# Formulation of effective theories for dark matter direct detection

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Based on [J. Hisano, K. Ishiwata, N. N., 1004. 4090, 1007. 2601, and 1210. 5985]
[J. Hisano, K. Ishiwata, N. N., M. Yamanaka, 1012. 5455]
and [J. Hisano, K. Ishiwata, N. N., T. Takesako, 1104. 0228]

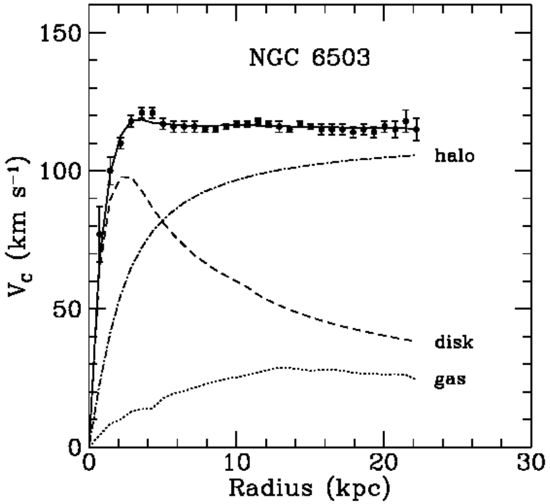
# Outline

1. Introduction
2. The method of Effective theory
3. Some results
   a) Pure bino DM
   b) Wino/higgsino DM (high-scale SUSY scenario)
   c) KK photon DM in the MUED model
4. Summary

# 1. Introduction

# Introduction | Evidence for dark matter (DM)

## Galactic scale

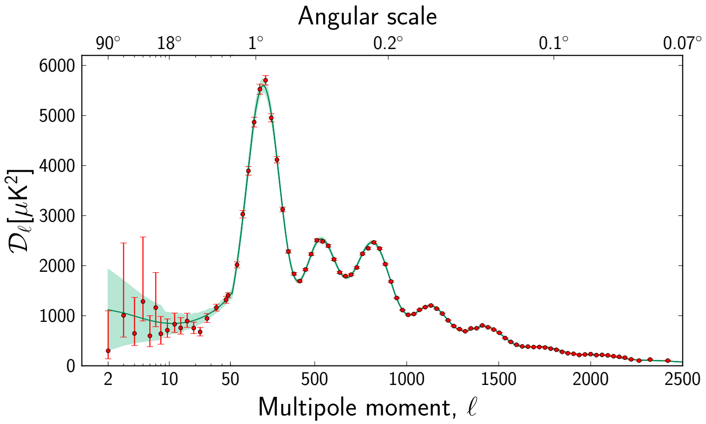



Begeman et. al. (1991)

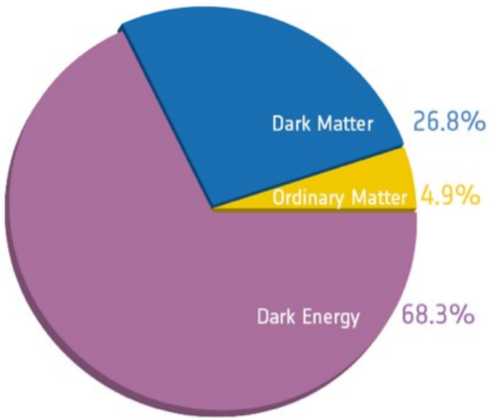
## Scale of galaxy clusters

Clowe et. al. (2006)

## Cosmological scale




Planck (2013)



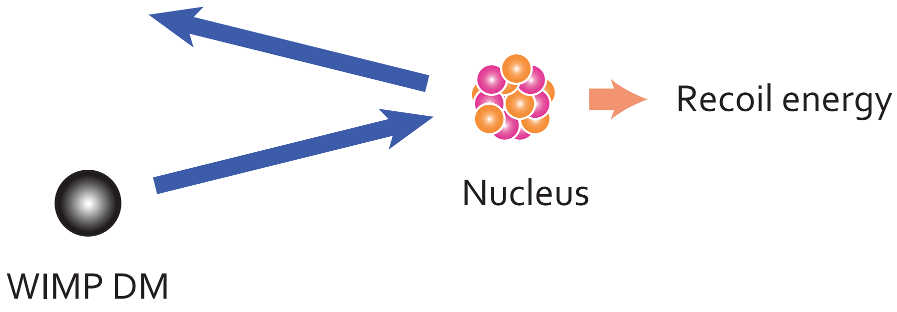

Planck (2013)

