Measurement of Quark Masses
Top Quark Mass at D0

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Methods of quark masses determination

- QCD and Lagrangian:

$$\mathcal{L} = \sum_{k=1}^{N_F} \overline{q}_k \left(i\not{D} - m_k \right) q_k - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$

- has a chiral symmetry in limit that quark masses vanish
- dynamical chiral symmetry breaking Λχ = 1 GeV:

  heavy *m* > Λχ, explicit chiral symmetry breaking

  light *m* < Λχ, spontaneous chiral symmetry breaking
- leads to quark masses ratios:

  for example $\frac{m_u}{m_d} = 0{,}553 \pm 0{,}043$ $\frac{m_s}{m_d} = 18{,}9 \pm 0{,}8$

Sum Rules

- 1979 - Shifman, Vainshtein and Zakharov
- hadrons are represented by their interpolation in quark direction -> advantage of less dependence on model
- some hadronic parameters (hadron masses, momenta…) have to be experimentally measured
- use of τ decay (*s* quark mass), e^+e^- annihilation or *B* meson deacy (quarks *c* and *b*)
- relates experimental observables like $R(s)$ to QCD parametres like α_s or quark masses

Finite energy sum rule:

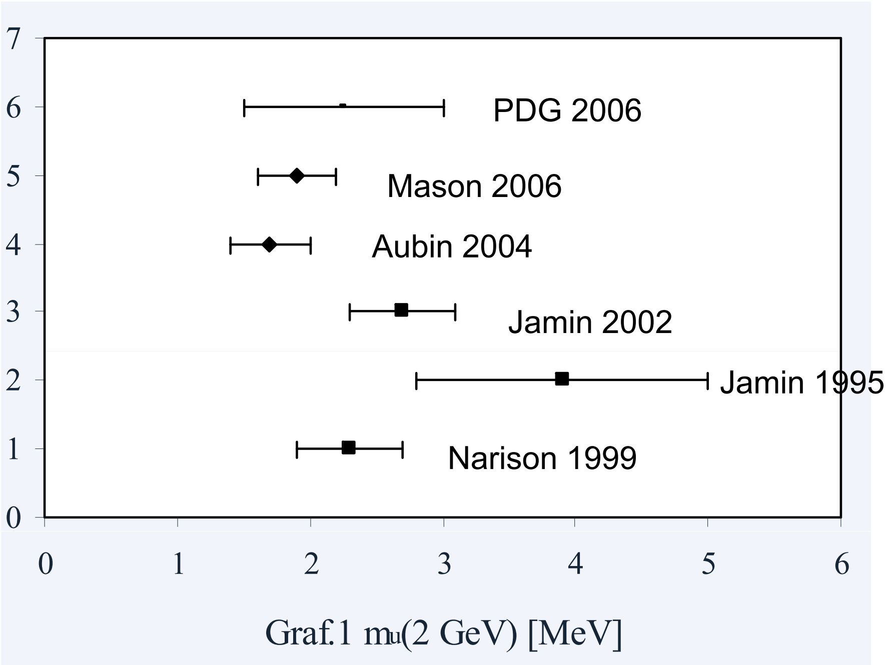
$$\int_0^{s_0} w(s)\,R(s)\,ds = 6\pi i \oint_{|s|=s_0} w(s)\,\Pi(s)\,ds$$

