CERN Accelerator School
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# Beam-Beam Interactions

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Thanks to the Beam-beam Team:
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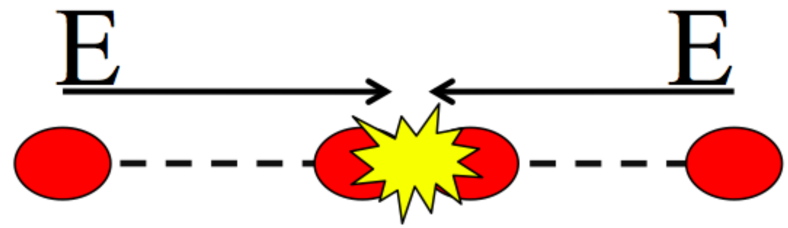
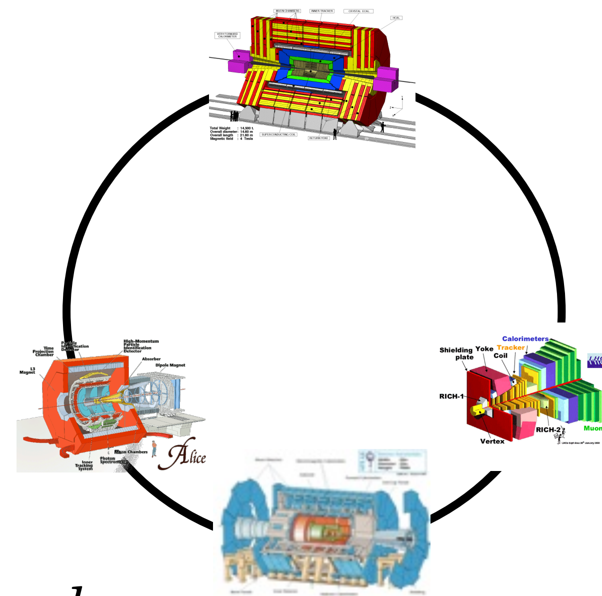
# Hadron Circular Colliders

$$E^* \approx 2 \times E$$




$$N_{event/s} = L \cdot \sigma_{event}$$

$$L \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b \cdot f_{rev}$$




**Bunch intensity:** $N_p = 1.15 - 1.65 \cdot 10^{11}\ ppb$

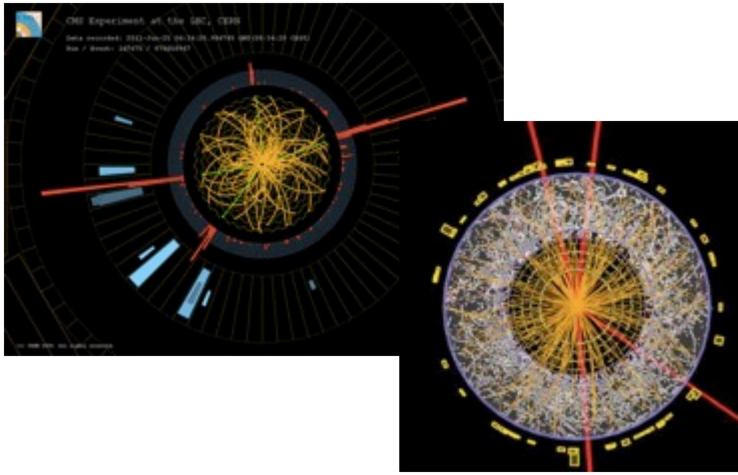
**Transverse Beam size:** $\sigma_{x,y} = 16 - 30\ \mu m$

**Number of bunches** $1370 - 2808$

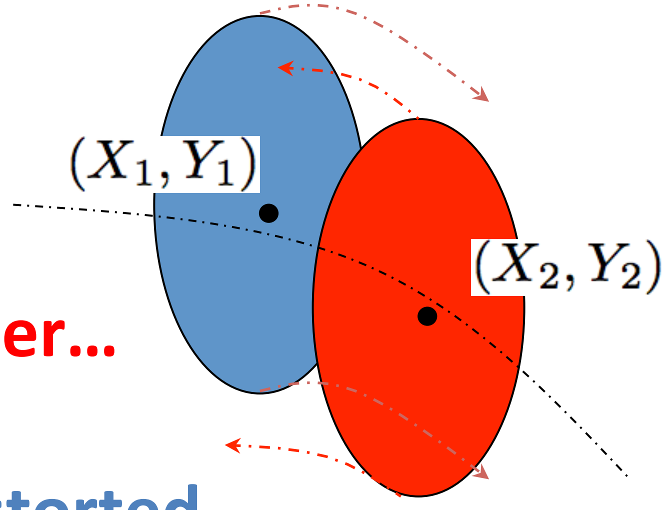
**Revolution frequency** $11\ kHz$

# When do we have beam-beam effects?

- They occur when two beams get closer and collide



- Two types
  - High energy collisions between two particles (wanted)
  - Distortions of beam by electromagnetic forces (unwanted)




- **Unfortunately: usually both go together…**
- **0.001% (or less) of particles collide**
- **99.999% (or more) of particles are distorted**

# Beam-beam effects: overview

- **Circular Colliders**: interaction occurs at every turn

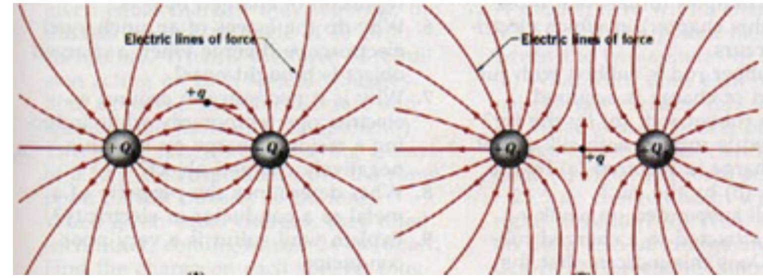
  - Many effects and problems
  - Try to understand some of them

- Overview of effects (single particle and multi-particle effects)
- Qualitative and physical picture of effects
- Observations from the LHC
- Mathematical derivations and more info in References or at

**Beam-beam webpage [http://lhc-beam-beam.web.cern.ch/lhc-beam-beam/](http://lhc-beam-beam.web.cern.ch/lhc-beam-beam/)**
**And CAS Proceedings**

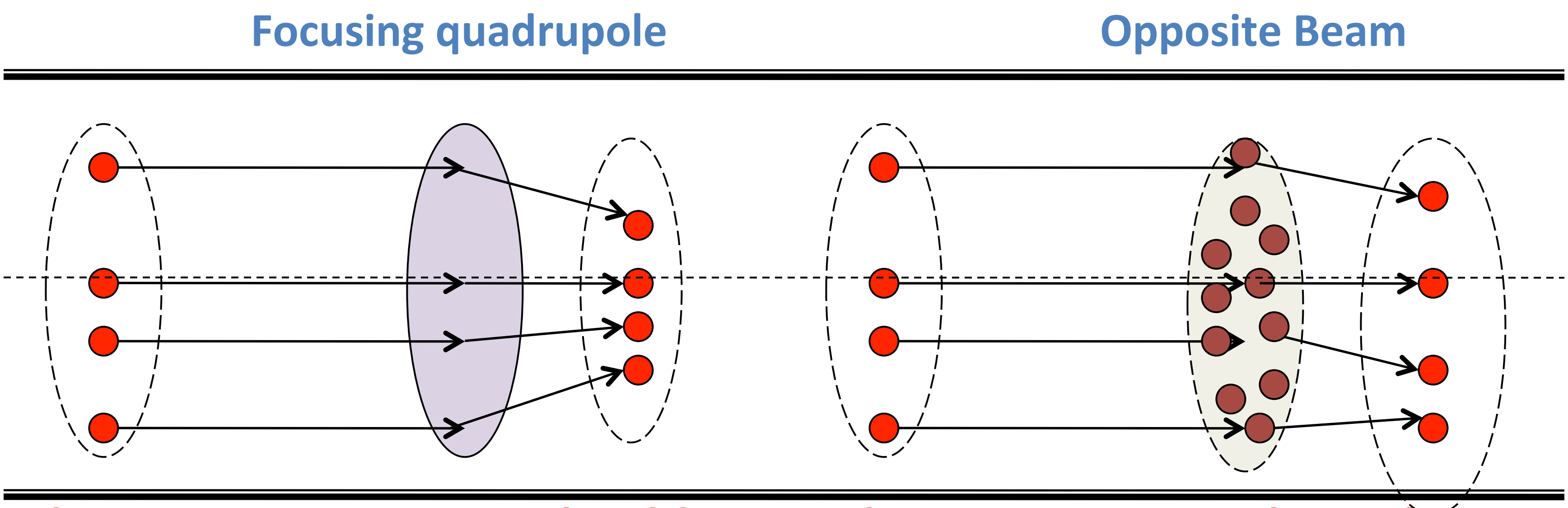
# Beams EM potential

- Beam is a collection of charges
- Beam is an electromagnetic potential for other charges




Force on itself (space charge) and opposing beam (beam-beam effects)

Single particle motion and whole bunch motion distorted




A beam acts on particles like an electromagnetic lens, but…

# Beam-beam Mathematics

General approach in electromagnetic problems **Reference[5]** already applied to beam-beam interactions in **Reference[1,3, 4]**

$$\Delta U = -\frac{1}{\epsilon_0}\rho(x,y,z)$$

Derive potential from Poisson equation for charges with distribution ρ

Solution of Poisson equation

$$U(x,y,z,\sigma_x,\sigma_y,\sigma_z) = \frac{1}{4\pi\epsilon_0}\int\int\int \frac{\rho(x_0,y_0,z_0)dx_0dy_0dz_0}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2)}}$$

$$\overrightarrow{E} = -\nabla U(x,y,z,\sigma_x,\sigma_y,\sigma_z)$$

Then compute the fields

$$\overrightarrow{F} = q(\overrightarrow{E} + \overrightarrow{v} \times \overrightarrow{B})$$

From Lorentz force one calculates the force acting on test particle with charge q

**Making some assumptions we can simplify the problem and derive analytical formula for the force…**

# Round Gaussian distributions:

Gaussian distribution for charges:
Round beams:
Very relativistic, Force has only radial component :

$$\sigma_x = \sigma_y = \sigma$$

$$\beta \approx 1 \qquad r^2 = x^2 + y^2$$

$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}}\right]$$

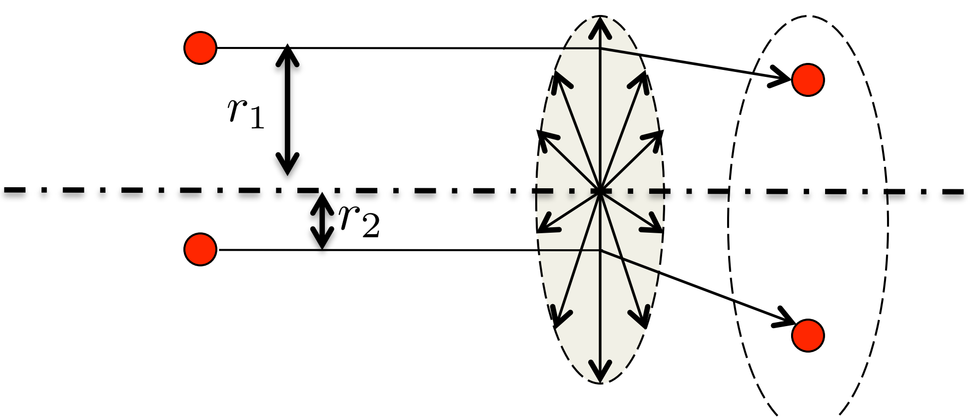
Beam-beam Force

$$\Delta r' = \frac{1}{mc\beta\gamma} \int F_r(r,s,t)\ dt$$

Beam-beam kick obtained integrating the force over the collision (i.e. time of passage)

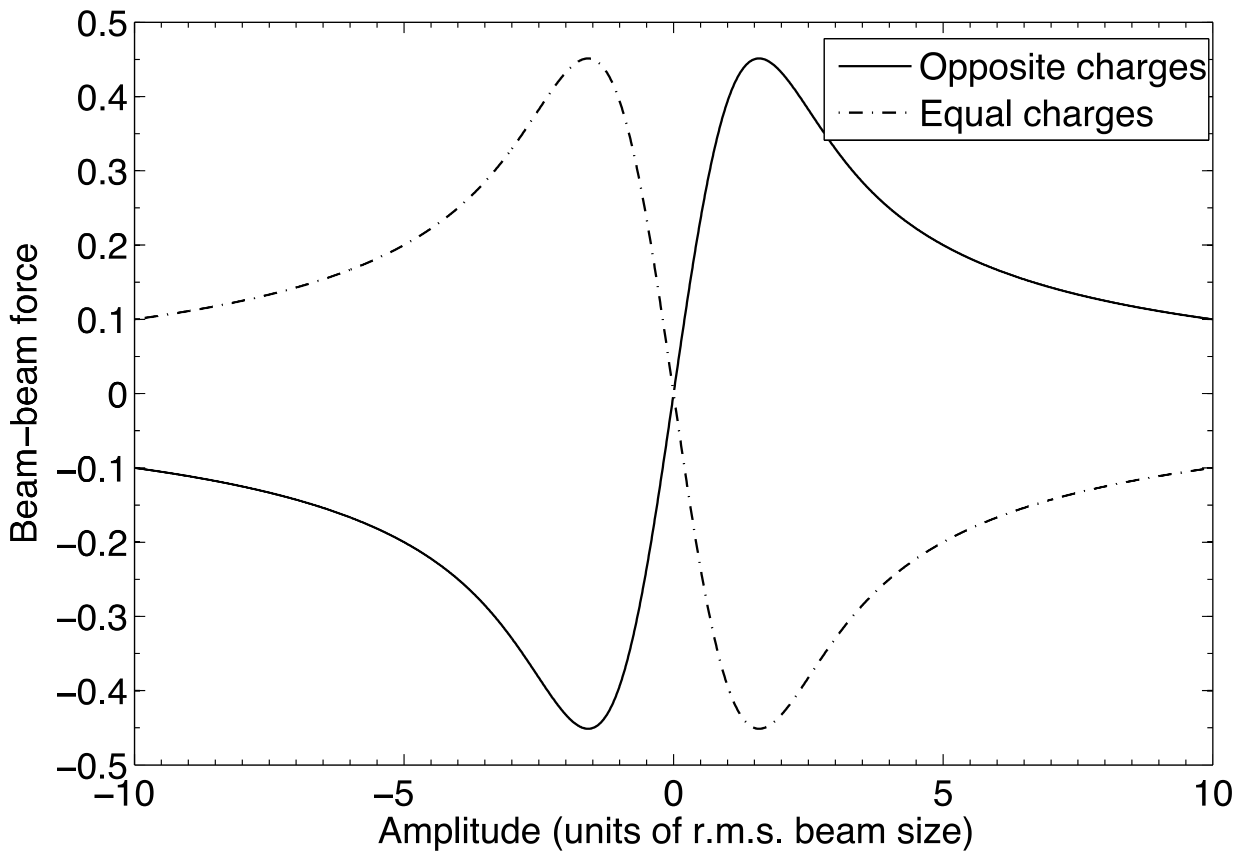
$$\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2}[1 - e^{-\frac{r^2}{2\sigma^2}}]$$

Only radial component in relativistic case




**How does this force looks like?**

# Beam-beam Force




$$F_r(r) = \pm \frac{ne^2(1+\beta_{rel}^2)}{2\pi\epsilon_0}\frac{1}{r}[1-\exp(-\frac{r^2}{2\sigma^2})]$$

# Why do we care?

## Pushing for luminosity means stronger beam-beam effects

$$\mathcal{L} \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b$$

$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}}\right]$$

**Physics fill lasts for many hours 10h – 24h**

**Strongest non-linearity in a collider YOU CANNOT AVOID!**


La Une | Mercredi 4 juillet 2012 | Dernière mise à jour 18:09

Mon journal numérique | Abonnements | Météo: Genève 26°

Tribune de Genève

Immo | Emploi | Auto | Petites Annonces

CARNET NOIR
L'acteur de télévision Andy Griffith est mort à 86 ans

CHAMP-DOLLON
Pour s'être plaint sur Facebook, un gardien est puni

FORMULE 1
La pilote Maria De Villota grièvement blessée

GENÈVE SUISSE MONDE ÉCONOMIE BOURSE SPORTS CULTURE PEOPLE VIVRE AUTO HIGH-TECH SAVOIRS SERVICES

PHYSIQUE

Une nouvelle particule a été découverte

Une nouvelle particule a été découverte par des chercheurs du CERN lancés sur la trace du boson de Higgs. Plus...

Mis à jour il y a 2 minutes

Bourse ›

| | | |
|---|---|---|
| SMI | 6'191.43 | -0.04% |
| Stoxx50 | 2'437.70 | -0.21% |
| DJIA | 12'943.82 | +0.56% |

Les plus lus ›

1. Le Conseil d'Etat a fait valser ses cadres. Une première
2. Les fontaines de Dubai pleurent Whitney Houston
3. Champ-Dollon: pour s'être plaint sur Facebook, un gardien est puni
4. La pilote Maria De Villota grièvement blessée
5. Sentier des Toblerones: Apple censure la ville de Gland

Genève au fil du temps

Aux origines du casino de Genève. Le Kursaal fut l'une des attractions de la ville pendant 80 ans.

Voir nos galeries photo

Cet été, bien informé rime avec mobilité

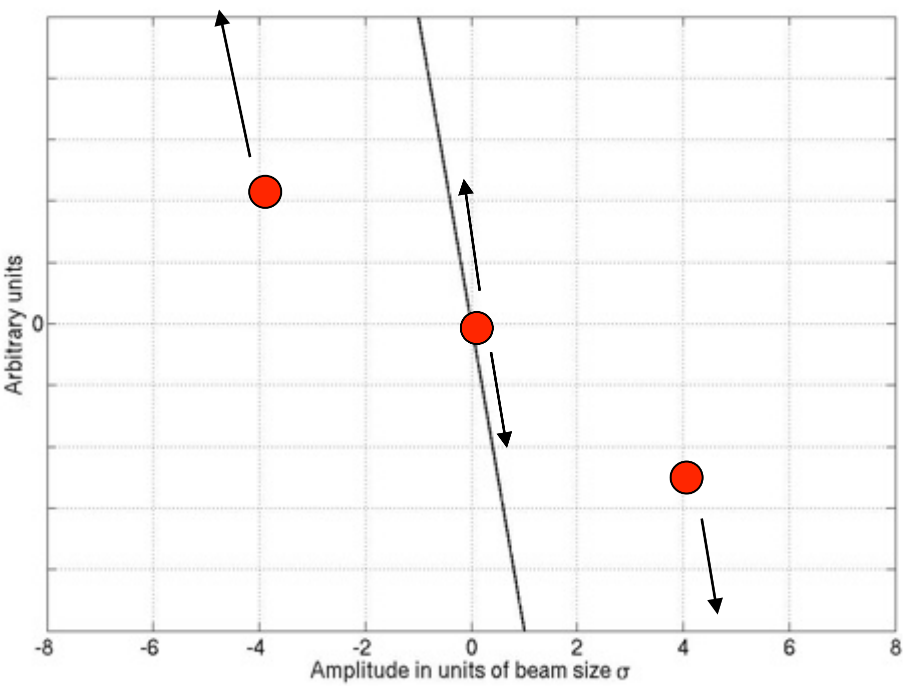
**Two main questions:**

**What happens to a single particle?**

**What happens to the whole beam?**

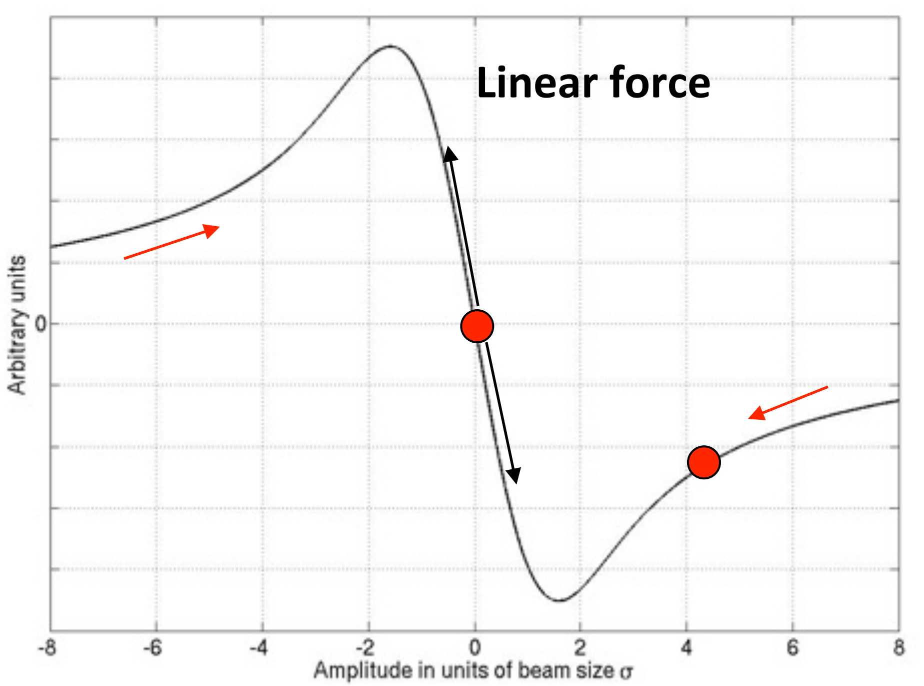
# Beam-Beam Force: single particle…

Lattice defocusing quadrupole




$$F = -k \cdot r$$

Beam-beam force




$$\mathrm{F} \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - \mathrm{e}^{-\frac{r^2}{2\sigma^2}}\right]$$

