qualitative similar results

$\sigma_{\mathrm{LO}}(\mathrm{fb})$

$\mathrm{pp} \to V_1 V_2 (+\gamma) + X$

$\sqrt{s} = 14 \text{ TeV}$

$W^- W^+$
ZZ
$W^+ Z$
$W^- Z$
$\gamma\gamma$

$p_{\mathrm{T}}^{\mathrm{cut}}(\mathrm{GeV})$

$\delta_{\mathrm{EW}}(\%)$

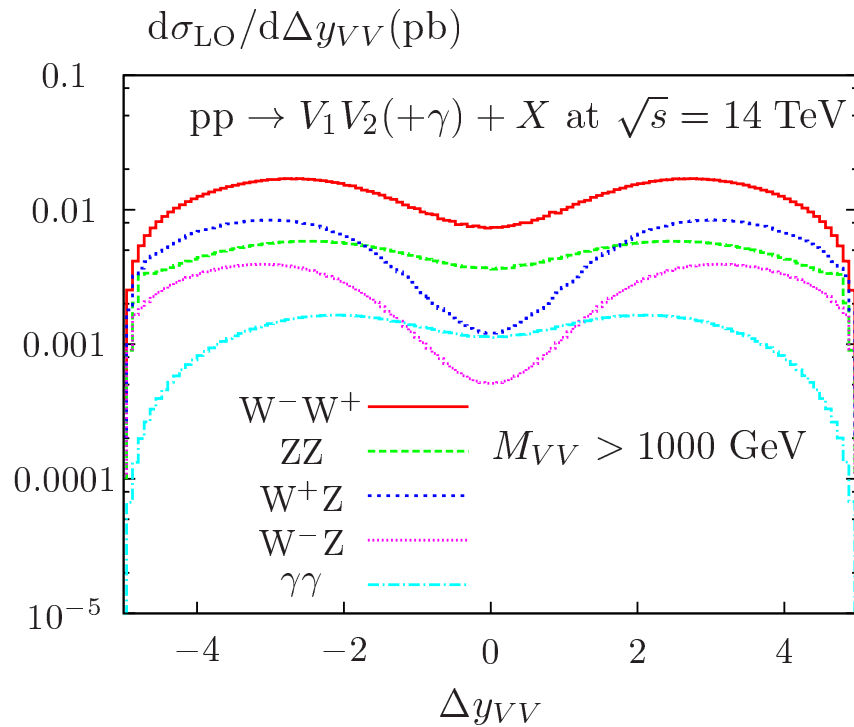$p_{\mathrm{T}}^{\mathrm{cut}}(\mathrm{GeV})$

particularly large corrections for ZZ at large p_{T}: $\sim -40\%$
(two-loop NNLL $\to$ IV)
small p_{T}: $\sim -4\%$

particularly large corrections for ZZ at large M_{ZZ}: $\sim -40\%$

small M_{ZZ}: $\sim -4\%$

large effects on $\Delta y \equiv y_{V_1} - y_{V_2}$ distribution for large $M_{V_1 V_2}$

$\frac{d\sigma}{d\Delta y}\big|_{M_{V_1 V_2}} \,\widehat{=}\,$ angular distribution in $V_1 V_2$ restframe

$\longrightarrow$ impact on search for anomalous couplings

Impact of NLO corrections?

	summed	LL	L+	++	+−
$pp \to ZZ + X$					
ZZ polarizations	summed	LL	L+	++	+−
LHC14					
default cuts σ_{LO}/pb $\delta\sigma_{\mathrm{weak}}$/pb	7.067 -0.338(-0.292)	0.402 -0.015(-0.014)	0.734 -0.029(-0.025)	0.100 -0.004(-0.003)	4.997 -0.257(-0.223)
$p_{\mathrm{T,Z}} > 500$ GeV σ_{LO}/pb $\delta\sigma_{\mathrm{weak}}$/pb	$10^{-2} \times$ [0.499 $-0.195(-0.148)$]	$10^{-7} \times$ [0.921 $-4.70(+5.577)$]	$10^{-4} \times$ [0.334 $-0.087(-0.067)$]	$10^{-7} \times$ [0.230 $-0.426(-0.185)$]	$10^{-2} \times$ [0.492 $-0.192(-0.147)$]
$p_{\mathrm{T,Z}} > 1000$ GeV σ_{LO}/pb $\delta\sigma_{\mathrm{weak}}$/pb	$10^{-3} \times$ [0.146 $-0.088(-0.062)$]	$10^{-9} \times$ [0.189 $-4.319(+30.04)$]	$10^{-6} \times$ [0.306 $-0.126(-0.090)$]	$10^{-10} \times$ [0.475 $-2.953(+2.295)$]	$10^{-3} \times$ [0.146 $-0.088(-0.062)$]

(second row: interference (interference+(1-loop)^2))

low p_{T}: universal corrections $\sim -4\%$
$\implies$ angular distribution unchanged

high p_{T}: large corrections, only one combination (+−) survives
$\implies$ angular distribution unchanged
$\implies$ **universal correction factor**

Assumption: factorization of EW and QCD corrections

$$\mathrm{d}\sigma_{\mathsf{QCD} \times \mathsf{EW}} = K_{\mathsf{weak}}(\widehat{s}, \widehat{t}) \times \mathrm{d}\sigma_{\mathsf{QCD}}$$

$\mathrm{d}\sigma_{\mathsf{QCD}} \widehat{=}$ best prediction for QCD-corrected cross section

- $\widehat{s}, \widehat{t}$ from matching of kinematics:
$$\widehat{s} = M^2_{V_1 V_2}; \qquad \widehat{t} = ?$$

  – small p_T of $V_1 V_2 \Longrightarrow \widehat{\Theta} =$ scattering angle in $V_1 V_2$ restframe
  (collinear or soft QCD corrections)

  – large p_T of $V_1 V_2$ (assosiated with hard jet!)
  define directions of incoming protons in $V_1 V_2$ restframe
  $$\mathbf{e}^*_{1,2} \equiv \frac{\mathbf{p}^*_{1,2}}{|\mathbf{p}^*_{1,2}|}$$
  scattering axis: $\widehat{\mathbf{e}}^* \equiv \frac{\mathbf{e}^*_1 - \mathbf{e}^*_2}{|\mathbf{e}^*_1 - \mathbf{e}^*_2|}$
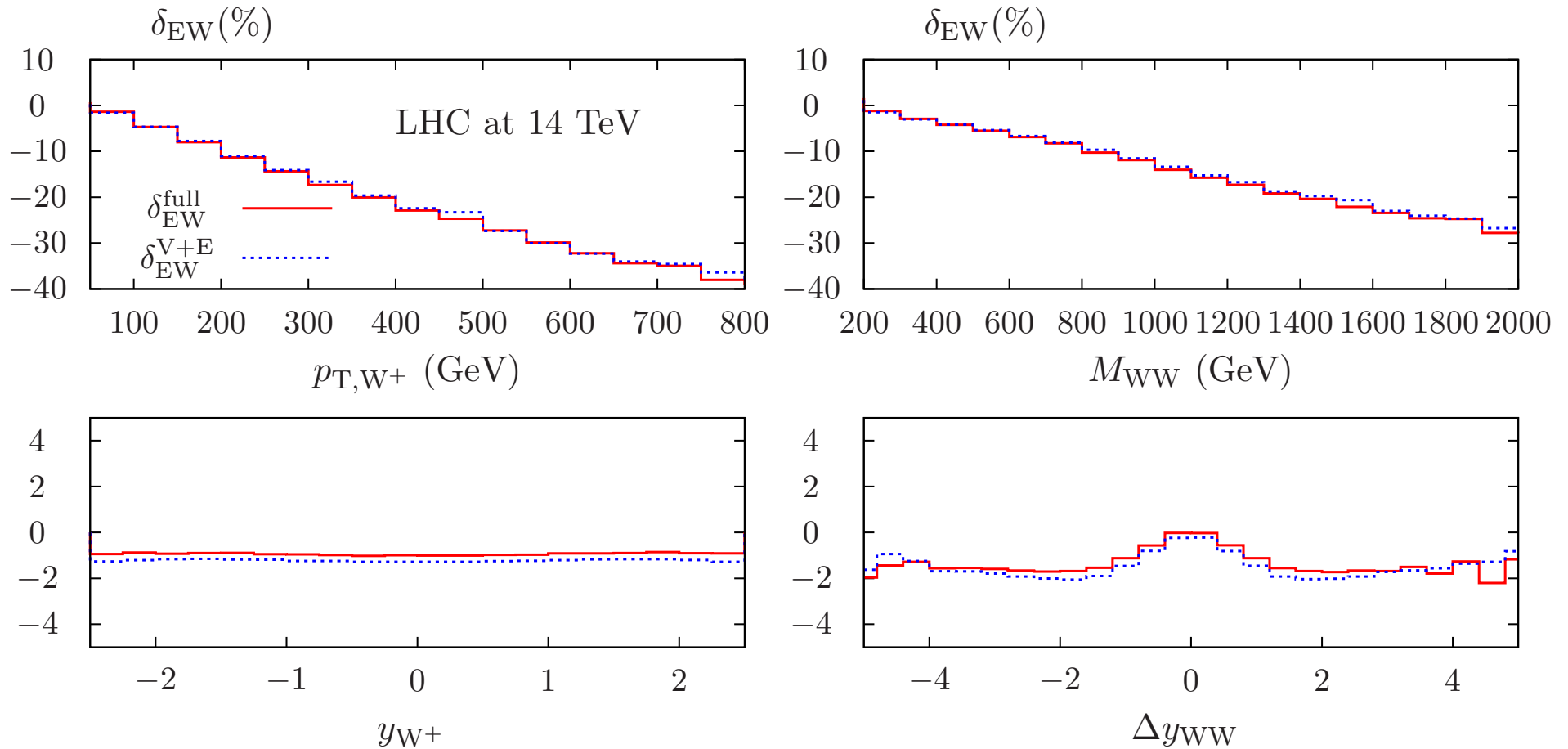 scattering angle: $\cos \widehat{\Theta} = \frac{\mathbf{p}^*_{V_1}}{|\mathbf{p}^*_{V_1}|} \cdot \widehat{\mathbf{e}}^*$

- or (HERWIG)
  $\widehat{s}, \widehat{t}$ provided by the generator

NLO corrections (electroweak) with full QED
replaced by virtual + endpoint subtraction (V+E). (Dittmaier)
(remainder = real radiation - endpoint subtraction < 1%)

$pp \to WW$ at 14 TeV:

HERWIG++ Results (preliminary)
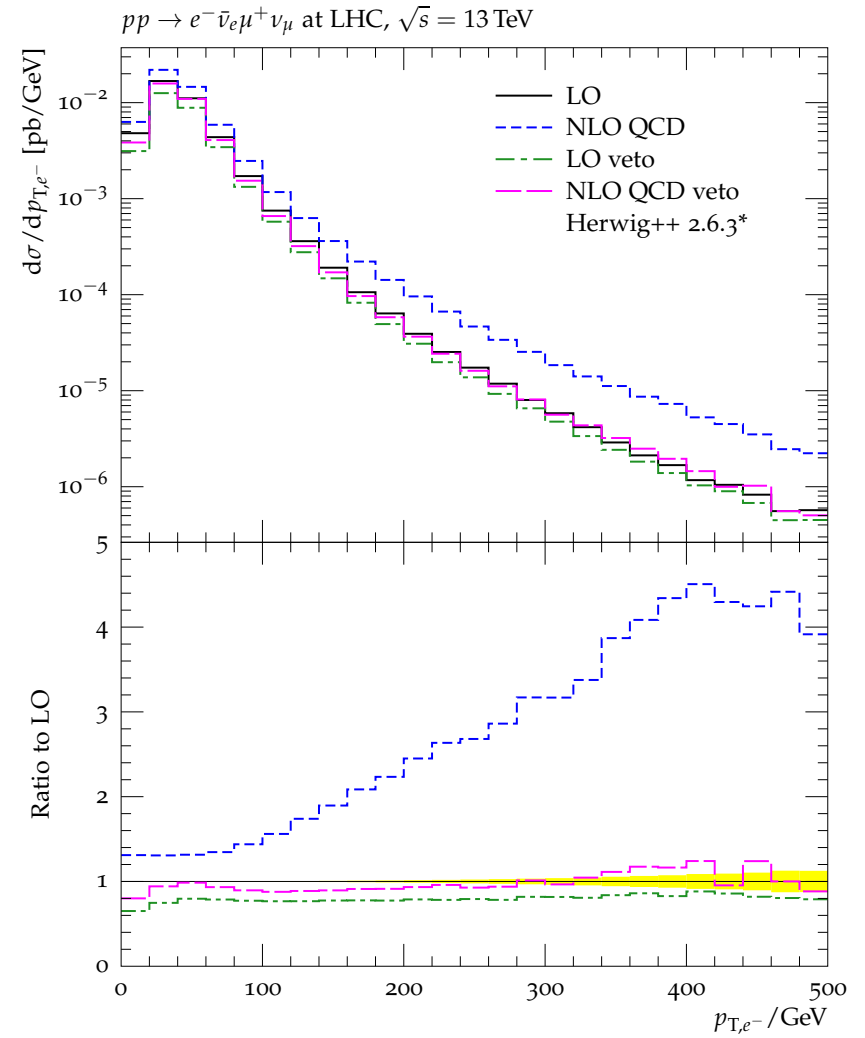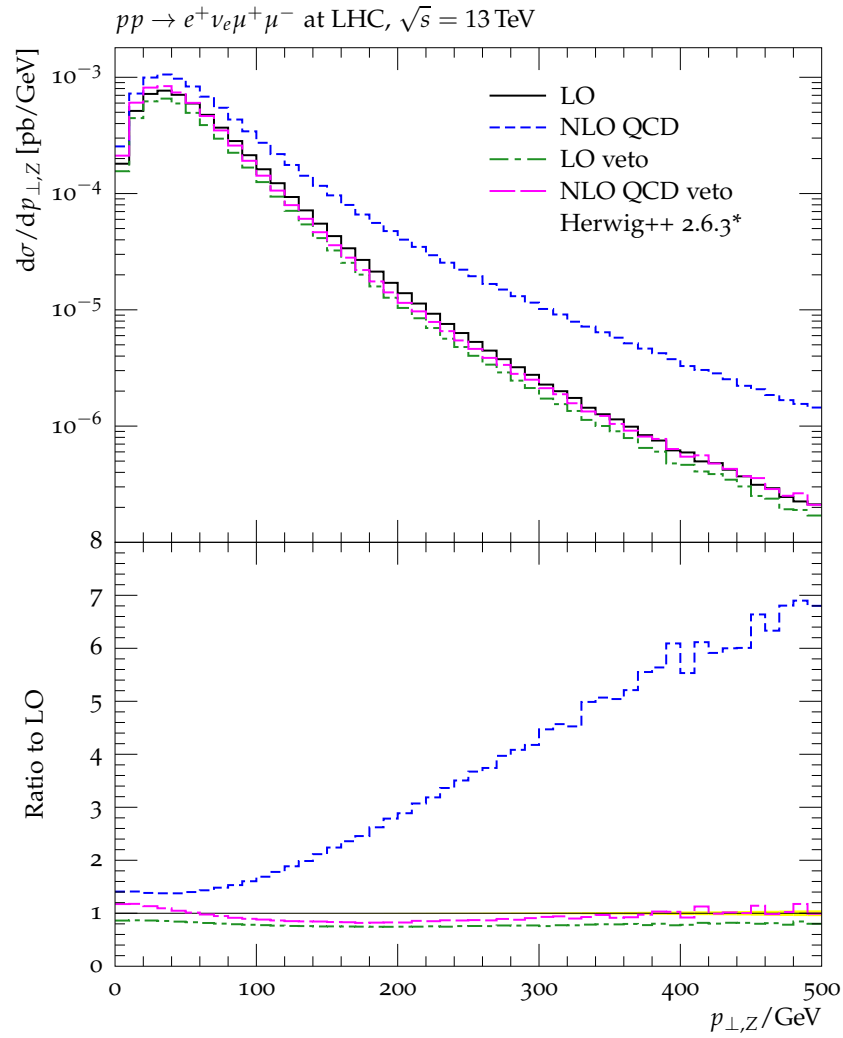
Event definition (jet veto)

- Large QCD corrections at large transverse momenta

- Events dominated by V + jet (+ soft V), require back-to-back vector bosons

$\Rightarrow$ $\left| \sum_i \mathbf{p}^i_{T,l} + \mathbf{p}_{\mathsf{T},\mathrm{miss}} \right| < \rho \left(\sum_i |\mathbf{p}^i_{T,l}| + p_{\mathsf{T},\mathrm{miss}} \right)$
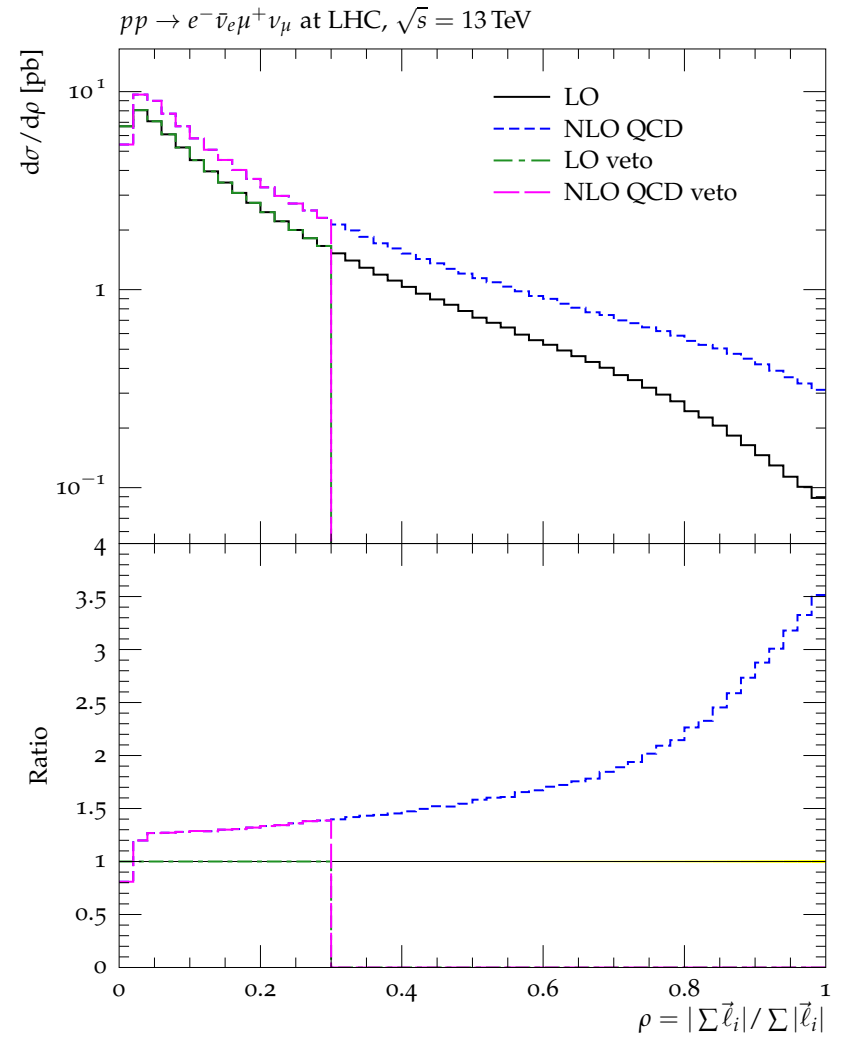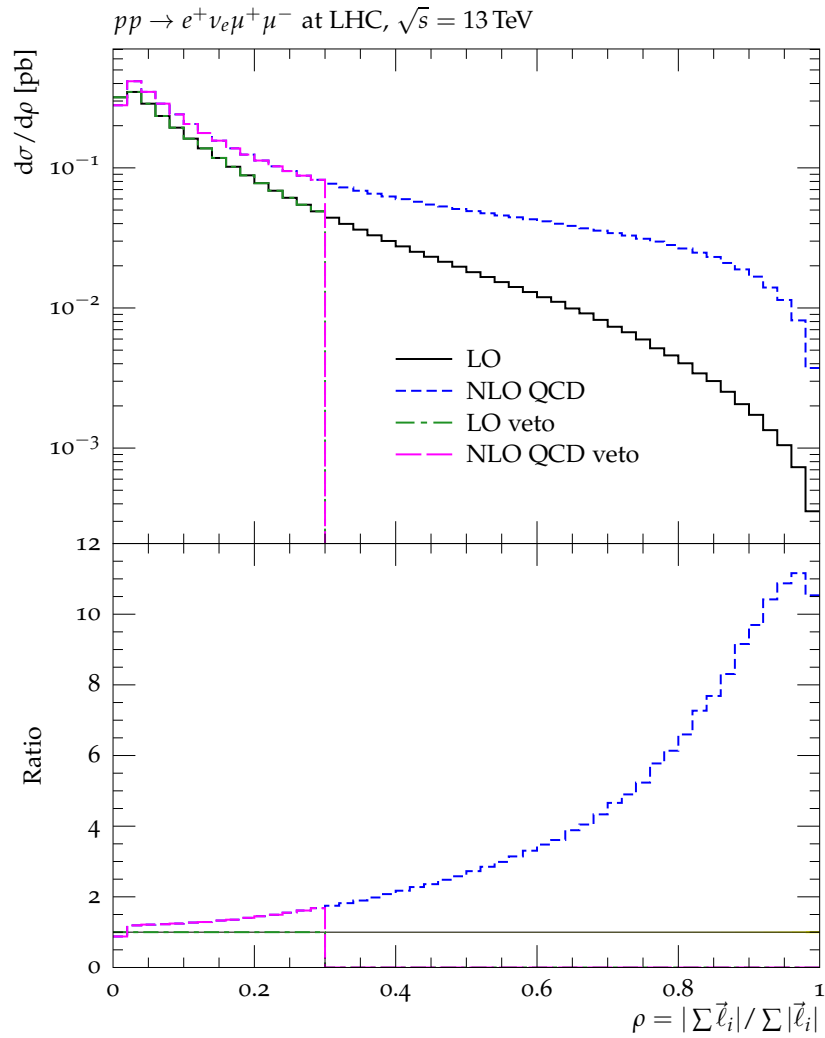($\rho = .3$ gives reasonable results)
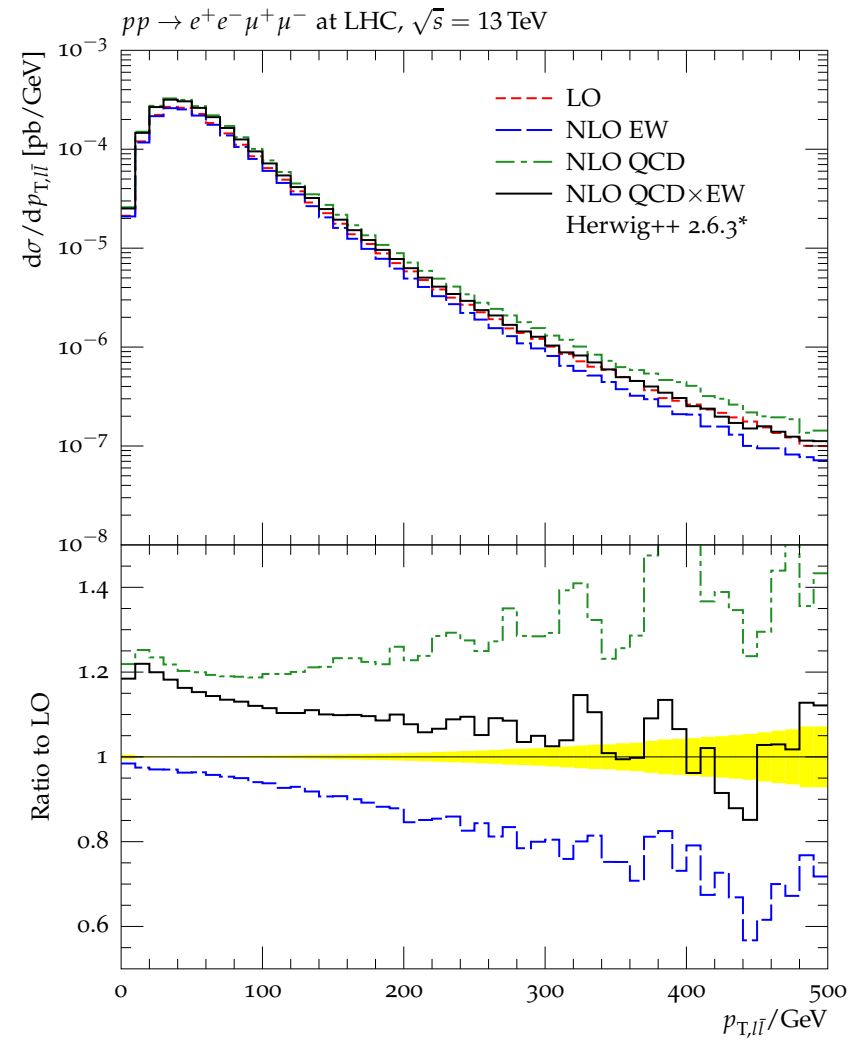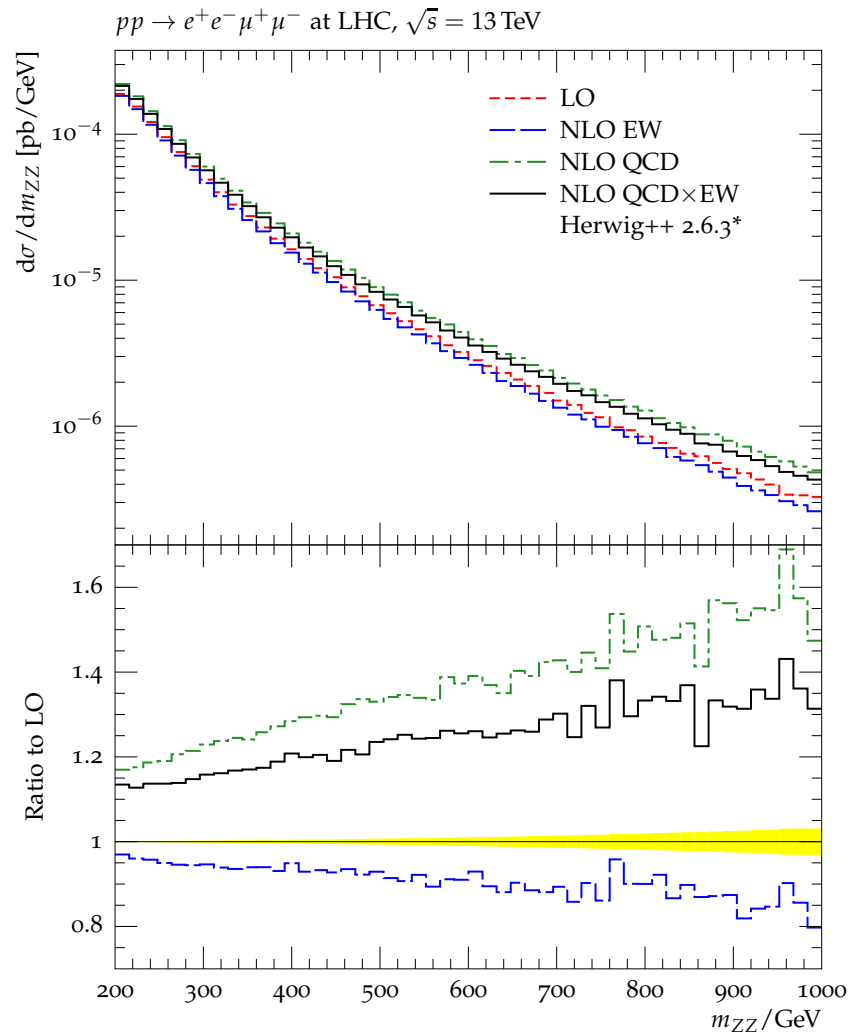
Effect of veto on WZ and WW production