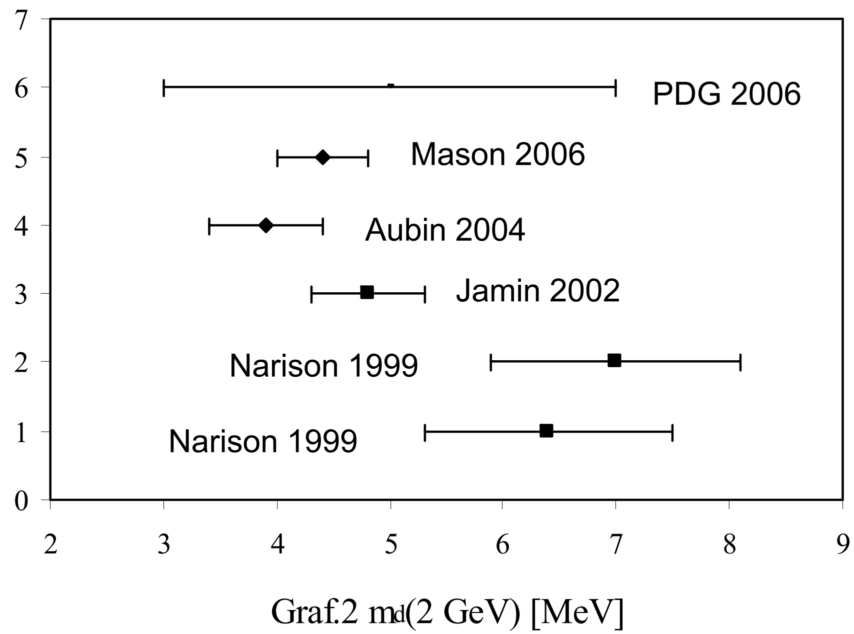
Lattice QCD

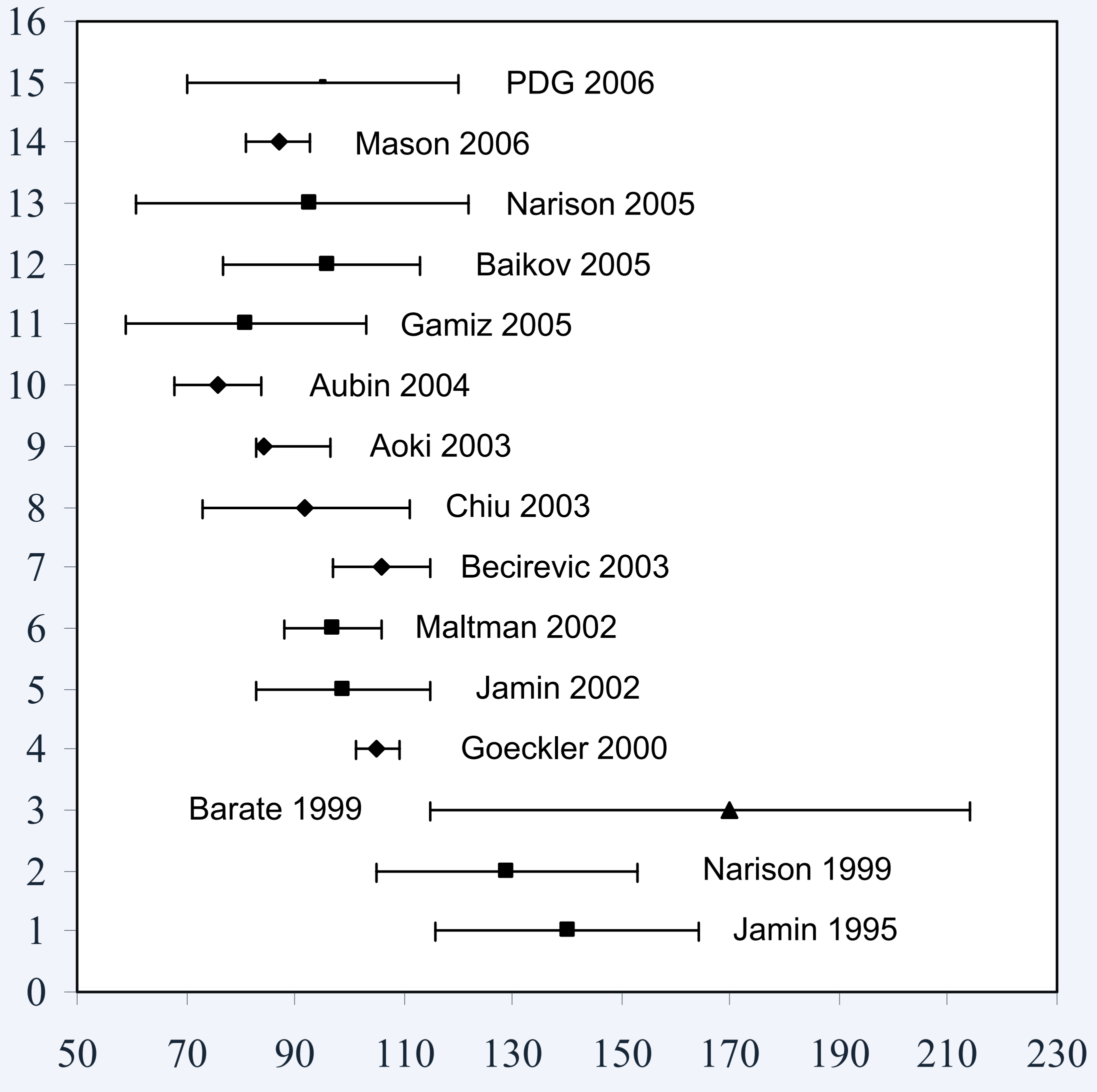
- investigates QCD non-pertubatively
- space-time is approximated by a finite, discrete lattice of points
- input parameters (hadron masses) are experimentally evalueted
- „physical“ quark masses are obtained after proper extrapolation to the continuum limit
- bare quark masses (with lattice spacing *a*) -> renormalisation to MS-bar scheme
- earlier quenched simulations are now replaced by unquenched simulation with sea quarks ($N_f = 3$)

Light quarks

- *u*, *d* and *s* quark
- ■ Sum rules, ◆ Lattice QCD, ▲ directly measured, · PDG