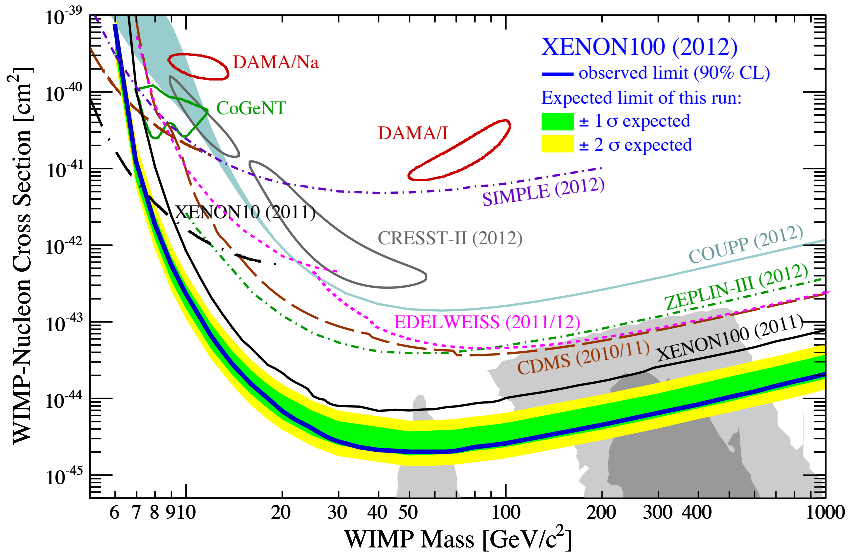
One of the most promising candidates for dark matter is

## Weakly Interacting Massive Particles (WIMPs)

- have masses roughly between 10 GeV ~ a few TeV.
- interact only through weak and gravitational interactions.
- Their thermal relic abundance is naturally consistent with the cosmological observations [thermal relic scenario].
- appear in models beyond the Standard Model.

# Introduction | Direct detection experiments




[XENON100 collaboration, arXiv: 1207. 5988]

- XENON 100 collaboration gives a stringent constraint on spin-independent WIMP-nucleon scattering cross section.

$$\sigma_{\rm SI} < 2.0 \times 10^{-45}\ {\rm cm}^2$$ (for WIMPs of mass 55 GeV)

- Ton-scale detectors for direct detection experiments are expected to yield significantly improved sensitivities.

# Motivation

To study the nature of dark matter based on direct detection experiments, the precise calculation of

**the WIMP-nucleon scattering cross section**

is required.

■ **Previous works**

- For Majorana DM
  e.g.) M. Drees and M. Nojiri, Phys. Rev. D **48** (1993) 3483.
- For vector DM
  H. C. P. Cheng, J. L. Feng and K. T. Matchev, Phys. Rev. Lett. **89**, 211301 (2002).
  G. Servant and T. M. P. Tait, New J. Phys. **4**, 99 (2002).

> - In these works, some of the leading contributions (especially those of gluon) to the scattering cross sections are not properly taken into account.
> - We study the way of evaluating the cross section systematically by using the method of effective field theory

## 2. The method of effective theory

# Method of effective theories

1. By integrating out heavy particles, we obtain the effective interactions of WIMP DM with quarks and gluons.

Operator Product Expansion (OPE)

$$\mathcal{L}_{\text{eff}} = \sum_i C_i(\mu)\mathcal{O}_i(\mu)$$

$C_i$ (μ) : Wilson coefficients

include short-distant effects

$O_i$ (μ) : Effective operators

Higer-dimensional operators. Their nucleon matrix elements contain the effects of long-distance.

μ : factorization scale (μ∼$m_Z$)

A scale at which a high-energy theory is matched with the effective theory.