vector-to-scalar ratio $\rho = \left| \sum_i \vec{p}^{\,i}_{\mathsf{T},l} + \vec{p}_{\mathsf{T},\mathrm{miss}} \right| / \left(\sum_i |\vec{p}^{\,i}_{\mathsf{T},l}| + p_{\mathsf{T},\mathrm{miss}} \right)$
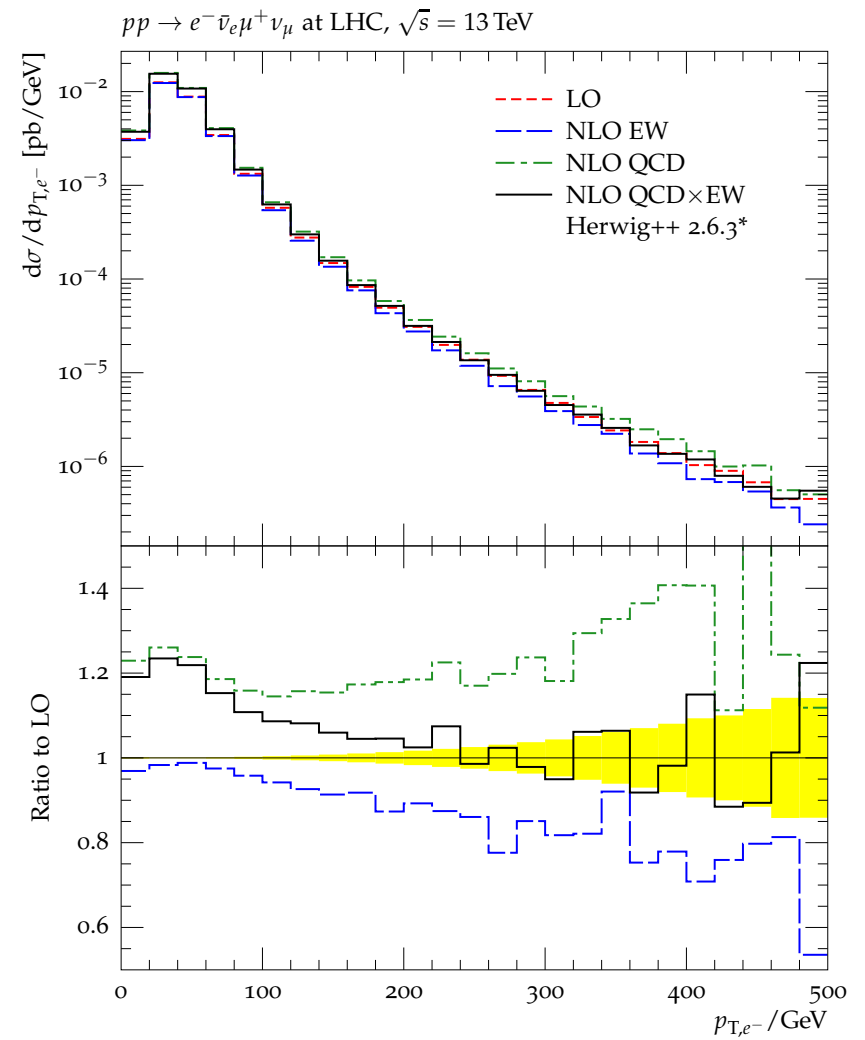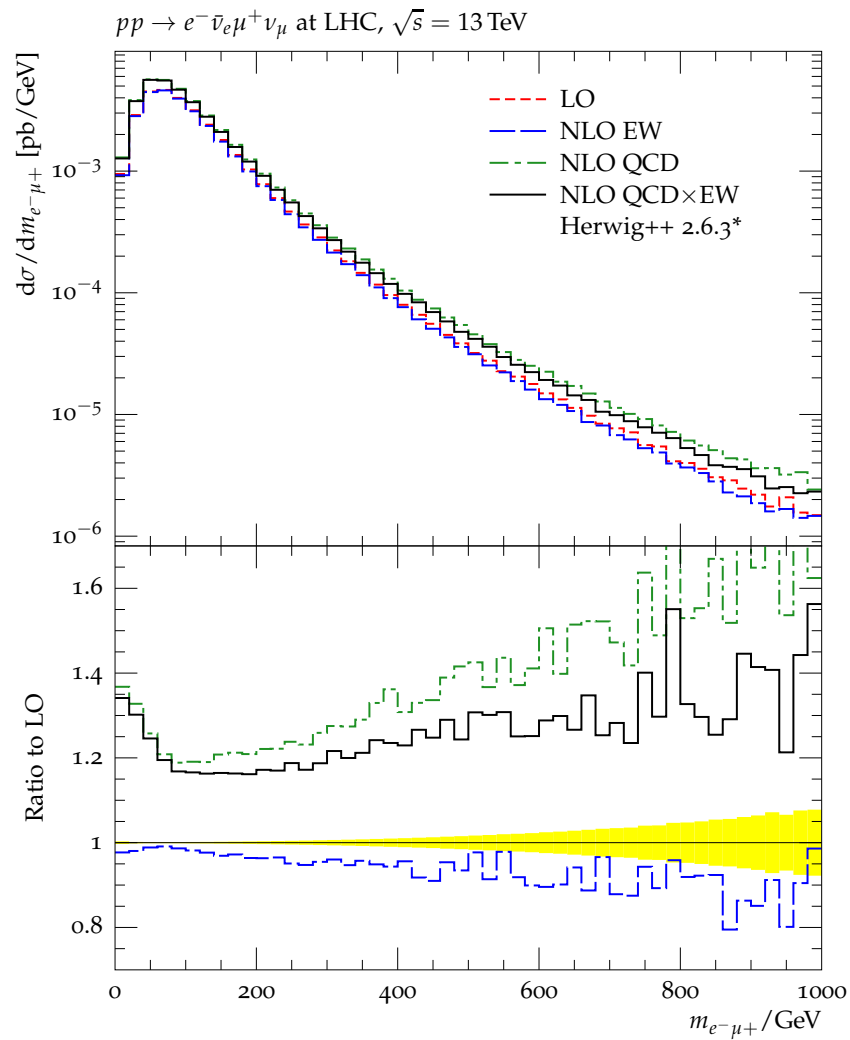
Simulation for $pp \to ZZ \to e^+e^-\mu^+\mu^- + X$ at 13 TeV, m_{ZZ} and $p_{\mathrm{T},Z}$ (leading Z) distributions
(standard event selection cuts)