**For small amplitudes: linear force**

**For large amplitude: very non-linear**

**The beam will act as a strong non-linear electromagnetic lens!**

# Can we quantify the beam-beam strenght?

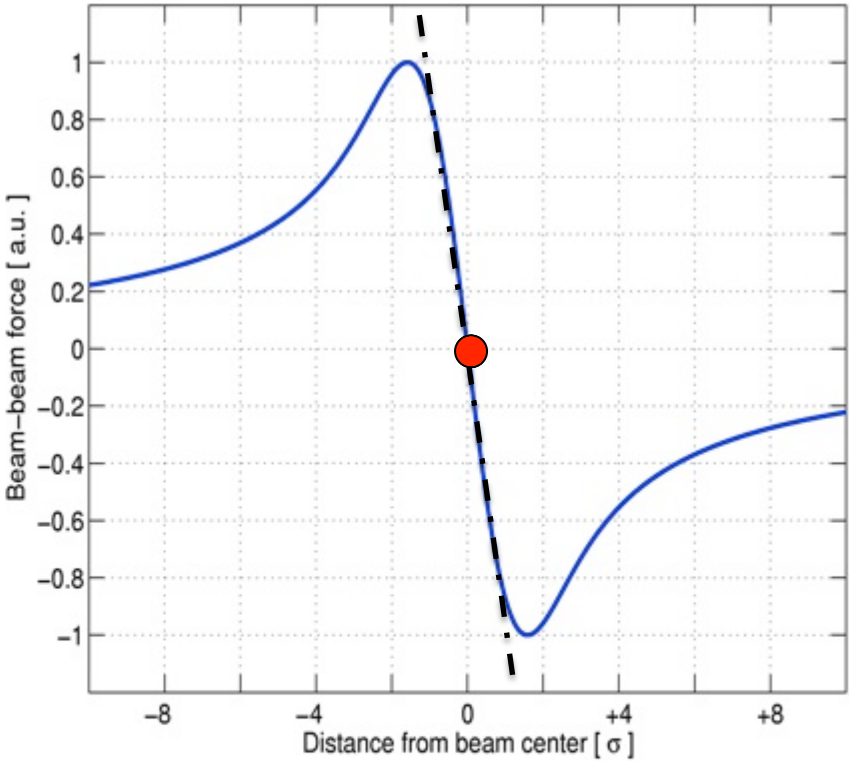
**Quantifies the strength of the force but does NOT reflect the nonlinear nature of the force**

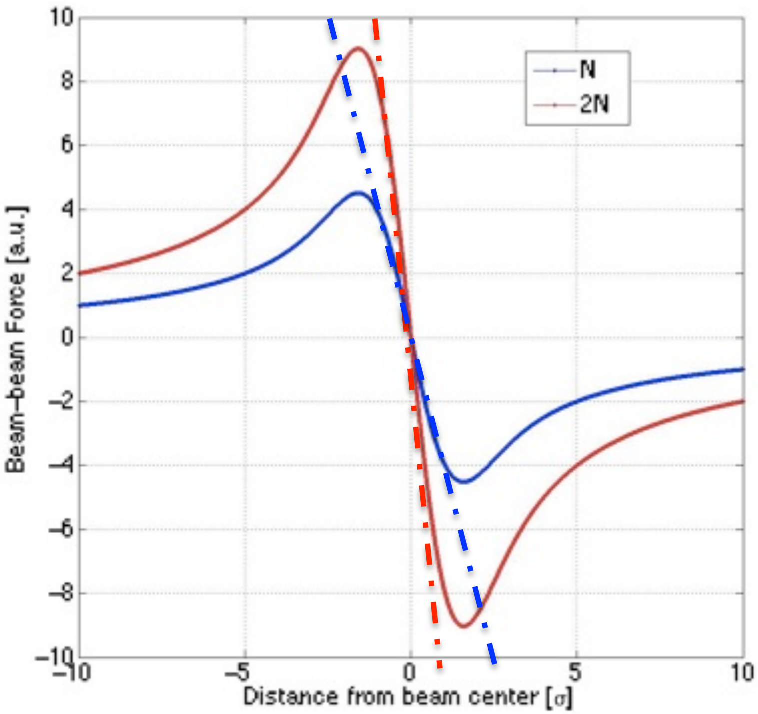
**For small amplitudes: linear force**

$$F \propto -\xi \cdot r$$

The slope of the force gives you the beam-beam parameter $\xi$

Beam-beam force




$$\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}}\right]$$

$$\Delta r' = \frac{2N_p r_0}{\gamma} \cdot \frac{1}{r} \cdot \left[1 - \left(1 - \frac{r^2}{2\sigma^2} + \ldots\right)\right.$$

# Colliders:

For round beams:

$$\xi = \frac{\beta^*}{4\pi} \cdot \frac{\delta(\Delta r')}{\delta r} = \frac{N r_0 \beta^*}{4\pi\gamma\sigma^2}$$

For non-round beams:

$$\xi_{x,y} = \frac{N r_0 \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

**Examples:**

| Parameters | LHC nominal | LHC 2012 |
|---|---|---|
| Intensity $N_{p,e}$/bunch | $1.15\ 10^{11}$ | $1.6\ 10^{11}$ |
| Energy GeV | 7000 | 4000 |
| Beam emittance | 3.75 μmrad | 2.2-2.5 μmrad |
| Crossing angle (μrad) | 285 | 290 |
| $\beta_{x,y}^*$ (m) | 1.25-0.05 | 0.60-0.60 |
| Luminosity | $1\ 10^{34}$ | $7.6\ 10^{33}$ |
| $\xi_{bb}$ | **0.0034** | **0.006** |

# Linear Tune shift

**For small amplitudes beam-beam can be approximated as linear force as a quadrupole**

$$F \propto -\xi \cdot r$$

**Focal length:**

$$\frac{1}{f} = \frac{\Delta x'}{x} = \frac{N r_0}{\gamma \sigma^2} = \frac{\xi \cdot 4\pi}{\beta^*}$$

**Beam-beam matrix:**

$$\begin{pmatrix} 1 & 0 \\ -\frac{\xi \cdot 4\pi}{\beta^*} & 1 \end{pmatrix}$$

**Perturbed one turn matrix with perturbed tune ΔQ and beta function at the IP β*:**

$$\begin{pmatrix} cos(2\pi(Q+\Delta Q)) & \beta^* sin(2\pi(Q+\Delta Q)) \\ -\frac{1}{\beta^*} sin(2\pi(Q+\Delta Q)) & cos(2\pi(Q+\Delta Q)) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \cdot \begin{pmatrix} cos(2\pi Q) & \beta_0^* sin(2\pi Q) \\ -\frac{1}{\beta_0^*} sin(2\pi Q) & cos(2\pi Q) \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix}$$

# Linear tune

Solving the one turn matrix one can derive the tune shift ΔQ and the perturbed beta function at the IP β*:

**Tune is changed**

$$cos(2\pi(Q + \Delta Q)) = cos(2\pi Q) - \frac{\beta_0^* \cdot 4\pi\xi}{\beta^*} sin(2\pi Q)$$

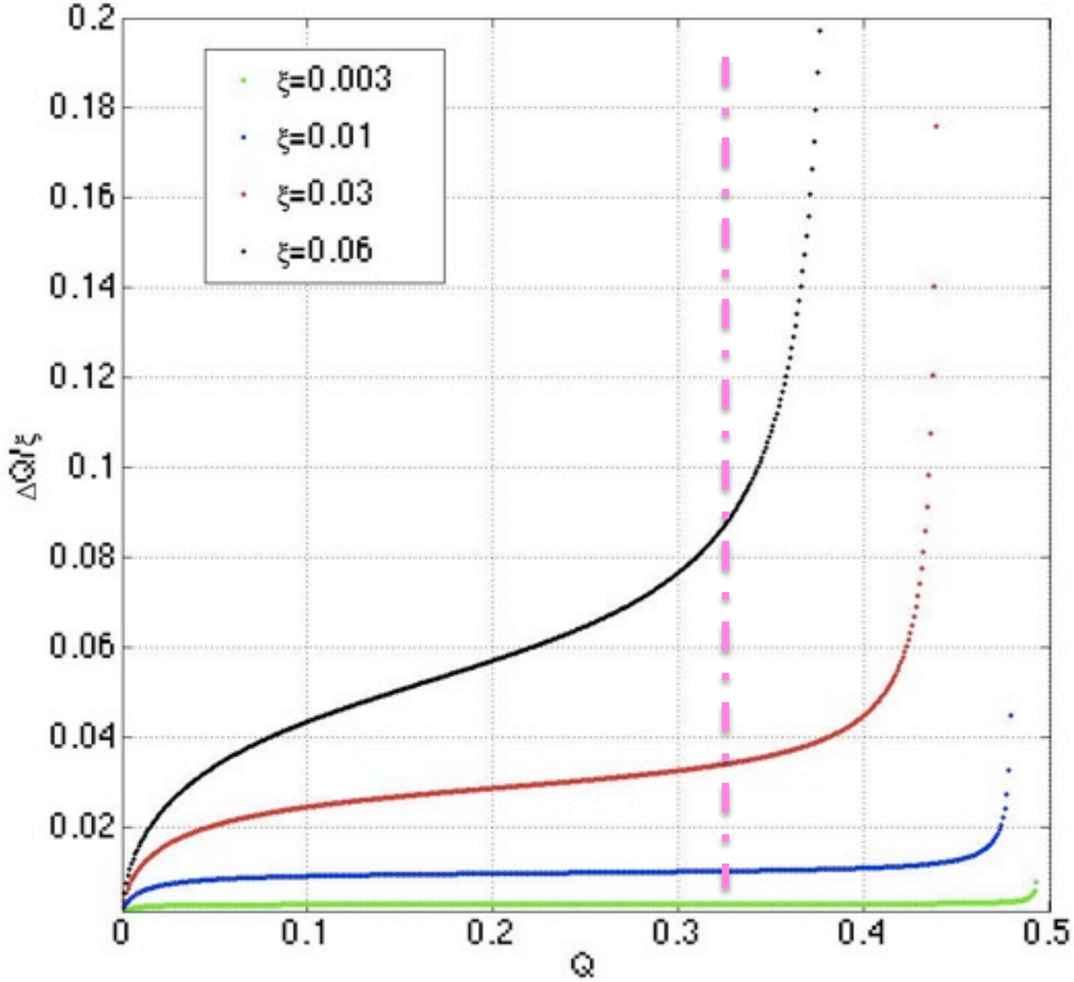
**β-function is changed:**

$$\frac{\beta^*}{\beta_0^*} = \frac{sin(2\pi Q)}{sin(2\pi(Q + \Delta Q))}$$

**…how do they change?**

# Tune dependence of tune shift and dynamic beta

**Tune shift as a function of tune**

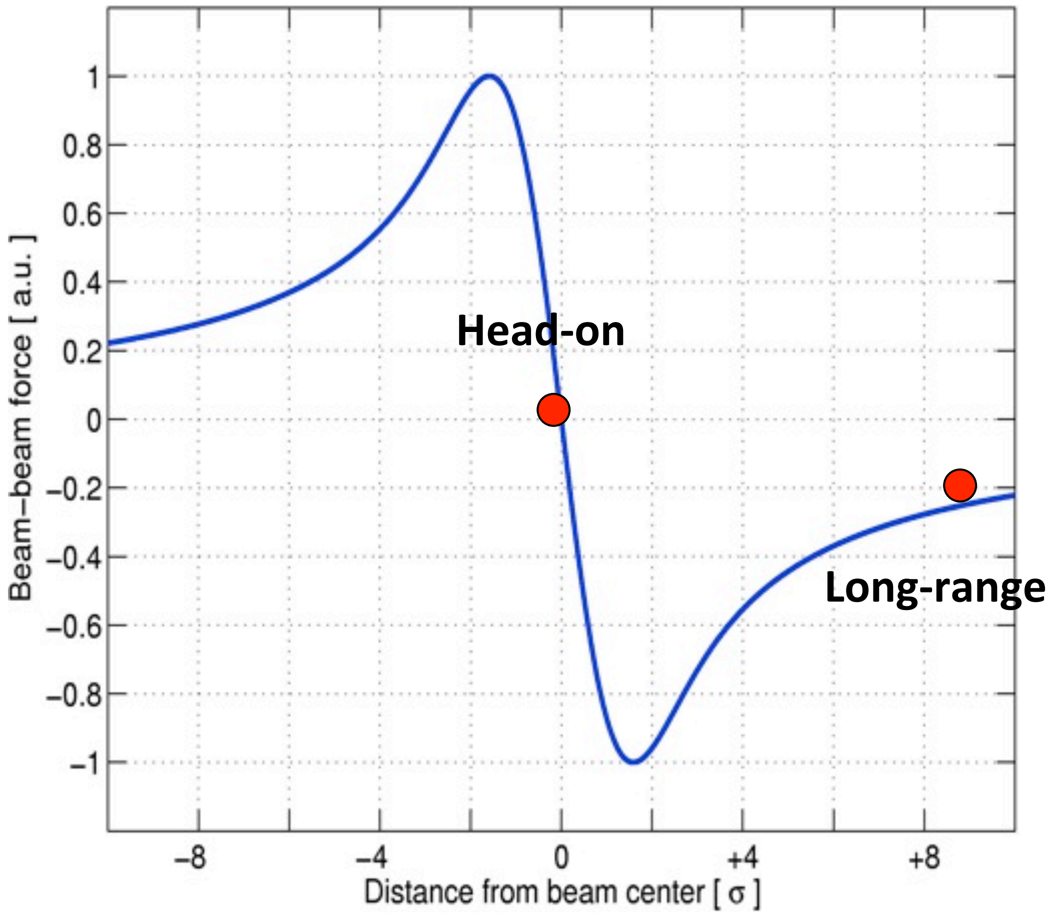



**Larger ξ → Strongest variation with Q**

# Head-on and Long-range interactions

$$L \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b \cdot f_{rev}$$

Beam-beam force




**Other beam passing in the center force: HEAD-ON beam-beam interaction**

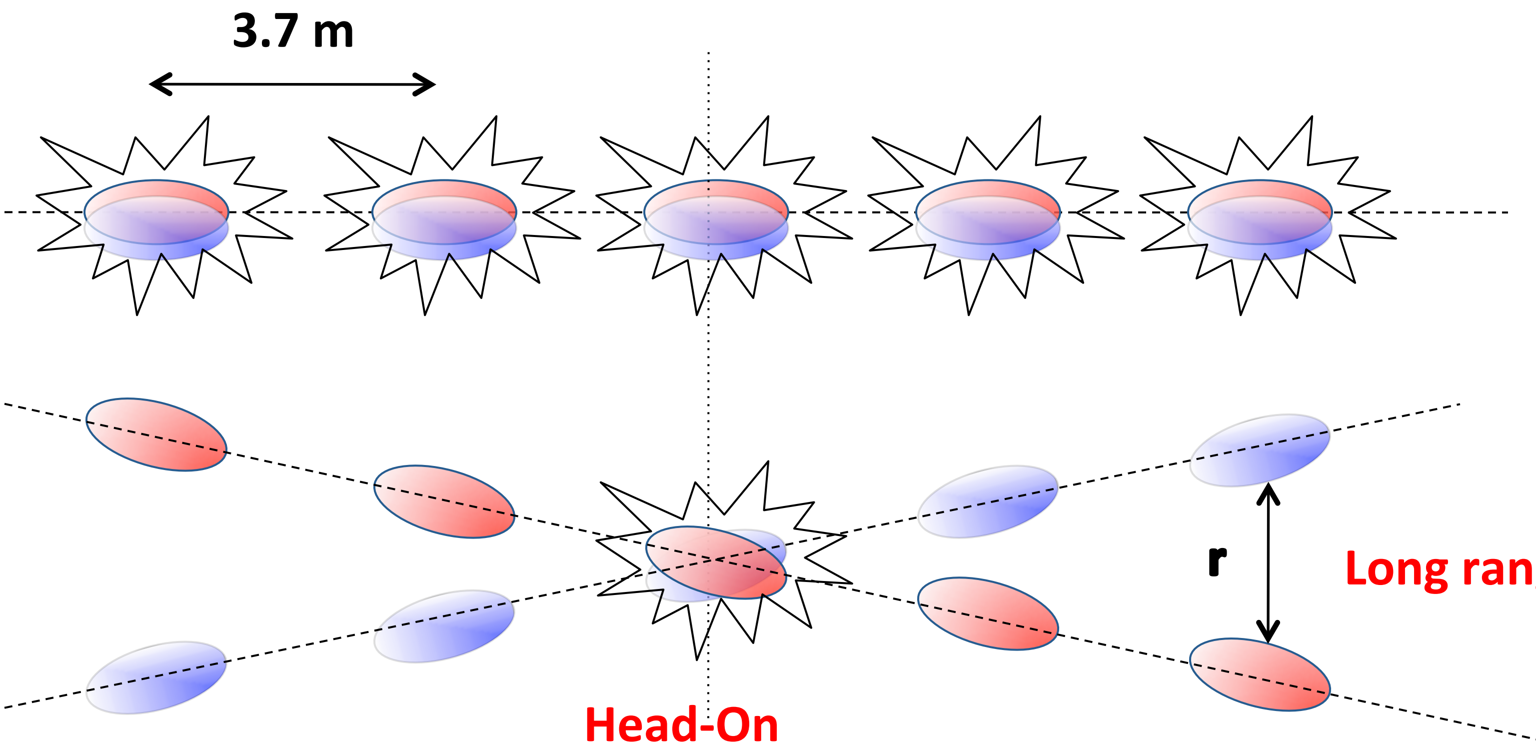
**Other beam passing at an offset of the force: LONG-RANGE beam-beam interaction**

# Multiple bunch Complications

## MANY INTERACTIONS

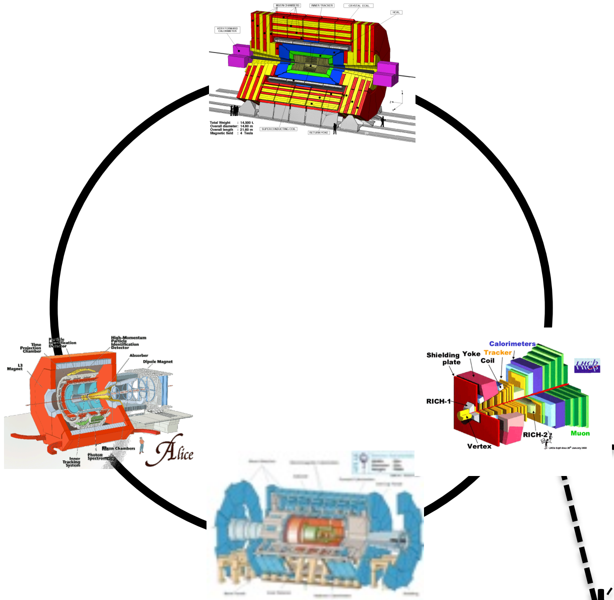
$$\mathcal{L} \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b$$

Num. of bunches : $n_b = 2808$

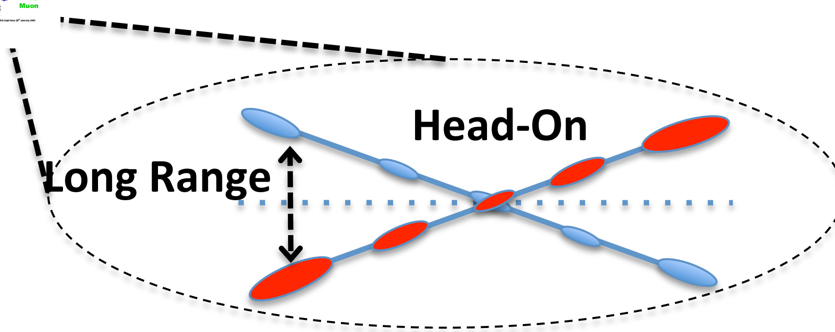


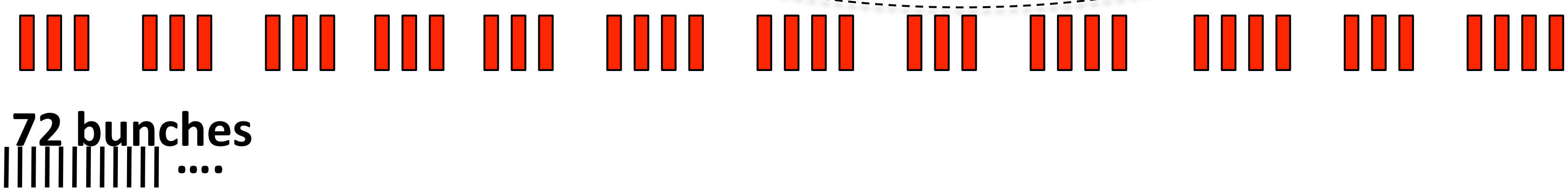

**For 25ns case 124 BBIs per turn: 4 HO and 120 LR**

# LHC, KEKB… colliders



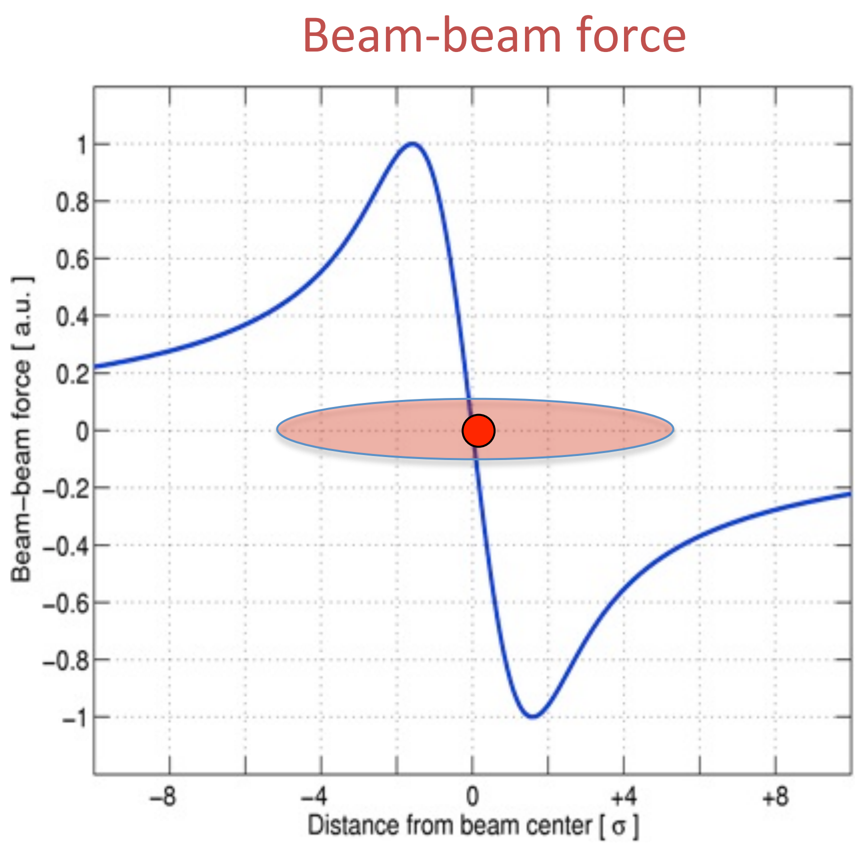

- Crossing angle operation
- High number of bunches in train structures




| | SppS | Tevatron | RHIC | LHC |
|---|---|---|---|---|
| Number Bunches | 6 | 36 | 109 | 2808 |
| LR interactions | 9 | 70 | 0 | 120/40 |
| Head-on interactions | 3 | 2 | 2 | 4 |

