Cosmological perturbations beyond linear order

Diego Blas

CERN

w/ R. Angulo, M. Crocce, M. Garny, T. Konstandin, R. Scoccimarro

JCAP 09 (2013) 024

arXiv: 1309.3308 [astro-ph.CO]

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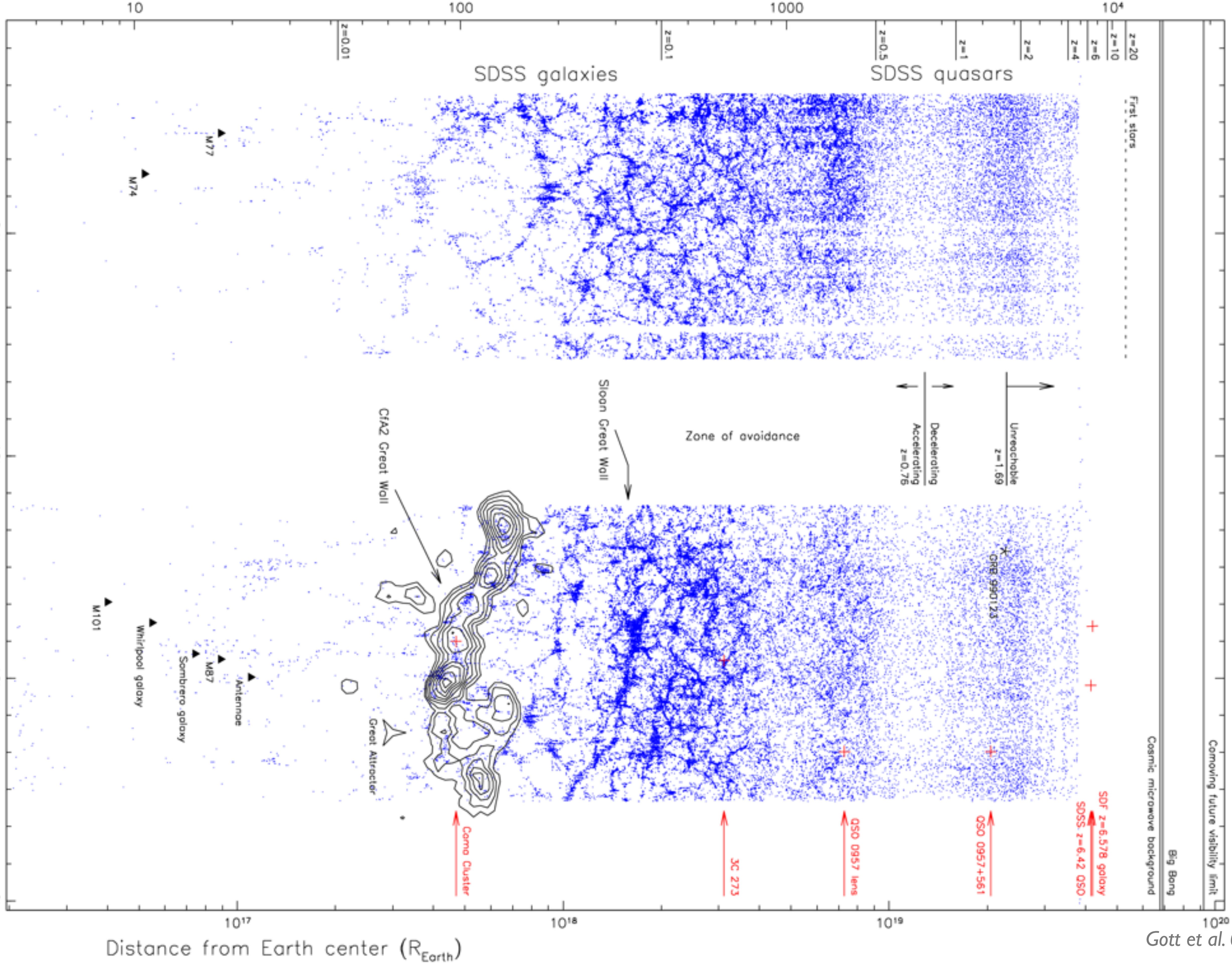
The Observed Universe

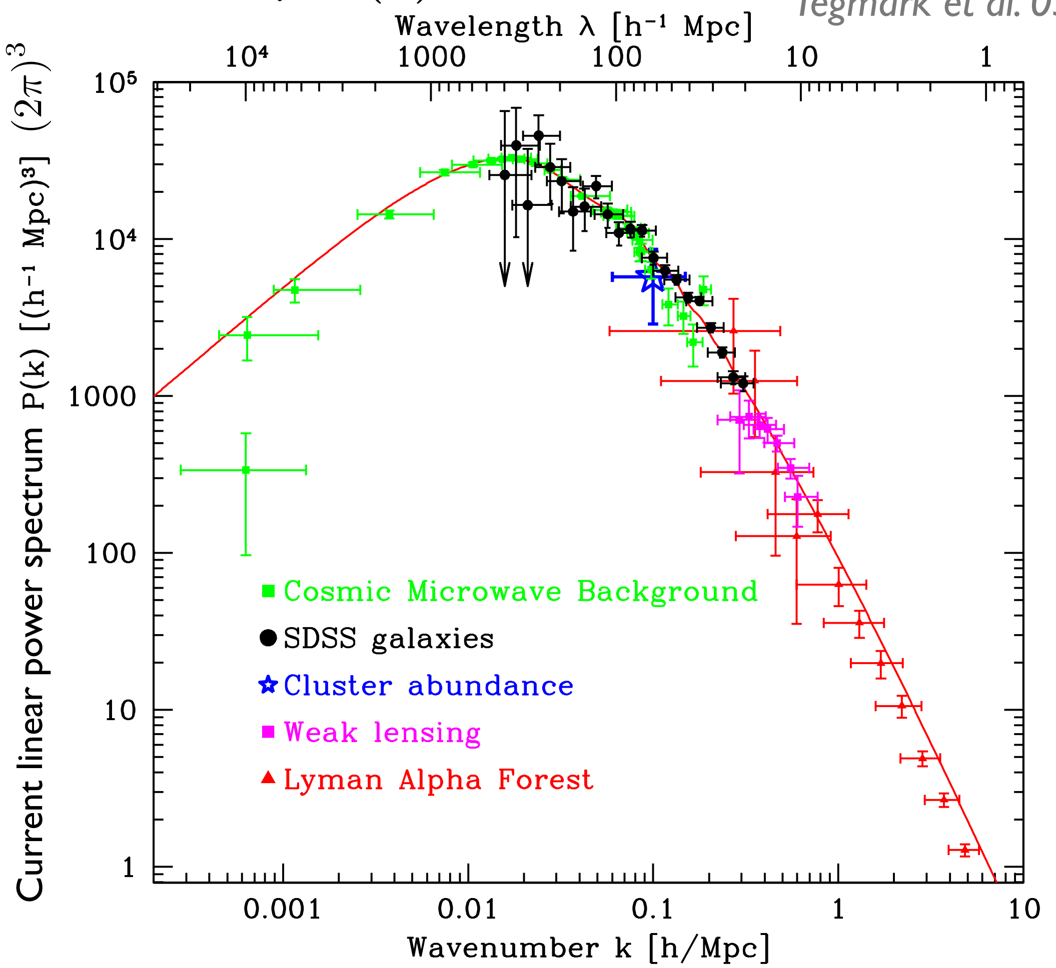


Gott et al. 05

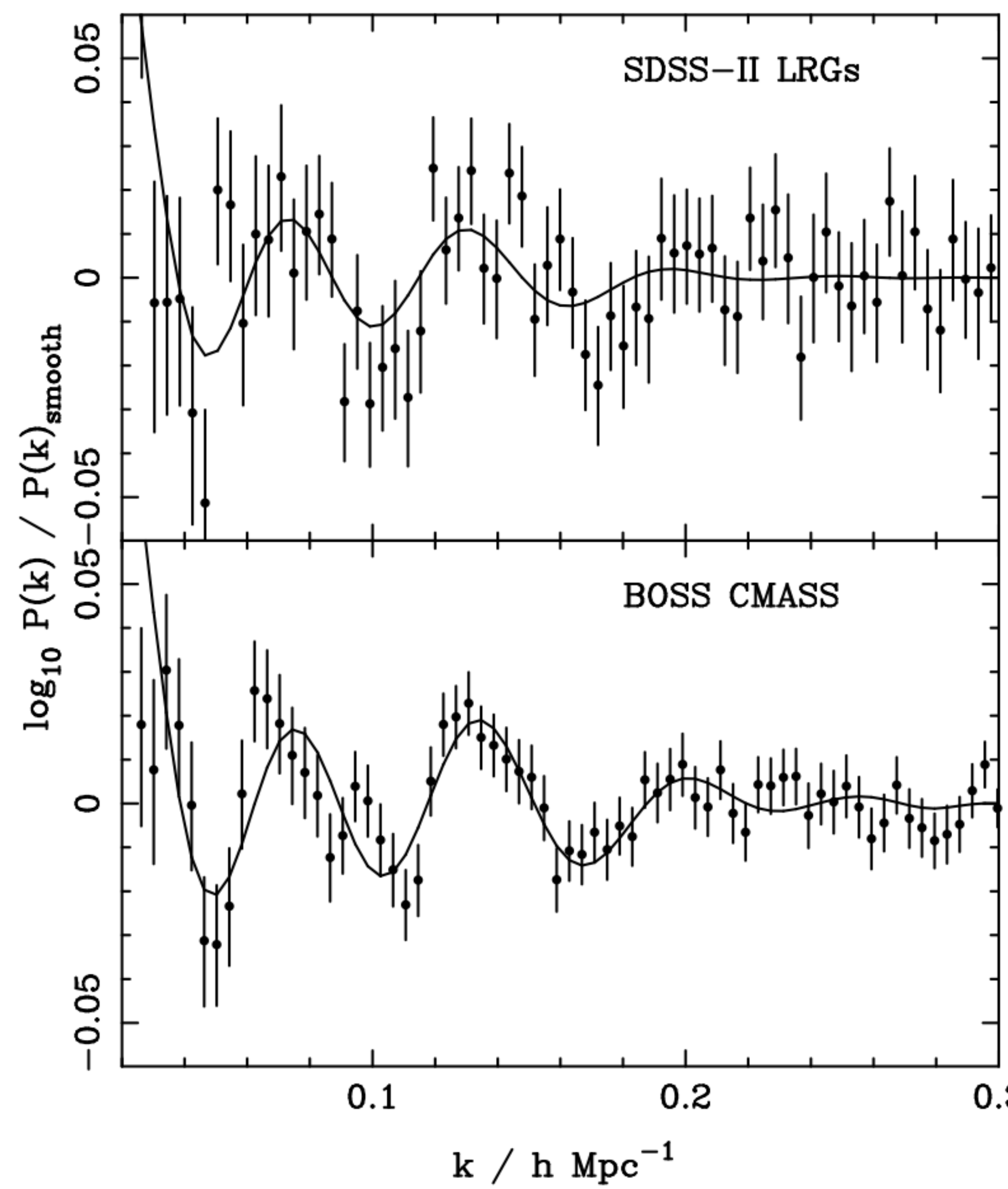
Observables: Matter power spectrum

$$\delta_n \equiv \frac{\rho_n(x,t)}{\bar{\rho}_n(t)} - 1 \qquad \langle \delta(k,t)\delta(k',t)\rangle = P(k,t)\delta^{(3)}(k+k')$$

