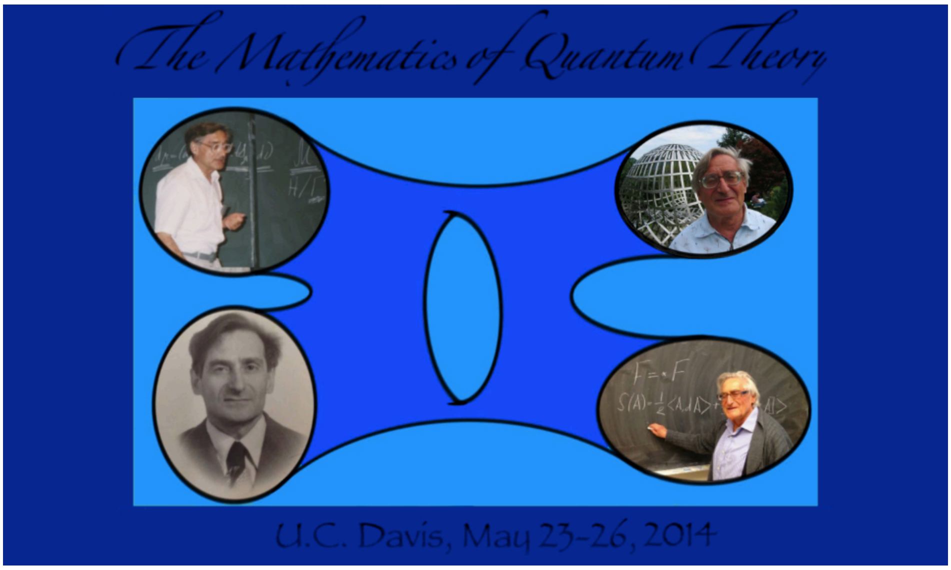
The Mathematics of Quantum Theory

Friday, May 23, 2014 - Monday, May 26, 2014

Buehler Alumni Center



Book of Abstracts

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The Hirzebruch-Riemann-Roch theorem in quantum K-theory

Author: Alexander Givental[1]

[1] *U.C. Berkeley*

The title theorem (which is a joint result of the speaker with Valentin Tonita)
expresses genus-0 K-theoretic Gromov-Witten invariants in terms of cohomological ones. The former are holomorphic Euler charactersistics of some interesting vector bundles over spaces of rational holomorphic curves in a given Kahler manifold, while the latter are suitable intersection indices in these spaces. The subject relies on many previous developments in Gromov-Witten theory, and is quite involved technically and conceptually. In this talk, we will focus on some relatively elementary aspect of the theory which, hopefully, has a general mathematical appeal. Namely, in contrast with the classical Hirzebruch-Riemann-Roch formula, the theorem in question is not a formula, but an example of what we call "adelic characterization". That is, generating functions for K-theoretic Gromov-Witten invariants (which happen to have the form of Laurent polynomials in one variable) are completely characterized by interpreting their Laurent series expansions near the poles at the roots of unity as generating functions for certain cohomological Gromov-Witten invariants.

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N=4 Super Yang-Mills Theory on the Coulomb Branch

Author: John Schwarz[1]

[1] *Caltech*

It is conjectured that the world-volume action of a probe D3-brane in an AdS_5XS^5 background of type IIB superstring theory, with one unit of flux, can be
reinterpreted as the exact effective action for U(2) N =4 super Yang-Mills theory
on the Coulomb branch. An analogous conjecture for ABJM theory
is also presented. The main evidence supporting these conjectures is that
the brane actions have all of the expected symmetries and dualities.

3

Categorification of quantum groups at a prime root of unity.

Author: Mikhail Khovanov[1]

[1] *Colombia*

Upon categorification quantum parameter q becomes a grading
shift. Root of unity requires a tricker setup, involving p-complexes
and characteristic p. We will explain the construction, some results
and conjectures for categorification of quantum groups, when the order
of q is a prime number.

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A mathematical approach to quantum curves

Author: Motohico Mulase[1]

[1] *U.C. Davis*

A quantum curve is an h-bar deformation family of D-modules on a complex analytic curve. It takes the form of a stationary Schroedinger equation in one dimension, quantizing the spectral curve, which is a ramified covering of the starting curve. The coordinate of the starting curve is a parameter of a generating function, and the spectral curve is the Riemann surface of holomorphy of this function. The quantum curve, as a differential equation, then characterizes this function, which is a generating function of quantum topological invariants. In this talk, I will present recent mathematical developments on this subject, obtained jointly with Dumitrescu, Dunin-Barkowski, Norbury, Popolitov, Shadrin, and Sulkowski.

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Towards the mathematics of AdS/CFT

Author: Kevin Costello[1]

[1] *North Western*

I'll discuss a conjectural framework for a twisted form of the AdS/CFT correspondence, which describes the operator product of a large N limit of a twisted form of N=4 super Yang-Mills in terms of a dual gravitational theory. The gravitational theory relevant for the twist we use is the 5-complex dimensional analog of the BCOV Kodaira-Spencer theory.