- Standard HERWIG++ setup used
  (v2.6.3, with simple add-on for EW corrections, 10M events), ZZ at NLO QCD matched with parton showers, hadronization included, underlying event switched off

- huge QCD corrections at large $p_{T,Z}$, factorized ansatz not justified
  $\rightarrow$ jet veto, cut on $p_{T,ZZ}$
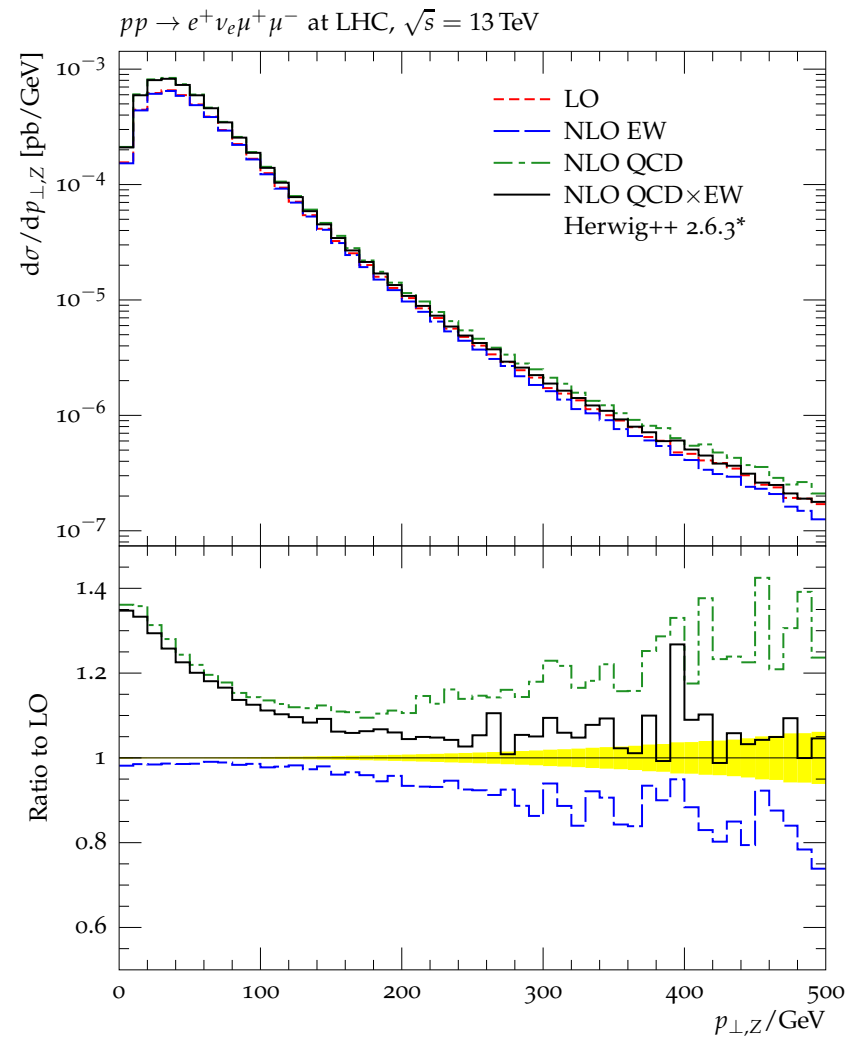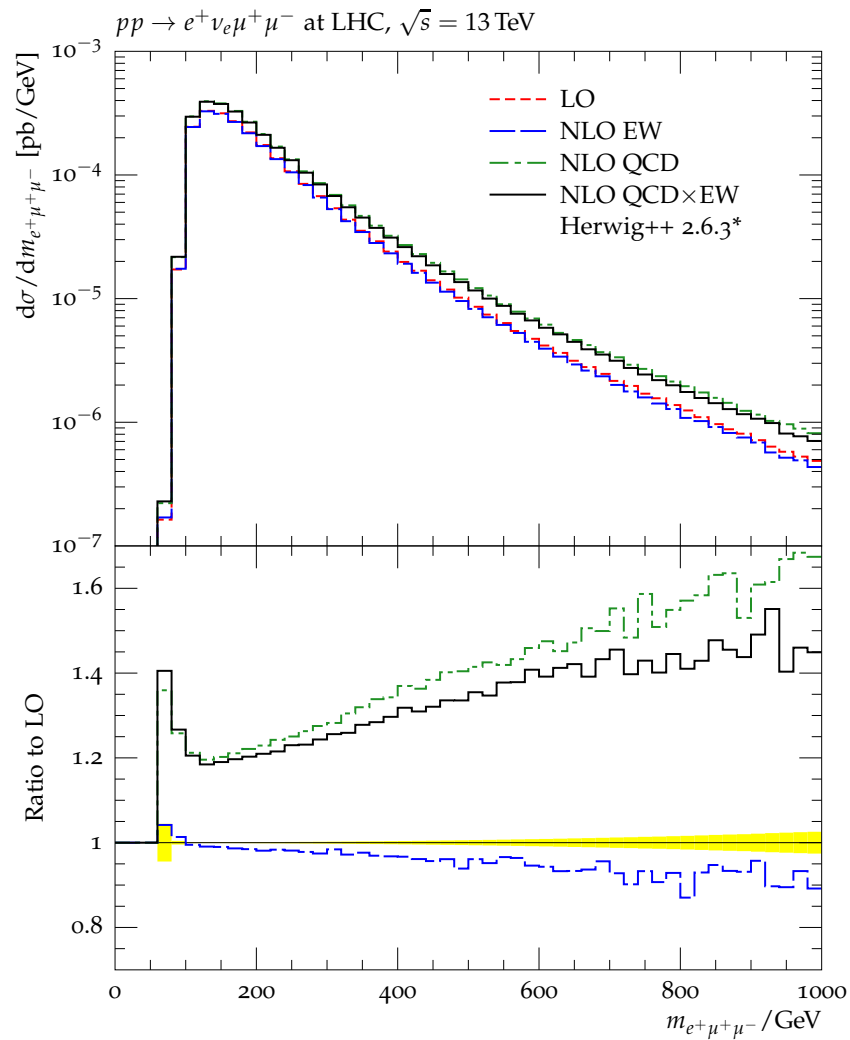 $|\sum_i \mathbf{p}_{T,l}^i| < 0.3 \sum_i |\mathbf{p}_{T,l}^i|$

Simulation for $\mathrm{pp} \to (\mathrm{W}^- \to)e^-\bar{\nu}_e\,(\mathrm{W}^+ \to)\mu^+\nu_\mu + X$ at 13 TeV, $m_{e\mu}$ and $p_{\mathrm{T},e}$ distributions
(standard event selection cuts)

- **Standard Herwig++ setup used**

  (v2.6.3, with simple add-on for EW corrections, 10M events), WW at NLO QCD $\oplus$ parton shower, hadronization included, underlying event switched off

- V+E approximate results consistent with arXiv:1208.3147

- **Large QCD corrections at large p_{T},** factorized ansatz not justified
  $\rightarrow$ jet veto, cut on $p_{\mathsf{T},\mathsf{leptons}}$
  $$\left| \sum_i \mathbf{p}^i_{T,l} + \mathbf{p}_{\mathsf{T},\mathsf{miss}} \right| < 0.3 \left(\sum_i |\mathbf{p}^i_{T,l}| + p_{\mathsf{T},\mathsf{miss}} \right)$$

Simulation for $\mathrm{pp} \to (\mathrm{W}^+ \to) e^+ \nu_e \, (\mathrm{Z} \to) \mu^- \mu^+ + X$ at 13 TeV, $m_{e\mu\mu}$ and $p_{\mathrm{T},Z}$ distributions
(standard event selection cuts)

- Standard Herwig++ setup used

  (v2.6.3, with simple add-on for EW corrections, 10M events), WZ at NLO QCD $\oplus$ parton shower, hadronization included, underlying event switched off

- Large QCD corrections at large p_T, factorized ansatz not justified
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  $$\left| \sum_i \mathbf{p}_{T,l}^i + \mathbf{p}_{T,\text{miss}} \right| < 0.3 \left(\sum_i |\mathbf{p}_{T,l}^i| + p_{T,\text{miss}} \right)$$

IV. Two Loop Results (Sudakov Logarithms)

one-loop: $\sim -40\%$

two-loop: $\sim ?$

(Vast amount of literature since ~ 2000)

Karlsruhe (Jantzen, J.K., Metzler, Penin, Smirnov, Uccirati)
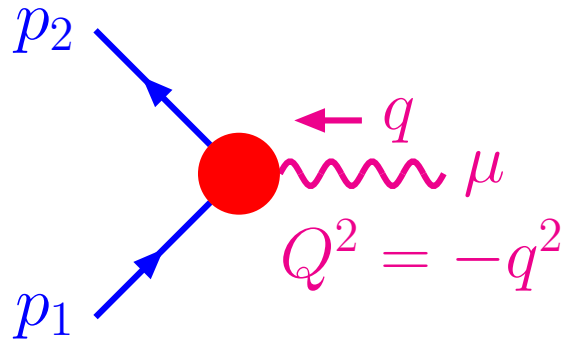
Fadin, Lipatov, Martin, Melles

PSI (Denner, Melles, Pozzorini, . . .)

Ciafaloni, . . .

Manohar, . . .