# A beam is a collection of particles

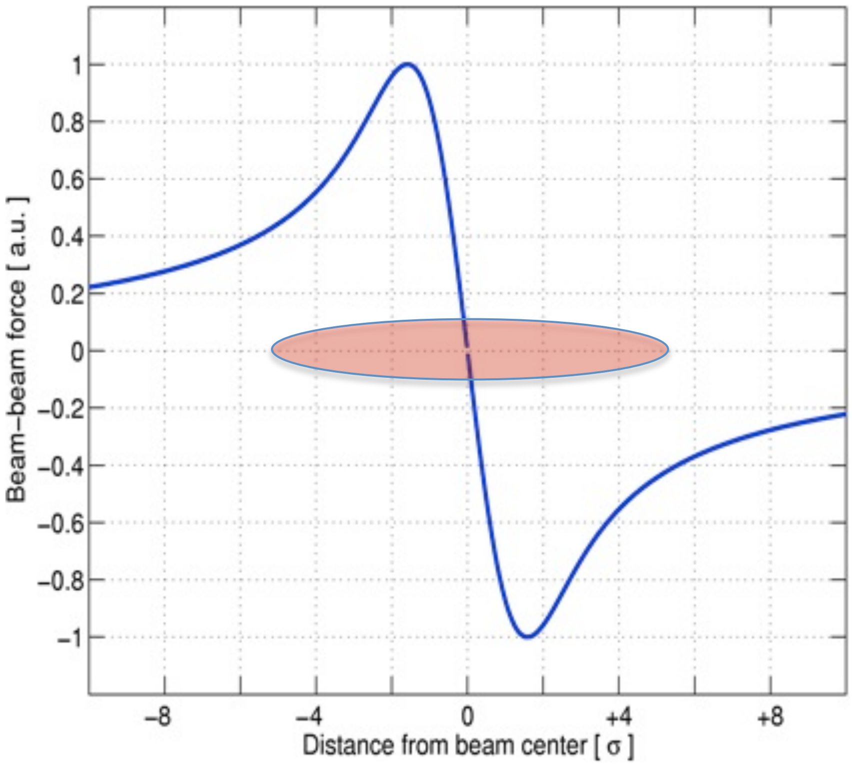



**Beam 2 passing in the center of force produce by Beam 1**
**Particles of Beam 2 will experience different ranges of the beam-beam forces**

**Tune shift as a function of amplitude (detuning with amplitude or tune spread)**

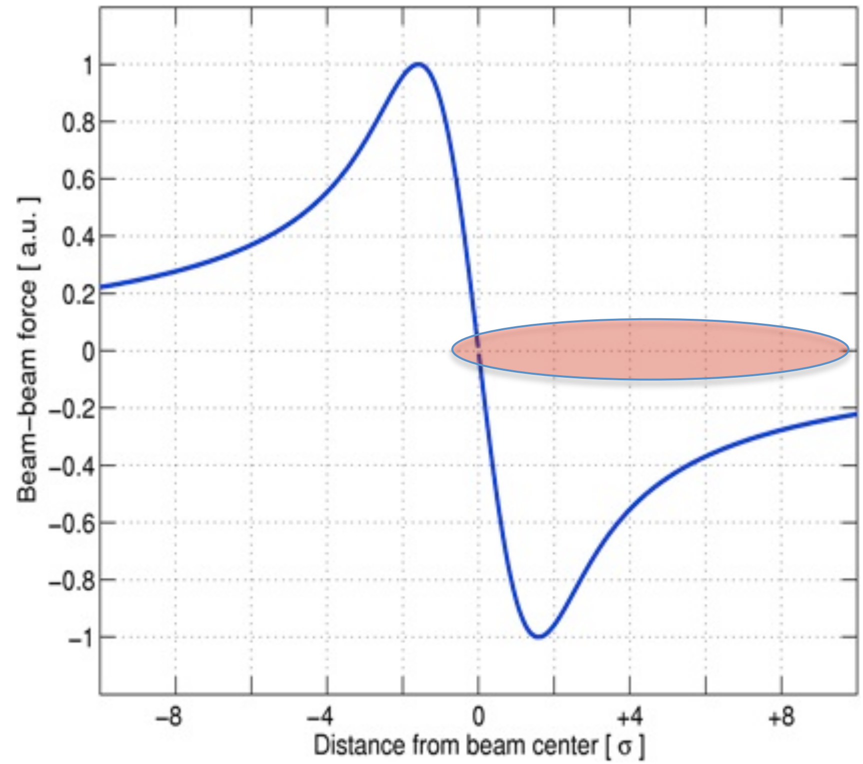
# A beam will experience all the force range

Beam-beam force




**Second beam passing in the center**
**HEAD-ON beam-beam interaction**

Beam-beam force



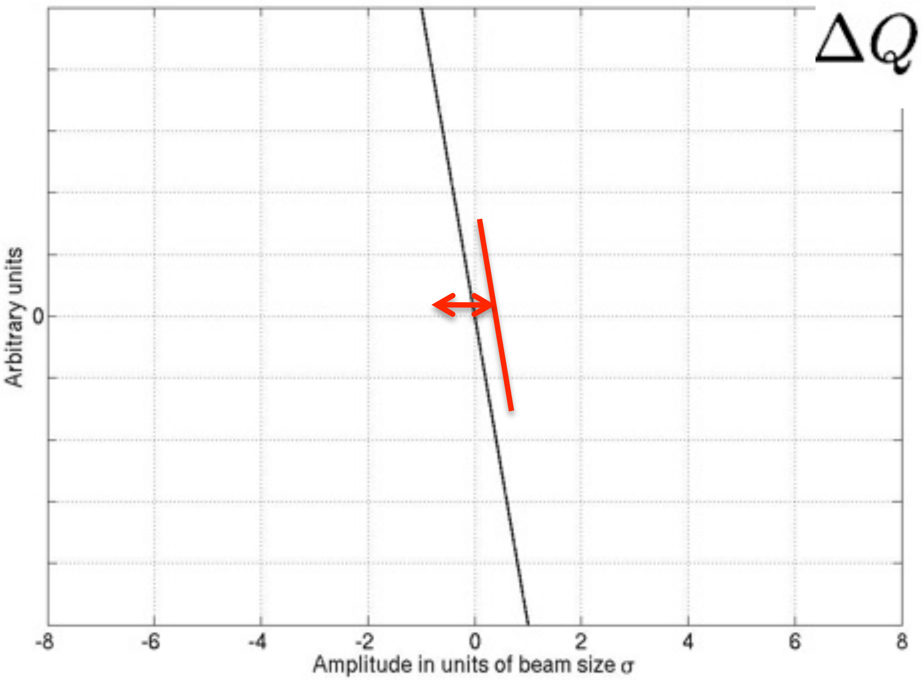

**Second beam displaced offset**
**LONG-RANGE beam-beam interaction**

**Different particles will see different force**

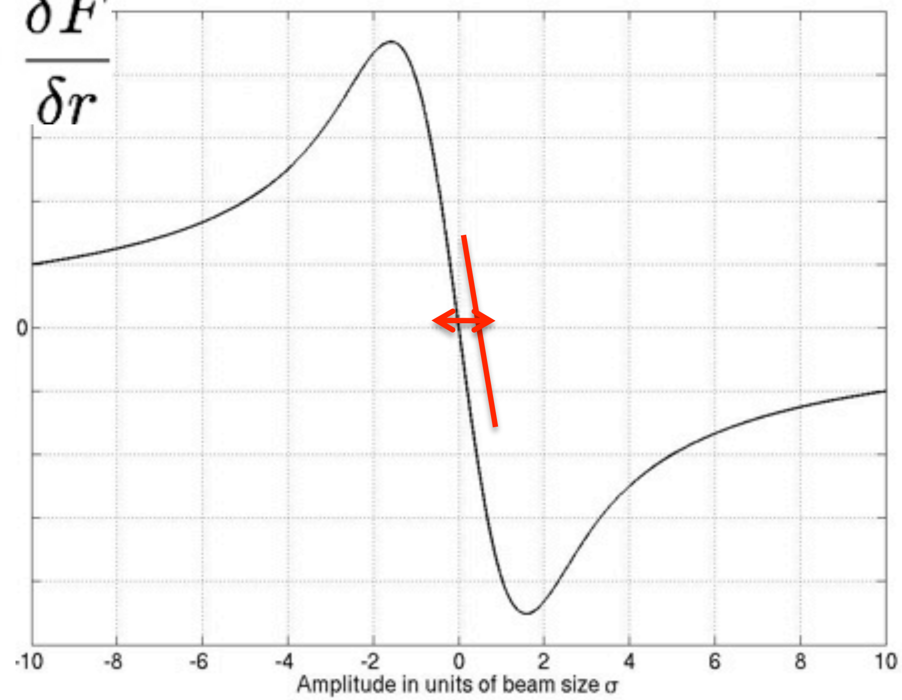
# Detuning with Amplitude for head-on

Instantaneous tune shift of test particle when it crosses the other beam is related to the derivative of the force with respect to the amplitude

$$\Delta Q \propto \frac{\delta F}{\delta r}$$




$$\Delta Q_{quad} = const$$




$$\Delta Q_{bb} \approx const$$

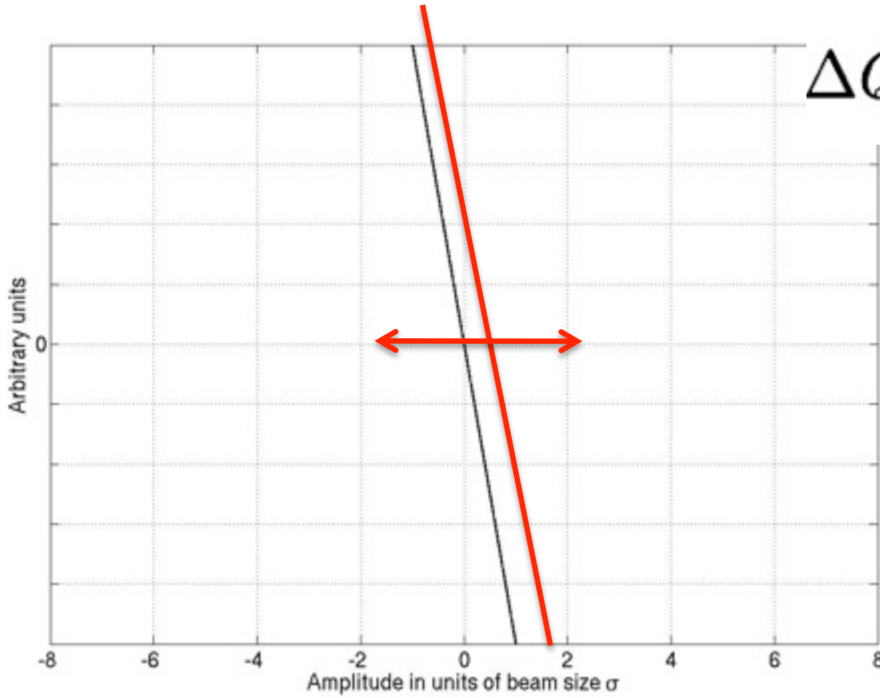
**For small amplitude test particle linear tune shift**

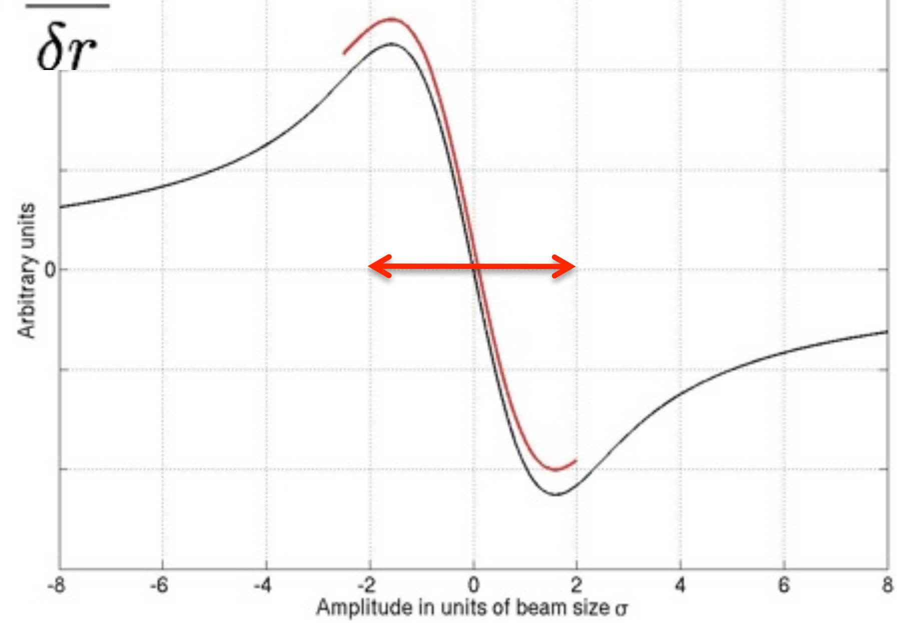
$$\lim_{r\to 0} \Delta Q(r) = -\frac{N r_0 \beta^*}{4\pi\gamma\sigma^2} = \xi$$

# Detuning with Amplitude for head-on

**Beam with many particles this results in a tune spread**

$$\Delta Q \propto \frac{\delta F}{\delta r}$$



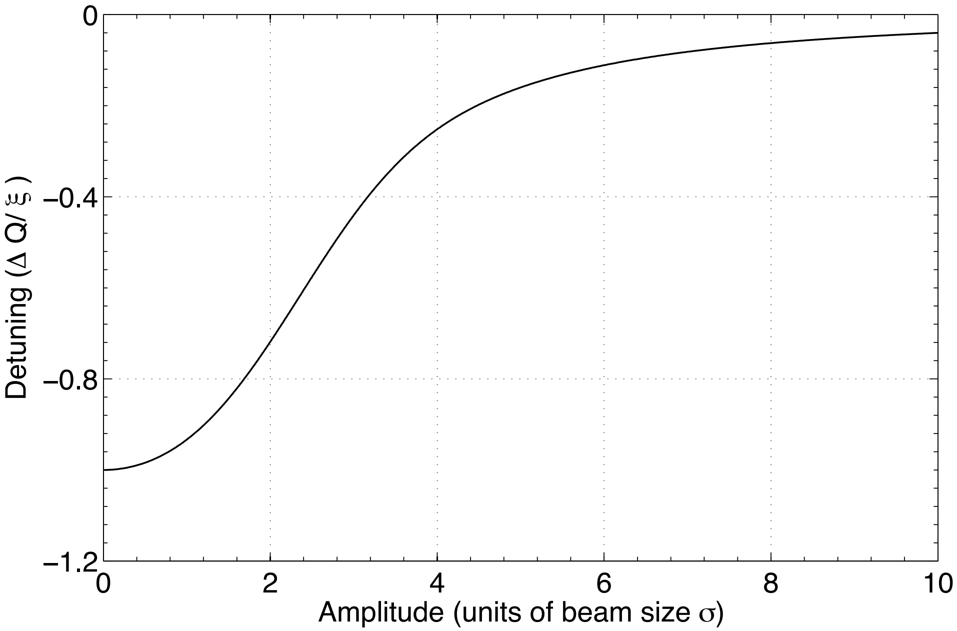

$$\Delta Q_{quad} = const \qquad \Delta Q_{bb} \neq const$$

$$\Delta Q(x) = \frac{N r_0 \beta}{4\pi\gamma\sigma^2} \cdot \frac{1}{(\frac{x}{2})^2} \cdot (\exp -(\frac{x}{2})^2 I_0 (\frac{x}{2})^2 - 1)$$

Mathematical derivation in Ref [3] using Hamiltonian formalism and in Ref [4] using Lie Algebra

# Head-on detuning with amplitude and footprints

## 1-D plot of detuning with amplitude

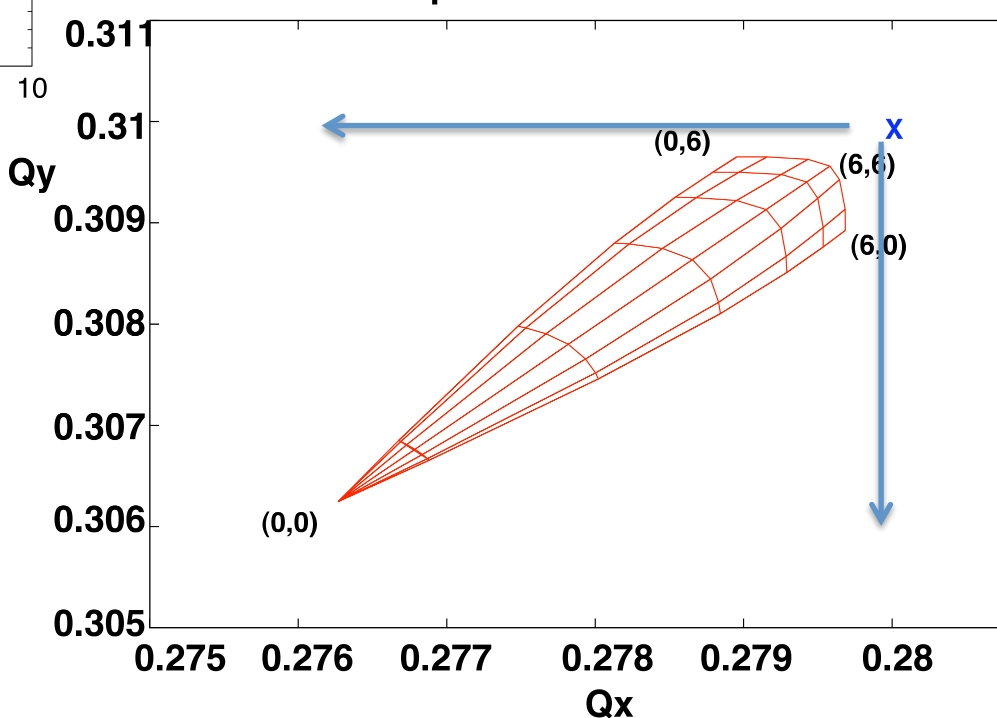



And in the other plane?

**THE SAME DERIVATION**

same tune spread

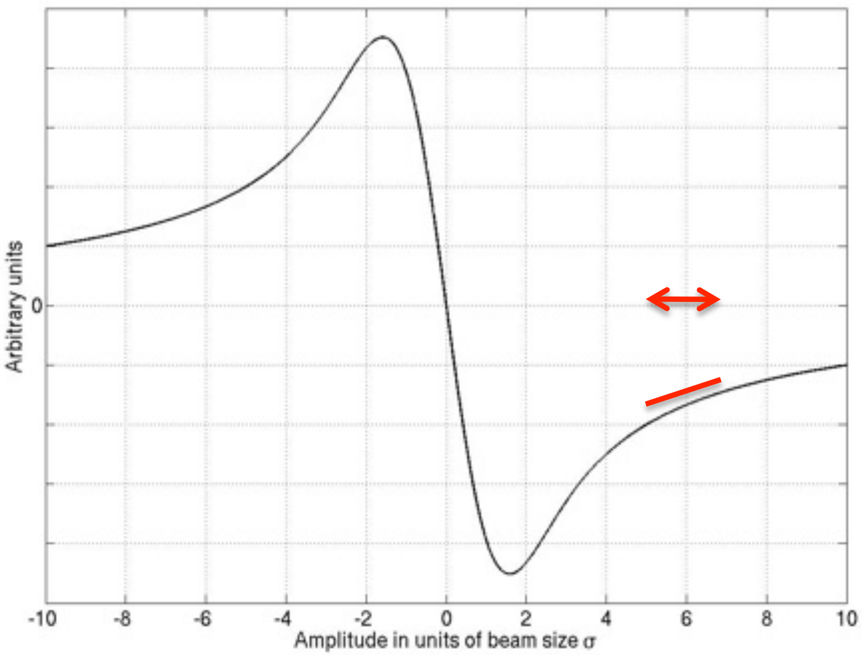
Tune footprint for head–on collision




**FOOTPRINT**

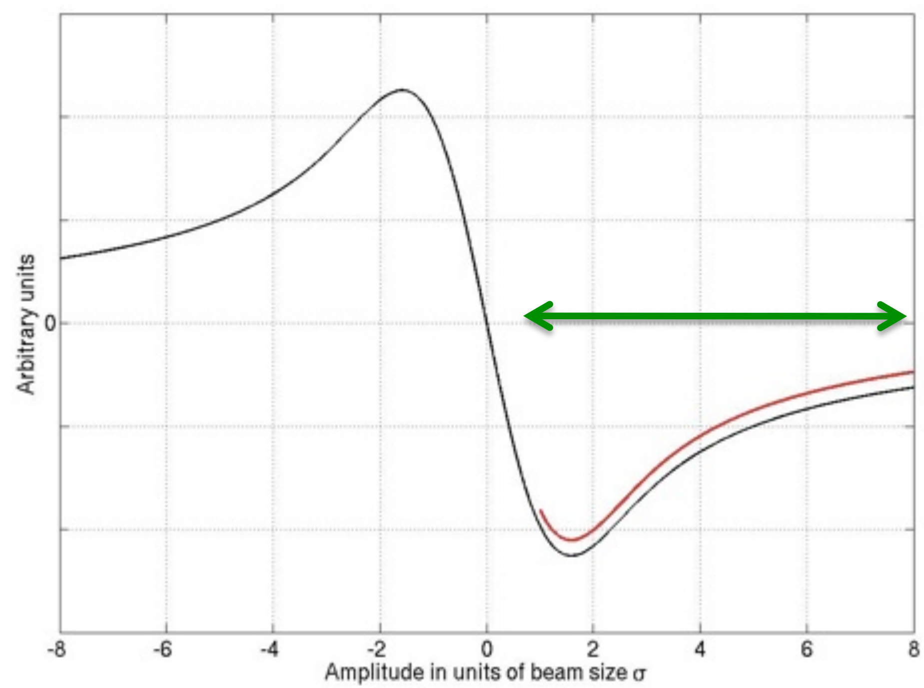
**2-D mapping of the detuning with amplitude of particles**