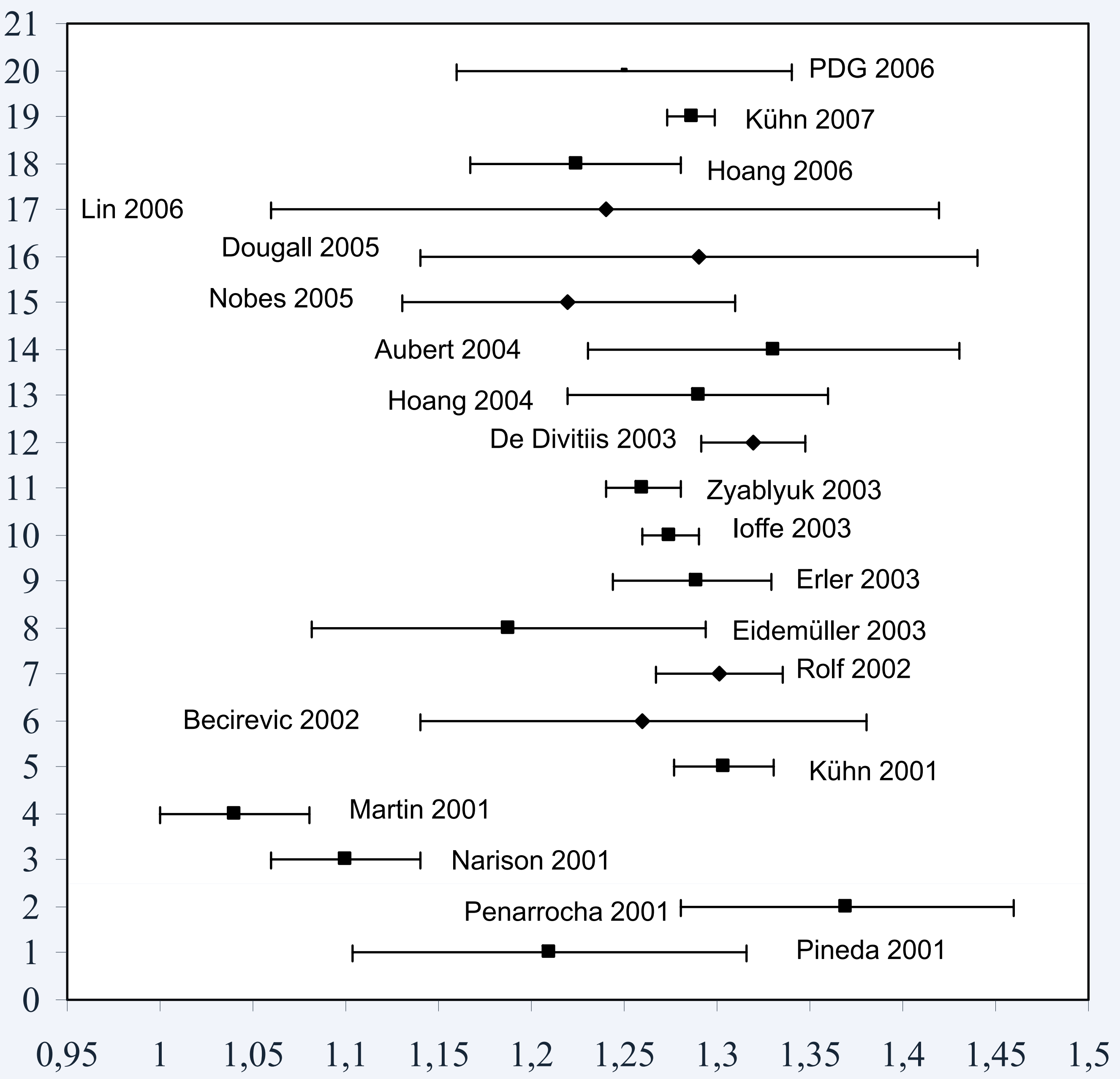




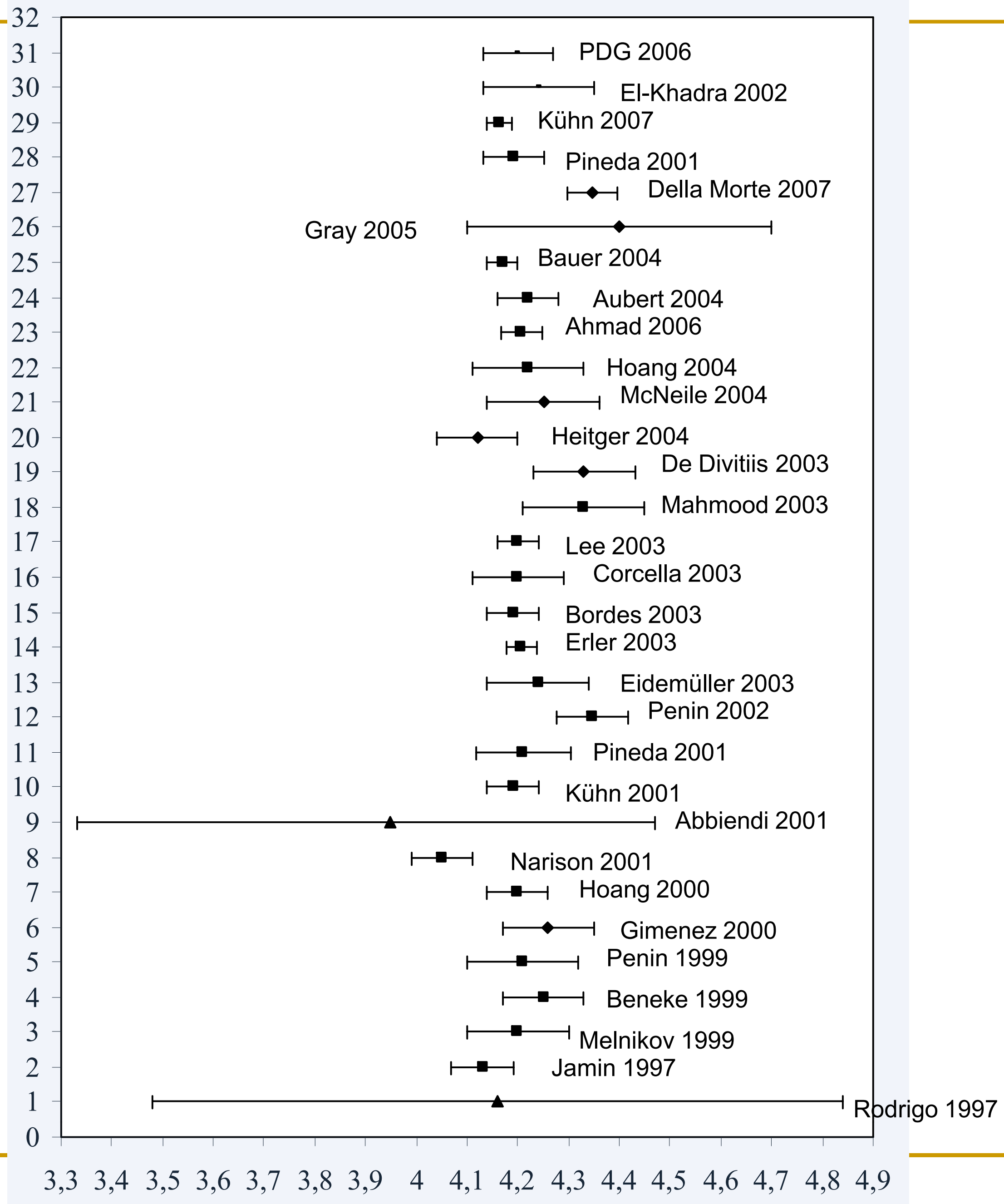
Graf.3 m_s(2 GeV) [MeV]