A) Form Factor and Evolution Equations

Born:

$$\mathcal{F}_{\text{Born}} = \bar{\psi}(p_2)\gamma_\mu\psi(p_1)$$

$$\frac{\partial}{\partial \ln Q^2}\mathcal{F} = \left[\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2))\right]\mathcal{F}$$

Collins, Sen

$$\Rightarrow \mathcal{F} = \mathcal{F}_{\text{Born}}\,F_0(\alpha(M^2))\exp\left\{\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\left[\int_{M^2}^{x}\frac{\mathrm{d}x'}{x'}\gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2))\right]\right\}$$

aim: N^{4}LL $\Rightarrow$ corresponds to all terms of the form:

$$\alpha^n \left[\underset{\text{LL}}{\mathcal{L}^{2n}} + \underset{\text{NLL}}{\mathcal{L}^{2n-1}} + \underset{\text{NNLL}}{\mathcal{L}^{2n-2}} + \underset{\text{N}^3\text{LL}}{\mathcal{L}^{2n-3}} + \underset{\text{N}^4\text{LL}}{\mathcal{L}^{2n-4}} \right]$$

$$\mathcal{L} \equiv \ln(Q^2/M^2)$$

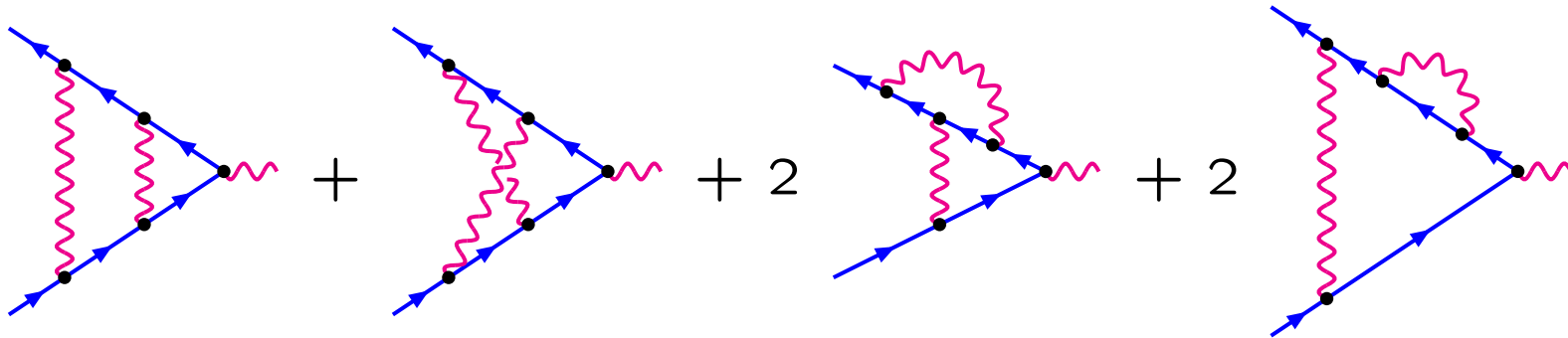NNLL requires running of α (i.e. β_0 and β_1) and:

$$\zeta(\alpha),\ \xi(\alpha),\ F_0(\alpha) \quad \text{up to } \mathcal{O}(\alpha) \quad \text{(one-loop)}$$
$$\gamma(\alpha) \quad \text{up to } \mathcal{O}(\alpha^2) \quad \text{(massless two loop)}$$

N^{3}LL requires two-loop calculation in high-energy limit including linear logarithms (available for non-abelian theory)

N^{4}LL requires complete two-loop calculation in high-energy limit (available for abelian theory)

B) Two-Loop Results: Massive U(1) Model

$$\mathcal{F}_\alpha(M, Q) = \mathcal{F}_{\text{Born}} \left[1 + \frac{\alpha}{4\pi} f^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 f^{(2)} + \ldots \right]$$
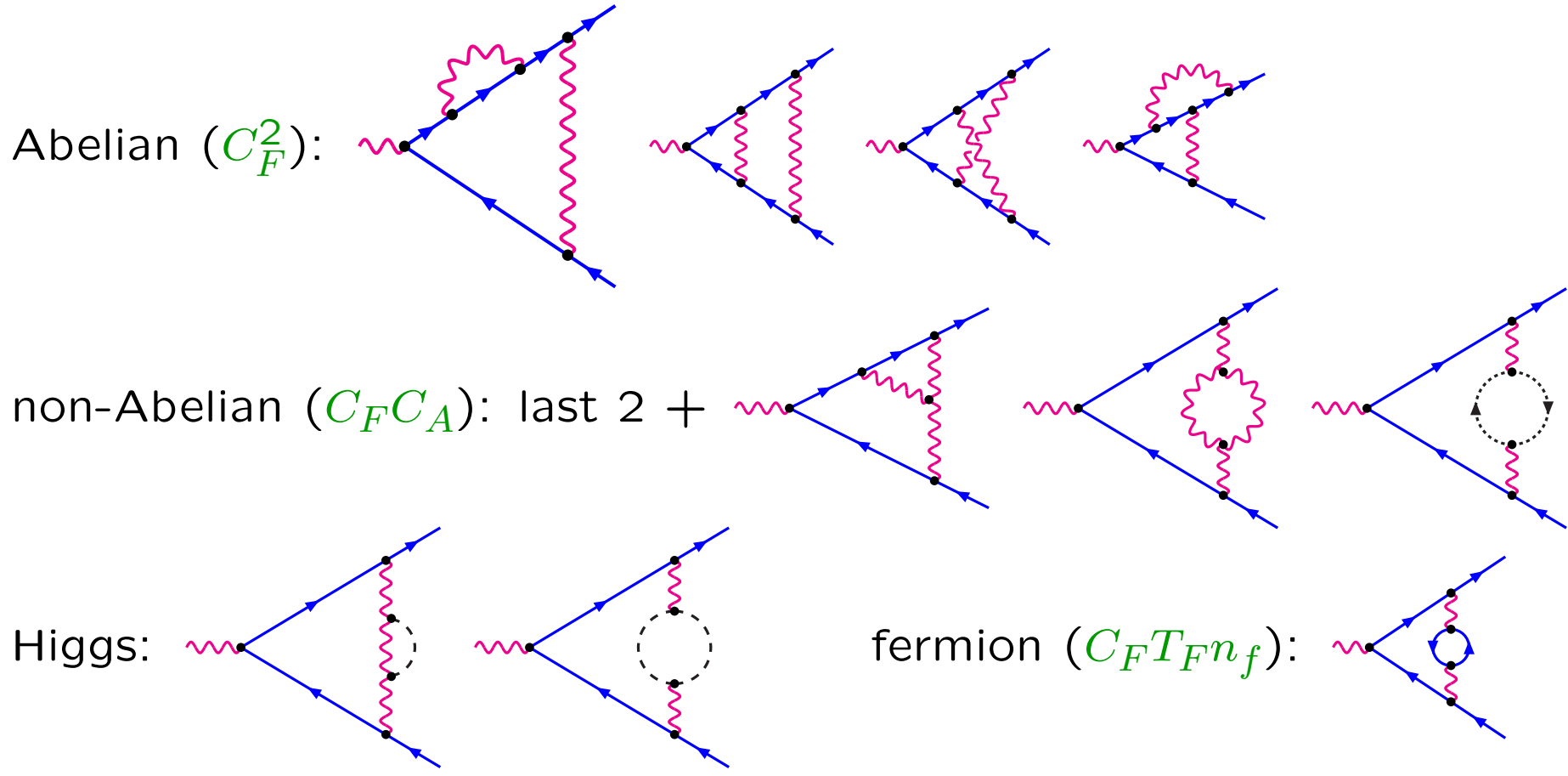

$$f^{(2)} = \frac{1}{2} \mathcal{L}^4 - 3 \mathcal{L}^3 + \left(8 + \frac{2}{3}\pi^2\right) \mathcal{L}^2 - \left(9 + 4\pi^2 - 24\zeta_3\right) \mathcal{L}$$

$$+ \frac{25}{2} + \frac{52}{3}\pi^2 + 80\zeta_3 - \frac{52}{15}\pi^4 - \frac{32}{3}\pi^2 \ln^2 2 + \frac{32}{3} \ln^4 2 + 256 \, \text{Li}_4\left(\frac{1}{2}\right)$$

$$\mathcal{L} \equiv \ln(Q^2/M^2)$$

C) Massive SU(2) form factor in 2-loop approximation

2-loop vertex diagrams (massless fermions, massive bosons):

Abelian (C_F^2):

non-Abelian ($C_F C_A$): last 2 +

Higgs: fermion ($C_F T_F n_f$):

+ 1-loop×1-loop corrections + renormalization

$$f_2 = +\frac{9}{32}\mathcal{L}^4 - \frac{19}{48}\mathcal{L}^3 - \left(-\frac{7}{8}\pi^2 + \frac{463}{48}\right)\mathcal{L}^2$$

$$+ \left(\frac{39}{2}\frac{\mathrm{Cl}_2\!\left(\frac{\pi}{3}\right)}{\sqrt{3}} + \frac{45}{4}\frac{\pi}{\sqrt{3}} - \frac{61}{2}\zeta_3 - \frac{11}{24}\pi^2 + 29\right)\mathcal{L}$$

Extensions to four-fermion scattering (Penin et al.) and W-pair production
(J.K., Metzler, Penin, Uccirati)
General form (for transverse W) :

$$\mathcal{A} = \alpha(\mu_{\mathsf{T}})\mathcal{Z}_{\Psi}\mathcal{Z}_W\widetilde{\mathcal{A}}$$