# And for long-range interactions?




**Second beam centered at d** (i.e. 6σ)

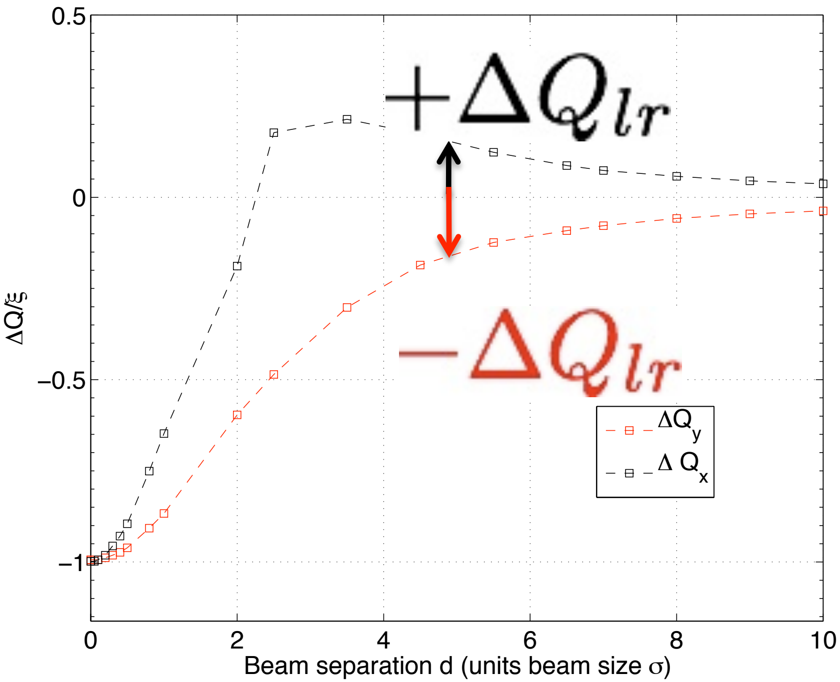
- Small amplitude particles **positive tune shifts**
- Large amplitude can go to **negative tune shifts**




**Long range tune shift scaling for distances** $d > 6\sigma$

$$\Delta Q_{lr} \propto -\frac{N}{d^2}$$

# Long-range footprints



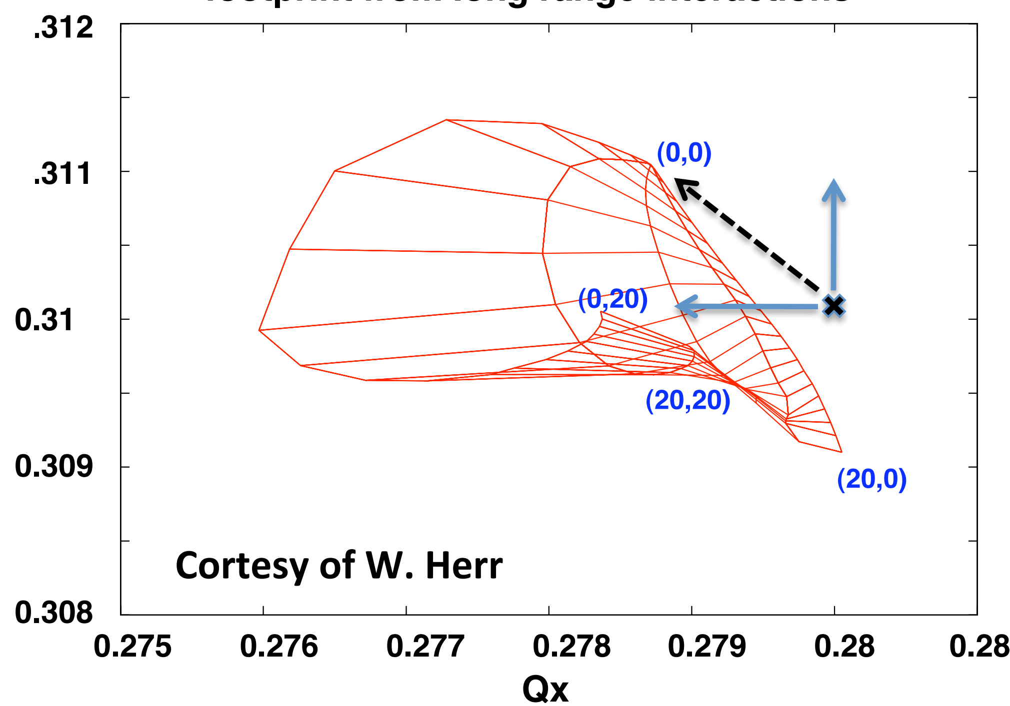

Separation in vertical plane!

And in horizontal plane?

The test particle is centered with the opposite beam

tune spread more like for head-on at large amplitudes

footprint from long range interactions




The picture is more complicated

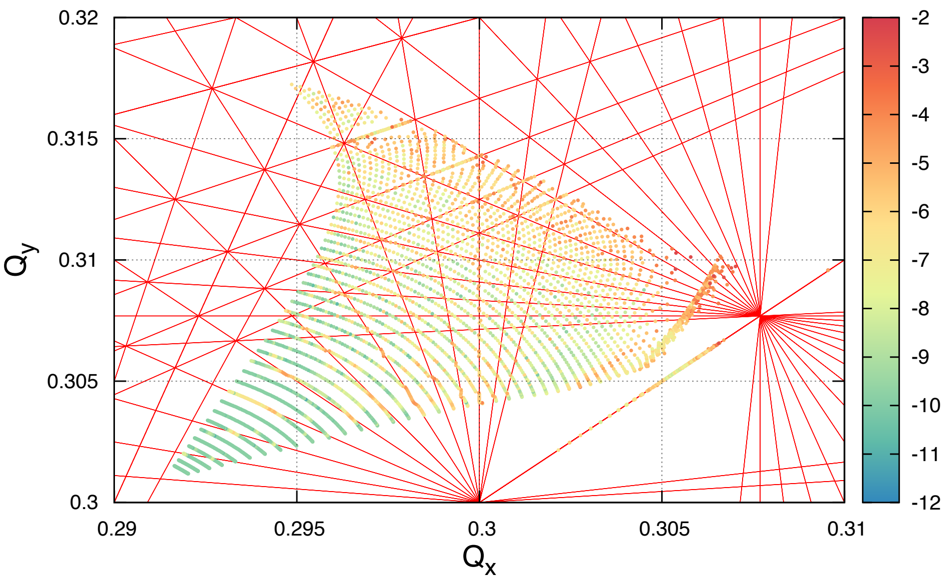
now the LARGE amplitude particles see the second beam and have larger tune shift

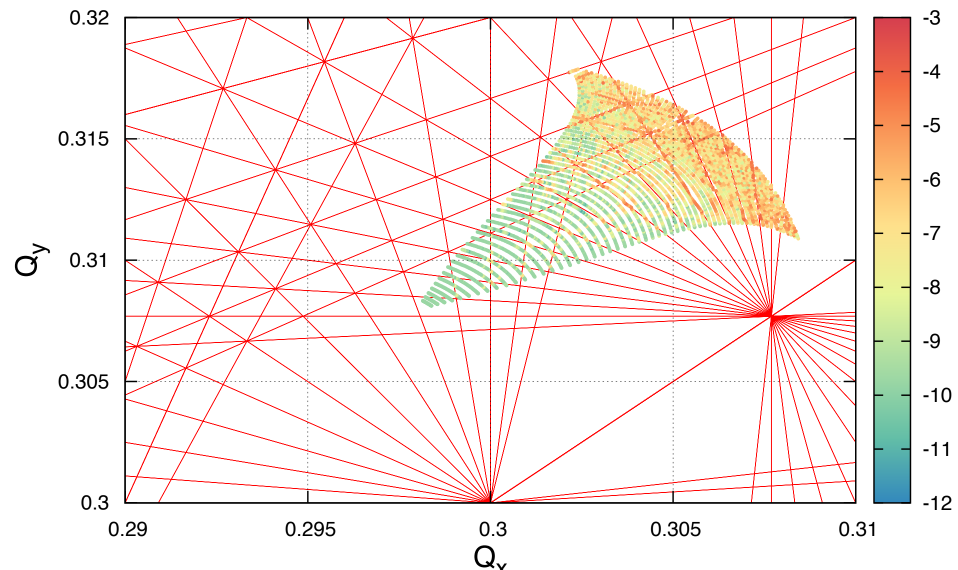
# Beam-beam tune shift and spread

**When long-range effects become important footprint wings appear (large amplitude particles deviate)**

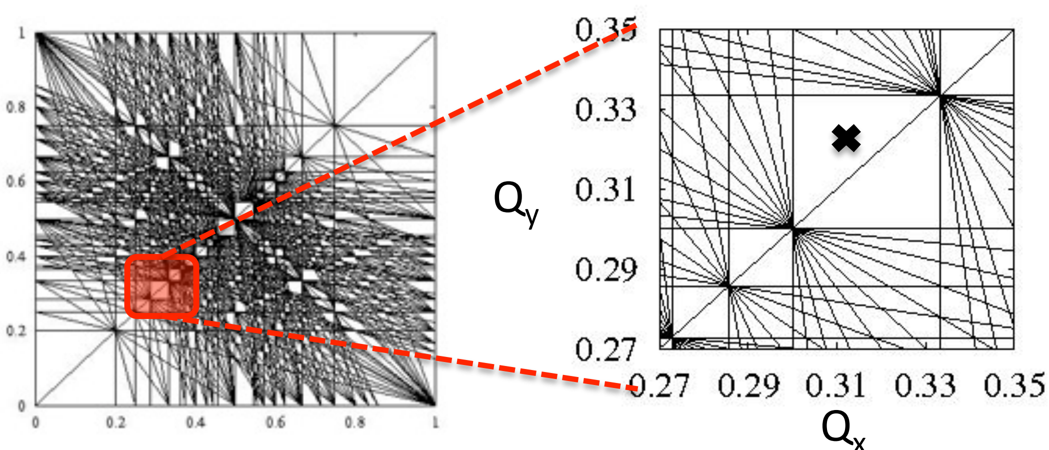
**Aim to reduce the area as much as possible and avoid chaotic motion!**

**296mrad** **390mrad**



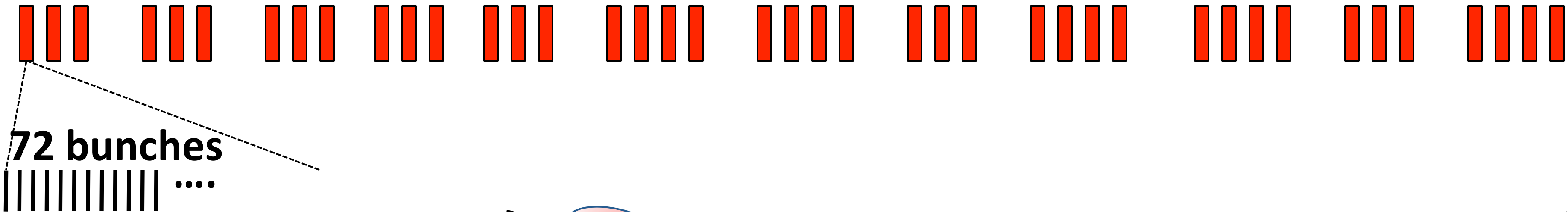




**Cortesy of J. Barranco**



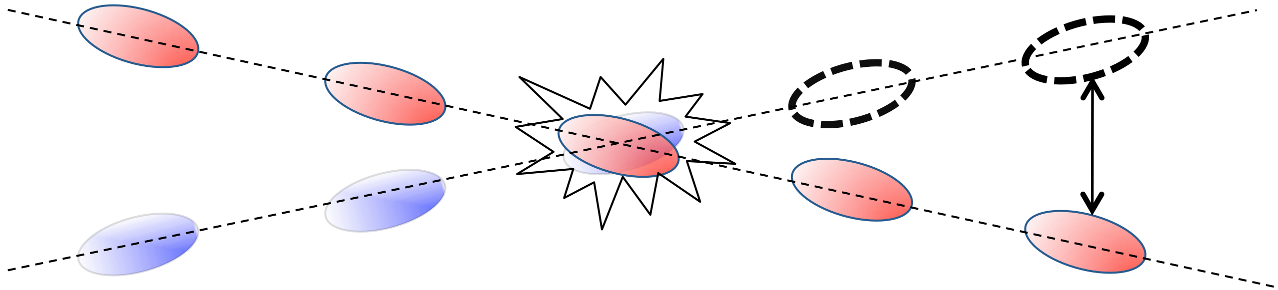
# Complications

## PACMAN and SUPER PACMAN bunches




**Pacman:**

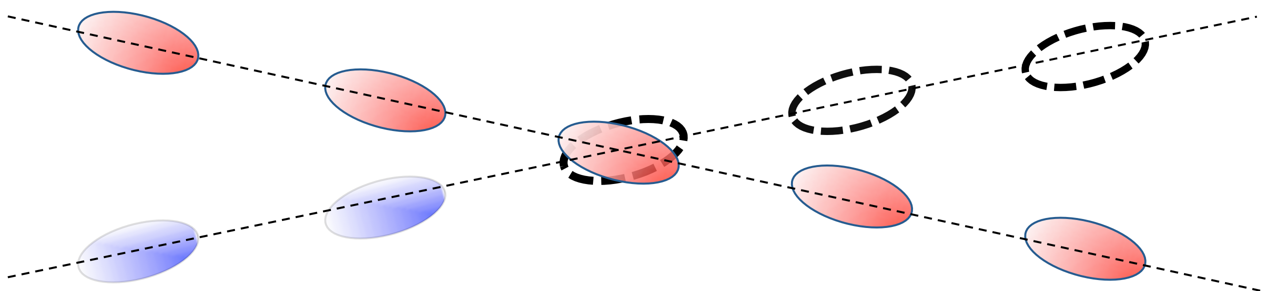
**miss long range BBI**



**Super Pacman:**

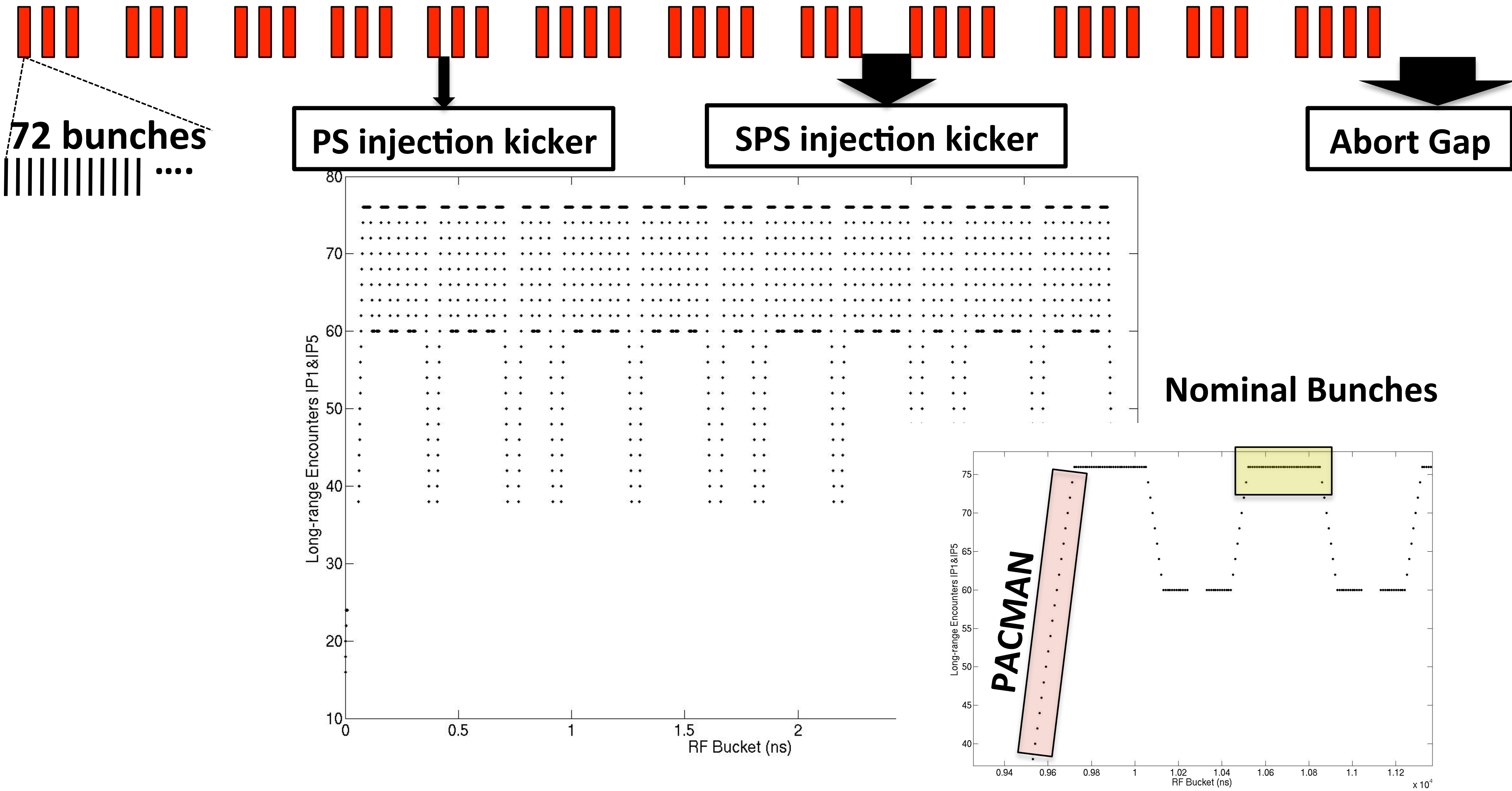
**miss head-on BBI**

**IP2 and IP8 depending on filling scheme**



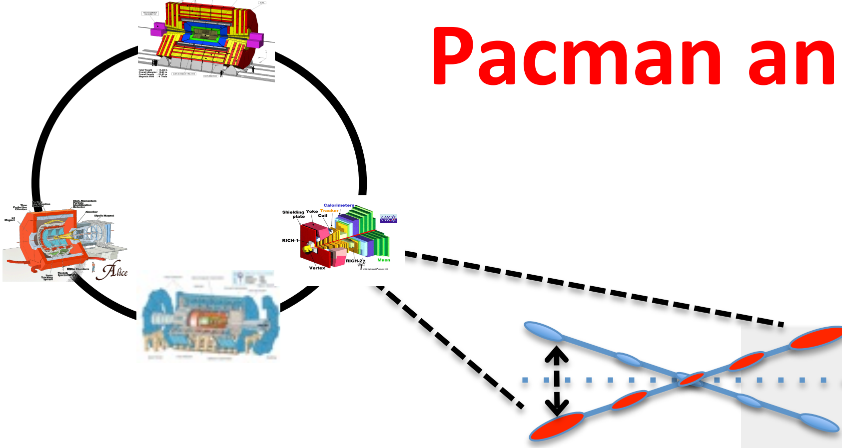
**Different bunch families: Pacman and Super Pacman**