Tegmark et al. 03



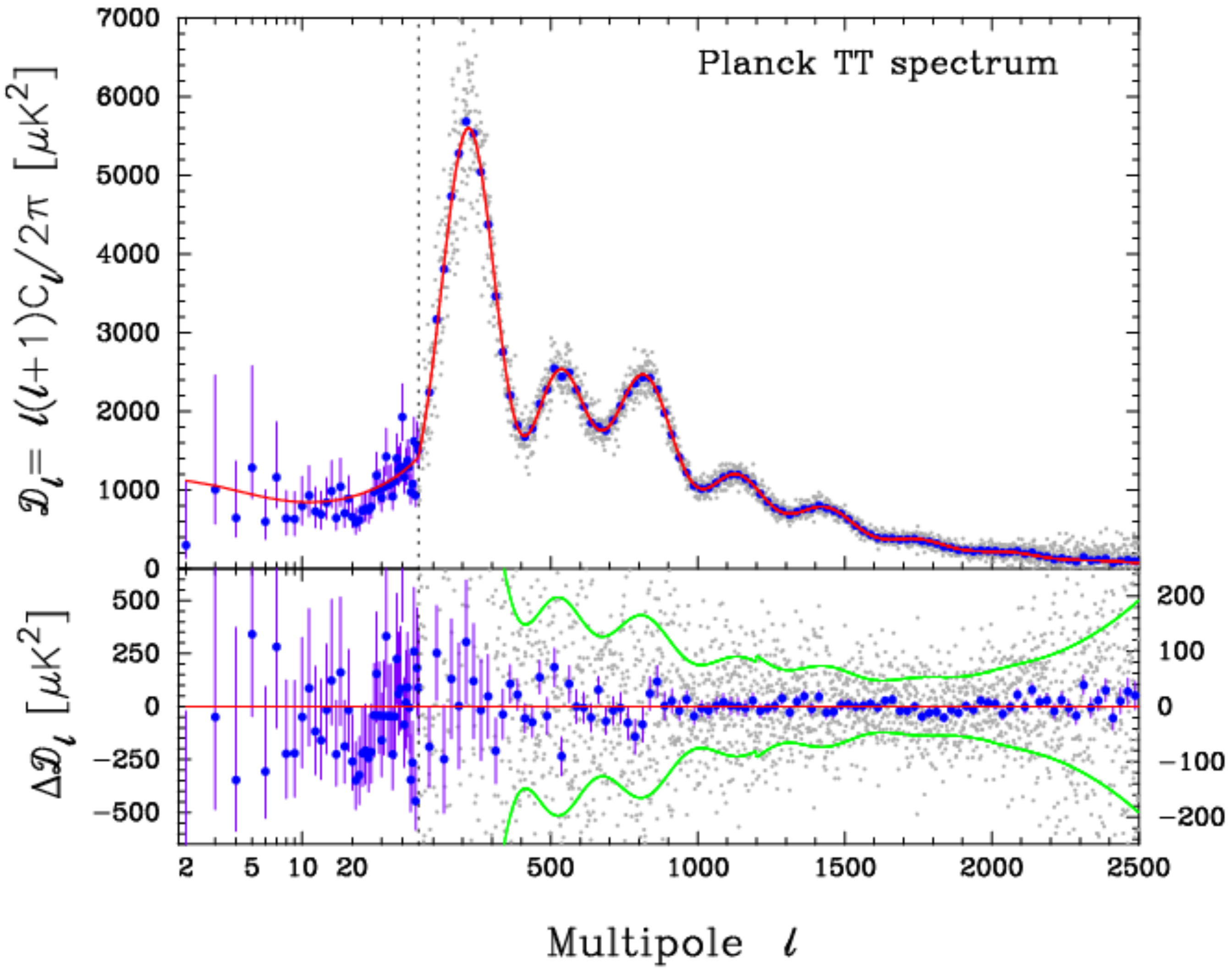
Anderson et al. 12



DES, Euclid, LSST,...: 1% level at different redshifts!
(also higher n-point correlation functions)

Early times information I: CMB

Planck Collaboration 13



	Best fit	68% limit
Ω_Λ	0.6825	0.686 ± 0.020
Ω_m	0.3175	0.314 ± 0.020
σ_8	0.8344	0.834 ± 0.027
z_{re}	11.35	$11.4^{+4.0}_{-2.8}$
H_0	67.11	67.4 ± 1.4
$10^9 A_s$	2.215	2.23 ± 0.16
...		

Early times information II

Very homogenous Universe at early times
+ **perturbations**

$$\mathcal{H}^2 = \frac{8\pi G}{3} a^2 \sum \rho_n$$

$$\mathcal{H} \equiv \frac{\dot{a}}{a}\,, \Omega_n \equiv \frac{\rho_n(t)}{\rho_c(t)}$$

SubH gravity: $$\Delta\phi = \frac{3}{2}\mathcal{H}^2 \sum \Omega_n \delta_n\,,$$

Matter:

- Plasma perturbations (baryons, photons) $$\delta_n \equiv \frac{\rho_n(x,t)}{\bar{\rho}_n(t)} - 1$$

Gravity increase collapse, pressure from photons prevents it

$$\ddot{\delta}_p + \mathcal{H}\dot{\delta}_p - c_p^2 k^2 \delta_p \supset k^2 \phi$$

Dominant before r-m equality: BAO

Imprint in the CMB

- DM (almost) decoupled

$$\ddot{\delta} + \mathcal{H}\dot{\delta} \supset k^2 \phi$$

Dominant after r-m equality
Collapsing!

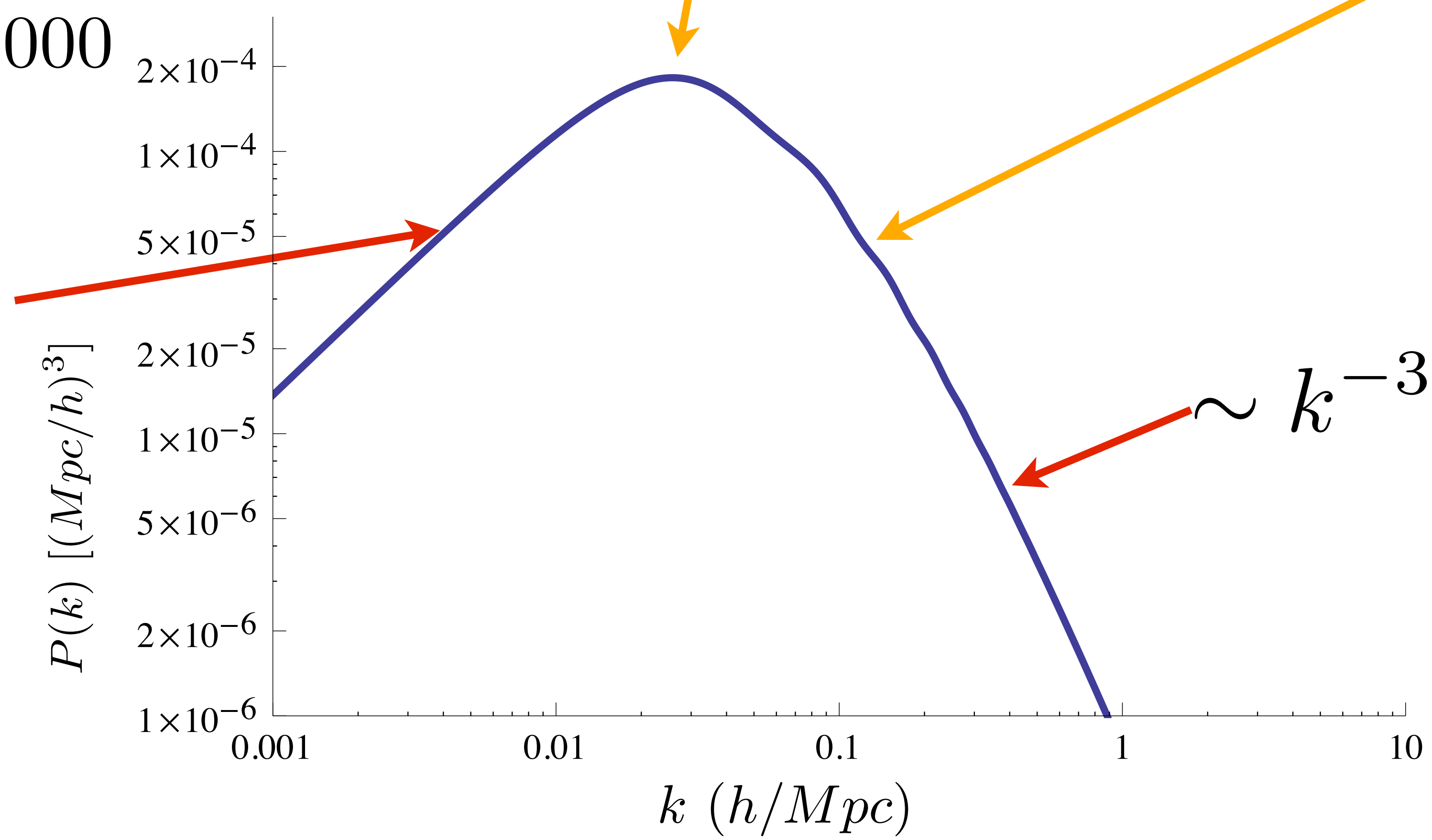
neutrinos, Λ

Matter power spectrum at decoupling

gaussian initial scale invariant PS +
radiation-matter transition + BAO imprint

CLASS CMB Code
Blas, Lesgourgues, Tram 11

$z = 1000$



$\sim k$

$\sim k^{-3}$

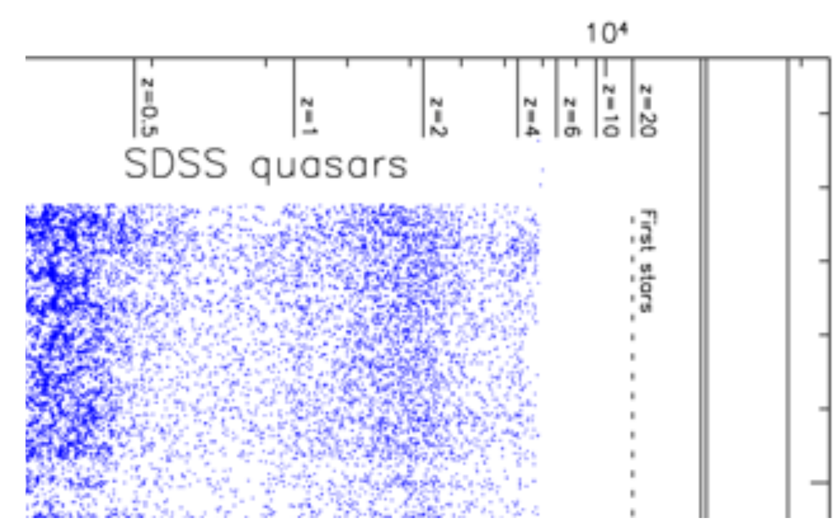
Eisenstein, Hu 97

$$P_k \sim \frac{k(log(e+1.84\beta k))^2}{(log(e+1.84\beta k)+Ck^2)^2}$$

Small quantity for PT: $\delta_k(z=1000)$

Gravity makes matter clump: $\delta_k \sim a(t)$

perturbations grow! PT will break down



what can we learn from PT?