Heavy quarks

- *c*, *b* and *t* quarks
- ■ Sum rules, ◆ Lattice QCD, ▲ directly measured,
  · PDG and El-Khadra
- for light quarks masses is the uncertainty about 20 %, for heavy masses a few percent
- EFT (Effective field theory) - HQET, NRQCD and Lattice QCD



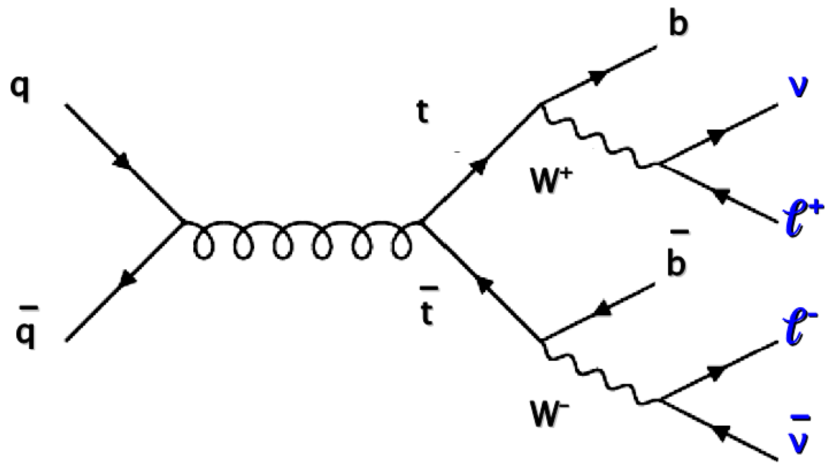
Graf.4 $m_c(m_c)$ [GeV]



Graf.5 $m_b(m_b)$ [GeV]

Top quark mass

- directly at Tevatron by the CDF and D0 groups
- top pairs mostly produced through *qantiq* annihilation, then decay:

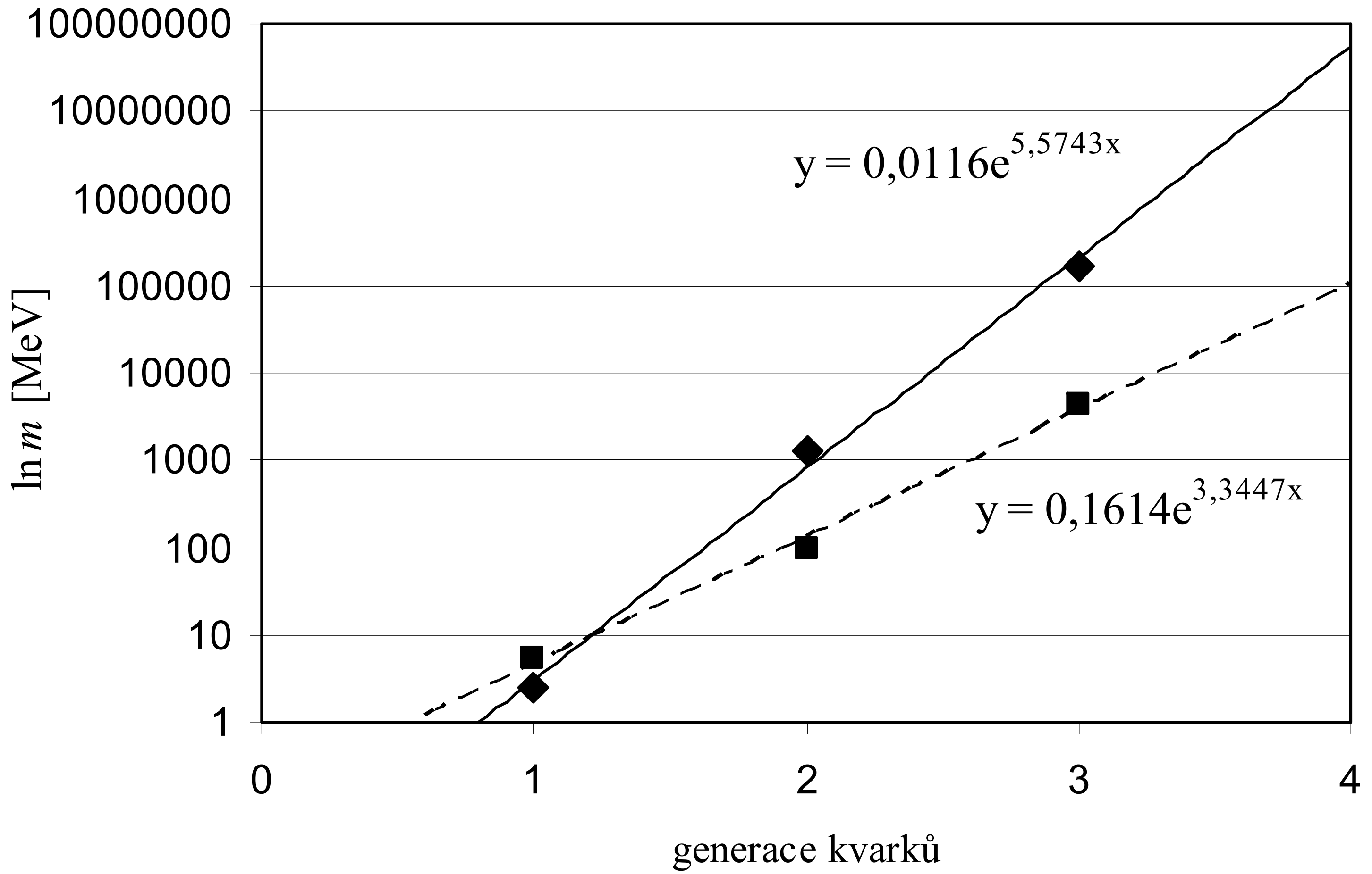


- world‘s average for pole mass by TEVEWWG (03/2007):