$\mathcal{Z}_{W,\Psi} =$ form factors of quark or W-field
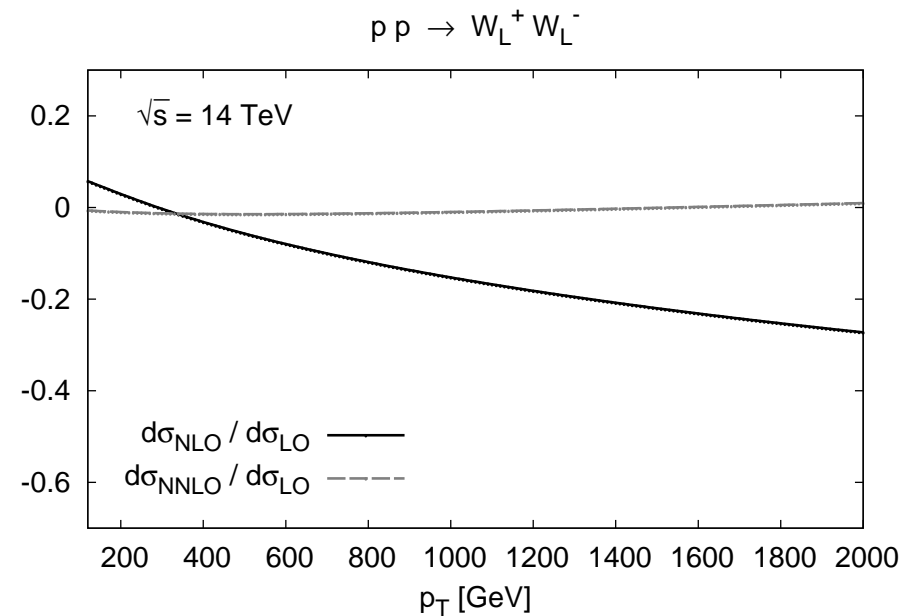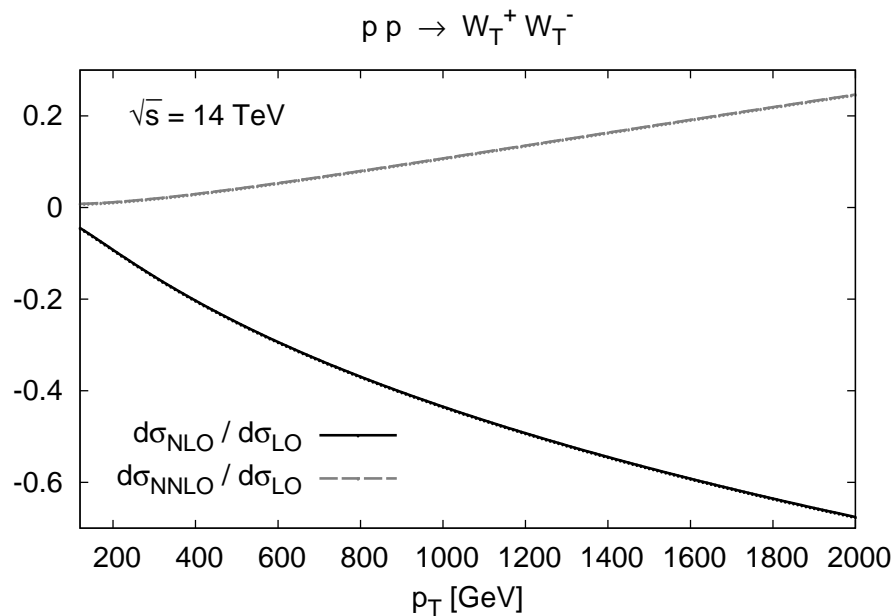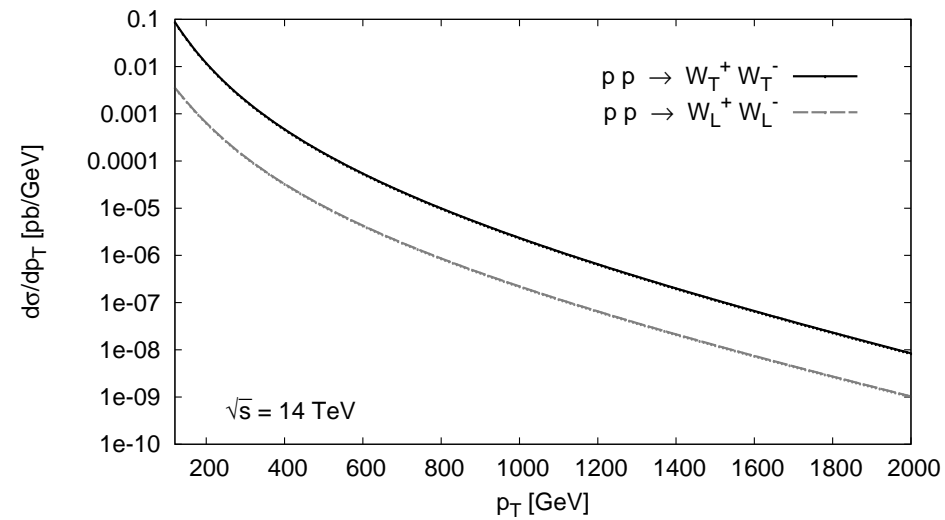$\widetilde{\mathcal{A}} =$ reduced amplitude (matrix in isospin space)

Solution of

$$\frac{\partial}{\partial \ln Q^2}\widetilde{\mathcal{A}} = \chi(\alpha(Q^2))\widetilde{\mathcal{A}}$$

($\chi \equiv$ anomalous dimension matrix)

up to quadratic logarithms only one-loop terms ($\ln^2, \ln^1, \ln^0$) of $q\bar{q} \longrightarrow WW$ and
the two-loop term $\gamma^{(2)}$ of the anomalous dimension (form factor!) required.

Complication: massless photon! $\Rightarrow$ QED subtracted

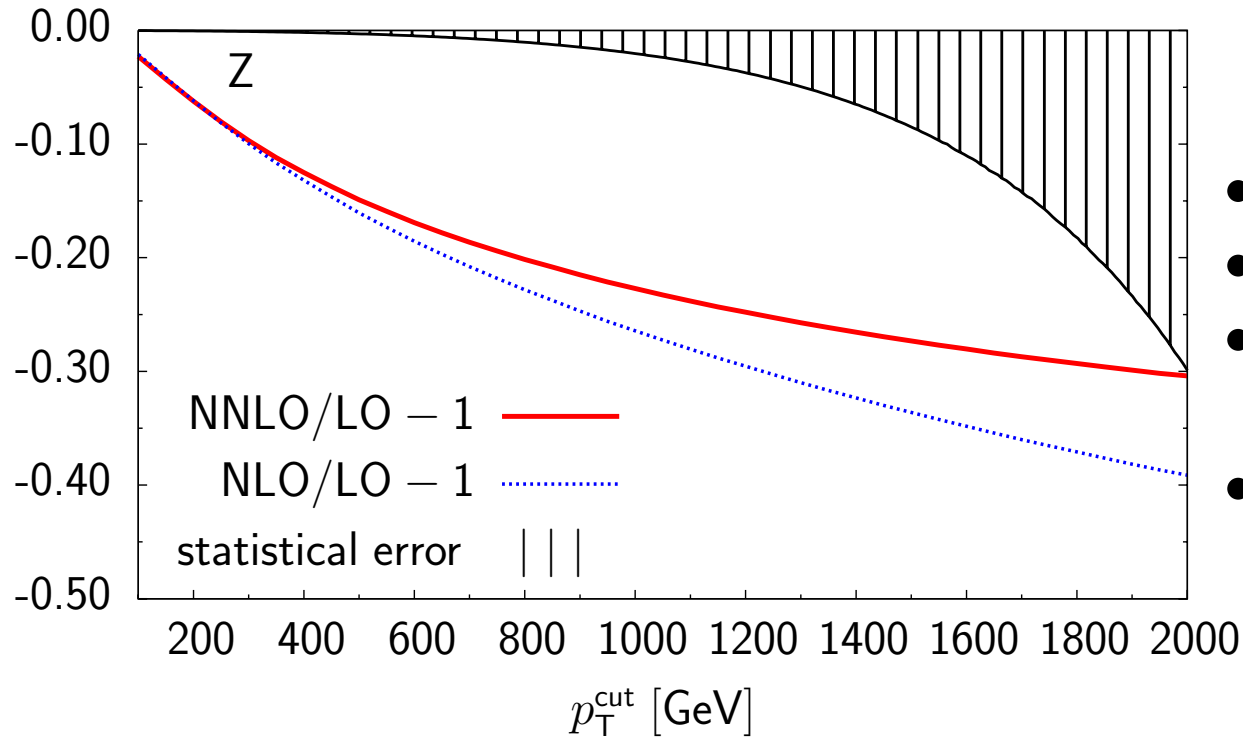$(W_{T,L}:$ Isospin $I = 1, 1/2; \Rightarrow$ relative factor 6/11$)$

V. Summary

- Weak corrections at LHC14 may reach -40%

- complicated impact on differential distributions
  $\Longrightarrow$ fake anomalous couplings; sensitive to cuts

- first steps toward combination of QCD and EW corrections in Monte Carlo generator

- dominant two-loop corrections (partially) available

Addendum: V + jet, V=W,Z,γ

JK, Kulesza, Pozzorini, Schulze
Denner, Dittmaier, Kasprzik, Mück

Complete one loop calculation
NLL approximation at two loops

- one-loop $\sim 30\%$ at $p_\mathrm{T} \sim 1\,\mathrm{TeV}$
- two-loop relevant above 1 TeV
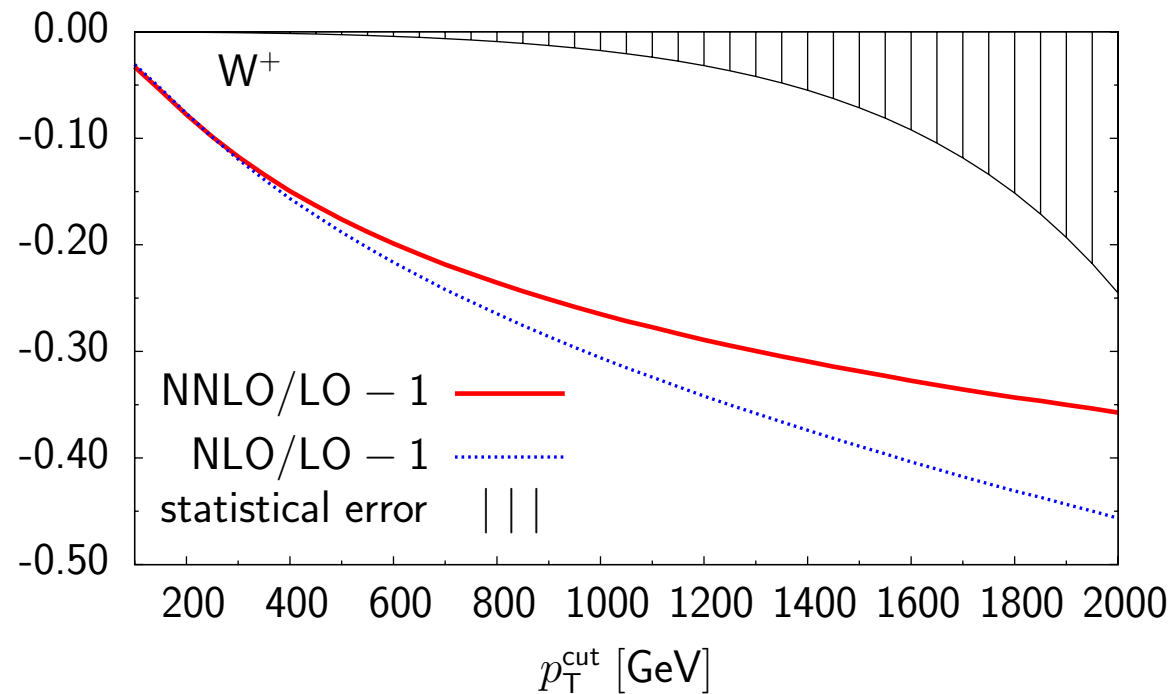- important angular-dependent logarithmic terms
- experiment: p_T up to 2 TeV

Relative **NLO** and **NNLO** corrections w.r.t. the **LO** and statistical error for the unpolarized integrated cross section for $pp \to Zj$ at $\sqrt{s} = 14$ TeV.