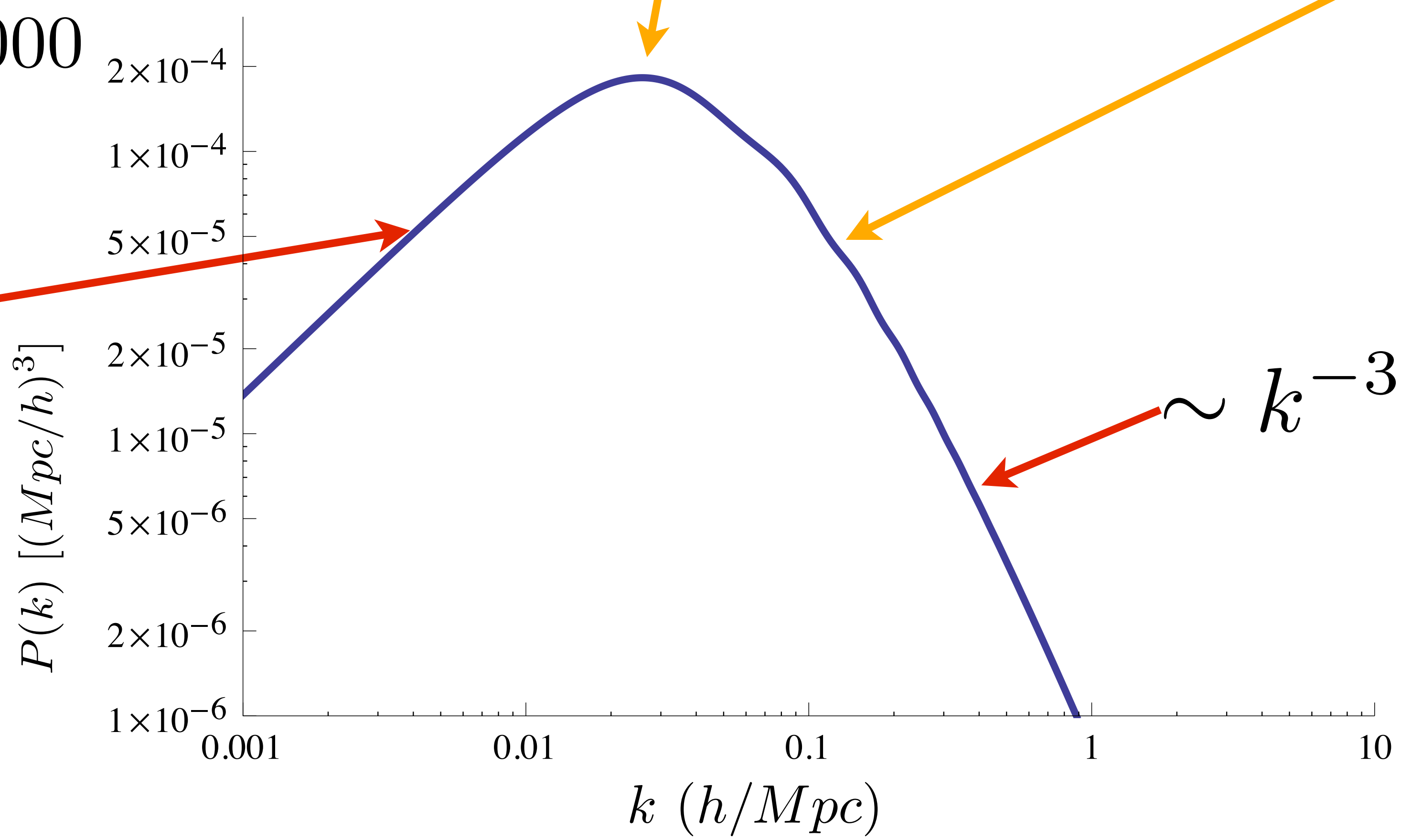
Matter power spectrum at decoupling

gaussian initial scale invariant PS +
radiation-matter transition + BAO imprint

CLASS CMB Code
Blas, Lesgourgues, Tram 11

$z = 1000$

$\sim k$



$\sim k^{-3}$

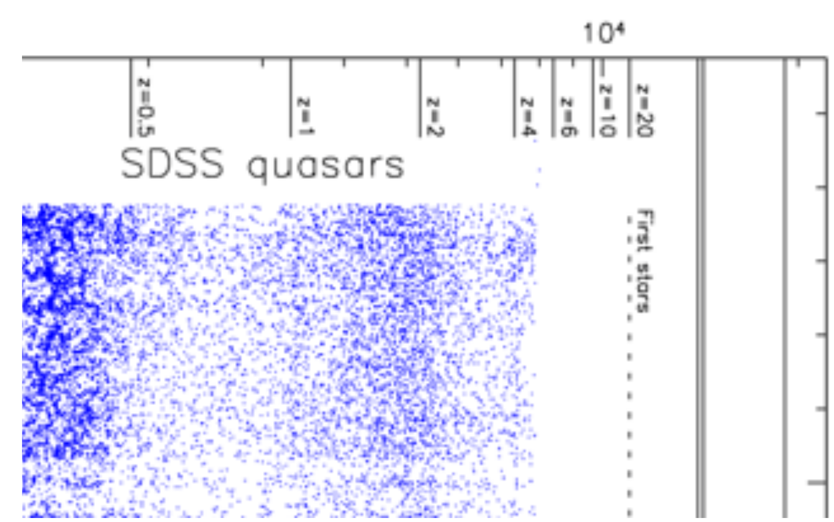
Eisenstein, Hu 97

$$P_k \sim \frac{k}{(1 + k^2/k_0^2)^2}$$

Small quantity for PT: $\delta_k(z = 1000)$

Gravity makes matter clump: $\delta_k \sim a(t)$

perturbations grow! PT will break down



what can we learn from PT?

Theoretical framework

$$8\pi G T^m_{\mu\nu} = G_{\mu\nu}$$

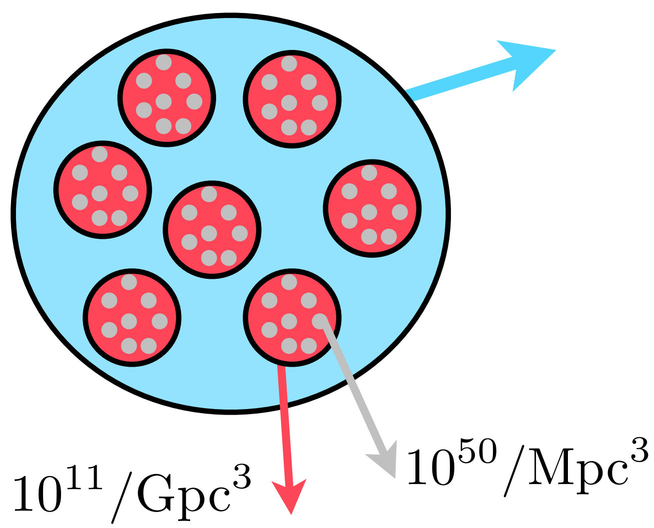
Matter   Non-relativistic and small ϕ

Fluid: collapsing matter (DM+b) matches very well a pressureless medium interacting through gravity

Continuity $\dot{\delta}_{DM} + \partial_i([1+\delta_{DM}]v^i_{DM}) = 0$

Euler $\dot{v}^i_{DM} + \mathcal{H} v^i_{DM} + v^j_{DM}\partial_j v^i_{DM} = -\partial_i \phi$

GR $\longrightarrow$ $\Delta\phi = \frac{3}{2}\mathcal{H}^2 \sum \Omega_n \delta_n$



Vlasov: ‘Particles’ (sampling δ_{DM}) interacting through gravity

$$p^i_A \equiv a m_A v^i_A \,, \qquad \frac{\mathrm{d}p^i_A}{\mathrm{d}t} = -a m_A \partial_i \phi$$

$$\frac{\mathrm{d}f(x,p,t)}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \frac{p^i_A}{a m_A}\partial_i f - a m_A \partial_i \phi \frac{\partial f}{\partial p^i_A} = 0$$

Validity of the fluid description

Taking moments:

Particles per volume $\int \mathrm{d}^3p f(x,p,t) \equiv \rho(x,t)$, $\delta_n \equiv \frac{\rho_n(x,t)}{\bar{\rho}_n(t)} - 1$

Velocity field $\int \mathrm{d}^3p \frac{p^i}{am} f(x,p,t) \equiv \rho(x,t)\, v^i(x,t)$

...

$$\frac{\mathrm{d}f(x,p,t)}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \frac{p_A^i}{am_A}\partial_i f - am_A \partial_i \phi \frac{\partial f}{\partial p_A^i} = 0 \ , \quad \Delta\phi = \frac{3}{2}\mathcal{H}^2 \sum \Omega_n \delta_n$$

$$\dot{\delta} + \partial_i([1+\delta]v^i) = 0$$

$$\dot{v}^i + \mathcal{H}v^i + v^j\partial_j v^i = -\partial_i\phi - \frac{1}{\rho}\partial_j \left[\int \mathrm{d}^3p \frac{p^i p^j}{(am)^2} f - \rho\, v^i v^j\right]$$

...

Deviation from single flow
(Shell crossing)

Decomposition of v:

Divergence $\theta \equiv \partial_i v^i$

Vorticity $w^i \equiv \epsilon^{ijk}\partial_j u^k$

Pressureless perfect fluid

Pueblas, Scoccimarro 09

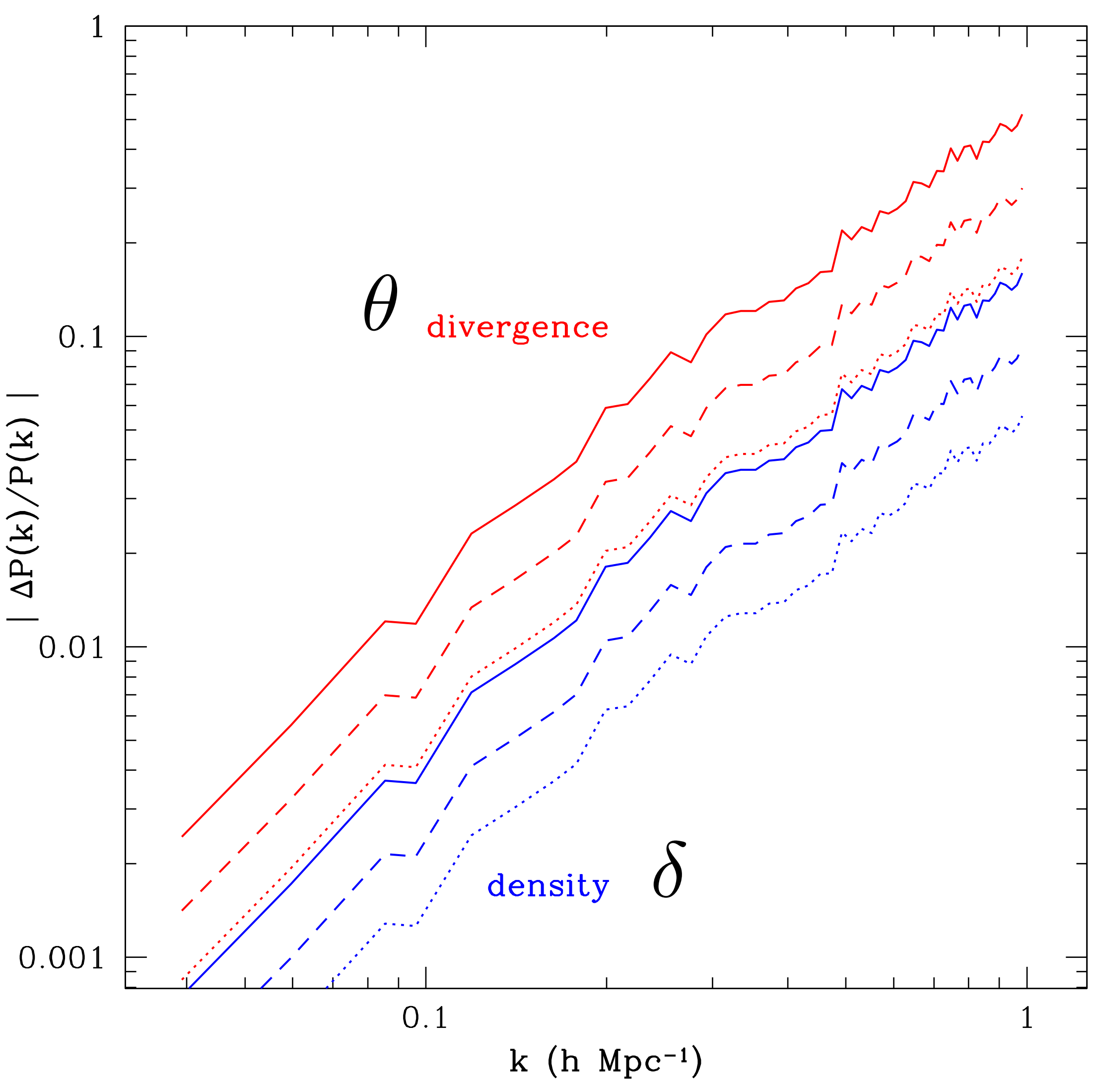
$$\dot{\delta}(k,t) + \theta(k,t) = 0$$

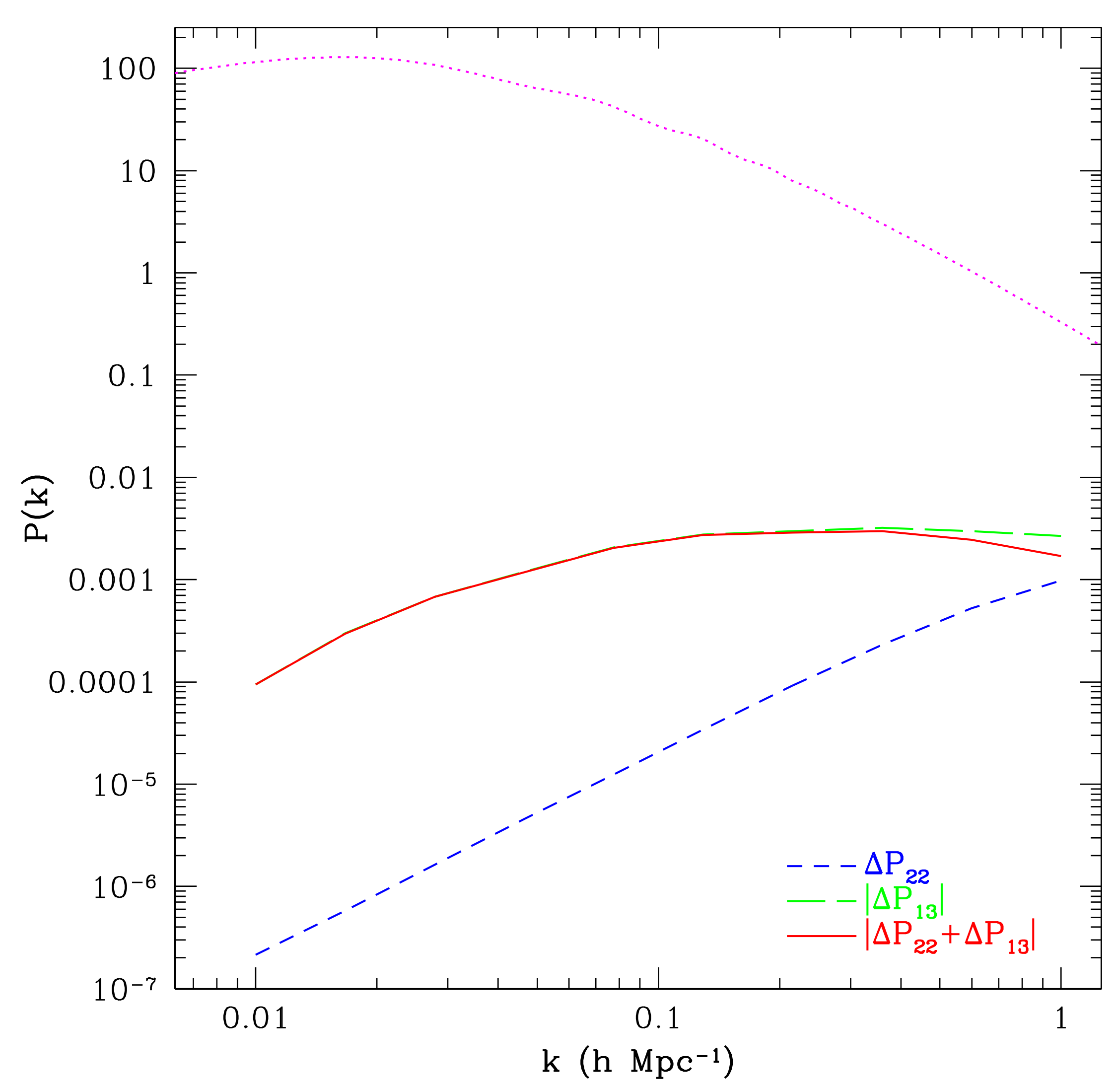
$$\dot{\theta}(k,t) + \mathcal{H}\theta(k,t) + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta(k,t) = q_\theta$$

Shell-crossing

$$\dot{w}^i + \frac{1}{2} w^i = q_w^i$$

Vorticity





Perturbation theory (PT)

$\theta \equiv \partial_i v^i$

Linear — Vertices (MC)

$$\dot{\delta}(k,t) + \theta(k,t) = -\alpha(k_1,k_2)\theta(k_1,t)\delta(k_2,t)\delta^{(3)}(k_1+k_2-k)$$

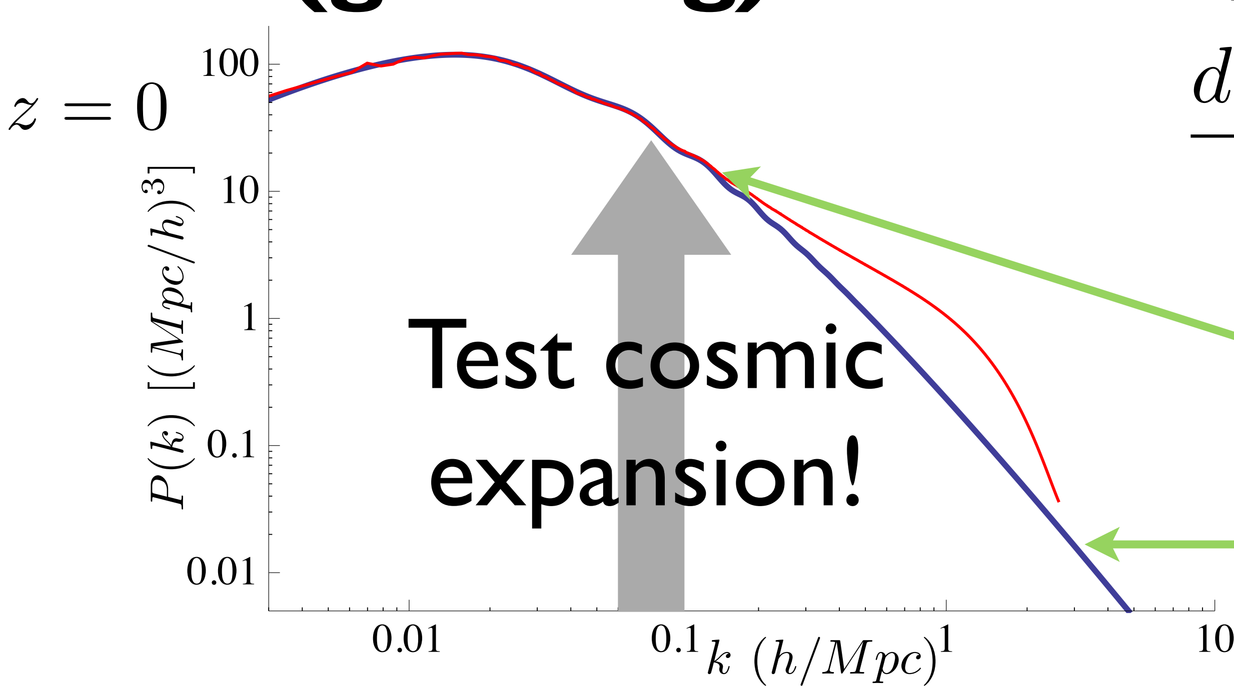
$$\dot{\theta}(k,t) + \mathcal{H}\theta(k,t) + \frac{3}{2}\Omega_m\mathcal{H}^2\delta(k,t) = -\beta(k_1,k_2)\theta(k_1,t)\theta(k_2,t)\delta^{(3)}(k_1+k_2-k)$$

$$\alpha(k_1,k_2) \equiv \frac{(k_1+k_2)\cdot k_1}{k_1^2}, \quad \beta(k_1,k_2) \equiv \frac{(k_1+k_2)^2 k_2\cdot k_1}{2k_1^2k_2^2}$$

$$\delta(k,t) = \sum_n \tilde{F}_n(t;k_1,...,k_n)\delta(k_1,t_0)\ldots\delta(k_n,t_0)\delta^{(3)}(k-\sum k_i)$$

Linear (growing) mode: $\delta_L(k,t) = D^{(+)}(t)\delta(k,t_0)$

$$\frac{d^2D^{(+)}}{d\tau^2} + \mathcal{H}\frac{dD^{(+)}}{d\tau} = \frac{3}{2}\Omega_m\mathcal{H}^2D^{(+)}$$



Horizon Run 2, Kim et al. 11

BAO weakly non-linear!

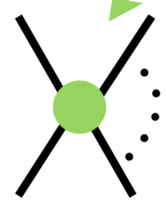
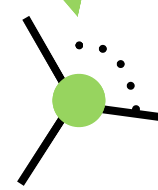
Linear prediction

NNLO formalism

$\Omega_m = 1$

$$\delta(k,t) = \sum_n a(t)^n F_n(k_1, ..., k_n)\delta(k_1,t_0)\ldots\delta(k_n,t_0)\delta^{(3)}(k - \sum k_i)$$

Power spectrum $\langle \delta(k,t)\delta(k',t)\rangle = P(k,t)\delta^{(3)}(k+k')$

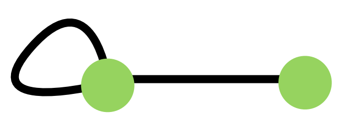
$F_i\,\delta^{(3)}(\sum k_i - k)$   $F_j\,\delta^{(3)}(\sum q_i + k)$

$F_1(k_1) = 1$

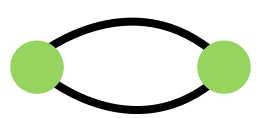
Gaussian IC

$\langle \delta(k,t_0)\delta(k',(t_0))\rangle = P_0(k)\delta^{(3)}(k+k')$

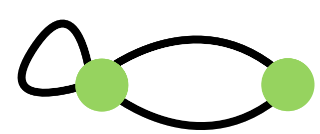
NLO (1L) : $\sim \delta(k,t_0)^4$



$$P_{13} = 3P_0(k)\int d^3q F_3^s(k,q,-q)P_0(q)$$



$$P_{22} = 2\int d^3q\,[F_2^s(q,k-q)]^2\,P_0(q)P_0(|k-q|)$$

NNLO (2L): $\sim \delta(k,t_0)^6$  $+ \ldots$

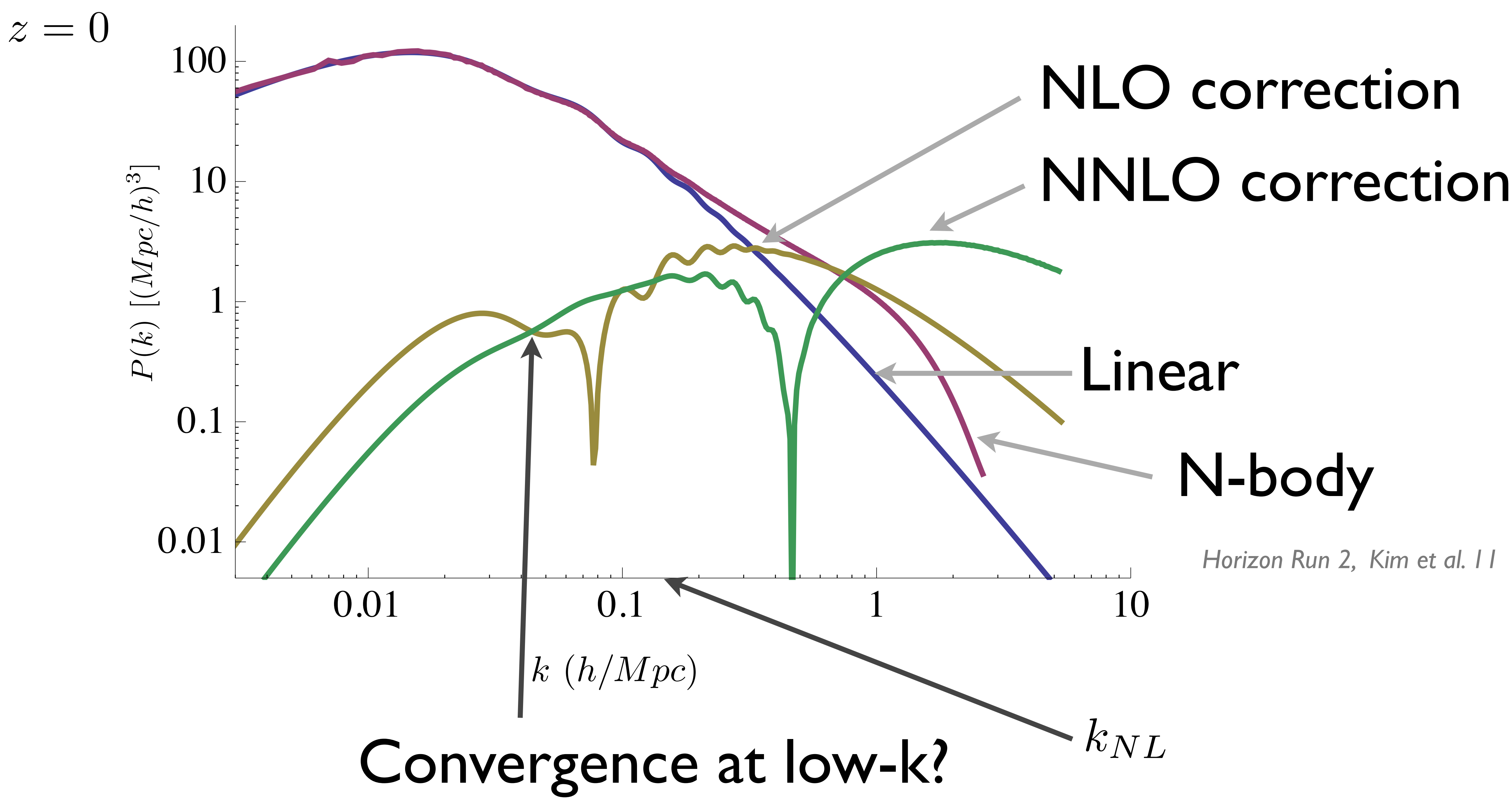
$$P(k,t) = a^2 P_0 + a^4(2P_{13} + P_{22}) + a^6(2P_{15} + 2P_{24} + P_{33}) + \ldots$$

NLO NNLO

NNLO results

e.g. Taruya et al. 12

DB, Garny and Konstandin 13A



Horizon Run 2, Kim et al. 11

Enhanced contribution from soft modes?