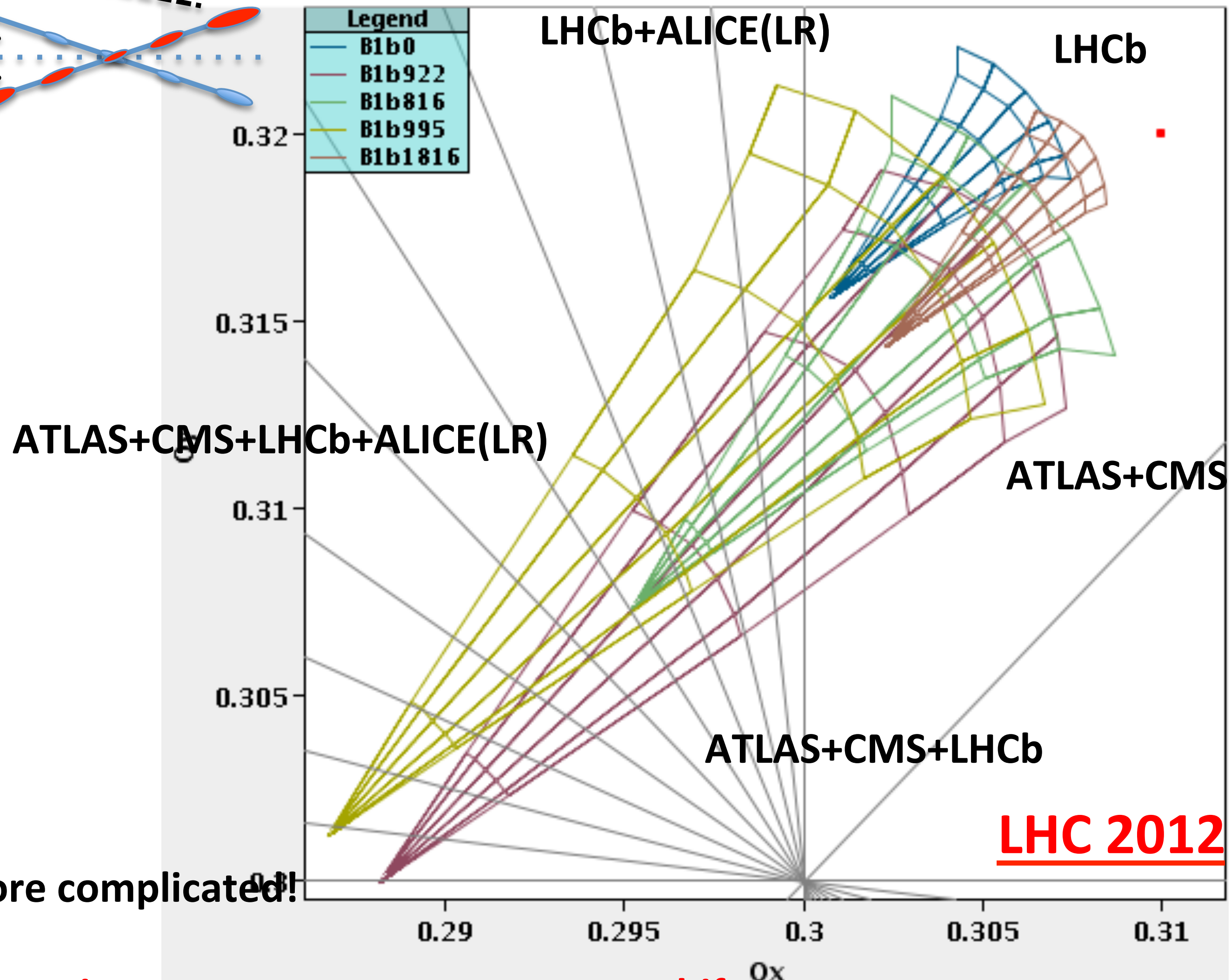
# LHC Complications: filling schemes




Pacman Bunches: different number of long-range interactions

# Pacman and Super-pacman






**…operationally it is even more complicated!**

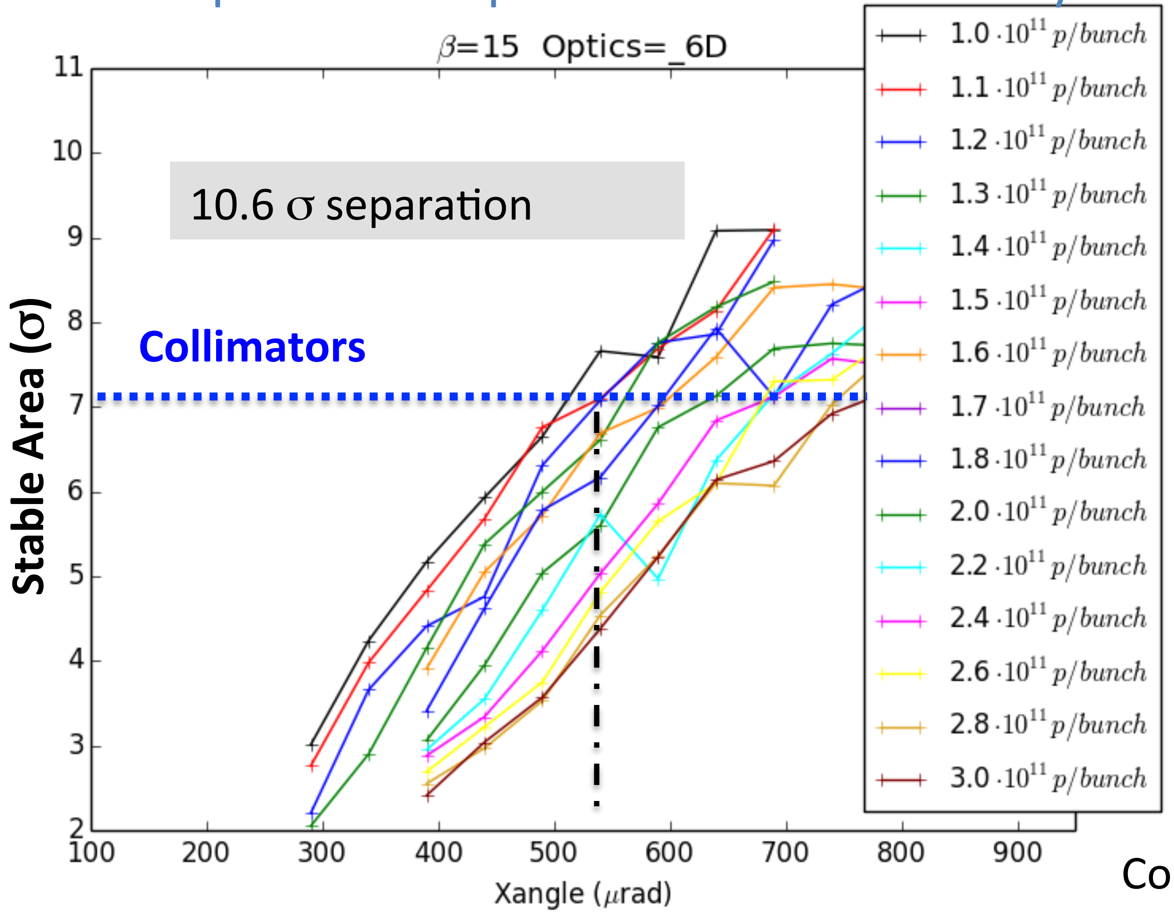
**…intensities, emittances…**

**Alternating crossing IP1 and IP5 is to compensate pacman tune shifts**

# Particles stability

**Dynamic Aperture: area in amplitude space with stable motion**

Stable area of particles depends on beam intensity and crossing angle




$$d_{lr} \propto \sqrt{\frac{\beta^* \alpha^2 \gamma}{\epsilon_n}}$$

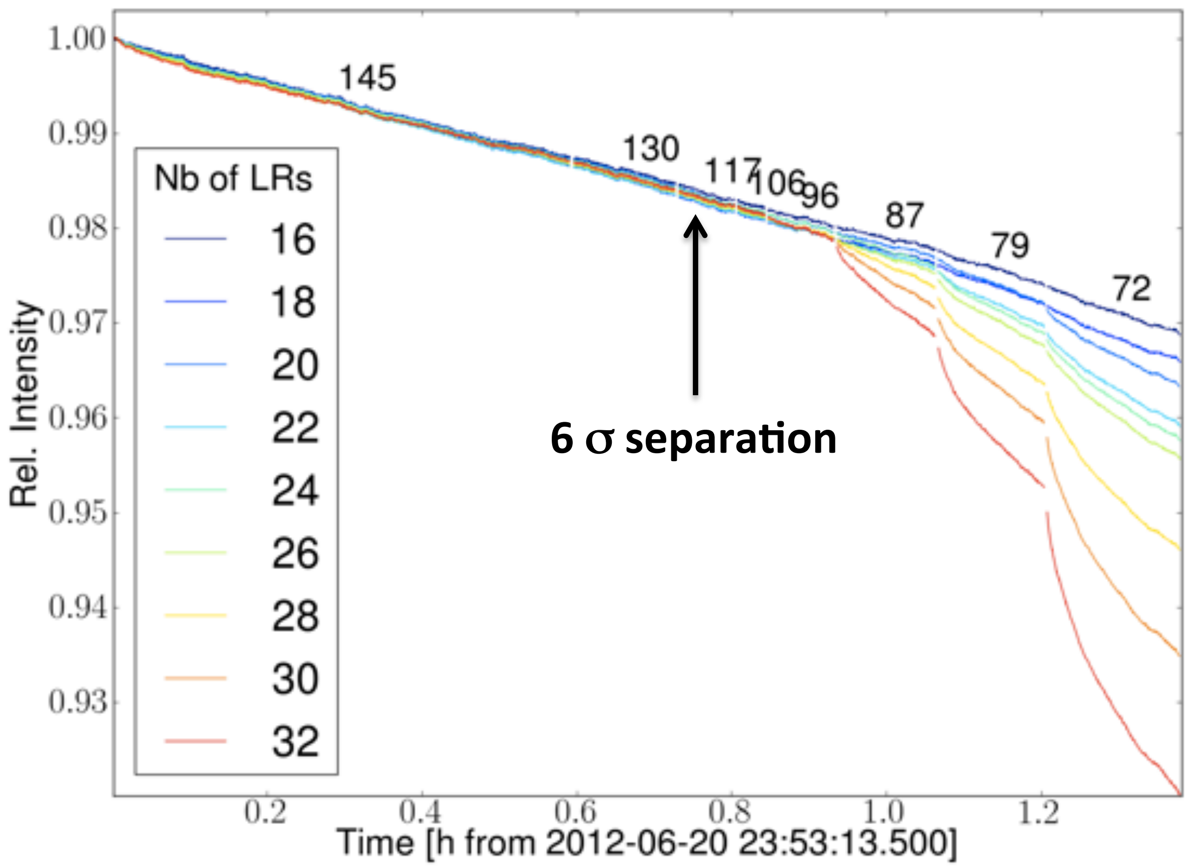
$$F_{bb} \propto N_p$$

Cortesy D. Banfi

Stable area depends on beam-beam interactions therefore the choice of running parameters (crossing angles, β*, intensity, working points) is the result of careful study of different effects!

# DO we see the effects of LR in the LHC?

Courtesy X. Buffat




**Small crossing angle = small separation**

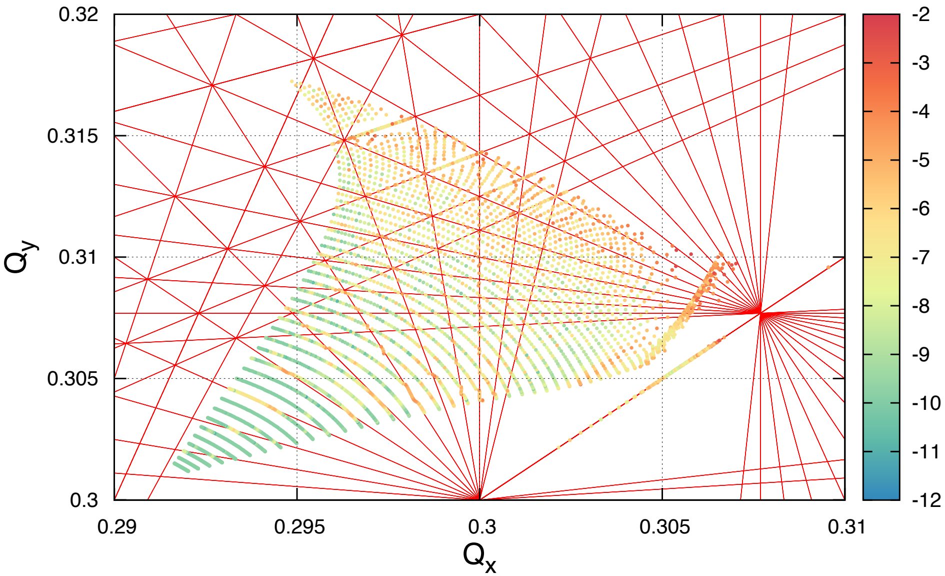
**If separation of long range too small particles become unstable and get lost**

**Bunches start loosing large amplitude particles**
**Particle losses follow number of Long range interactions**
**Nominal LHC will have twice the number of interactions**

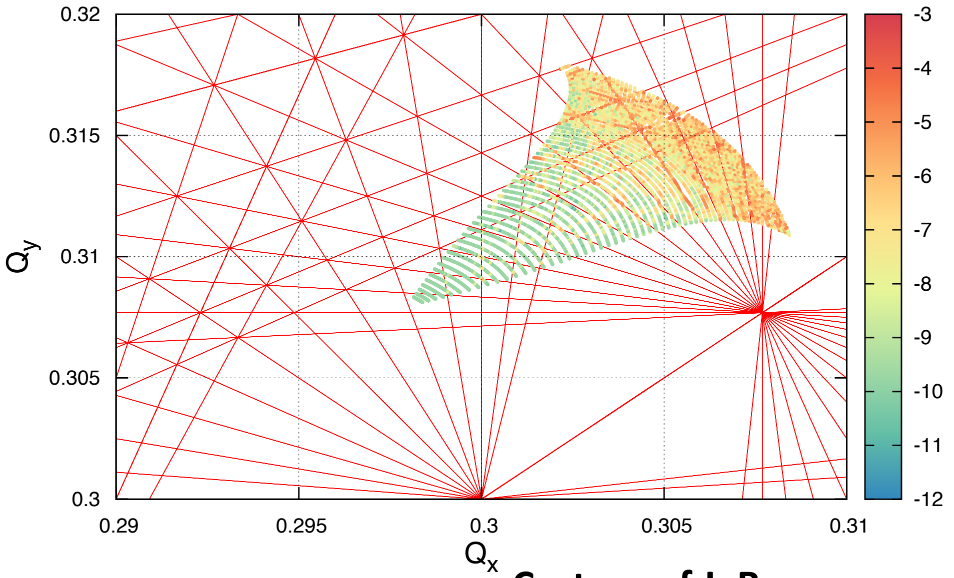
# Beam-beam tune shift and spread

**When long-range effects become important footprint wings appear**
**Aim to reduce the area as much as possible and avoid chaotic motion!**

**296mrad**




**390mrad**




**Cortesy of J. Barranco**

# Orbit Effects

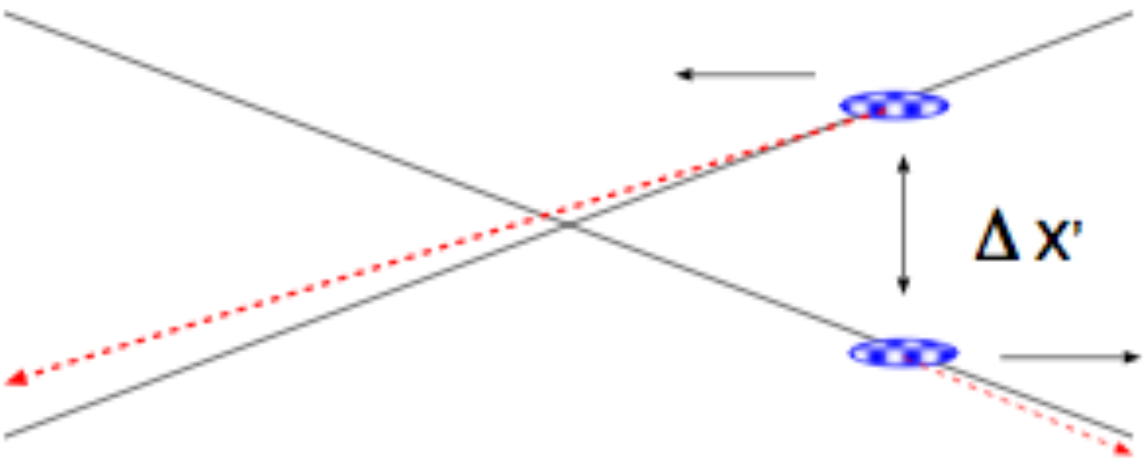
Long Range Beam-beam interactions lead to orbit effects

Long range kick $$\Delta x'(x+d,y,r) = -\frac{2Nr_0}{\gamma}\frac{(x+d)}{r^2}[1-\exp(-\frac{r^2}{2\sigma^2})$$

For well separated beams $d \gg \sigma$

The force has an amplitude independent contribution: **ORBIT KICK**

$$\Delta x' = \frac{const}{d}[1 - \frac{x}{d} + O(\frac{x^2}{d^2}) + \ldots]$$

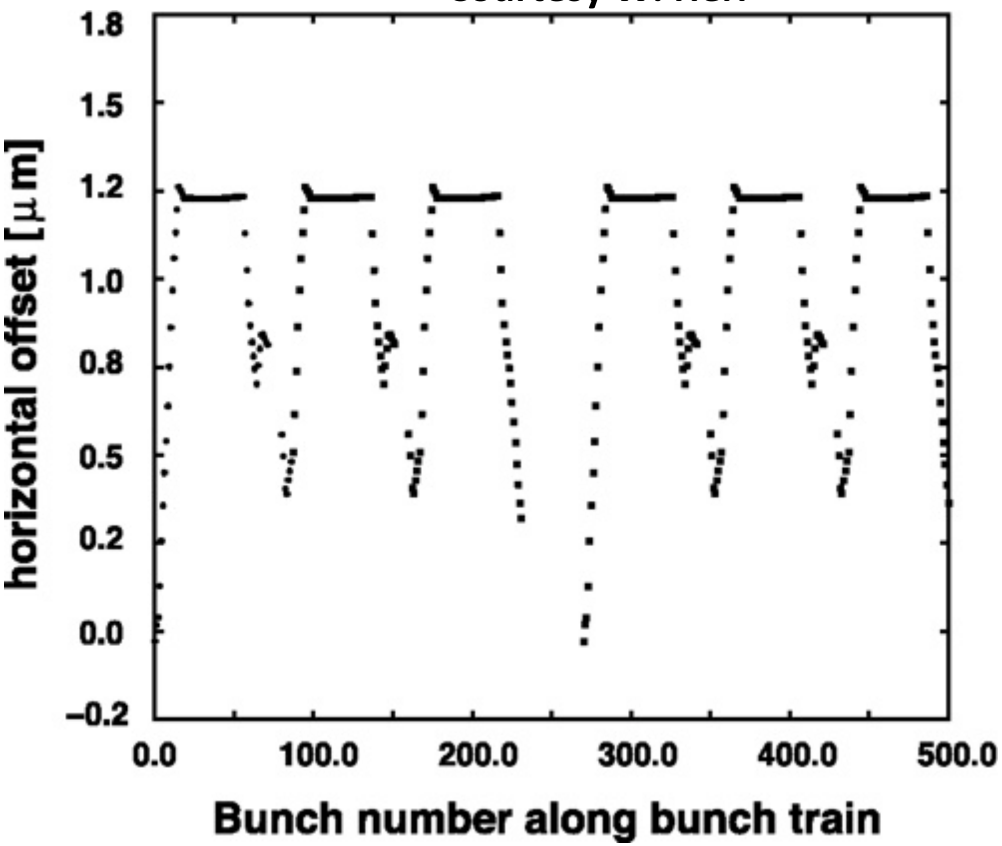



**Orbit can be corrected but we should remember PACMAN effects**

# LHC orbit effects

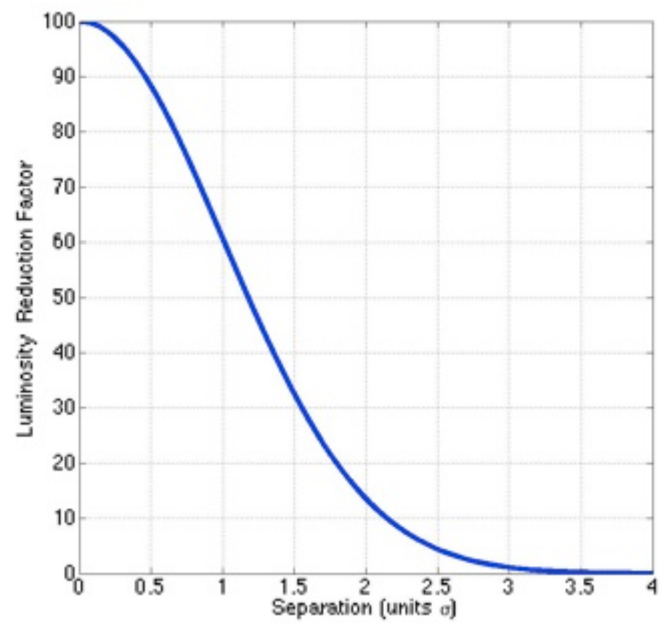
Orbit effects different due to Pacman effects and the many long-range add up giving a non negligible effect

Courtesy W. Herr



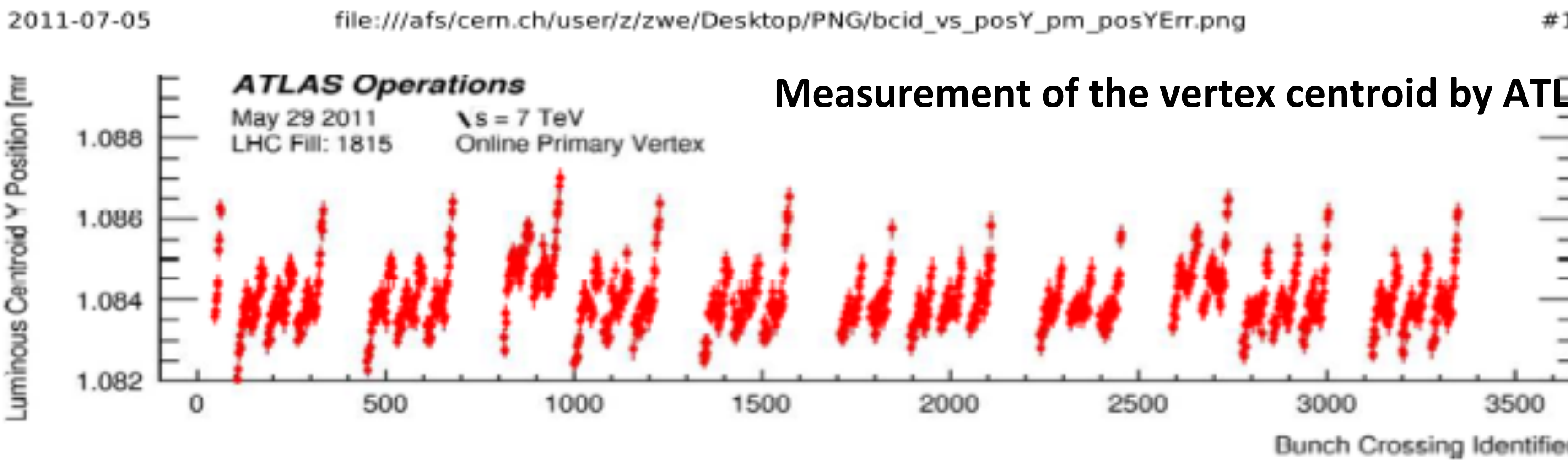

$$L = L_0 \cdot e^{-\frac{d^2}{4\sigma_x^2}}$$

d = 0 - 0.4 units of beam size



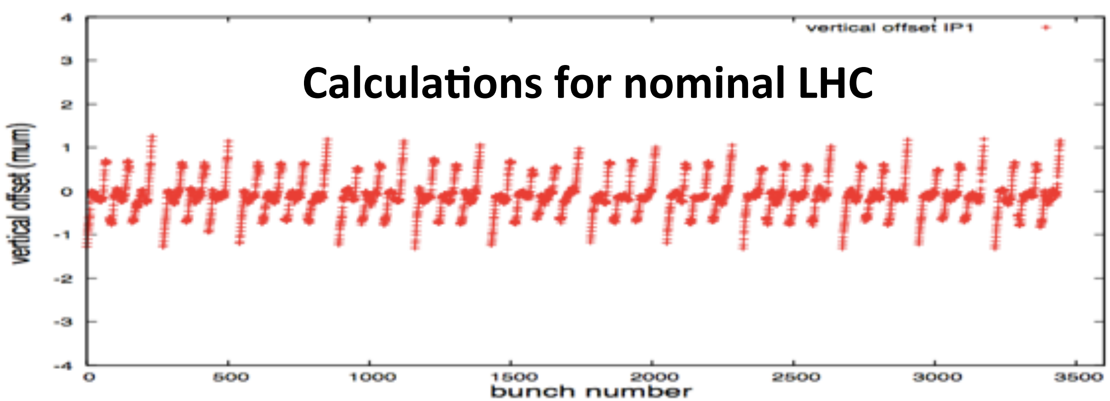
# Long range orbit effect

Long range interactions leads to orbit offsets at the experiment a direct consequence is deterioration of the luminosity