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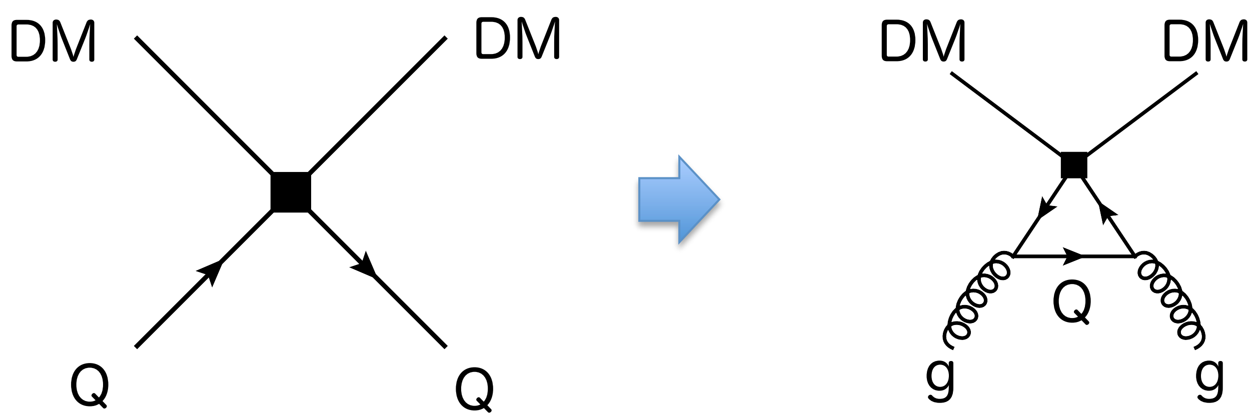
# Method of effective theories

2. Evaluate the nucleon matrix elements of the effective operators (at a certain scale).

When evolving the operators down to the scale, we need to match the effective theories above/below each quark threshold.




3. By using the nucleon matrix elements, we evaluate the scattering cross section of DM with a nucleon

# Effective Lagrangian for Majorana DM

$$\mathcal{L}_q = d_q \bar{\tilde{\chi}}^0 \gamma^\mu \gamma_5 \tilde{\chi}^0 \bar{q} \gamma_\mu \gamma_5 q$$ ← Spin-dependent (SD)

$$+ f_q m_q \bar{\tilde{\chi}}^0 \tilde{\chi}^0 \bar{q} q + \frac{g_q^{(1)}}{M} \bar{\tilde{\chi}}^0 i\partial^\mu \gamma^\nu \tilde{\chi}^0 \mathcal{O}^q_{\mu\nu} + \frac{g_q^{(2)}}{M^2} \bar{\tilde{\chi}}^0 i\partial^\mu i\partial^\nu \tilde{\chi}^0 \mathcal{O}^q_{\mu\nu}$$

$$\mathcal{L}_G = f_G \bar{\tilde{\chi}}^0 \tilde{\chi}^0 G^a_{\mu\nu} G^{a\mu\nu} + \frac{g_G^{(1)}}{M} \bar{\tilde{\chi}}^0 i\partial^\mu \gamma^\nu \tilde{\chi}^0 \mathcal{O}^g_{\mu\nu} + \frac{g_G^{(2)}}{M^2} \bar{\tilde{\chi}}^0 i\partial^\mu i\partial^\nu \tilde{\chi}^0 \mathcal{O}^g_{\mu\nu}$$

Spin-independent (SI)

$\tilde{\chi}^0$ : DM    $m_q$ : quark mass    $M$ : DM mass

## Majorana condition

$$\bar{\tilde{\chi}}^0 \gamma^\mu \tilde{\chi}^0 = 0$$

$$\bar{\tilde{\chi}}^0 \sigma^{\mu\nu} \tilde{\chi}^0 = 0$$

## Twist-2 operator

$$\mathcal{O}^q_{\mu\nu} \equiv \frac{1}{2} \bar{q} i \left( D_\mu \gamma_\nu + D_\nu \gamma_\mu - \frac{1}{2} g_{\mu\nu} \not{D} \right) q$$

$$\mathcal{O}^g_{\mu\nu} \equiv G^{a\rho}_\mu G^a_{\rho\nu} + \frac{1}{4} g_{\mu\nu} G^a_{\alpha\beta} G^{a\alpha\beta}$$

# Effective Lagrangian for vector DM

$$\mathcal{L}_q = \frac{d_q}{M}\epsilon_{\mu\nu\rho\sigma}B^\mu i\partial^\nu B^\rho \bar{q}\gamma^\sigma\gamma_5 q$$ ← Spin-dependent (SD)

$$+f_q m_q B^\mu B_\mu \bar{q}q \; + \frac{g_q}{M^2}B^\rho i\partial^\mu i\partial^\nu B_\rho \mathcal{O}^q_{\mu\nu}$$

$$\mathcal{L}_G = f_G B^\mu B_\mu G^a_{\mu\nu}G^{a\mu\nu} \; + \frac{g_G}{M^2}B^\rho i\partial^\mu i\partial^\nu B_\rho \mathcal{O}^g_{\mu\nu}$$