Expectation for NLO

Soft modes $k \gg q$

$$P_{13} = 3P_0(k) \int d^3q F_3^s(k,q,-q) P_0(q) \sim P_0(k)\, k^2 \int dq P_0(q)$$

~ 13 at $k \sim k_{NL}$

Similar for all loops: it can be resummed! (RPT, eikonal)

Crocce, Scoccimarro, 05
Bernardeau et al. 11

$$P_{NL} = e^{-k^2 \int dq P_0(q)/2} P_0(k) + P_{MC}$$

BUT it is cancelled by P_{22} !

DB, Garny and Konstandin 13A

(spurious scale for PS, important for other quantities)

Follows from Galilean invariance!

$$x^i \mapsto x^i + V^i T\,, \qquad \delta_k \mapsto \delta_k e^{ik\cdot V\, t(T)}$$

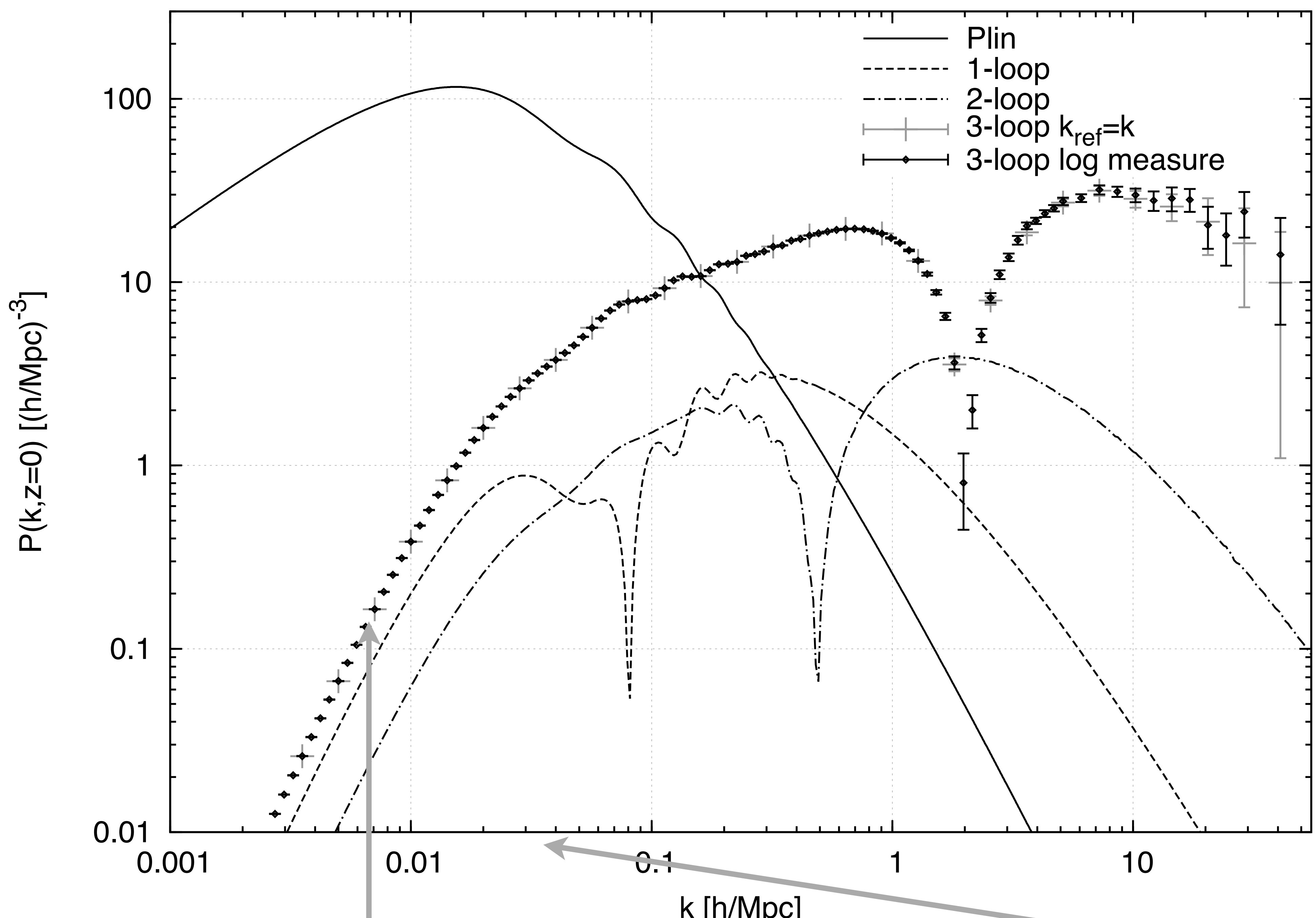
Jain, Bertschinger 95
Scoccimarro, Frieman 95
Bernardeau et al 12

For numerics, the cancellation is challenging:
make it explicit by an **IR safe integrand**

DB, Garny and Konstandin 13A
Carrasco et al. 13

NNNLO (3 Loop)

DB, Garny and Konstandin 13B



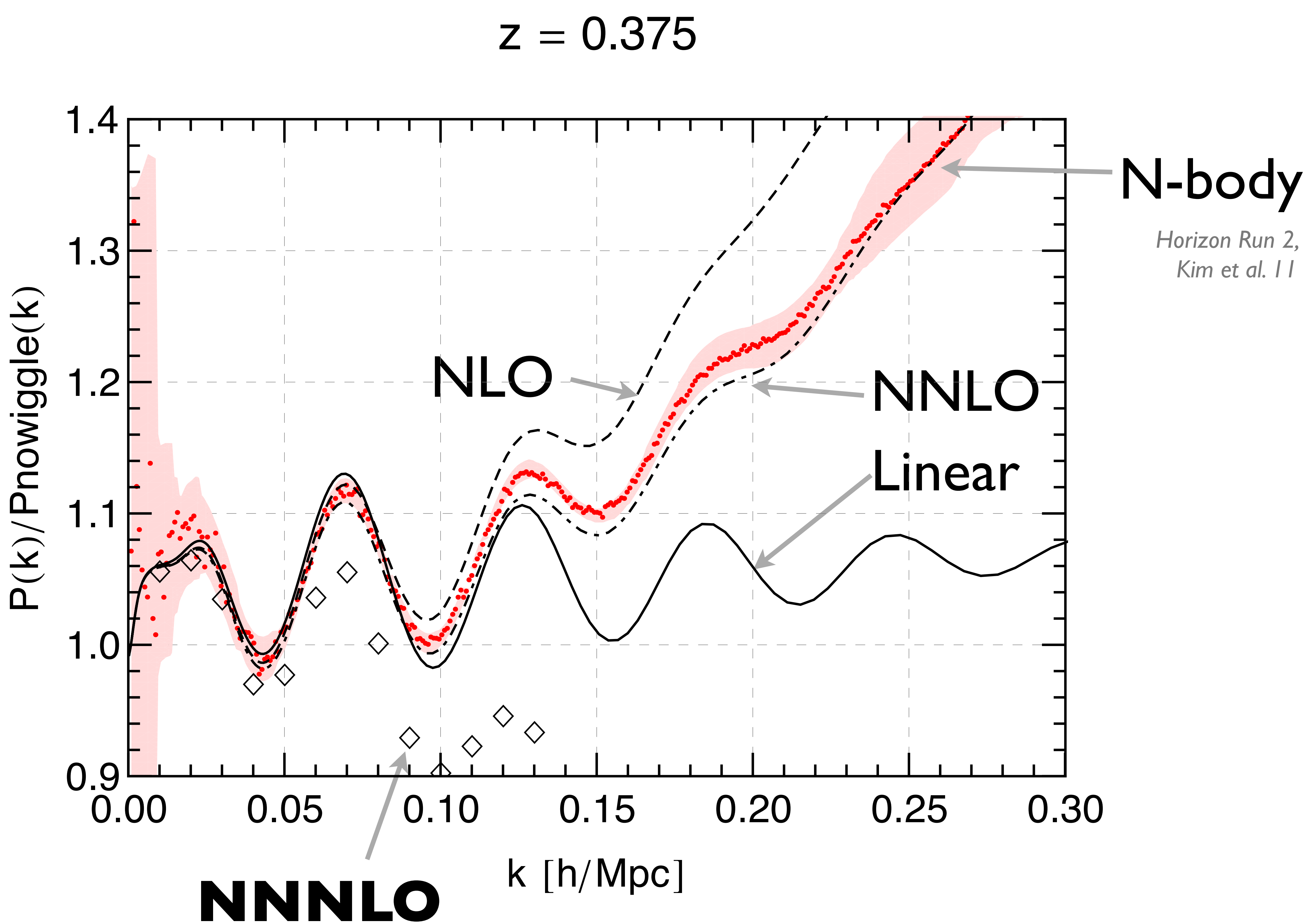
3L surpasses 1L at low k!

k_{NL}

shifted to lower k

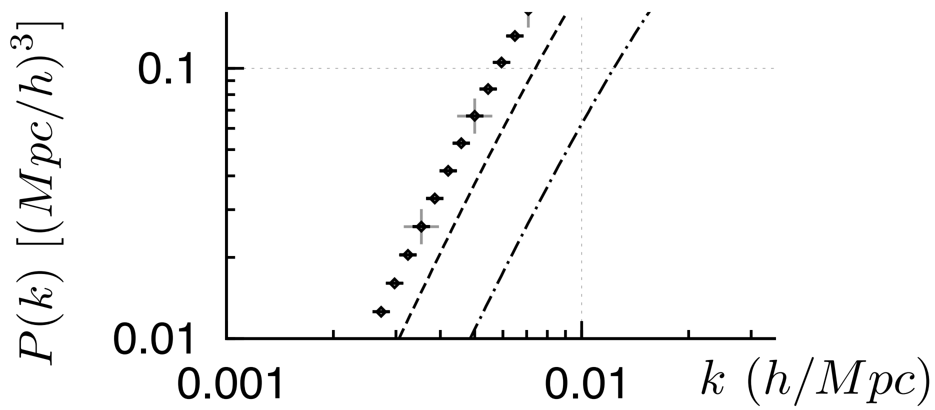
3 Loop ‘disaster’

DB, Garny and Konstandin 13B



Expectations for 3 Loop

Small k



Fry 94
Goroff et al 86

$$P_{L-loop} \to \frac{(2L+1)!}{2^{L-1}L!} a^{2(L+1)} P_0(k) \int d^3q_1 ... \int d^3q_L F^s_{2L+1}(k, q_1, -q_1, ..., q_L, -q_L) P_0(q_1)...P_0(q_L)$$

DB, Garny and Konstandin 13B

For $k^2 \ll q_i^2$, $\quad F^s_{2L+1}(k, q_1, -q_1, ..., q_L, -q_L) \sim k^2/q^2$

$$P_{L-loop} \to -C_L \frac{244\pi}{325} a^{2(L+1)} k^2 P_0(k) \int dq P_0(q) \sigma_l(q)^{2L-2}$$

with $C_1 = 1, \ C_2 \sim 0.71, \ C_3 = 1.05$ and $\sigma_l(q) \equiv 4\pi \int_0^q dp\, p^2 P_0(p)$

for realistic ΛCDM $\quad P_k \sim \dfrac{k}{(1 + k^2/k_0^2)^2}$

$$P_{L-loop} \to -k^2 P_0(k) \left(C_L \frac{(3L-1)!}{2^{3L}} \right) a^{2(L+1)}$$

non-convergent!