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Degeneration of algebraic varieties and K-theory

Author: Vadim Vologodsky[1]

[1] *University of Oregon*

The talk is based on our joint works with Kontsevich, Schwarz, and Walcher.
Let X be a smooth proper algebraic variety over the formal punctured disk, maximally degenerated at the origin. Generalizing Mumford's construction in the case of abelian varieties I will attach to X a certain mixed Tate motive over the punctured disk and explain some application of this construction to the Mirror Symmetry.

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Do all subfactors admit attendant conformal field theories?

Author: Vaughan Jones[1]

[1] *Berkeley*

A subfactor is functional analytic object with highly combinatorial structure theory.
Subfactors arise in various ways in conformal field theory via monodromy of n-point
functions or more simply via commutation of local observable algebras. Subfactor
technology has undergone many advances recently with a classification program

for subfactors of small index. We meet subfactors that are do not arise from any currently know conformal field theory but there seems to be no reason that such CFT's do not exist, indeed Evans and Gannon give some evidence that such CFT's do exist in the context of Vertex operator algebras. We will describe some of these "exotic" subfactors and suggest ways in which CFT's might be made out of them.

8

Symmetry Protected Topological Phases and Cobordisms"

Author: Anton Kapustin[1]

[1] *Caltech*

Recently a new and rather unexpected connection between physics and algebraic topology has been noted. Namely, it appears that phases of matter with an energy gap, no long-range entanglement, and fixed symmetry can be classified using cobordism theory. I will exhibit several examples of this connection and describe a possible explanation.

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Convex polytopes and infrared categories.

Author: Yan Soibelman[1]

[1] *Kansas*

In a recent work of Gaiotto,Moore and Witten the "algebra of the infrared" for certain massive 2d theories with (2,2) supersymmetry was introduced. For Landau-Ginzburg models it gives a Morse-theoretical description of the corresponding A_∞-category of A-branes. It turns out that the combinatorial part of their work admits a higher-dimensional generalization. I am going to discuss that generalization, its relation to Gaiotto-Moore-Witten's work and speculate about possible applications.

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2-functions, L-functions, and mirror symmetry

Author: Johannes Walcher[1]

[1] *McGill*

I will review extended mirror symmetry, explain the notion of 2-functions that we introduced (as an arithmetic generalization of di-logarithm) in recent work with Schwarz and Vologodsky, and the calculation of special values of L-functions that arise in this context.

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Is Quantization Unique?

Author: Sergei Gukov[1]

[1] *Caltech*

Quantization of planar algebraic curves naturally enters many branches of physics and mathematics. For example, quantization of the zero locus of the A-polynomial of a knot gives the best known way to package an infinite set of colored Jones invariants of that knot into a single equation. Quantization of such A-polynomial curves is similar and, in some examples even identical, to quantization of spectral curves of trigonometric integrable systems. The latter lead to Baxter equations. Surprisingly, however, if the same planar curve appears as the A-polynomial of a knot and the spectral curve of a trigonometric integrable system, the result of its "quantization" in these two frameworks will be different.

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Holomorphicity in QFT

Author: Maxim Kontsevich[1]

[1] *IHES*

TBA

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AGT and Triality

Author: Mina Aganagic[1]

[1] *Berkeley*

The AGT correspondence relates a class of gauge theories in four dimensions with two dimensional CFT's. I will describe a simple proof of the correspondence when the CFT admits a free field representation. In those cases, vortex defects of the gauge theory play a crucial role, extending the correspondence to a triality.

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Quantum curves and the infinite-dimensional Grassmannian

Author: Albert Schwarz[1]

[1] *UC Davis*

One says that a pair (P, Q) of ordinary differential operators specify a quantum curve if $[P, Q] = \hbar$. If a pair of difference operators (K, L) obey the relation $KL = \lambda LK$ where $\lambda = e^{\hbar}$ we say that they specify a discrete quantum curve.

This terminology is prompted by well known results about commuting differential and difference operators , relating pairs of such operators with pairs of meromorphic functions on algebraic curves obeying some conditions.

Our methods are based on the interpretation of quantum curves in terms of infinite-dimensional Grassmannian; in particular, it follows from this interpretation that (discrete) KP-hierarchy can be used to deform a (discrete) quantum curve.
The main goal is to study the moduli spaces of quantum curves. We will relate the moduli spaces for different $\hbar$. We will show how to quantize a pair of commuting differential or difference operators (i.e. to construct the corresponding quantum curve or discrete quantum curve)

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Naturalness of Slow Nambu-Goldstone Modes and Graph Theory

Author: Petr Horava[1]

[1] *University of California, Berkeley*

TBA

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M-theory and DT-theory

Author: Andrei Okounkov[1]

[1] *Colombia*

This will be a report on a joint work with Nikita Nekrasov (arXiv:1404.2323), the goal of which is to find an exact match between the M2-branes contributions to the M-theory index and computations in K-theoretic Donaldson-Thomas theory of 3-folds.

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Moduli of super Riemann surfaces

Author: Ron Donagi[1]

[1] *UPenn*

Albert Schwarz has made some of the most decisive early contributions to the theory of super Riemann surfaces and its connections with perturbative superstring theory. This subject has been revisited in recent works of Witten, and is rapidly developing in the form of super algebraic geometry. In this talk I will survey some of these recent developments.