**Measurement of the vertex centroid by ATLAS**

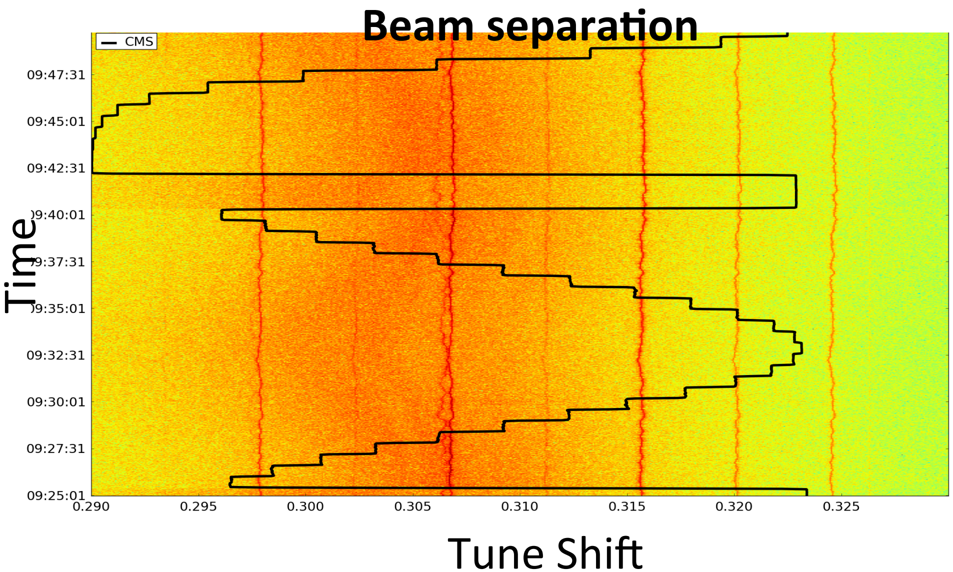
**Courtesy W. Kozanecki**

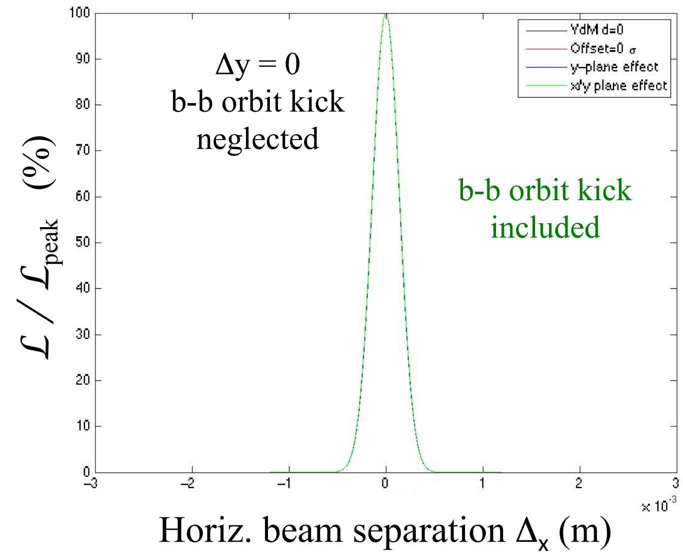



**Calculations for nominal LHC**

Effect is already visible with reduced number of interactions

# Van der Meer Scans: tune shift and orbit

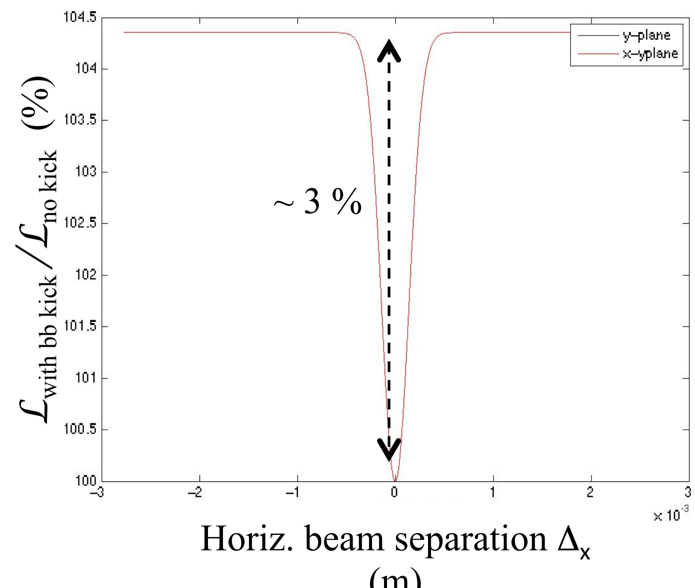



Van der Meer scan is a scan of the beam-beam force

- Tune shift
- Orbit effect
- Dynamic beta

**Beam-beam effects need to be taken into account for precision measurements!**



# Coherent dipolar beam-beam modes

Coherent beam-beam effects arise from the forces which an exciting bunch exerts on a whole test bunch during collision

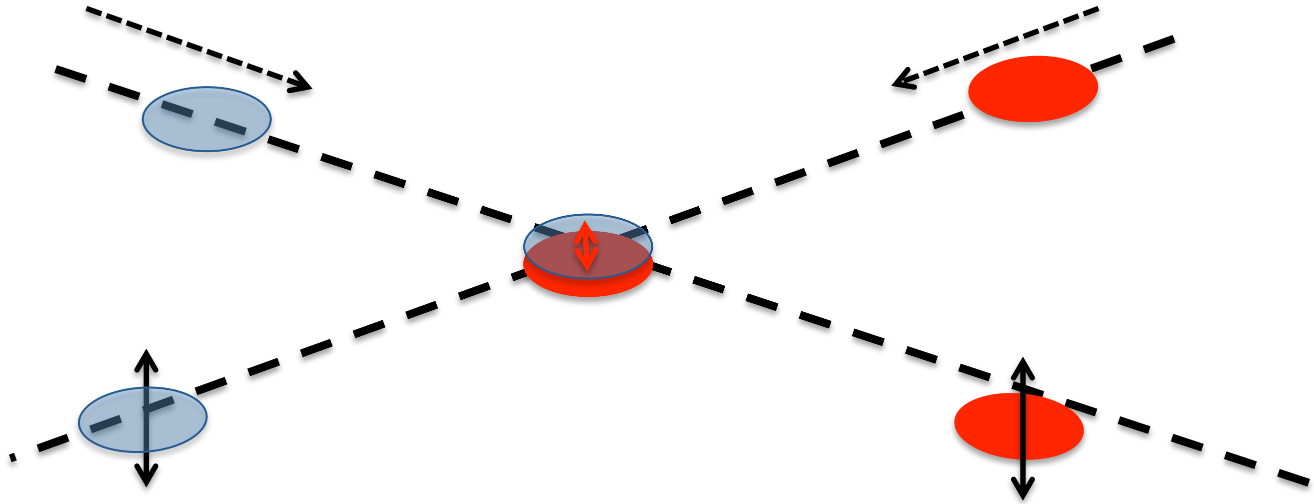
We study the collective behaviour of all particles of a bunch

Coherent motion requires an organized behaviour of all particles of the bunch

**Coherent beam-beam force**

- Beam distributions $\Psi_1$ **and** $\Psi_2$ mutually changed by interaction
- Interaction depends on distributions
    - Beam 1 $\Psi_1$ solution depends on beam 2 $\Psi_2$
    - Beam 2 $\Psi_2$ solution depends on beam 1 $\Psi_1$
- Need a self-consistent solution

# Coherent beam-beam effects



- Whole bunch sees a kick as an entity (coherent kick)
- Coherent kick seen by full bunch different from single particle kick
- Requires integration of individual kick over particle distribution

$$\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2} \cdot \left[1 - \mathrm{e}^{-\frac{r^2}{4\sigma^2}}\right]$$

- Coherent kick of separated beams can excite coherent dipolar oscillations
- All bunches couple because each bunch “sees” many opposing bunches(LR): many coherent modes possible!

# Coherent effects

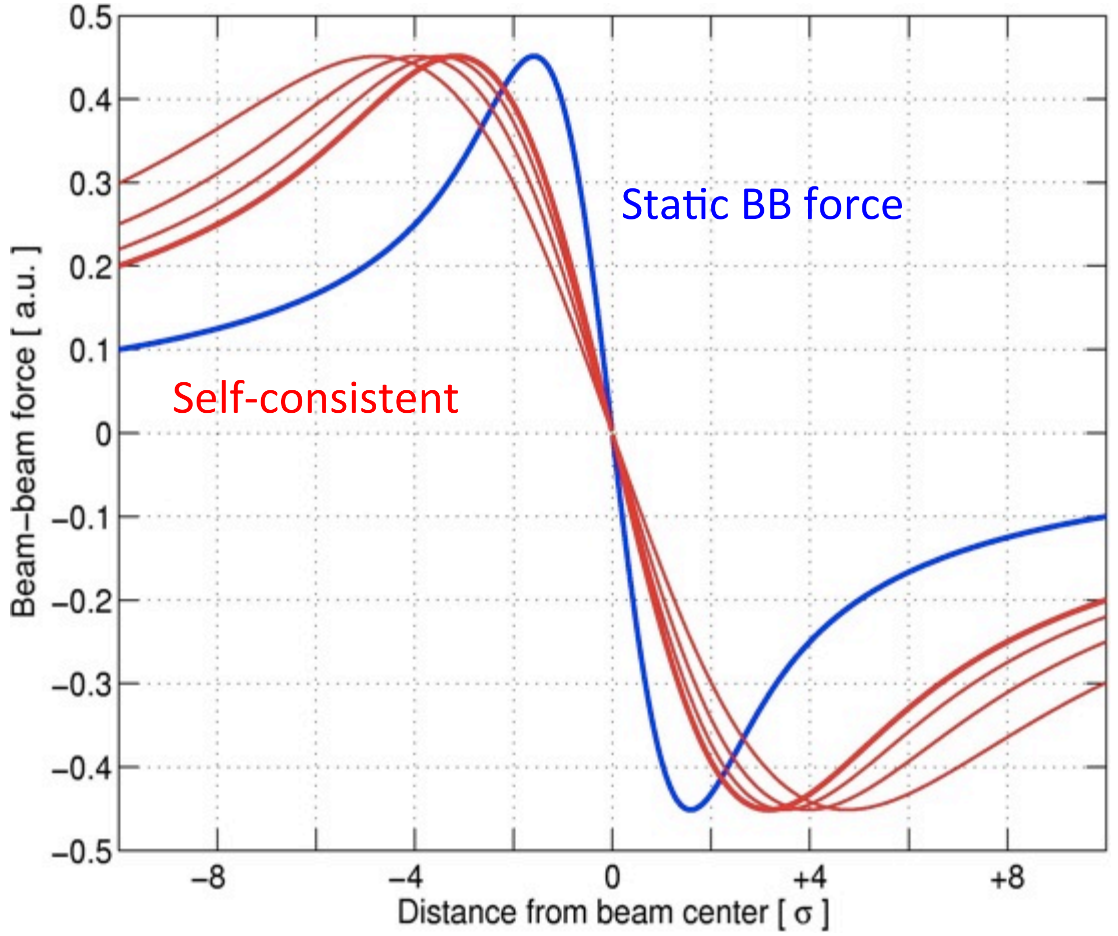
## Self-consistent treatment needed

**Perturbative methods**

static source of distortion:
example magnet

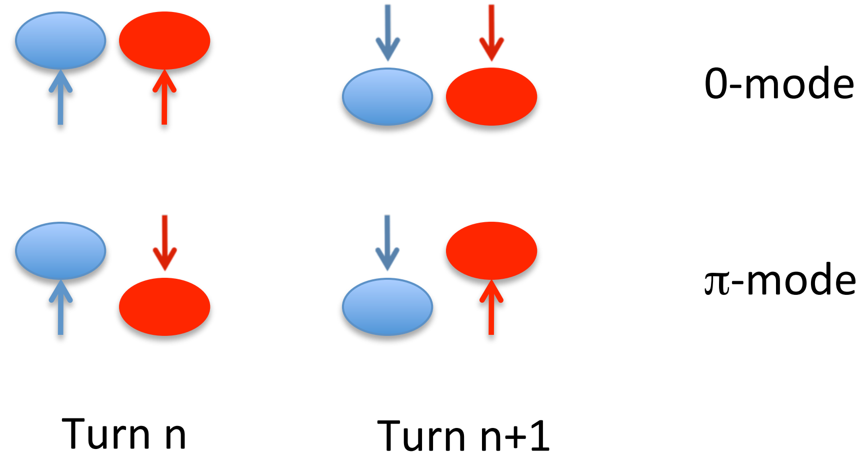
**Self-consistent method**

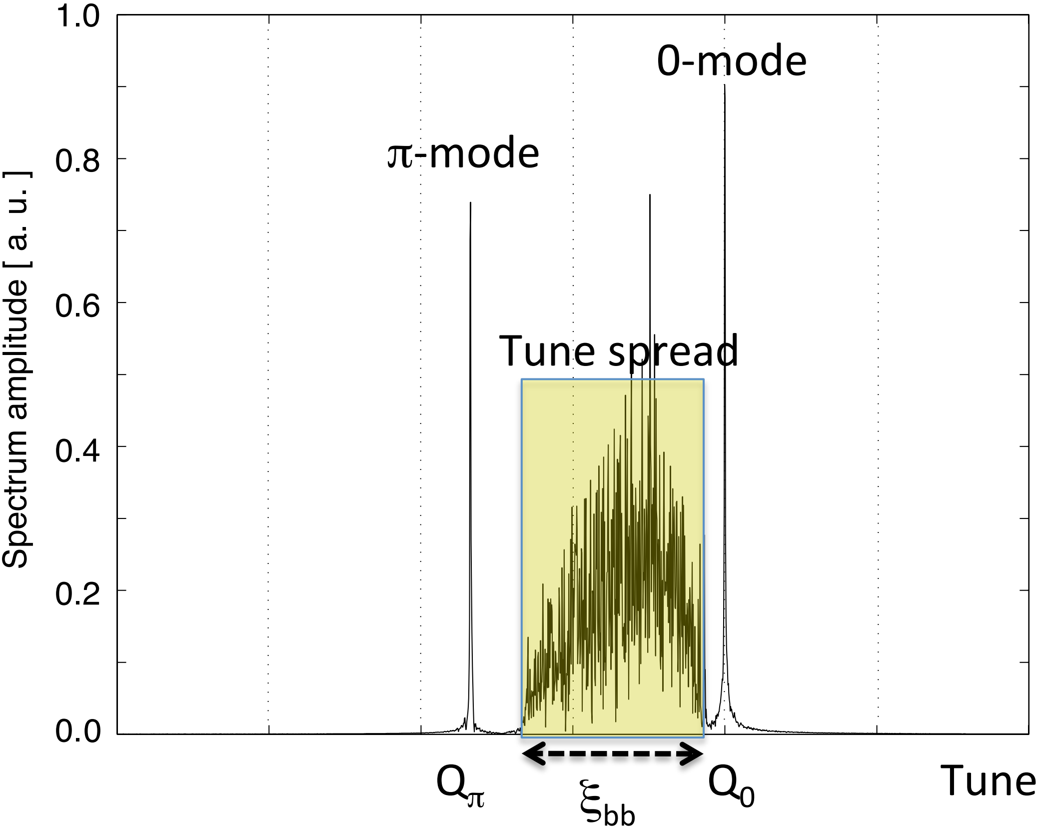
source of distortion changes
as a result of the distortion




**For a complete understanding of BB effect a self-consistent treatment should be used**

# Simple case: one bunch per beam




MOVIE

0-mode at unperturbed tune $Q_0$

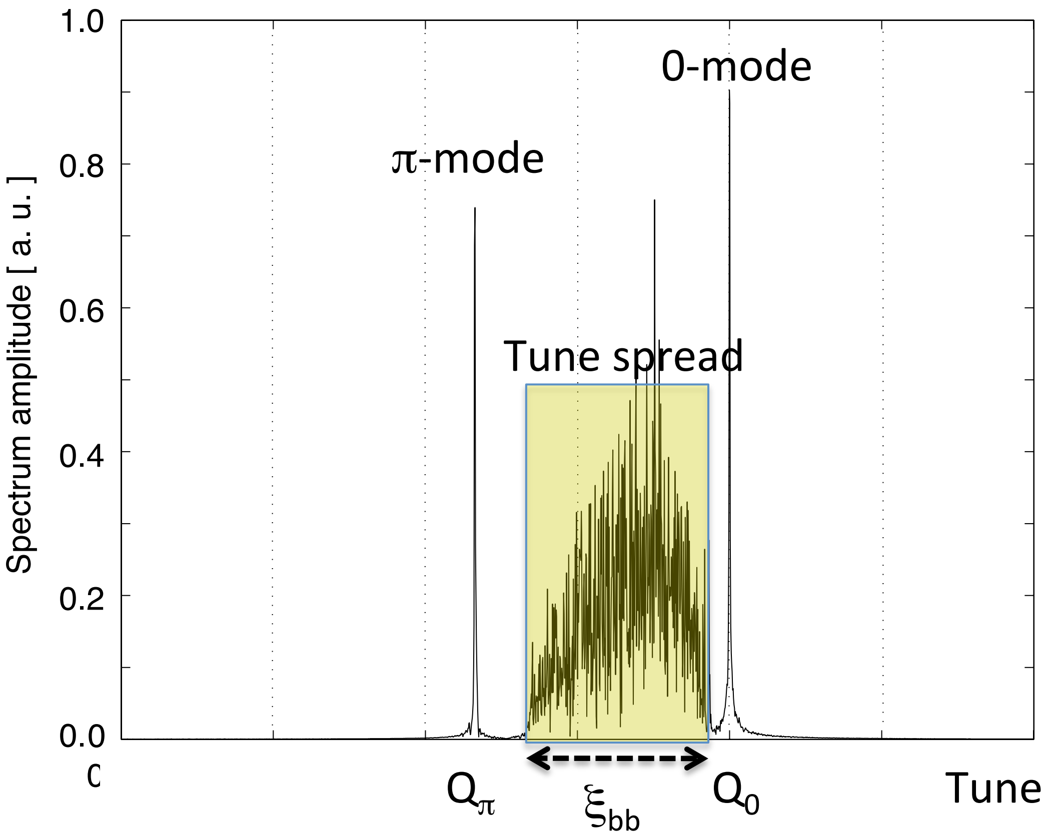
π-mode is shifted at $Q_\pi$ =1.1-1.3 $\xi_{bb}$

Incoherent tune spread range [0,-ξ]

$$\Delta Q = Y \cdot \xi$$

- Coherent mode: two bunches are "locked" in a coherent oscillation
- 0-mode is stable (mode with NO tune shift)
- π-mode can become unstable (mode with largest tune shift)

# Simple case: one bunch per beam and Landau damping

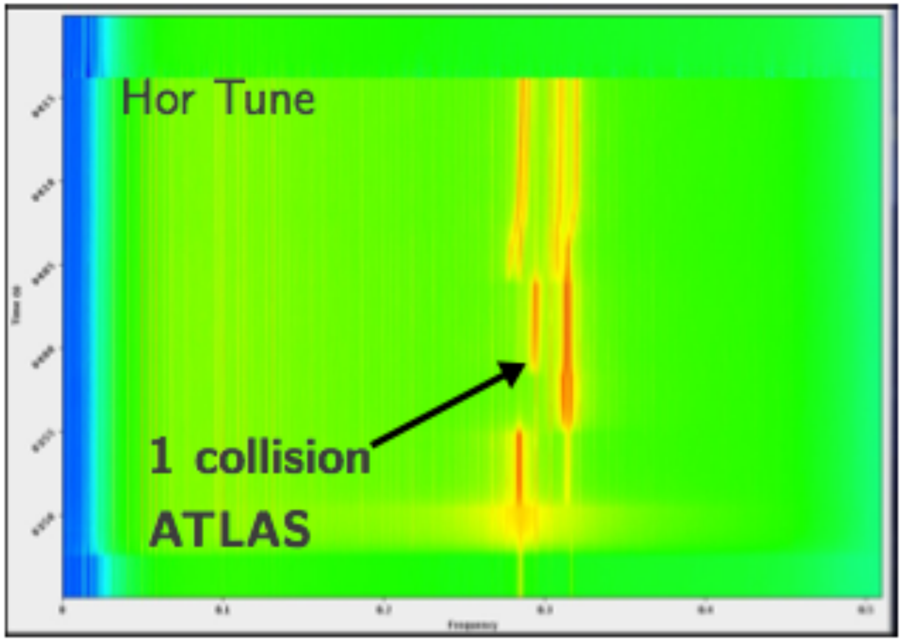


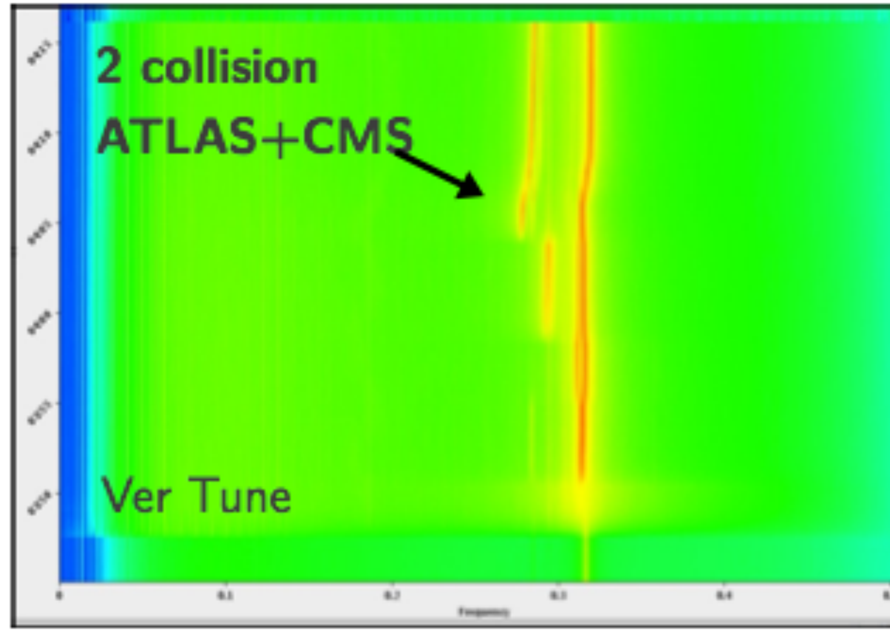

**Incoherent tune spread is the Landau damping region any mode with frequency laying in this range should not develop**

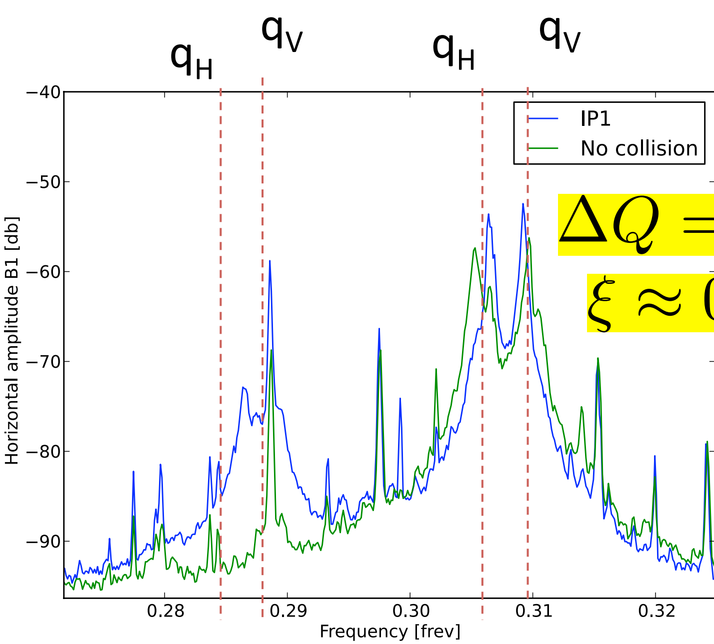
- π-mode has frequency out of tune spread (Y) so it is not damped!