Padé resummation

DB, Garny and Konstandin 13B

Goal: produce a convergent series!

Small k $P_{L-loop} \to -k^2 P_0(k)\left(C_L \frac{(3L-1)!}{2^{3L}}\right) a^{2(L+1)}$ non-convergent!

Treat it as an **asymptotic** series

$$P_{L-loop} \to -C_L \frac{244\pi}{325} a^{2(L+1)} k^2 P_0(k) \int dq P_0(q) \sigma_l^{2L-2}(q)$$

$$P_{2-loop}/P_{lin} \sim 6\% \qquad z=0,\ k=0.1\ h/Mpc$$

Resummation to get 1%!

$$P_{low-k} = -\frac{244\pi}{315} a^4 P_0(k) \int dq P_0(q) K(a^2 \sigma_l^2(q))$$

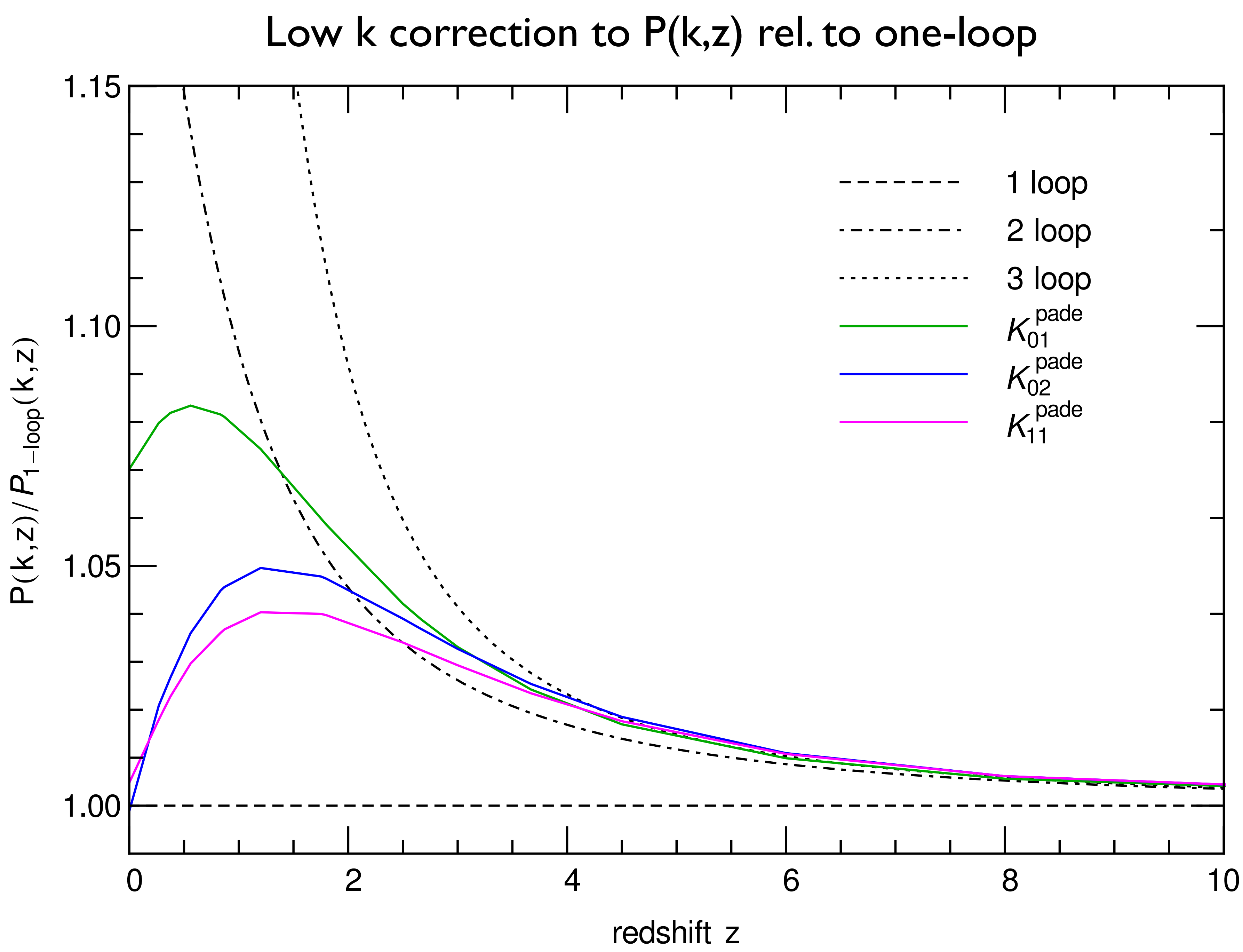
$$K(x) = \sum C_L x^{L-1} \quad \Rightarrow \quad K(x)^{Padé}_{nm} = \frac{1+\sum_{i=1}^n a_i x^i}{1+\sum_{j=1}^m b_j x^j}$$

Padé ansatz: expand and match!

e.g. 3 loops: $n,m = 0,2$ or $n,m = 1,1$

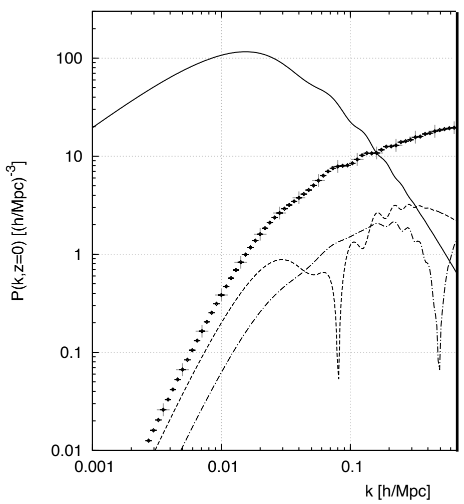
Padé results: convergence at low k

DB, Garny and Konstandin 13B



Padé results: perturbation theory

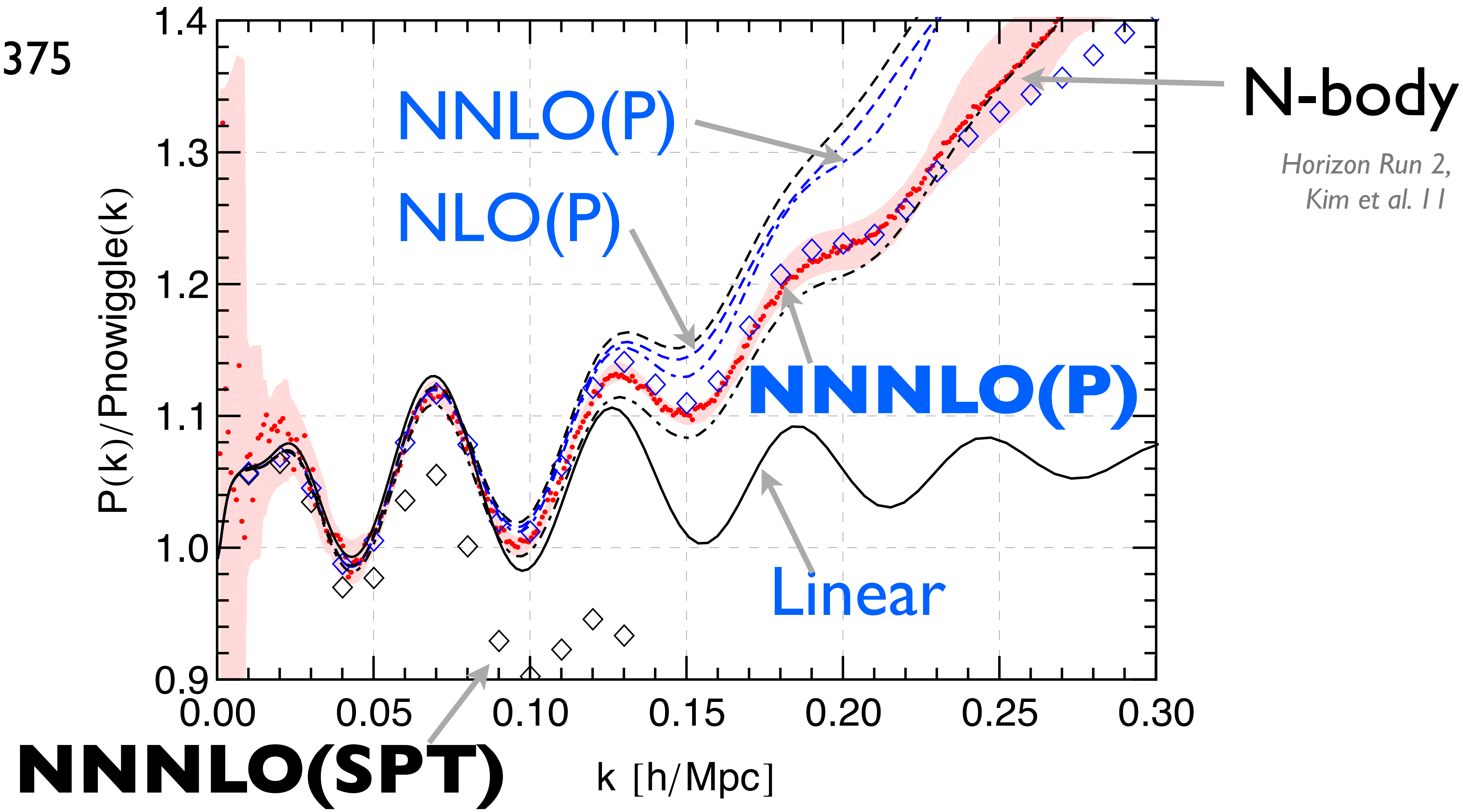
DB, Garny and Konstandin 13B



$$P(k,t) = P_l + P_{1-loop} + \ldots = P_l + P^{Padé}_{low-k} + \Delta P_{1-loop} \cdots$$