  M_t = (170,9 ± 1,8) GeV

Results for quark masses

m_u (2 GeV) 2,46 ± 0,54 MeV	m_d (2 GeV) 5,25 ± 0,93 MeV	m_s (2 GeV) 98,6 ± 15,8 MeV
m_c (m_c) 1,25 ± 0,07 GeV	m_b (m_b) 4,22 ± 0,08 GeV	M_t 170,9 ± 1,8 GeV



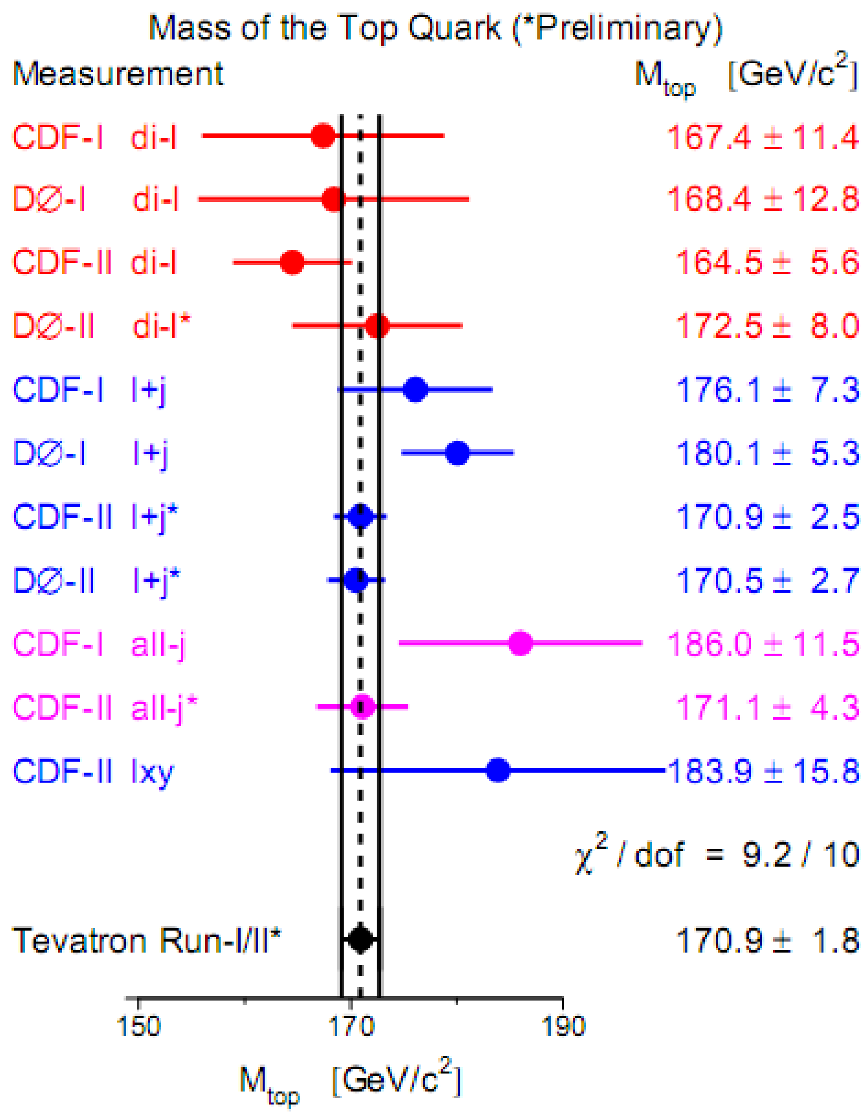
Exponential dependence between quark masses according to their generation (■ *d*, *s* and *b* a ◆ *u*, *c* and *t*)