Spin-independent (SI)

$B^\mu$ : DM  $m_q$ : quark mass  $M$ : DM mass

Some conditions

$$(\Box + M^2)B^\mu = 0$$

$$\partial_\mu B^\mu = 0$$

$$B^0 \to 0 \quad \text{(N.R.)}$$

Twist-2 operator

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## Nucleon matrix elements | Quark (scalar-type)

Nucleon matrix elements of scalar-type quark operators are evaluated by using the QCD lattice simulations.

mass fractions

$$\langle N|m_q\bar{q}q|N\rangle/m_N \equiv f_{T_q}$$

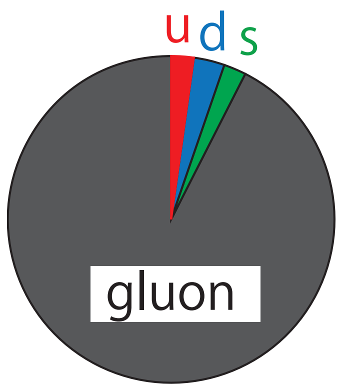
($m_N$ : Nucleon mass)

| For proton | |
|---|---|
| $f_{Tu}$ | 0.023 |
| $f_{Td}$ | 0.034 |
| $f_{Ts}$ | 0.025 |
| For neutron | |
| $f_{Tu}$ | 0.019 |
| $f_{Td}$ | 0.041 |
| $f_{Ts}$ | 0.025 |

H. Ohki et al. (2008)

Gluon contribution

$$1-\sum_{q=u,d,s} f_{Tq} \equiv f_{TG}$$




Mass fractions for proton

Remarks.

Strange quark content is much smaller than those evaluated with the chiral perturbation theory.

# Nucleon matrix elements | Gluon (scalar-type)

Nucleon matrix element of scalar-type gluon operator is evaluated by using the trace anomaly of the energy-momentum tensor.

■ Trace anomaly of the energy-momentum tensor in QCD （$N_f = 3$）

$$\Theta^\mu_\mu = \frac{\beta(\alpha_s)}{4\alpha_s} G^a_{\mu\nu} G^{a\mu\nu} + \sum_{q=u,d,s} m_q \bar{q} q$$

$$m_N \qquad \left( \beta(\alpha_s) = -\frac{9\alpha_s^2}{2\pi} \quad (\text{for} \quad N_F = 3) \right) \qquad \left( \sum_{q=u,d,s} m_N f_{Tq} \right)$$

$$m_N f_{TG} = -\frac{9\alpha_s}{8\pi} \langle N | G^a_{\mu\nu} G^{a\mu\nu} | N \rangle$$

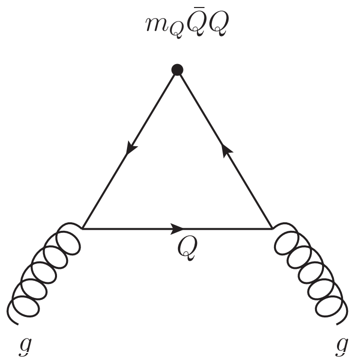
M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Phys. Lett. B **78** (1978) 443.

Heavy quark contribution

↓

Gluon contribution

$$m_Q \bar{Q} Q \to -\frac{\alpha_s}{12\pi} G^a_{\mu\nu} G^{a\mu\nu}$$



# Nucleon matrix elements | Twist-2 operators

Nucleon matrix elements of twist-2 operators are evaluated by using the parton distribution functions (PDFs).

$$\langle N(p)|\mathcal{O}^q_{\mu\nu}|N(p)\rangle = \frac{1}{m_N}(p_\mu p_\nu - \frac{1}{4}m_N^2\eta_{\mu\nu})(q(2)+\bar{q}(2))$$
$$\langle N(p)|\mathcal{O}^g_{\mu\nu}|N(p)\rangle = \frac{1}{m_N}(p_\mu p_\nu - \frac{1}{4}m_N^2\eta_{\mu\nu})\ G(2)$$