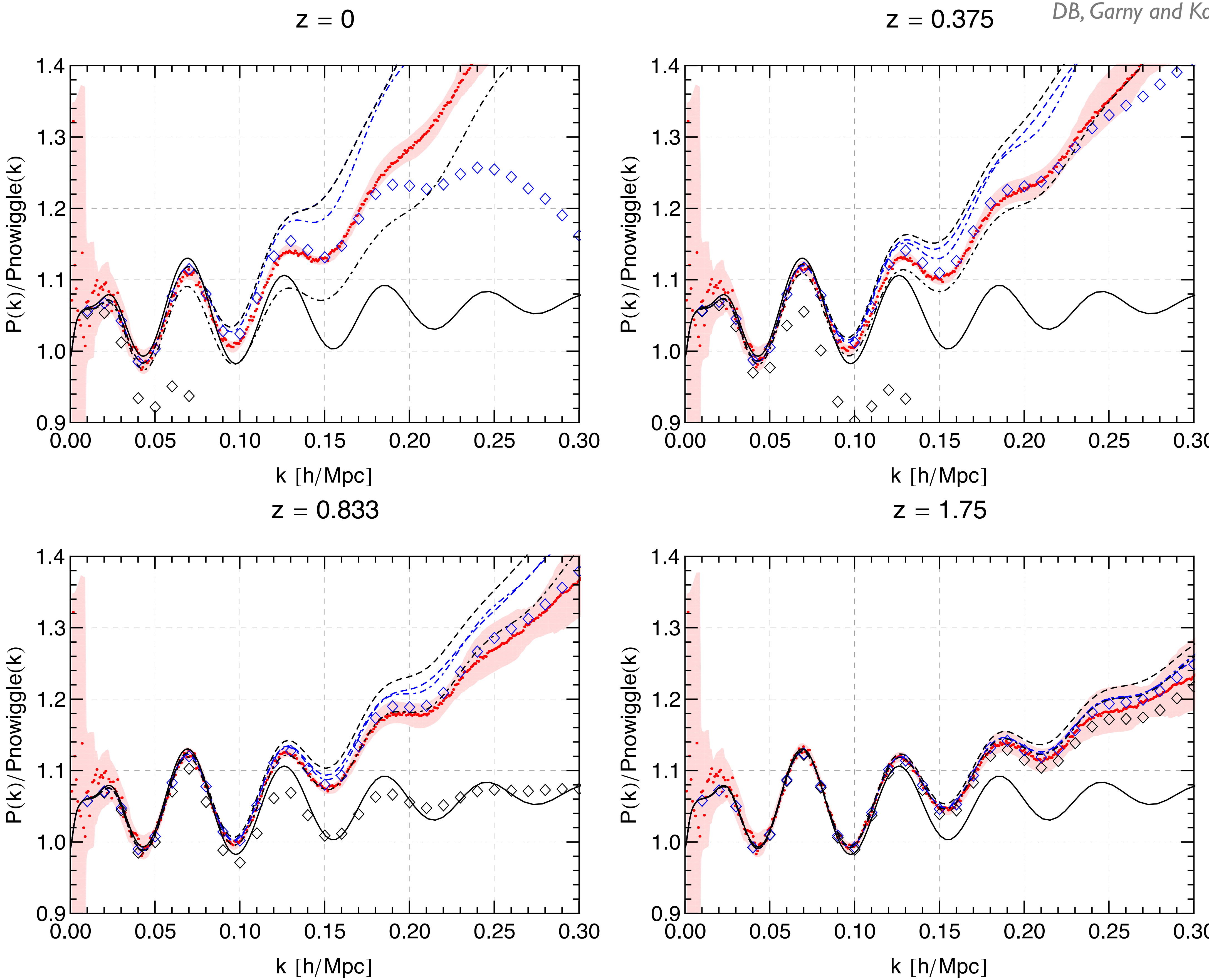
$$\Delta P_{L-loop} \equiv P_{L-loop} - P^{small-k}_{L-loop}\,, \quad P^{Padé}_{low-k} \equiv \sum P^{small-k}_{L-loop}$$

z=0.375



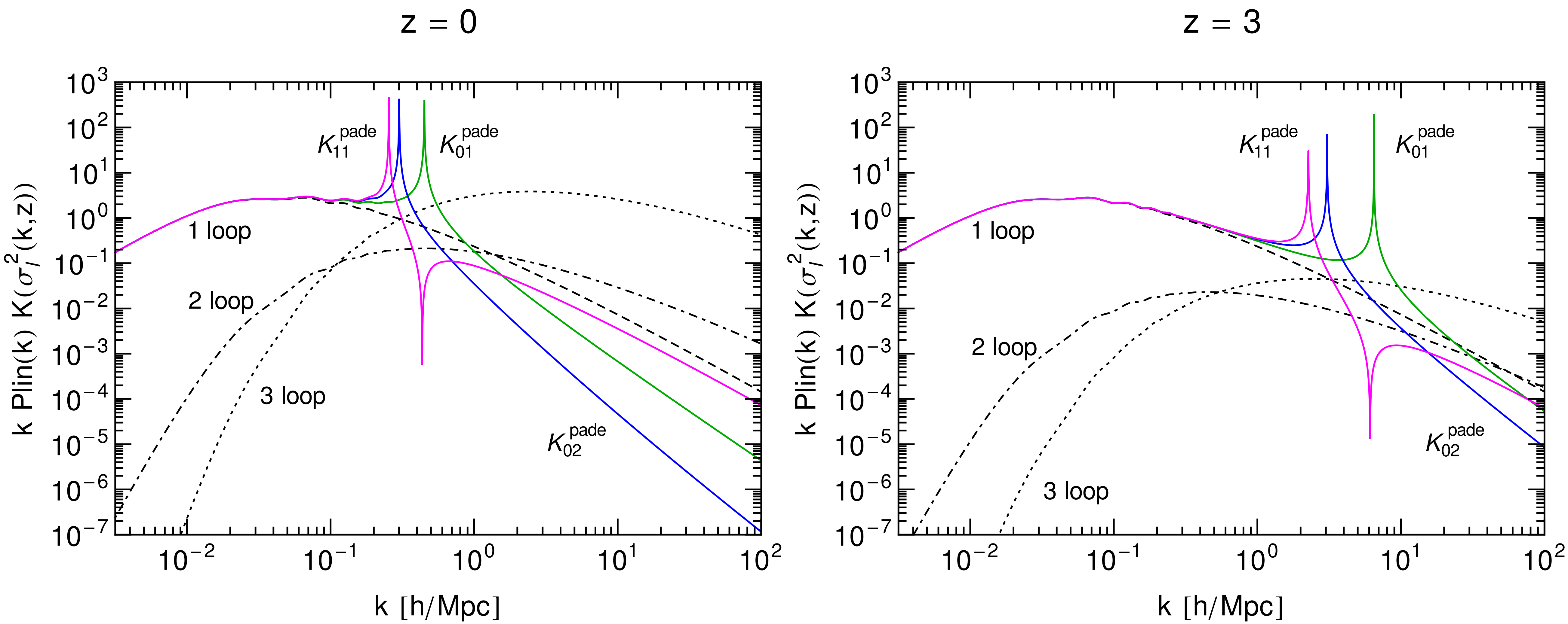
Padé results: redshift dependence

DB, Garny and Konstandin 13B



What's going on? Padé integrands

$$P_{L-loop} \to -C_L \frac{244\pi}{325} a^{2(L+1)} k^2 P_0(k) \int dq P_0(q) \sigma_l(q)^{2L-2}$$



The resummation **damps** the UV dependence!

This may made the series **convergent**!
(and the 1% target attainable)

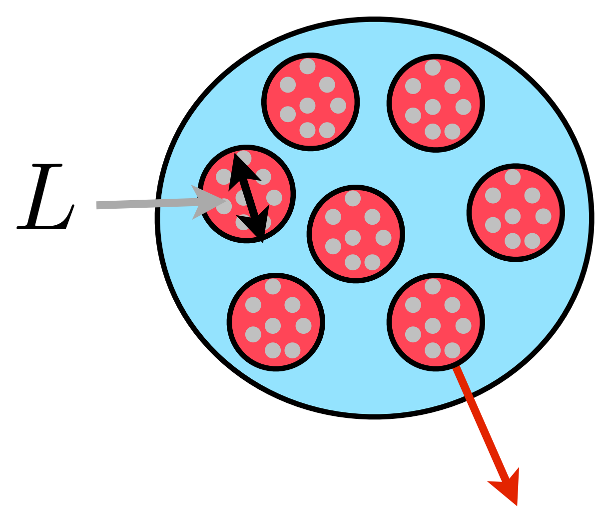
Conclusions

- Future surveys will test cosmological expansion and structure formation to percent level.
- At this precision, the Universe at large scales behaves like a pressureless perfect fluid.
- Perturbation theory in $\delta_k(t_0)$, but this grows with time.
- PT series is not convergent! (seems asymptotic) (result at 3 loop).
- Padé ansatz: parameter free resummation. Much better convergence properties and agreement with N-body.

(percent accuracy at BAO scales and $z = 0$ reachable)

For the future

- More analytical understanding.
- Other observables ($P_{\theta\theta}$, bispectrum,...), other IC (NG).
- Predictions for observations: results in redshift space, parametrization of BAOs, bias...
- Other ways of organizing (resum) the series? E.g. RPT or EFTofLSS: coarse-grained fluid to get rid of the influence of high-k at mildly non-linear k .



Pietroni et al. 11

Carrasco et al. 12

Primordial NG

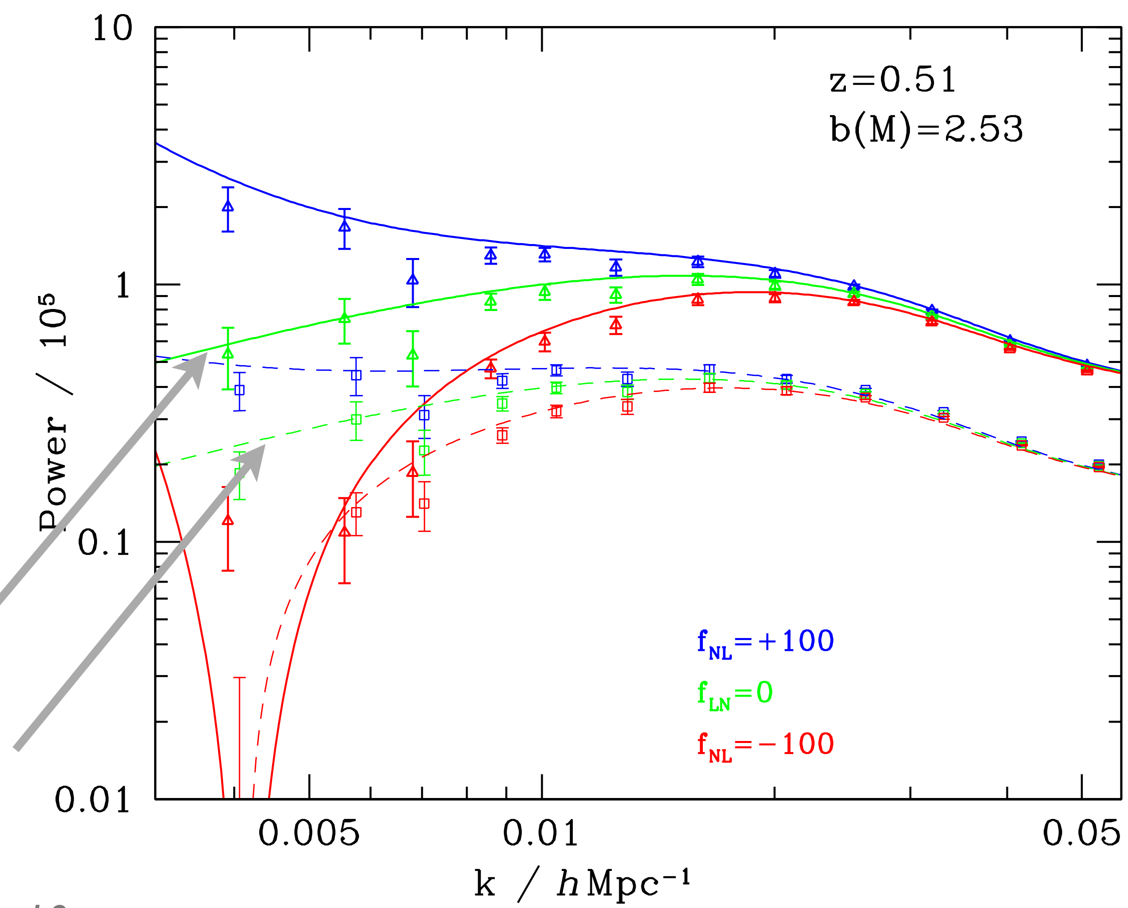
$$\phi = \phi_G + f_{NL}^{loc}\,\phi_G^2$$

PLANCK 13

$$f_{NL}^{loc} = 2.7 \pm 5.8$$

LSS may arrive to percent!