Top quark mass at Fermilab

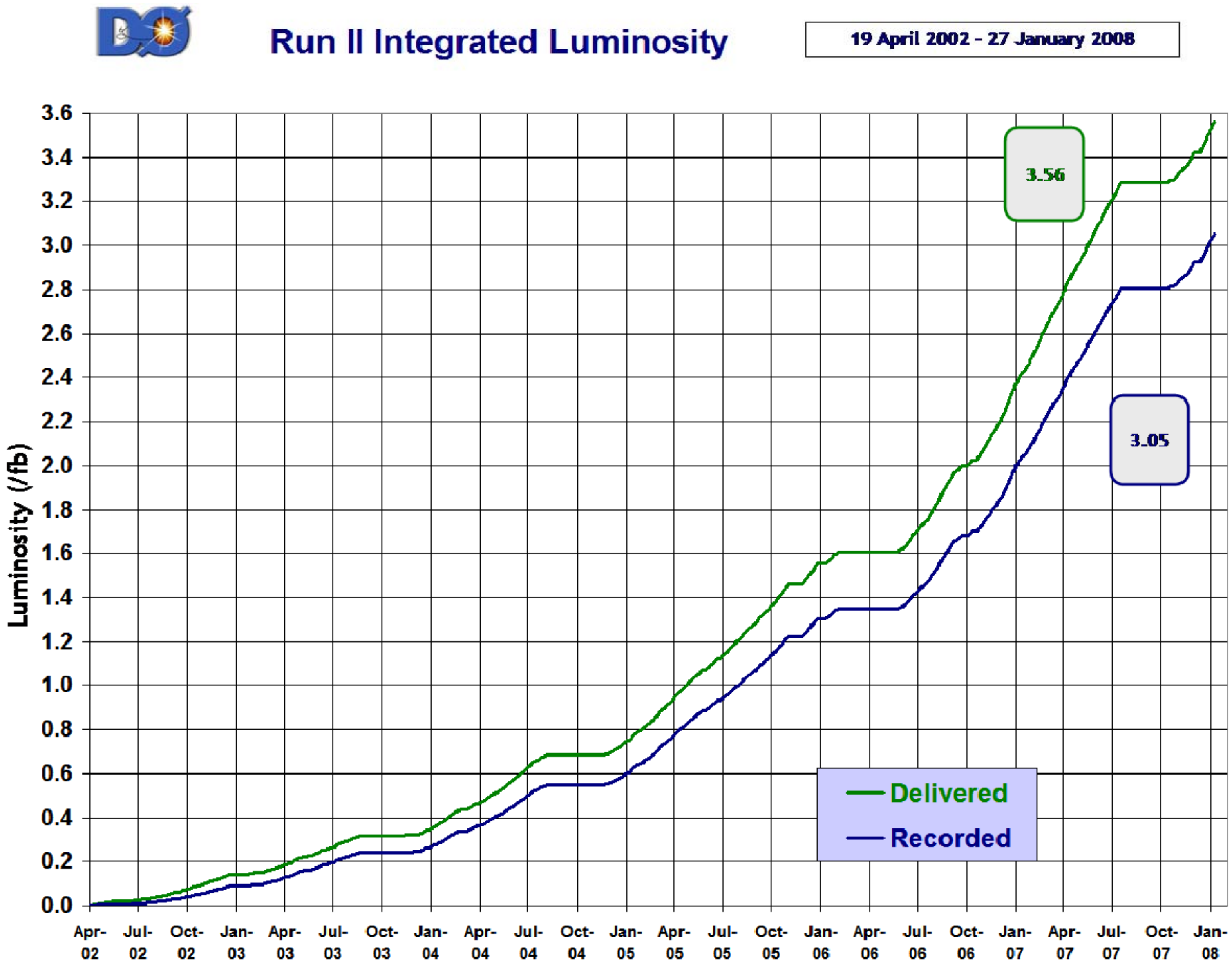
- Channels: - dilepton
  - lepton+jets
  - all-jets
- Methods: - Template Method (TM)
  - Matrix Element Method (MEM)
  - Ideogram Method (IM)

For dilepton channel - use weighting algorithms

- Neutrino Weighting Method
- Matrix Weighting Method



Mass of the Top Quark (*Preliminary)
Measurement
M_top [GeV/c^2]
CDF-I di-l
167.4 ± 11.4
DØ-I di-l
168.4 ± 12.8
CDF-II di-l
164.5 ± 5.6
DØ-II di-l*
172.5 ± 8.0
CDF-I l+j
176.1 ± 7.3
DØ-I l+j
180.1 ± 5.3
CDF-II l+j*
170.9 ± 2.5
DØ-II l+j*
170.5 ± 2.7
CDF-I all-j
186.0 ± 11.5
CDF-II all-j*
171.1 ± 4.3
CDF-II lxy
183.9 ± 15.8
χ^2 / dof = 9.2 / 10
Tevatron Run-I/II*
170.9 ± 1.8
150
170
190
M_top [GeV/c^2]



Run II Integrated Luminosity
19 April 2002 - 27 January 2008
Luminosity (/fb)
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Apr-02
Jul-02
Oct-02
Jan-03
Apr-03
Jul-03
Oct-03
Jan-04
Apr-04
Jul-04
Oct-04
Jan-05
Apr-05
Jul-05
Oct-05
Jan-06
Apr-06
Jul-06
Oct-06
Jan-07
Apr-07
Jul-07
Oct-07
Jan-08
3.56
3.05
Delivered
Recorded

My analysis

- just starting on dilepton channel
- all *e*+*e*, *e*+*μ* and *μ*+*μ* events
- loose skims from Common Sample Group, one skim for each channel (*ee*, *eμ, μμ)*
- Template method based on older reconstruction program by K. Smolek and P. Homola
- reconstruction of top mass by solving the set of equations describing kinematic constraints of this decay
- hope of gettting good statistical sensitivity on 3 fb^-1, although it will probably lead to a worse sensitivity than the matrix element method and the matrix weighting method

Kinematic constraints in dilepton decay:

$$p_x^b + p_x^{\bar{b}} + p_x^\nu + p_x^{\bar{\nu}} + p_x^l + p_x^{\bar{l}} = 0$$

$$p_y^b + p_y^{\bar{b}} + p_y^\nu + p_y^{\bar{\nu}} + p_y^l + p_y^{\bar{l}} = 0$$

$$(E_{l^+} + E_\nu)^2 - (p_x^{l^+} + p_x^\nu)^2 - (p_y^{l^+} + p_y^\nu)^2 - (p_z^{l^+} + p_z^\nu)^2 - M_{\mathrm{W}^+}^2 = 0$$

$$(E_{l^-} + E_{\bar{\nu}})^2 - (p_x^{l^-} + p_x^{\bar{\nu}})^2 - (p_y^{l^-} + p_y^{\bar{\nu}})^2 - (p_z^{l^-} + p_z^{\bar{\nu}})^2 - M_{\mathrm{W}^-}^2 = 0$$

$$(E_{l^+} + E_\nu + E_b)^2 - (p_x^{l^+} + p_x^\nu + p_x^b)^2 - (p_y^{l^+} + p_y^\nu + p_y^b)^2 - (p_z^{l^+} + p_z^\nu + p_z^b)^2 - M_t^2 = 0$$

$$(E_{l^-} + E_{\bar{\nu}} + E_{\bar{b}})^2 - (p_x^{l^-} + p_x^{\bar{\nu}} + p_x^{\bar{b}})^2 - (p_y^{l^-} + p_y^{\bar{\nu}} + p_y^{\bar{b}})^2 - (p_z^{l^-} + p_z^{\bar{\nu}} + p_z^{\bar{b}})^2 - M_{\bar{t}}^2 = 0$$