# Head-on beam-beam coherent mode: LHC

## BBQ Signals



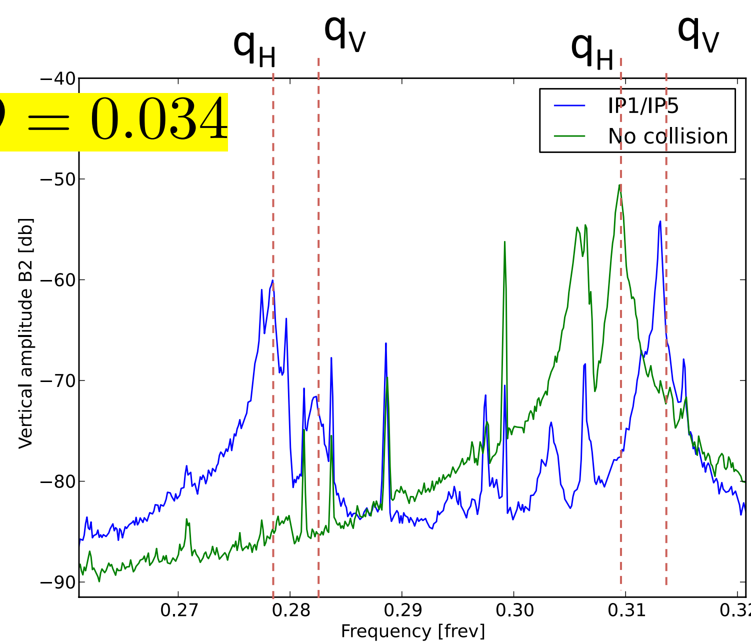



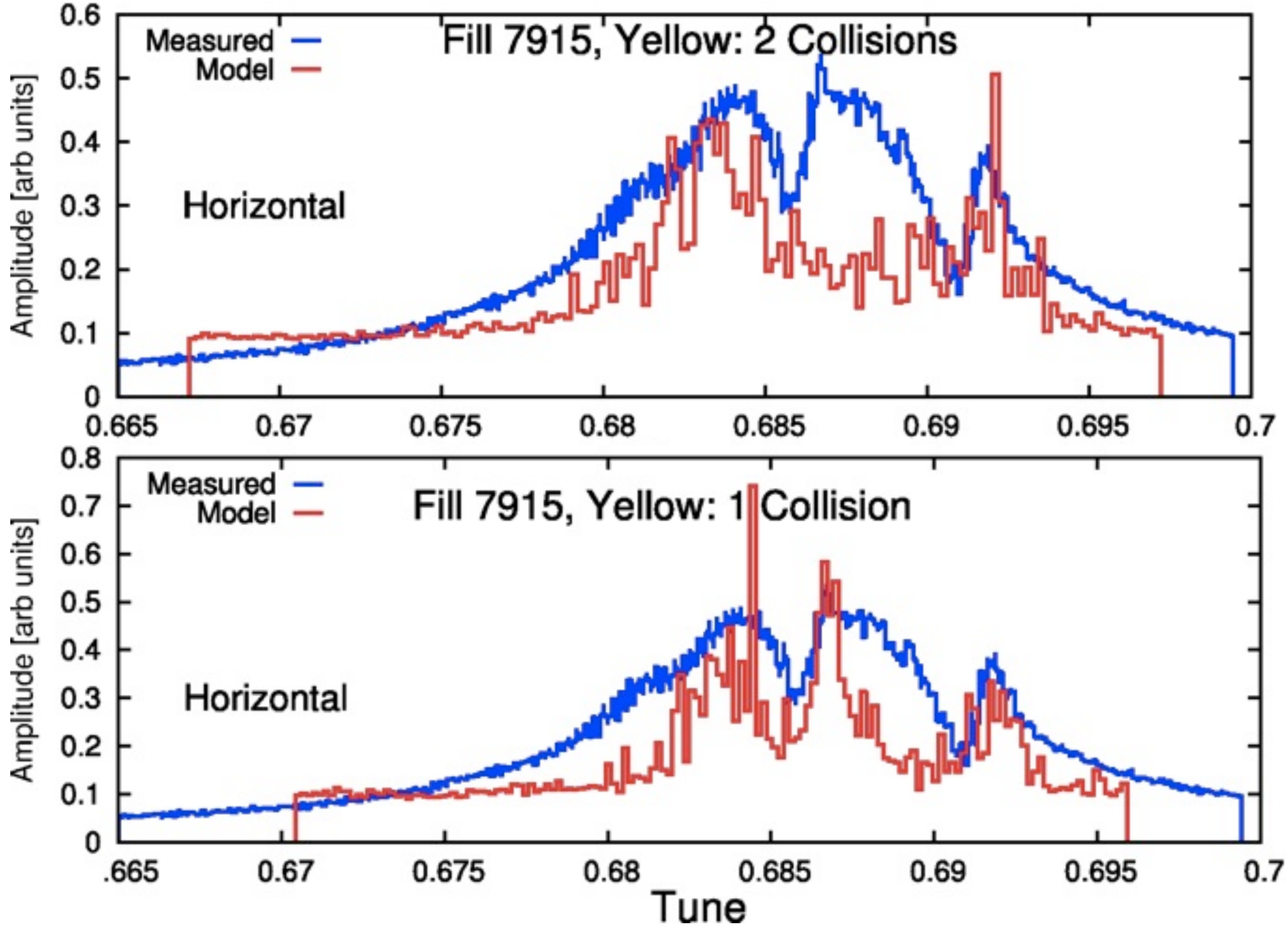

$\Delta Q = 0.019$

$\xi \approx 0.016$

$\Delta Q = 0.034$



# Beam-beam coherent modes and Landau Damping

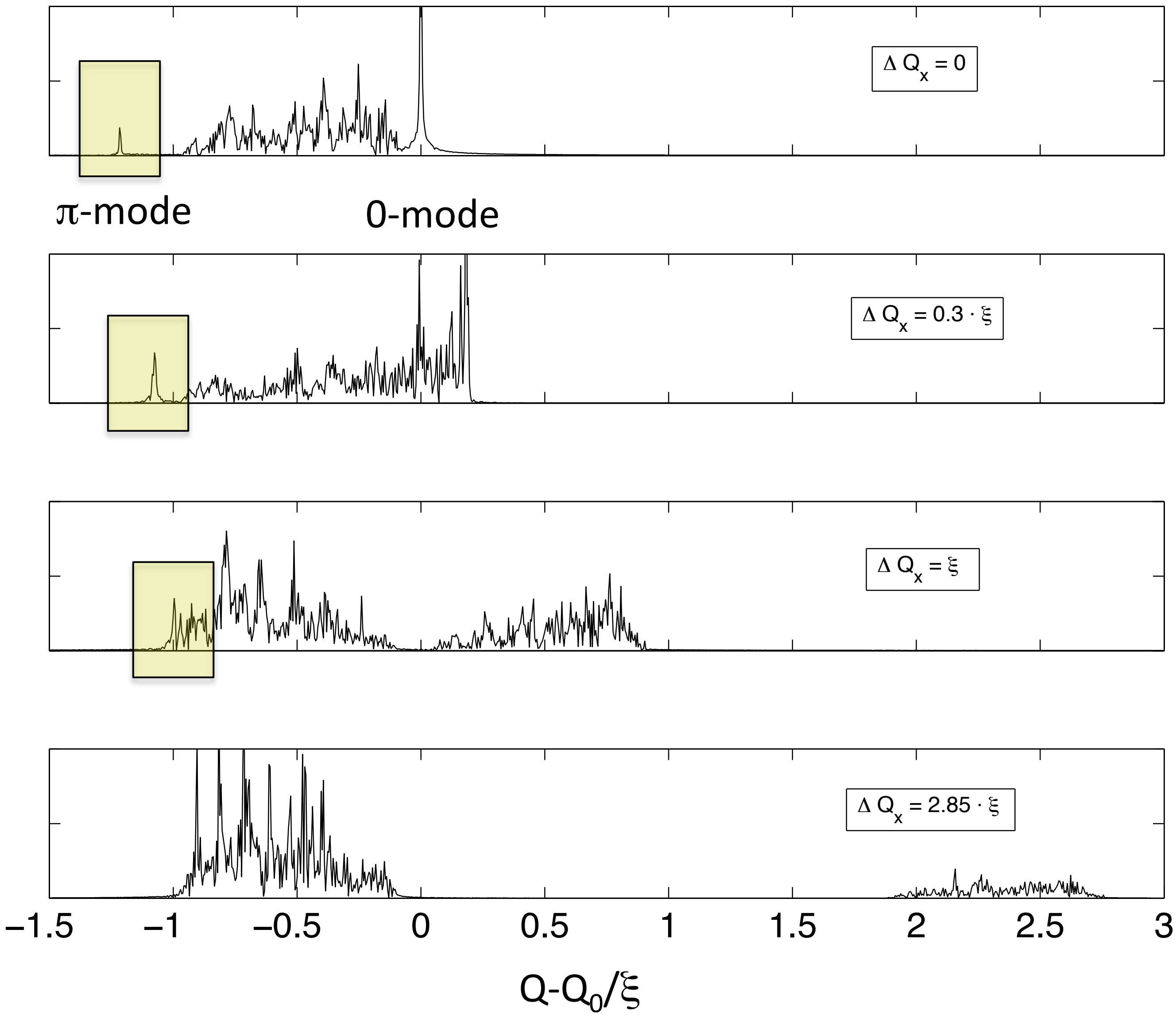



**If you have an exciting force (i.e. impedance) at this frequency: fast instabilities could occur**

**Pacman effect on coherent modes**

**Single bunch diagnostic so important**

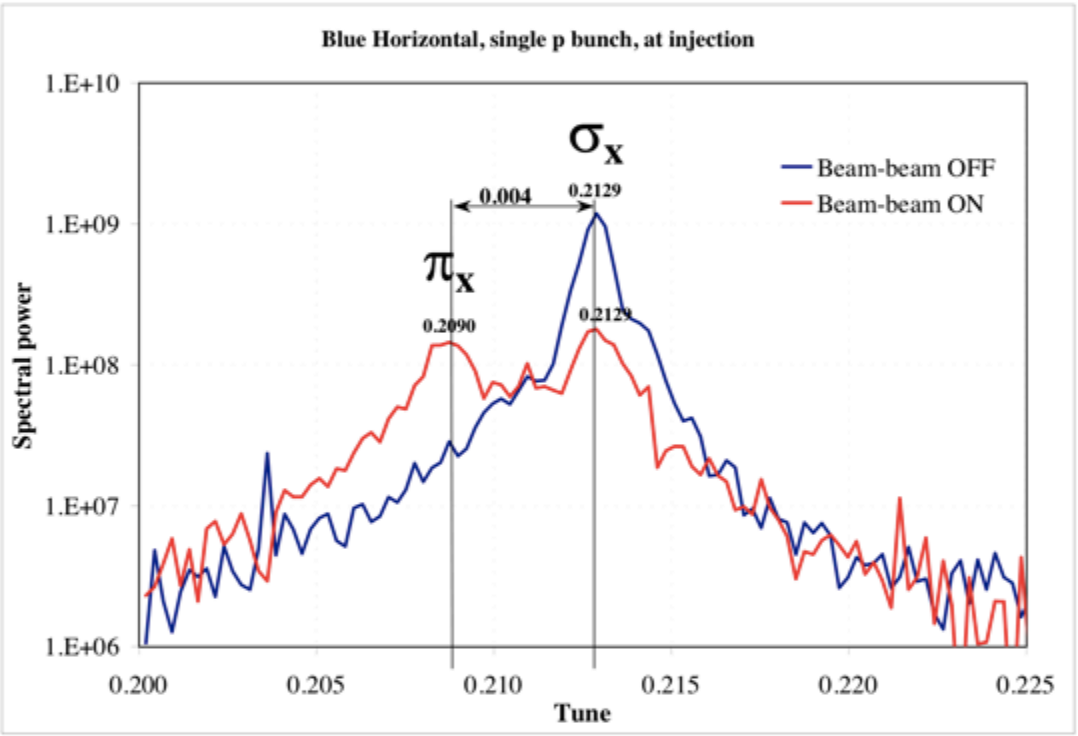
# Coherent modes and tune split




**Tune split breaks symmetry and coherent mode cannot develop**

**LHC has used a TUNE-SPLIT in 2010 physics run!**

# Coherent modes at RHIC

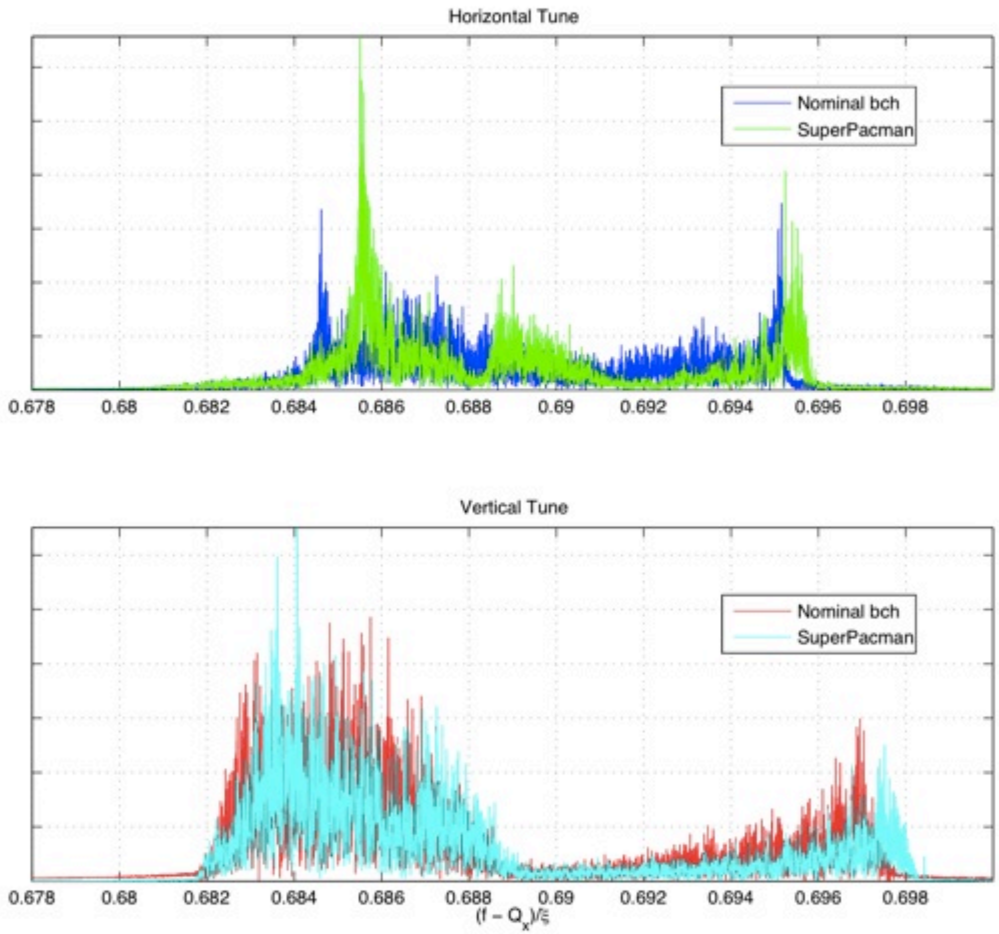


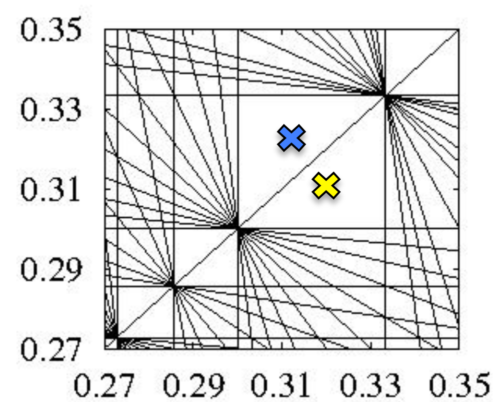

Courtesy W. Fischer (BNL)

**Tune spectra before collision and in collision two modes visible**

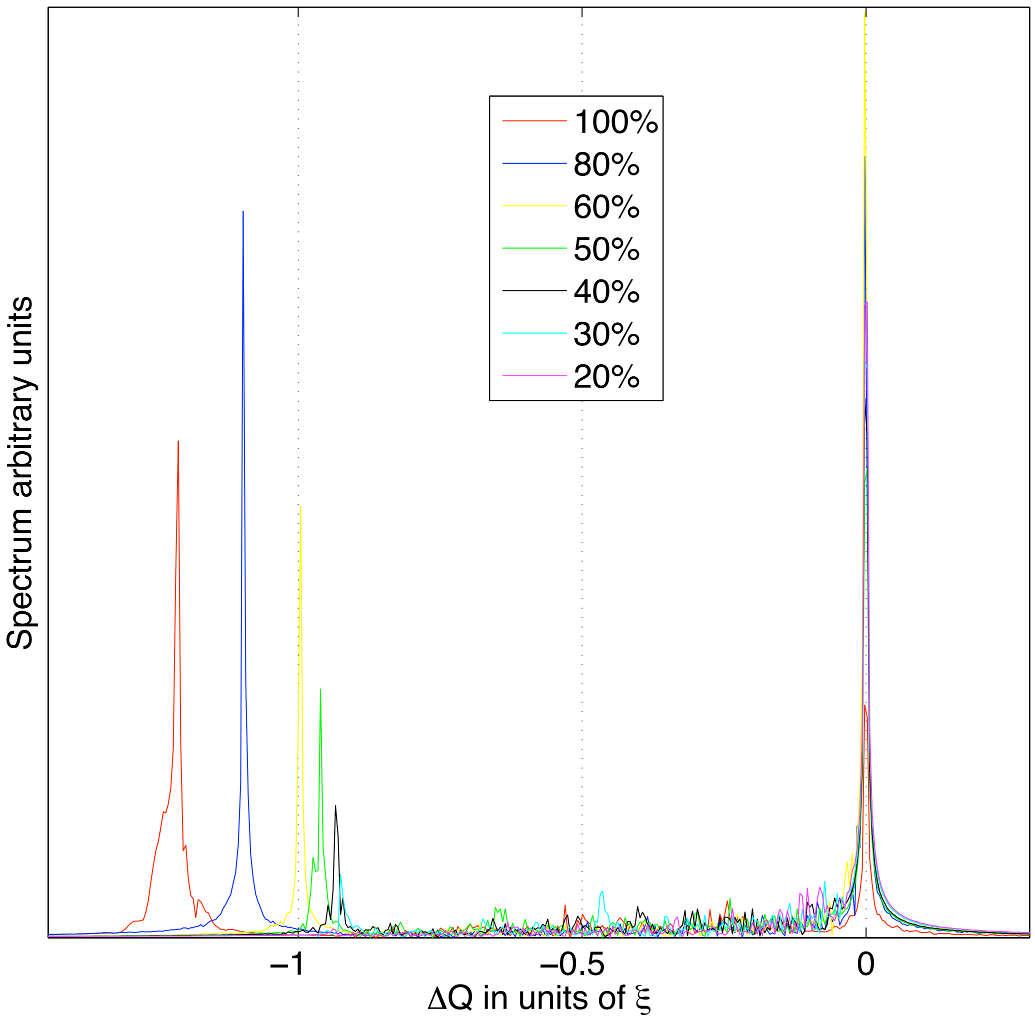
# Different tunes or intensities

**RHIC running with “mirrored” tune for years to break coherent oscillations**



# Different bunch intensities




For two bunches colliding head-on in one IP the coherent mode disappears if intensity ratio between bunches is 55%

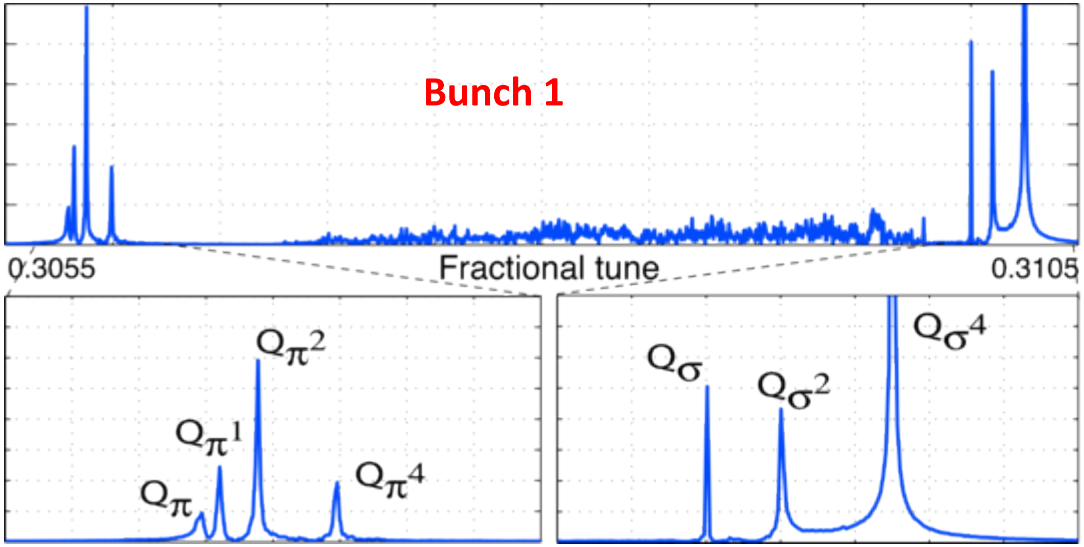
We assumed:

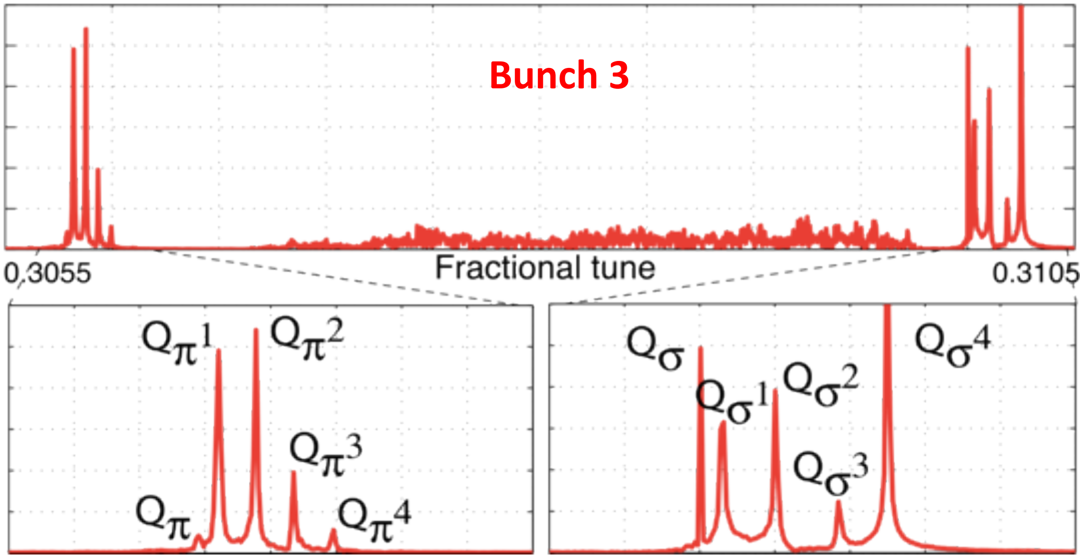
- equal emittances
- equal tunes
- NO PACMAN effects