Desjacques, Seljak 10

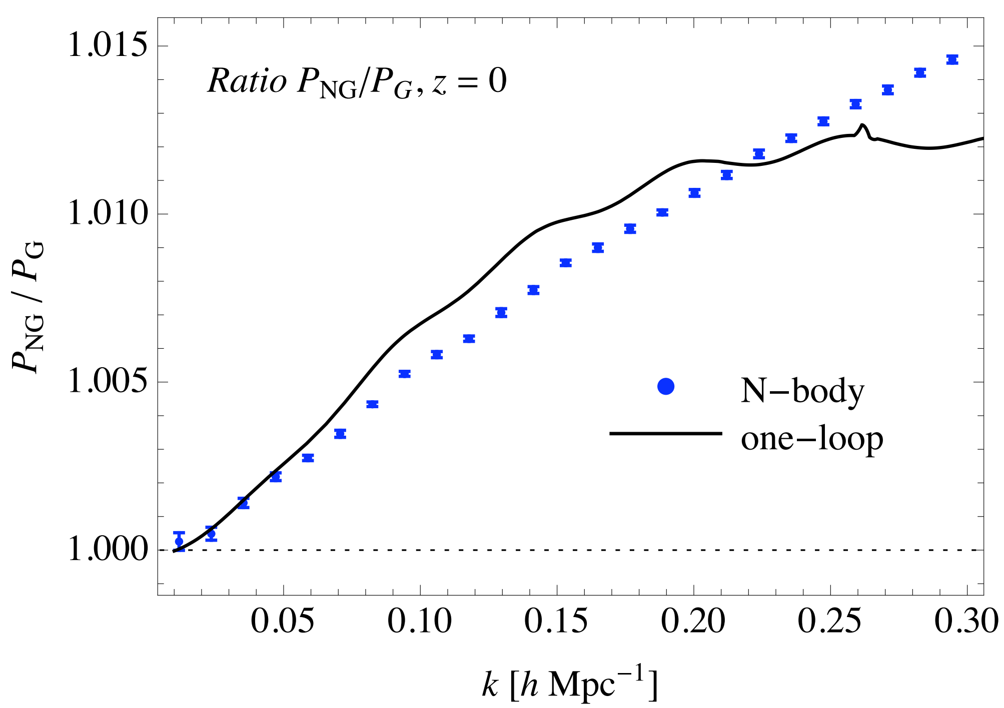


Halo-Halo

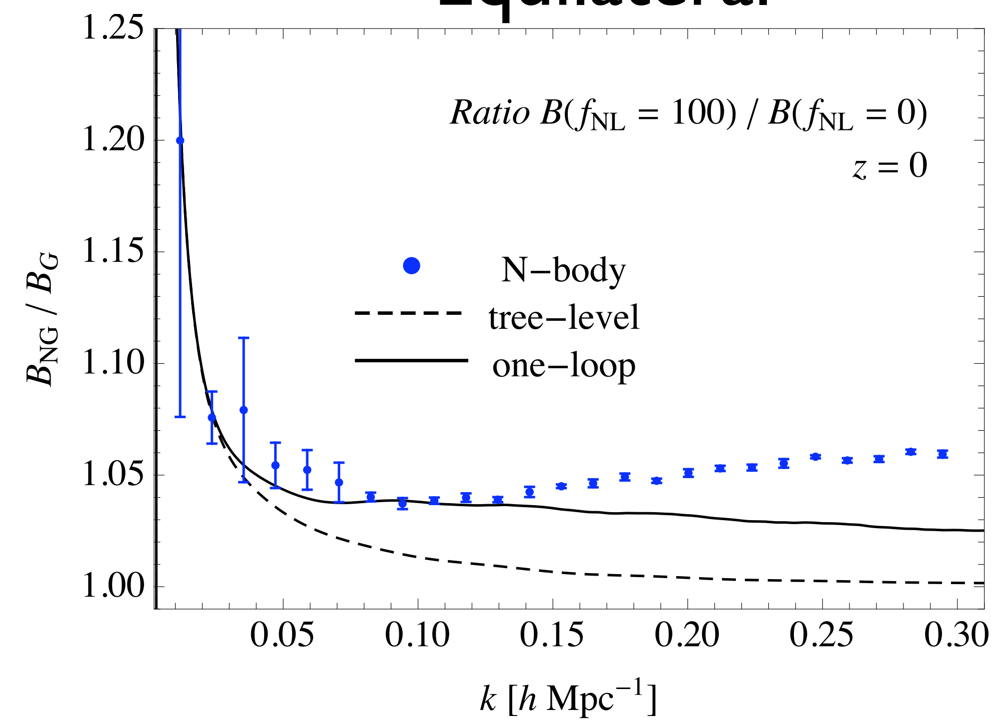
Halo-Matter

Matter PS

Sefusatti, Crocce, Desjacques 10

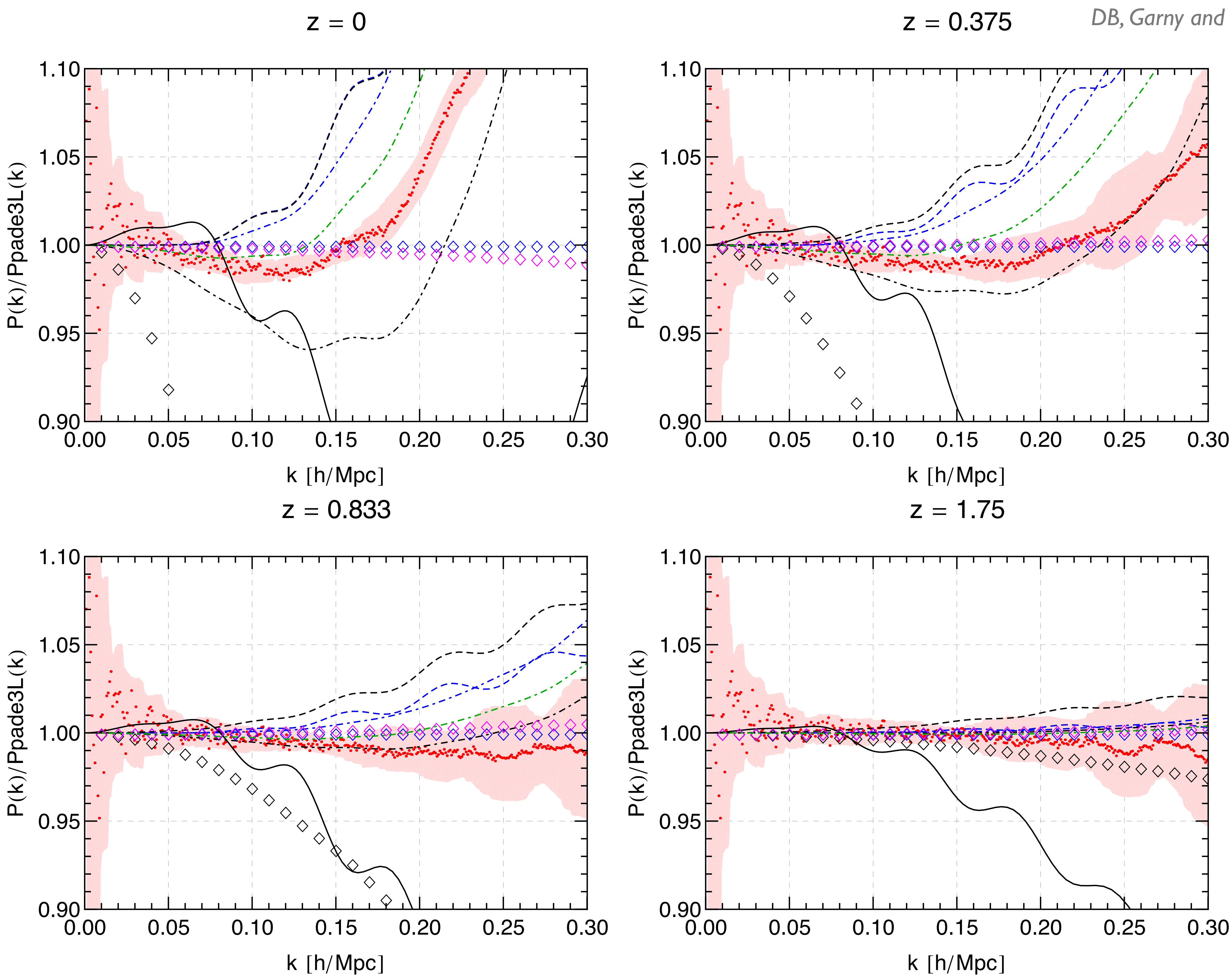


Equilateral



Percent accuracy

DB, Garny and Konstandin 13B

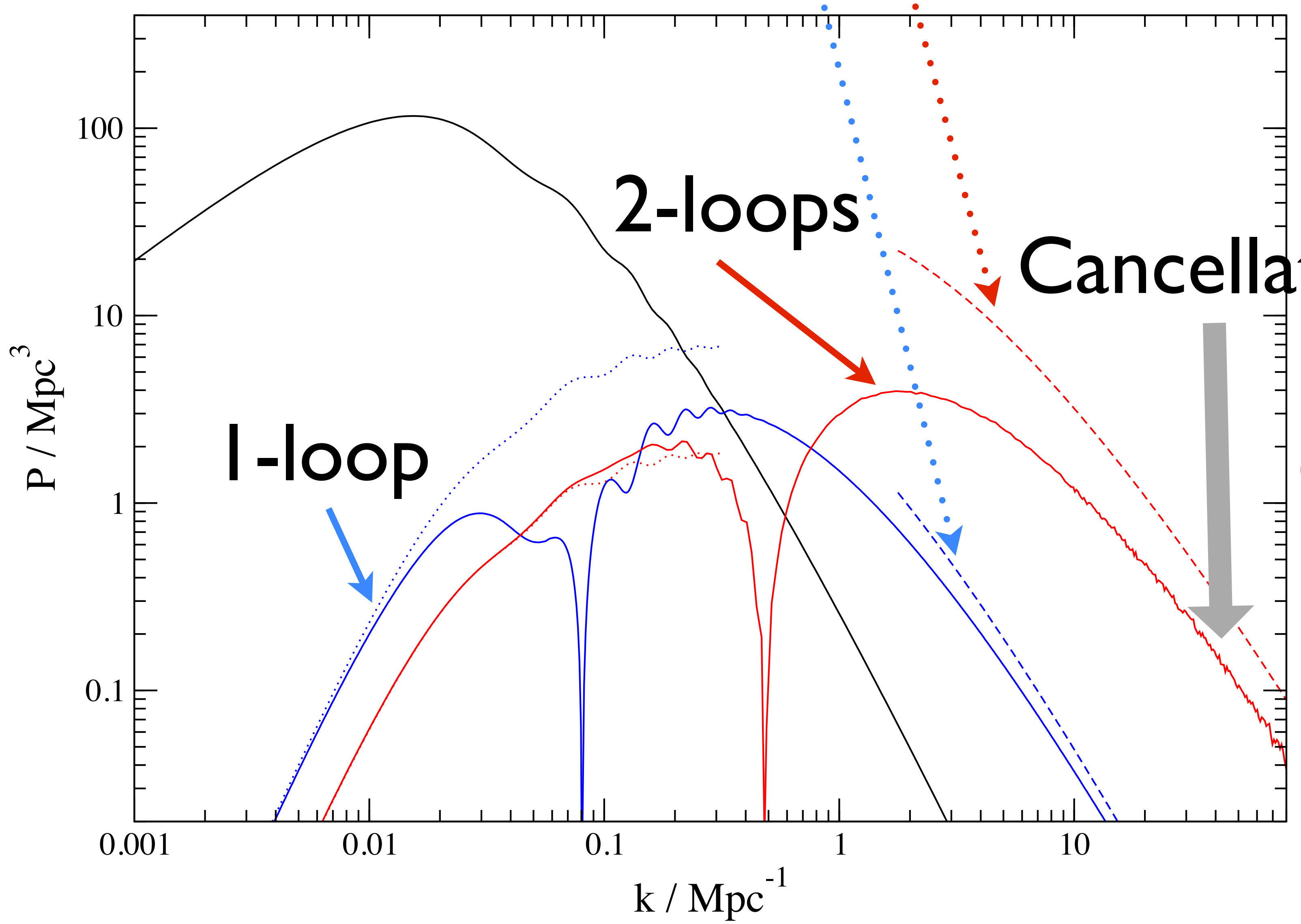


PS and other non-linearities

$$P_{1-loop}(k) \sim \left(1.14P^L(k) - 0.55k\partial_k P^L(k) + 0.1[k\partial_k]^2 P^L(k)\right)\sigma_l^2(k)\,,$$

$$P_{2-loop}(k) \sim \left(2.14P^L(k) - 1.62k\partial_k P^L(k) + 0.55[k\partial_k]^2 P^L(k) - 0.082[k\partial_k]^3 P^L(k) + 0.005[k\partial_k]^4 P^L(k)\right)\sigma_l^4(k)$$

$$\sigma_l^2(\Lambda,\eta) \equiv 4\pi \int^{\Lambda} dq\, q^2 P^L(q,\eta),$$



Cancellation of factor
4000^4
(same trick for the integrand)

LSS and Cosmological Parameters

Anderson et al. 12