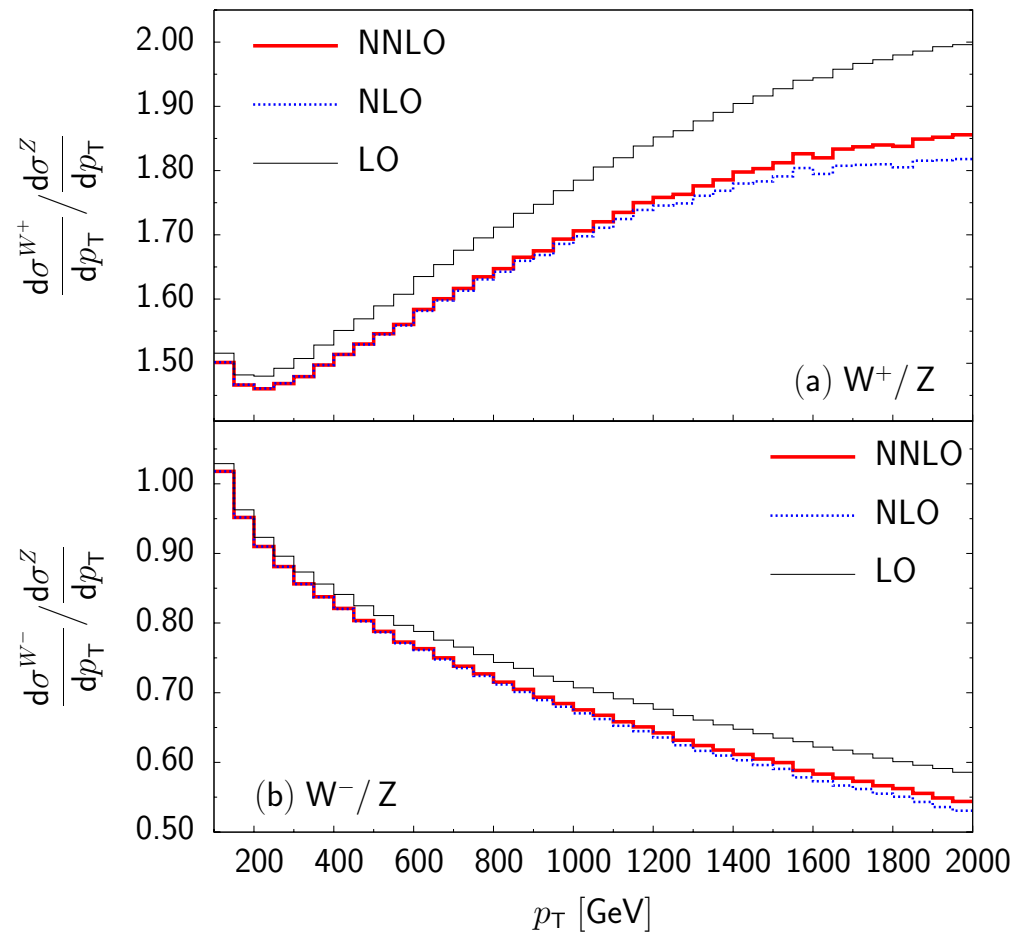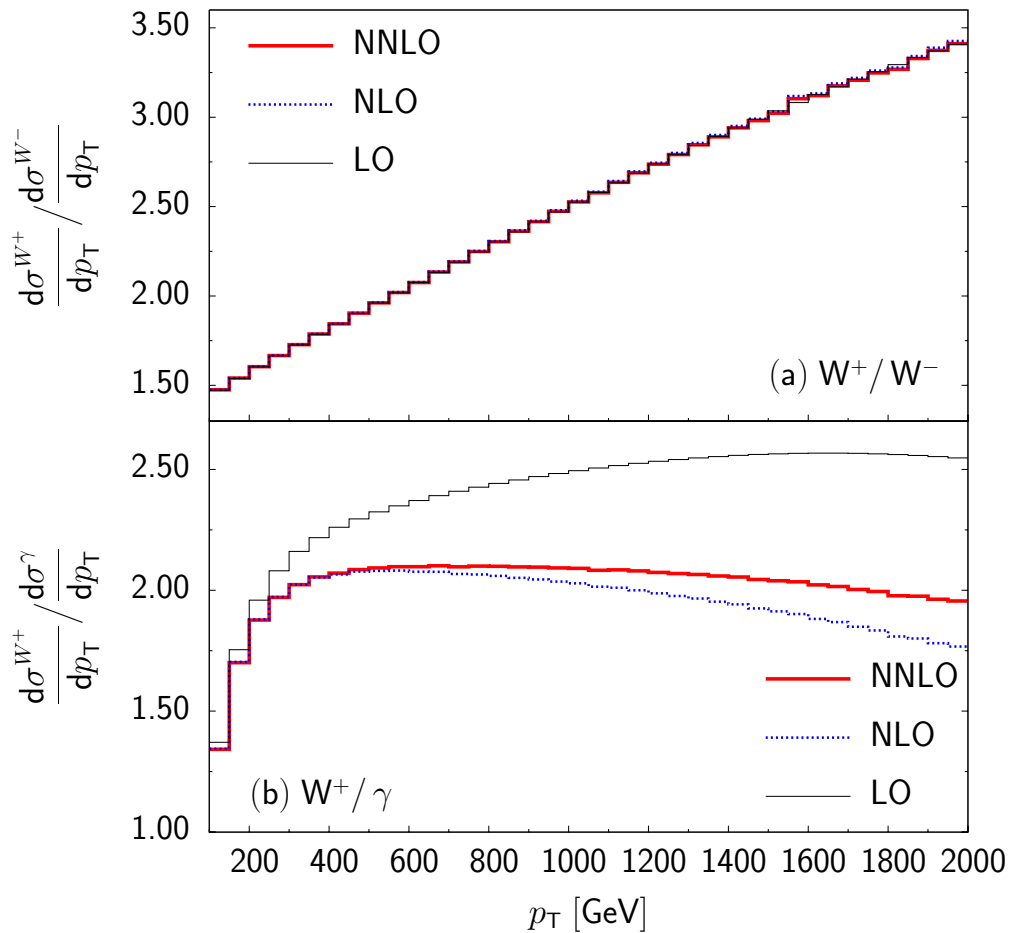(Similarly, but smaller by a factor 2 for jet$+\gamma$)

W production

additional complications:

- photon radiation as necessary part of virtual corrections (gauge invariance)

- IR singularities must be compensated by real radiation
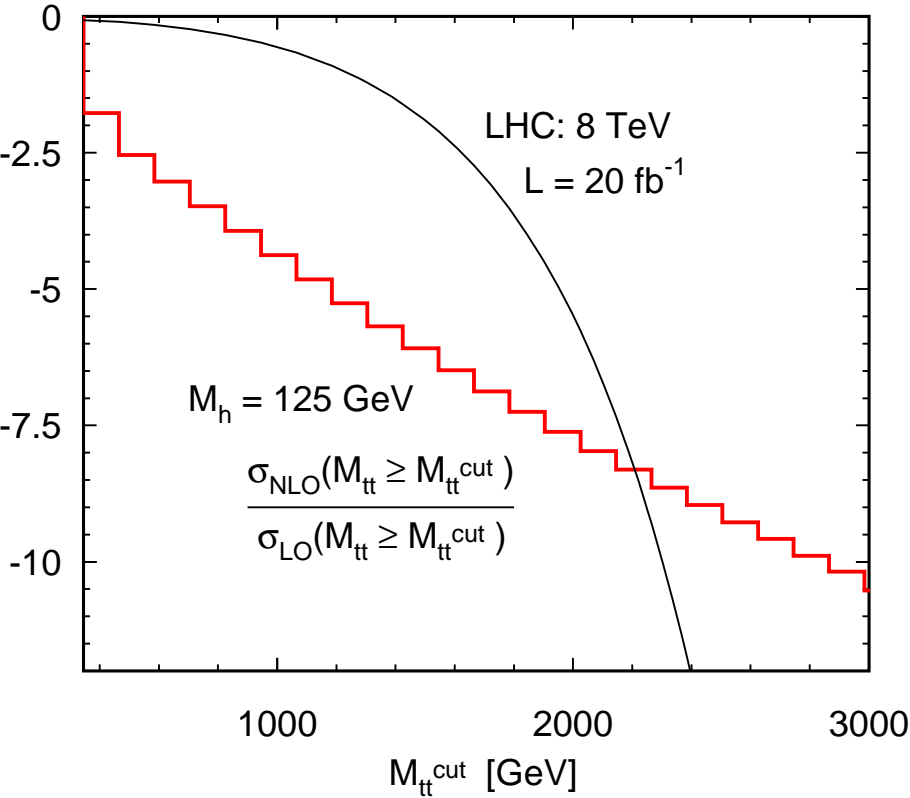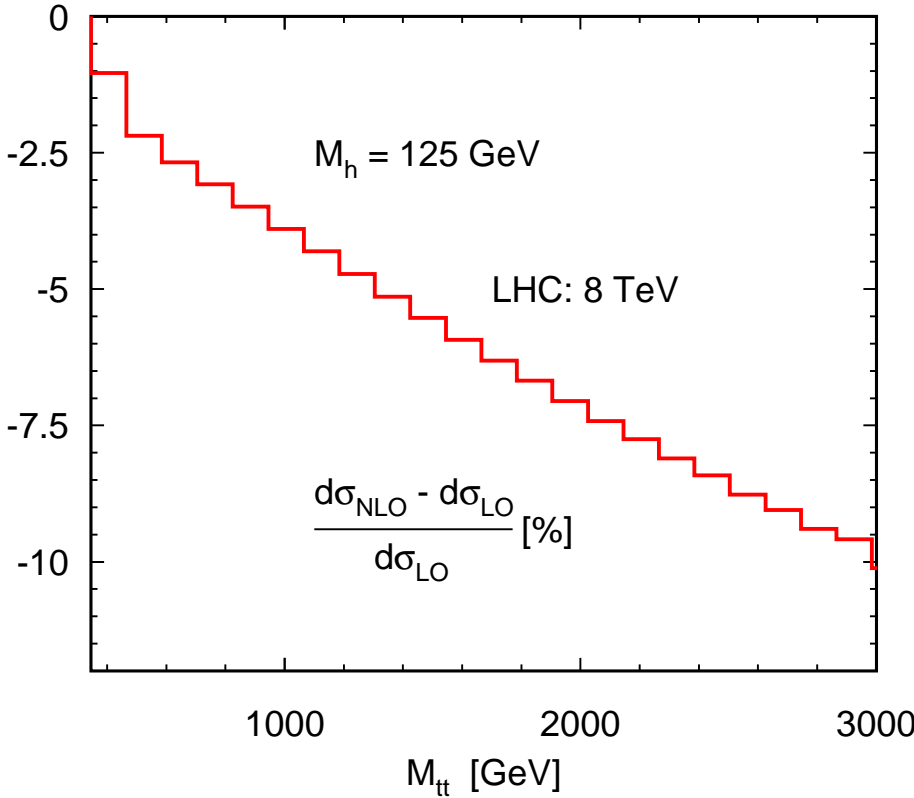
- $p_{\mathsf{T}}(W) = p_{\mathsf{T}}(\mathsf{jet}) + p_{\mathsf{T}}(\gamma)$
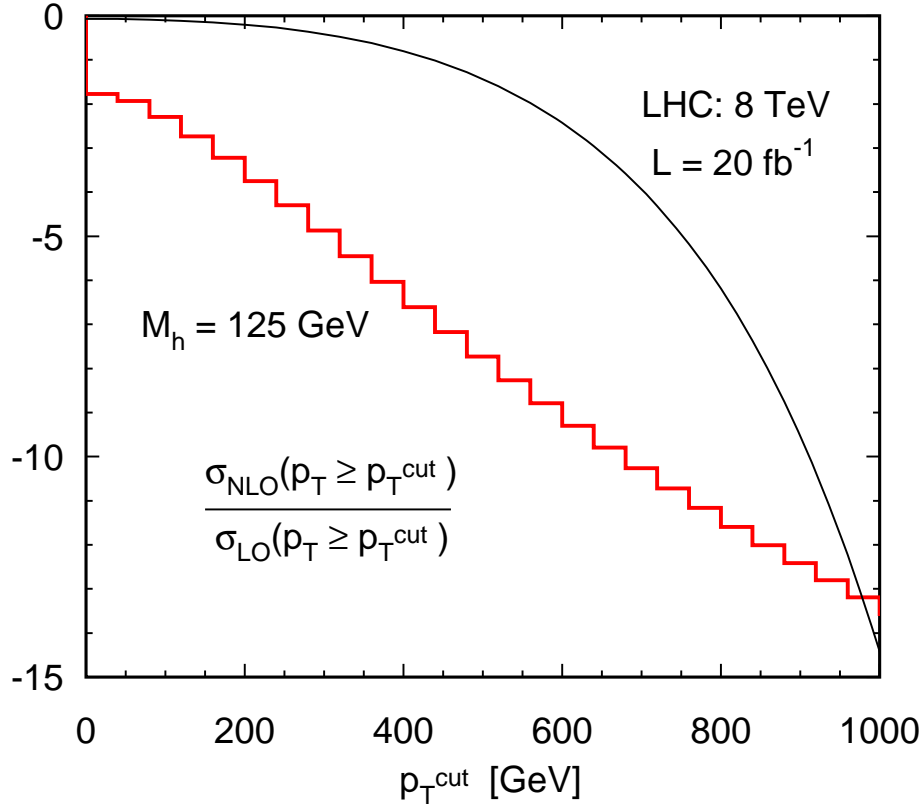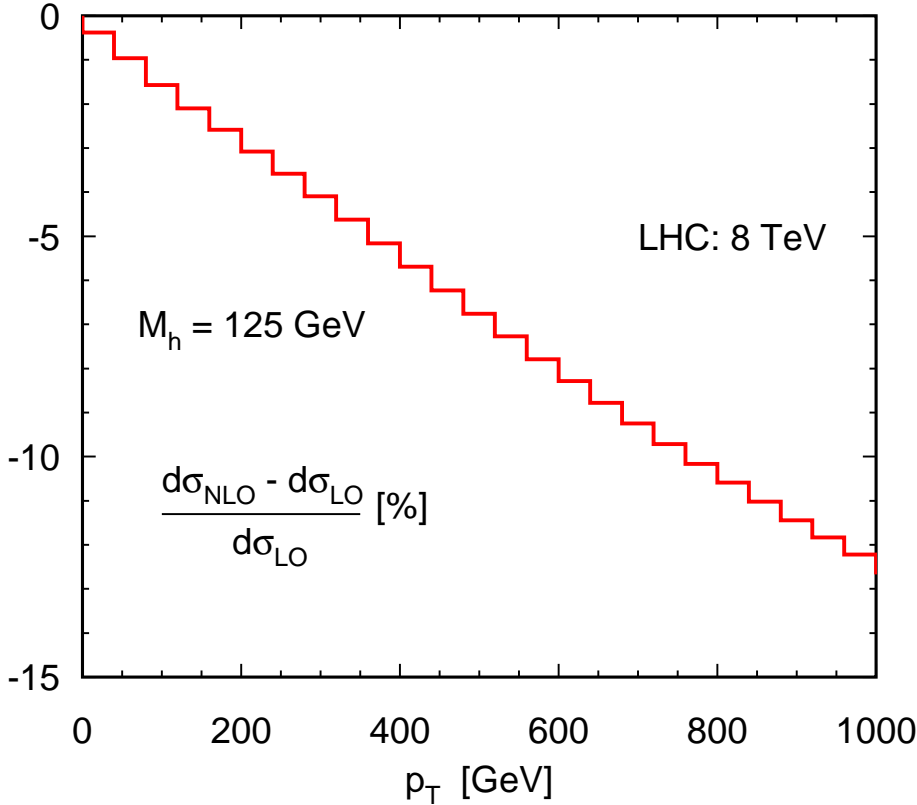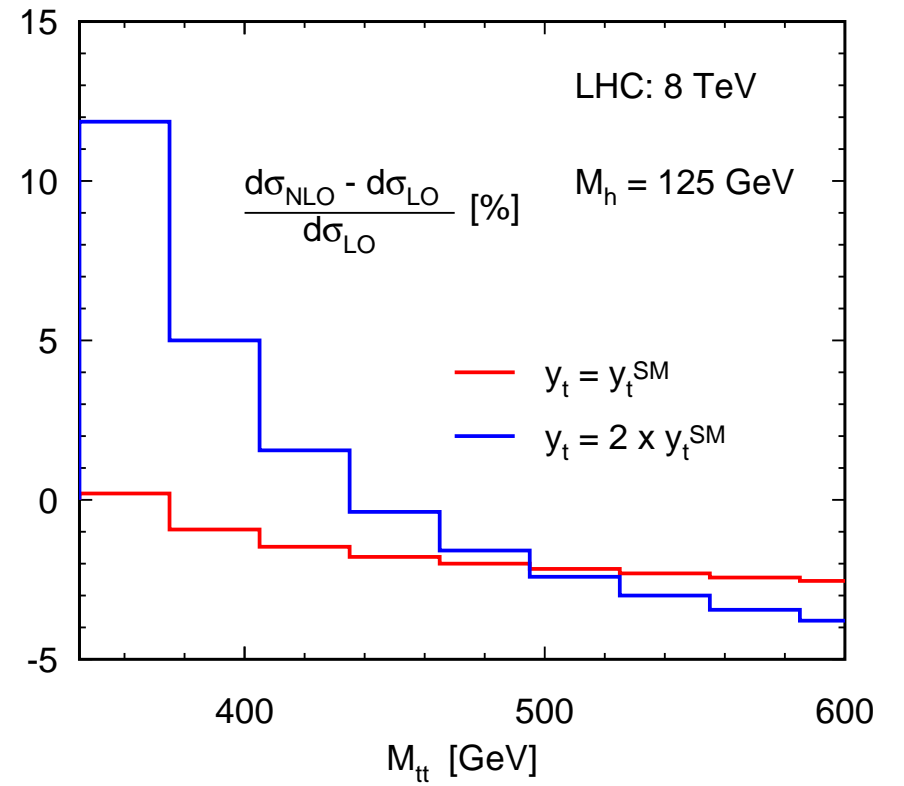