Set of four nonlinear equations:

$$K_3\sqrt{p_x^{\bar{\nu}2} + p_y^{\bar{\nu}2} + p_z^{\bar{\nu}2}} - (p_x^{l^-} p_x^{\bar{\nu}} + p_y^{l^-} p_y^{\bar{\nu}} + p_z^{l^-} p_z^{\bar{\nu}}) + M_1 = 0$$

$$K_4\sqrt{(p_x^{\bar{\nu}} - K_1)^2 + (p_y^{\bar{\nu}} - K_2)^2 + p_z^{\nu 2}} + p_x^{l^+}(p_x^{\bar{\nu}} - K_1) + p_y^{l^+}(p_y^{\bar{\nu}} - K_2) + p_z^{l^+} p_z^\nu + M_2 = 0$$

$$\left(C_1 + \sqrt{p_x^{\bar{\nu}2} + p_y^{\bar{\nu}2} + p_z^{\bar{\nu}2}}\right)^2 - (p_x^{\bar{\nu}} + K_5)^2 - (p_y^{\bar{\nu}} + K_6)^2 - (p_z^{\bar{\nu}} + K_7)^2 + M_3 = 0$$

$$\left(C_2 + \sqrt{(p_x^{\bar{\nu}} - K_1)^2 + (p_y^{\bar{\nu}} - K_2)^2 + p_z^{\nu 2}}\right)^2 - (p_x^{\bar{\nu}} + K_5)^2 - (p_y^{\bar{\nu}} + K_6)^2 - (p_z^{\nu} + K_8)^2 + M_4 = 0$$

for unknowns $p_x^{\bar{\nu}}, p_y^{\bar{\nu}}, p_z^{\bar{\nu}}, p_z^\nu$

• Transform to solvable set of two polynomical equations – one of the fourth and of the third order. It leads to decreasing dimension of the problem – remainings unknows - $p_x^{\overline{\nu}}$ and $p_y^{\overline{\nu}}$.

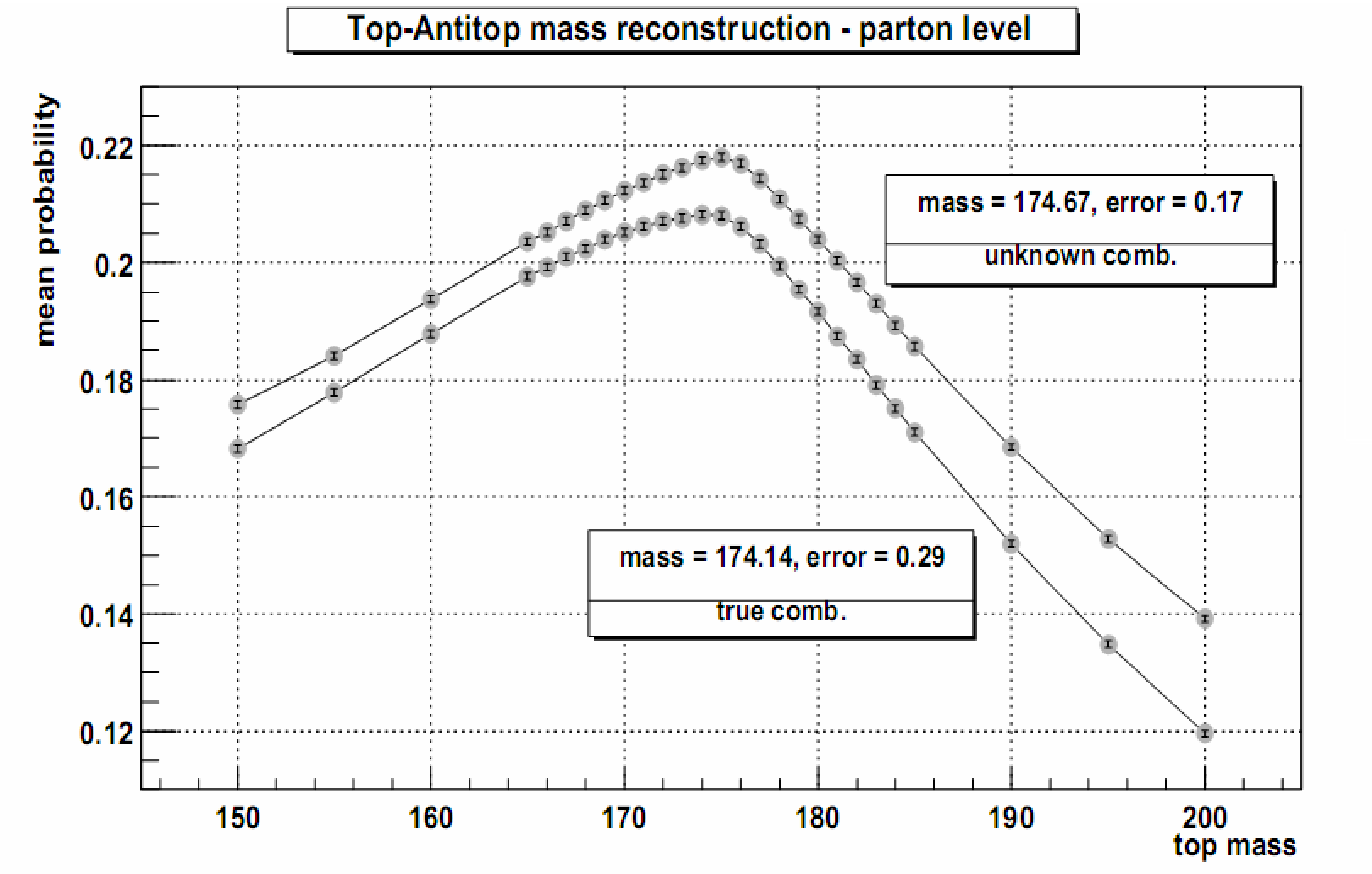
• Principle idea is to reduce step-by-step walking to one-dimensional interval and analytically compute other unknow from cubic equation which si made by multiplying previous two polynomical equations