(bunches will have different tunes)

**For coherent modes the key is to break the simmetry in your coupled system…(tunes, intensities, collision patters…)**

# And Long range interactions?

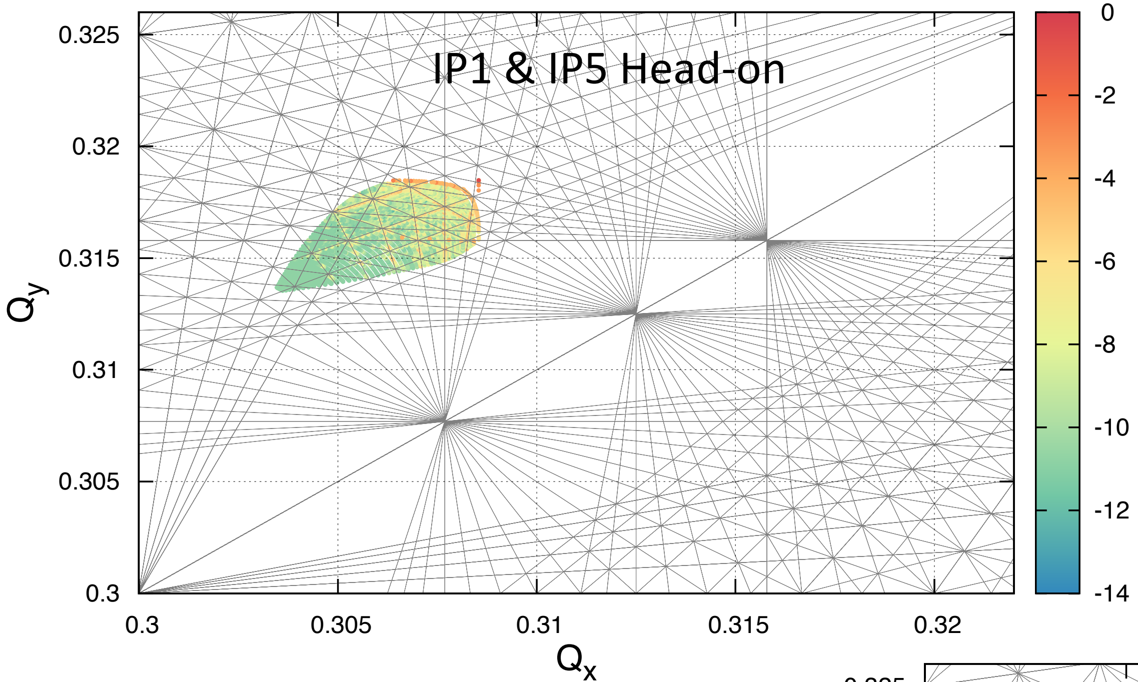


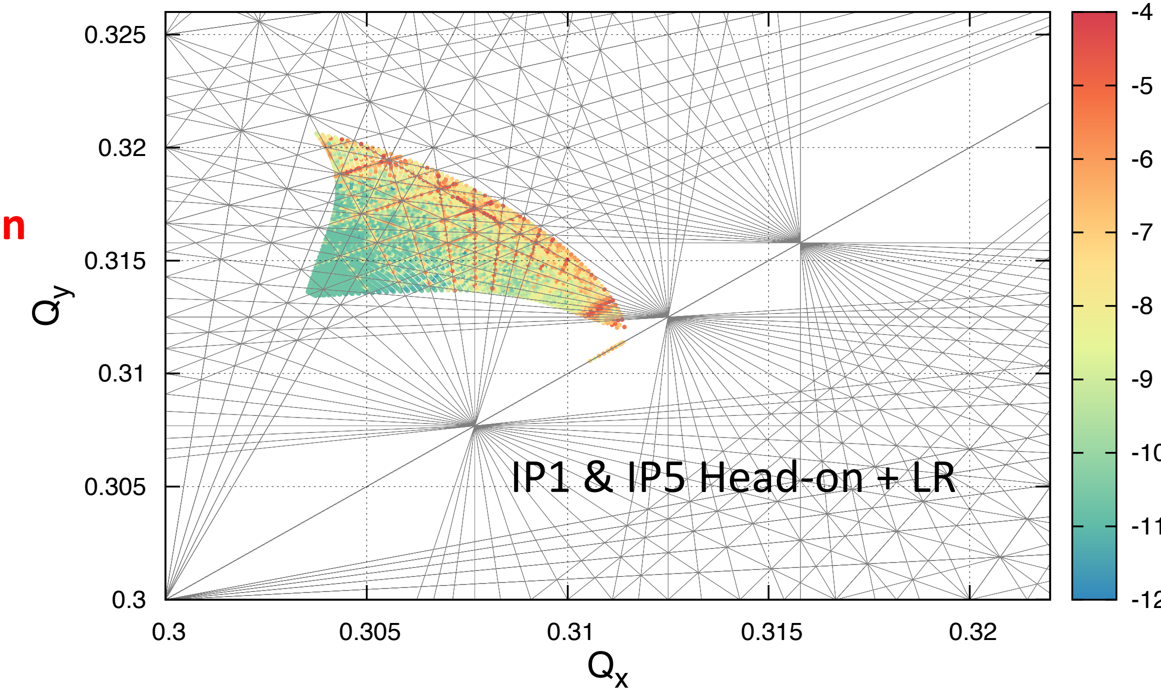

- Each bunch will have different number of modes and tune spectra
- No Landau damping of long-range coherent modes

**Single bunch diagnostic can make the difference**

# Beam-Beam compensation ideas




15 cm round Optics HL-LHC
590 μrad crossing angle

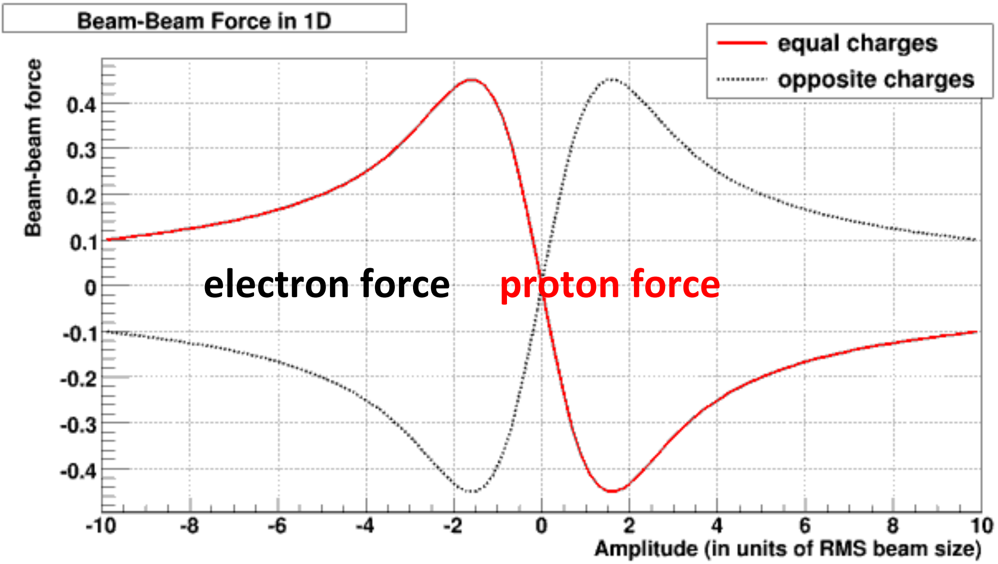


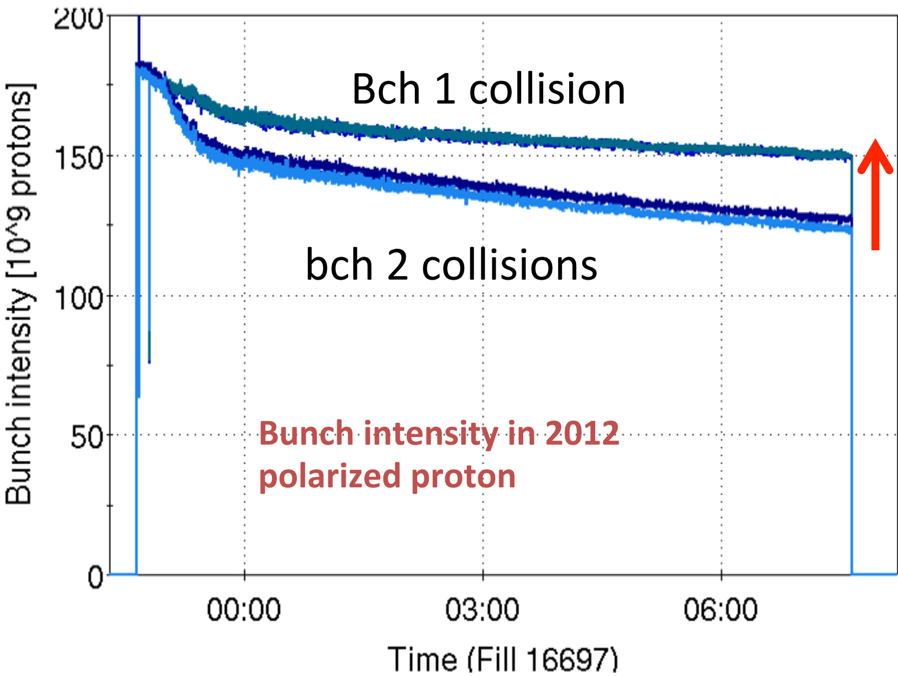

**Head-on interaction shift down in tune diagram!**
**Long range interactions open wings!**

# Beam-beam compensations:

## Head-on

- Linear e-lens, suppress head-on shift
- Non-linear e-lens, suppress tune spread



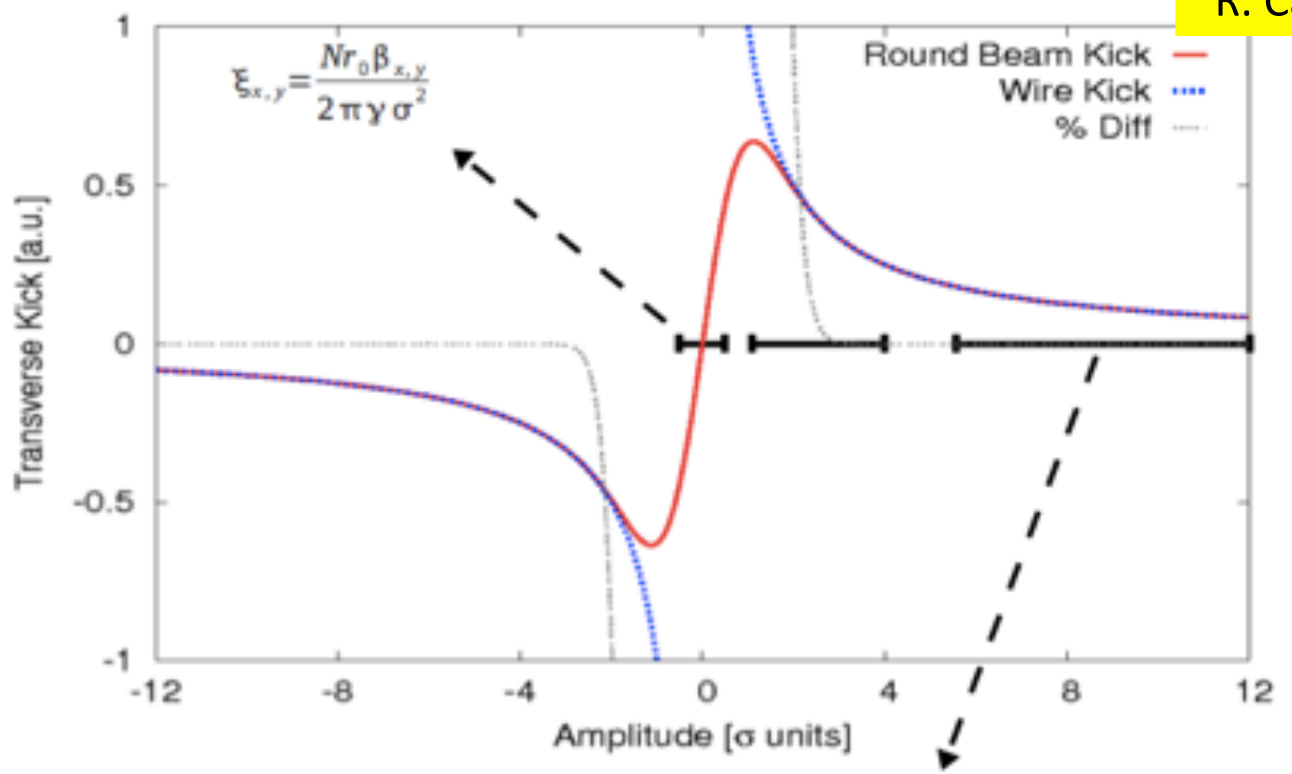




- Past experience: at Tevatron linear and non-linear e-lenses, also hollow…
- Present: test for half compensation at RHIC with non-linear e-lens

# Beam-beam compensations: long-range

Beam-beam wire compensation

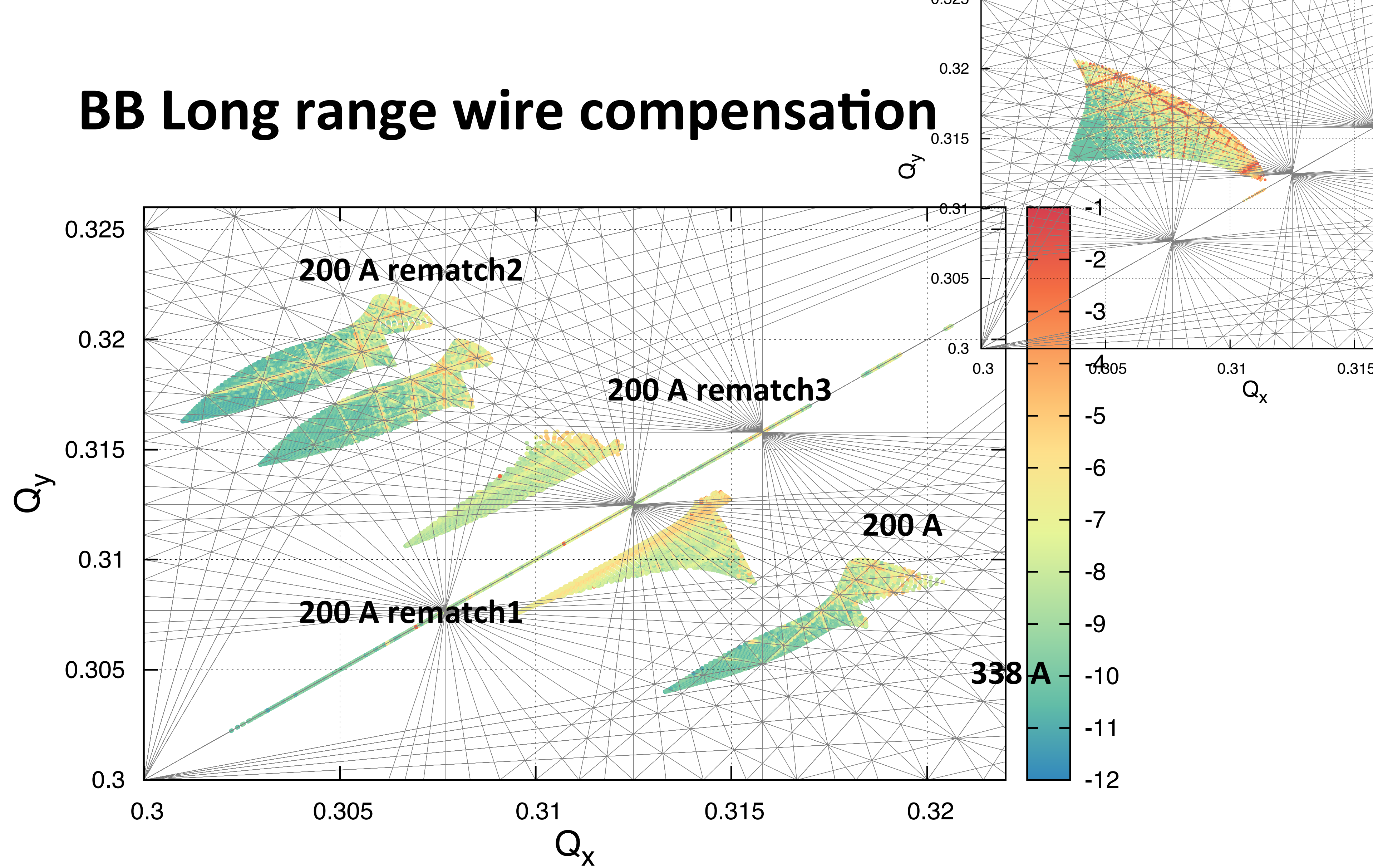
R. Calaga




σ << d: $\Delta x'(x,d)=-\frac{K}{d}.(1+\frac{x}{d}+\frac{x^2}{d^2}+...)$

- Past experience: at RHIC several tests till 2009…
- Present: simulation studies on-going for possible use in HL-LHC…

# BB Long range wire compensation




**Many studies from F. Zimmerman and first proposal J. P. Koutchouk**

**Studies on-going for LHC Upgrade scenarios where crossing angle reduction is fundamental!**

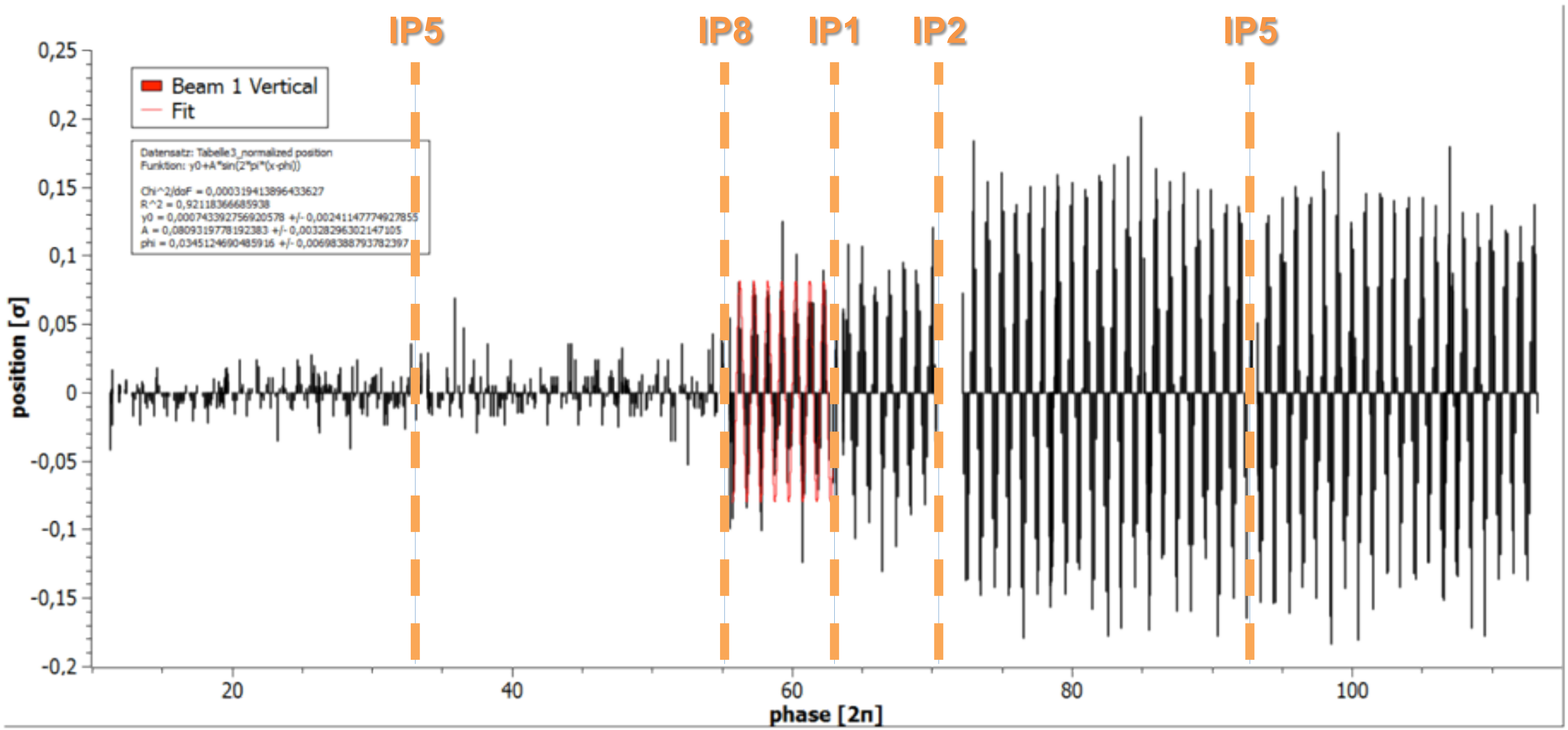
**Crossing angle impact LHC nominal 15% HL-LHC 70-80%!**

# …not covered here…

- *Linear colliders special issues*
- *Asymmetric beams effects*
- *Coasting beams*
- *Beamstrahlung*
- *Synchrobetatron coupling*
- *Beam-beam experiments*
- *Beam-beam and other collective effects (impedance)*
- *…*

Thank You!

# Long range orbit effect observations:




**Courtesy T. Baer**

**Vertical oscillation starts when one beam is ejected and dumped**

# References:


*[1] http://cern.ch/Werner.Herr/CAS2009/proceedings/bb_proc.pdf*

*[2] V. Shiltsev et al, "Beam beam effects in the Tevatron", Phys. Rev. ST Accel. Beams 8, 101001 (2005)*

*[3] Lyn Evans "The beam-beam interaction", CERN 84-15 (1984)*

*[4] Alex Chao "Lie Algebra Techniques for Nonlinear Dynamics" SLAC-PUB-9574 (2002)*

*[5] J. D. Jackson, "Classical Electrodynamics", John Wiley & Sons, NY, 1962.*

*[6] H. Grote, F. Schmidt, L. H. A. Leunissen,"LHC Dynamic Aperture at Collision", LHC-Project-Note 197, (1999).*

*[7] W. Herr,"Features and implications of different LHC crossing schemes", LHC-Project-Note 628, (2003).*

*[8] A. Hofmann,"Beam-beam modes for two beams with unequal tunes", CERN-SL-99-039 (AP) (1999) p. 56.*

*[9] Y. Alexahin, "On the Landau damping and decoherence of transverse dipole oscillations in colliding beams ", Part. Acc. 59, 43 (1996).*


…much more on the LHC Beam-beam webpage:

*http://lhc-beam-beam.web.cern.ch/lhc-beam-beam/*