(related results: Dittmaier, Kasprzik, ...)

ratios are less sensitive to QCD corrections

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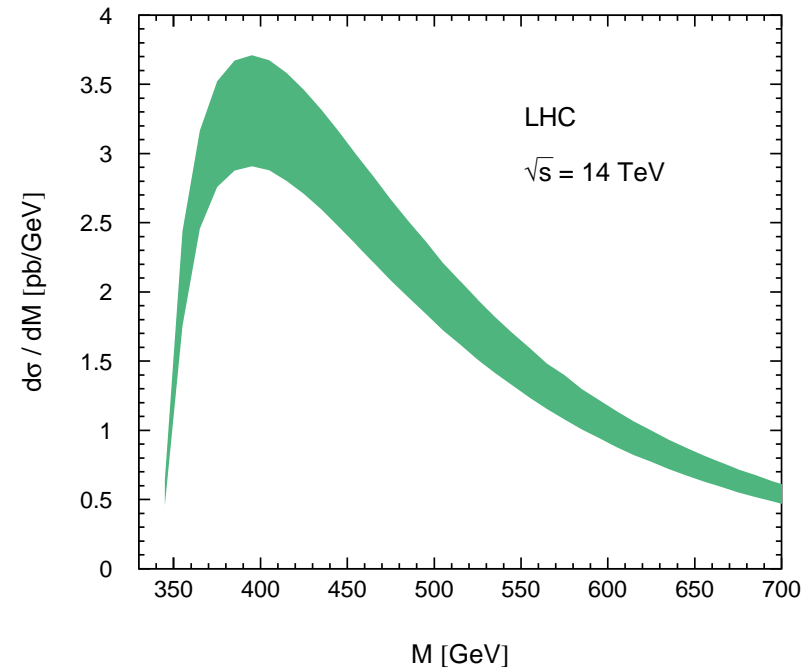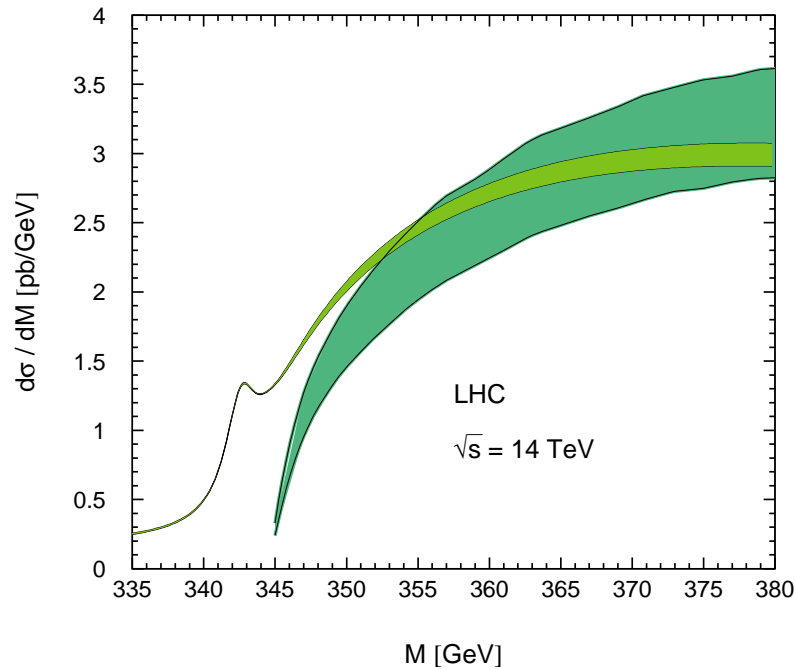# LHC at 8 TeV, top-pair invariant mass

$M_h = 125$ GeV

LHC: 8 TeV

$$\frac{d\sigma_{NLO} - d\sigma_{LO}}{d\sigma_{LO}}\ [\%]$$

M_{tt} [GeV]

LHC: 8 TeV

$L = 20$ fb^{-1}

$M_h = 125$ GeV

$$\frac{\sigma_{NLO}(M_{tt} \geq M_{tt}{}^{cut})}{\sigma_{LO}(M_{tt} \geq M_{tt}{}^{cut})}$$

$M_{tt}{}^{cut}$ [GeV]

LHC at 8 TeV, transverse momentum

Left panel axes: vertical $0, -5, -10, -15$; horizontal $0, 200, 400, 600, 800, 1000$; x-label p_T [GeV].

LHC: 8 TeV

$M_h = 125$ GeV

$$\frac{d\sigma_{NLO} - d\sigma_{LO}}{d\sigma_{LO}} \ [\%]$$

Right panel axes: vertical $0, -5, -10, -15$; horizontal $0, 200, 400, 600, 800, 1000$; x-label $p_T{}^{cut}$ [GeV].

LHC: 8 TeV

$L = 20$ fb^{-1}

$M_h = 125$ GeV

$$\frac{\sigma_{NLO}(p_T \geq p_T{}^{cut})}{\sigma_{LO}(p_T \geq p_T{}^{cut})}$$

dependence on Yukawa coupling: $Y_{\text{top}} \Rightarrow 2 * Y_{\text{top}}$

Threshold behaviour

Left: Invariant mass distribution $d\sigma/dM$ from NRQCD and for a fixed NLO for LHC with $\sqrt{s} = 14$ TeV. The bands are due to scale variation for $\mathcal{L} \otimes F$ from m_t to $4m_t$.

Right: Invariant mass distribution $d\sigma/dM$ from NLO calculation for LHC with $\sqrt{s} = 14$ TeV.