• Input parametres:

top mass (assume), W mass, px – py – pz and E od jet1, px – py – pz and E od jet2, px – py – pz of lepton, px – py – pz of antilepton, px – py missing pT

• cannot distinguish b-jet and b-bar jet – computing for for both possibilities

•Idea for top mass reconstruction: solve the equations for various fixed masses of top and observe the dependency of numbers of solutions found and probabilities for the best solution on this mass



Top-Antitop mass reconstruction - parton level
mean probability
0.22
0.2
0.18
0.16
0.14
0.12
150
160
170
180
190
200
top mass
mass = 174.67, error = 0.17
unknown comb.
mass = 174.14, error = 0.29
true comb.

Matrix Weighting Method at D0

- new preliminary results from D. Boline and U.Heintz
- algorithm: assume m_t, calculate $p_x^{\nu}, p_y^{\nu}, p_x^{\bar{\nu}}, p_y^{\bar{\nu}}$ using assumptions and kinematic constraints and $metx, mety$

  calculate probability density function for a given m_t as a function of the lepton energy *E* in the top quark rest frame

$$p(E|m_t) = 4 m_t E \frac{m_t^2 - m_b^2 - 2 m_t E}{(m_t^2 - m_b^2)^2 + m_W^2 (m_t^2 + m_b^2) - 2 m_W^4}$$

  sum over all solutions and jet-lepton combinations

- weight assigned to each solution

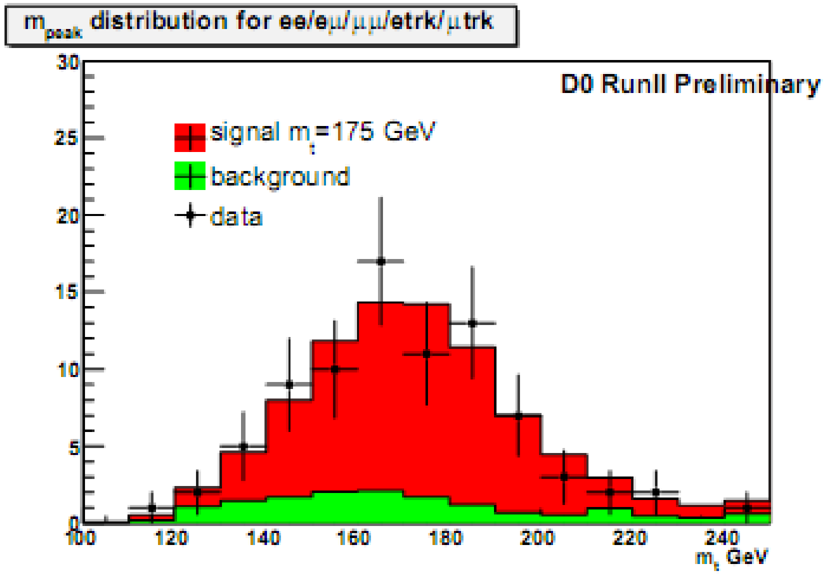
$$w = f(x)f(\overline{x})p(E_\ell^*|m_t)p(E_{\overline{\ell}}^*|m_t)$$

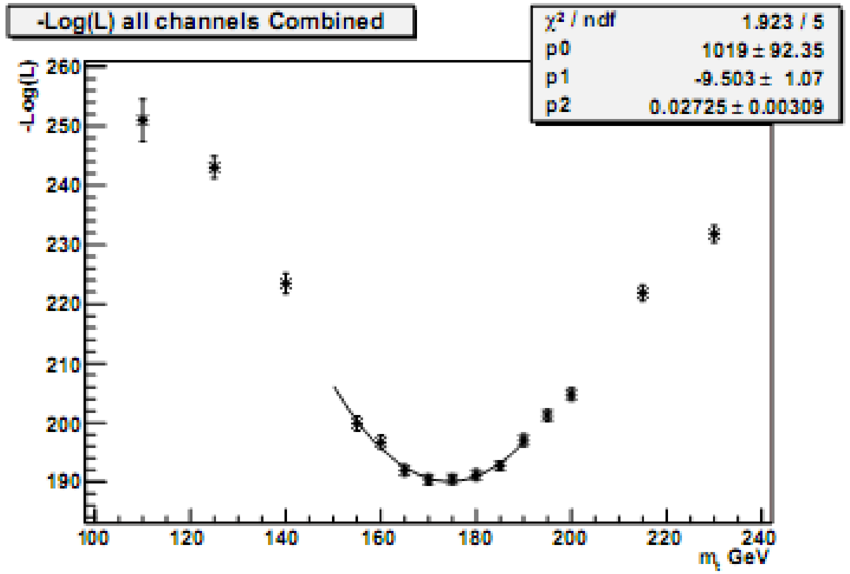
where *f*(*x*) is the Parton Distribution Function (PDF) and p is probability for hypothesized top mass

- likelihood for each value of the top quark mass is given by the sum of the weights over all the possible solutions

$$W_0(m_t) = \sum_{\text{sol}} \sum_{\text{assign}} w_{ij}$$

- measurement in the dilepton channels (*ee*, *eμ*, *μμ*, *e*track, *μ*track) on about 1 fb^-1 of data





Plots of –ln L versus top quark mass (left) and comparsion of peak masses in data and MC (right). For all five channels

The calibrated result for the combination of all channels with systematics is:

$$m_t = 174.9 \pm 4.2(stat)^{+2.3}_{-3.3}(syst)\ \mathrm{GeV}$$