We will study various aspects of supergeometry, including obstruction,
Atiyah, and super-Atiyah classes. This will be applied to the geometry of the moduli space of super Riemann surfaces. We prove that for genus greater than or equal to 5, this moduli space is not projected (and in
particular is not split): it cannot be holomorphically projected to its

underlying reduced manifold. Physically, this means that certain approaches to superstring perturbation theory that are very powerful in low orders have no close analog in higher orders. Mathematically, it means that the moduli space of super Riemann surfaces cannot be constructed in an elementary way starting with the moduli space of ordinary Riemann surfaces. It has a life of its own. If time allows, we will describe some of the other new features of this space. (This is based on joint with E. Witten)

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Non-perturbative Dyson-Schwinger equations and qq-characters

Author: Nikita Nekrasov[1]

[1] *IHES*

TBA

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A hyperholomorphic line bundle in $calN = 2$ theories

Author: Andy Neitzke[1]

[1] *U. Texas*

Compactifying $calN = 2$ supersymmetric field theory from four to three dimensions on a circle gives rise to a complex integrable system carrying a hyperkahler metric. In many cases this integrable system carries in addition a canonical hyperholomorphic line bundle, conjecturally related to the physics of the theory compactified on Taub-NUT space. I will describe the construction of this line bundle, its connection to complex Chern-Simons theory, and a closely related new smooth generating function for BPS state counts / Donaldson-Thomas invariants. The bundle in some cases coincides with one introduced by Haydys and Alexandrov-Persson-Pioline.

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Chern-Simons theory, S-duality, and a Tridiagonal Determinant Identity

Author: Ori Ganor[1]

[1] *Berkeley*

An equivalence between two Hilbert spaces will be discussed: (i) the space of states of $U(1)^n$ Chern-Simons theory on T^2 with coupling constants given by a certain class of tridiagonal matrices (with corners); and (ii) the space of ground states of strings on an associated mapping torus with T^2 fiber. The equality of dimensions of the two Hilbert spaces (i) and (ii) is equivalent to a known identity on determinants of tridiagonal matrices with corners. The equivalence of operator algebras acting on the two Hilbert spaces follows from a relation between the Smith normal form of the Chern-Simons coupling constant matrix and the isometry group of the mapping torus, as well as the torsion part of

its first homology group. The equivalence follows by studying the space of ground states of SL(2,Z)-twisted circle compactifications of U(1) gauge theory, connected with a Janus configuration, and further compactified on a torus. I will also discuss generalizations to U(n) gauge theory.

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On regularized geometry of loop spaces.

Author: Michael Movshev[1]

[1] *Stony Brook*

The $O(N+1)$-model or a sigma model whose target is a round N-dimensional sphere is a well established (by physical standards) subject. It attracts attention because the theory exhibit spontaneous mass generation-a feature that is also expected in a more realistic but also more complicated four-dimensional gauge theories. In addition, the $O(N+1)$-model is believed to be completely integrable. In particular, an explicit formula for the mass gap is know.

I will discuss mathematical aspects of quantum Hamiltonian formalism for the $O(N+1)$-model such as a precise statement of the mass gap conjecture and a possible definition of the renormalization group that goes beyond perturbation theory.

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Open String Hodge Theory

Author: Alexander Goncharov[1]

[1] *Yale*

TBA

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Public Lecture-String Theory: Past, Present and Future

Author: John Schwarz[1]

[1] *Caltech*

String theory connects the microscopic quantum world of elementary particles to the large-scale world of gravity and geometry. Physicists believe it may have the potential to achieve two very ambitious goals: (1) to provide a complete mathematical description of the physical laws that determine the properties of elementary particles and the forces that act on them and (2) to describe the origin and evolution of the universe. Much has been achieved, but string theory is still a work in progress. This talk will give a historical overview of the subject and discuss (without technical details) some of the problems that remain to be overcome.

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Topological recursion, cohomological field theories and quantization

Author: Nicolas Orantin[1]

[1] *Instituto Superior Tecnico*

The topological recursion method is a formalism developed in the context of random matrix theories in order to solve an associated problem of combinatorics consisting in the enumeration of discrete surfaces. This inductive procedure allows to enumerate such surfaces of arbitrary topology out of the only genus 0 data. This theory has further been formalized out of the context of random matrices and mysteriously solved many problem of enumerative geometry using a universal inductive procedure.

In this talk, I will present this topological recursion procedure and explain the reason why it solves many problems of enumerative geometry at once. I will show that, given a semi-simple Frobenius manifold, one can identify the formula of the ancestor Gromov-Witten potential derived by Givental with the correlation functions computed by a local version of the topological recursion. The role of mirror symmetry will be explained and exemplified in the computation of the Gromov-Witten invariants of the projective line. I will finally explain how this procedure produces a semi-classical approximation of a wave function obtained by quantizing the corresponding spectral curve.

Based on joint works with Chekhov, Dunin-Barkowski, Eynard, Norbury, Shadrin and Spitz.