Here, q(2) and G(2) are called the second moments of PDFs, which are defined by

$$q(2)+\bar{q}(2) = \int_0^1 dx\ x\ [q(x)+\bar{q}(x)]$$
$$G(2) = \int_0^1 dx\ x\ g(x)$$

| Second moment at $\mu = m_Z$ | | | |
|---|---|---|---|
| $G(2)$ | 0.48 | | |
| $u(2)$ | 0.22 | $\bar{u}(2)$ | 0.034 |
| $d(2)$ | 0.11 | $\bar{d}(2)$ | 0.036 |
| $s(2)$ | 0.026 | $\bar{s}(2)$ | 0.026 |
| $c(2)$ | 0.019 | $\bar{c}(2)$ | 0.019 |
| $b(2)$ | 0.012 | $\bar{b}(2)$ | 0.012 |

J. Pumplin et al., JHEP 0207:012

# Effective coupling of Majorana DM with nucleon

The SI coupling of Majorana DM with nucleon is given as

$$\mathcal{L}_{eff} = f_N \bar{\tilde{\chi}} \tilde{\chi} \bar{N} N$$

$$\begin{aligned} f_N/m_N &= \sum_{q=u,d,s} f_q f_{Tq} + \sum_{q=u,d,s,c,b} \frac{3}{4} \left( q(2) + \bar{q}(2) \right) \left( g_q^{(1)} + g_q^{(2)} \right) \\ &- \frac{8\pi}{9\alpha_s} f_{TG} f_G + \frac{3}{4} G(2) \left( g_G^{(1)} + g_G^{(2)} \right) . \end{aligned}$$

The gluon contribution turns out to be comparable to the quark contributions even if it is induced by higher loop diagrams.

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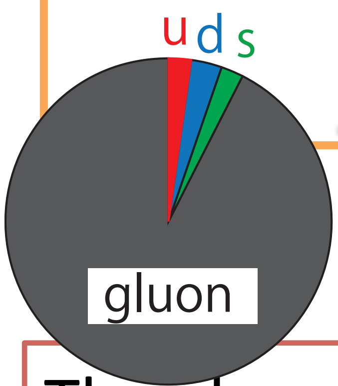


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suppressed by $\alpha_s$

The gluon contribution turns out to be comparable to the quark contributions even if it is induced by higher loop diagrams.

## SI elastic scattering cross section

By using the effective coupling, we eventually compute scattering cross sections of the DM with a nucleus.

$$\sigma_{\mathrm{SI}} = \frac{4}{\pi}\left(\frac{Mm_T}{M+m_T}\right)^2 |n_p f_p + n_n f_n|^2$$

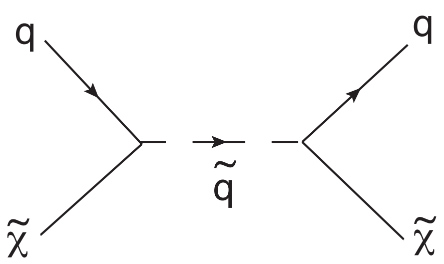
$m_T$ : the mass of the target nucleus
$n_p$ : the number of proton
$n_n$ : the number of neutron

In the following discussion, we show the SI cross sections of DM with a proton, as a reference value.

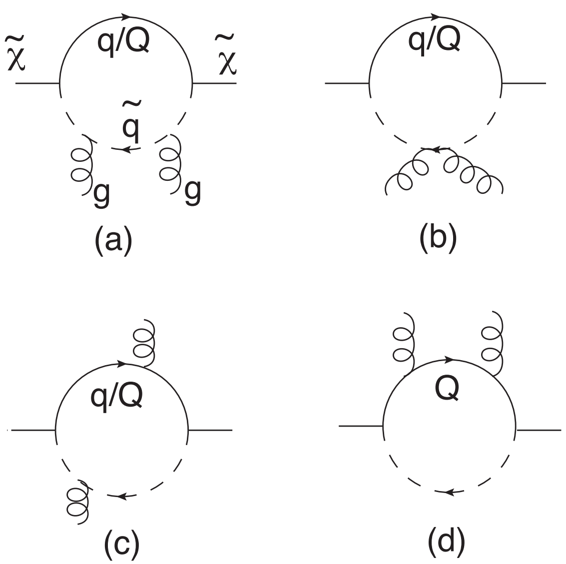
# 3. Some results

## Pure Bino DM

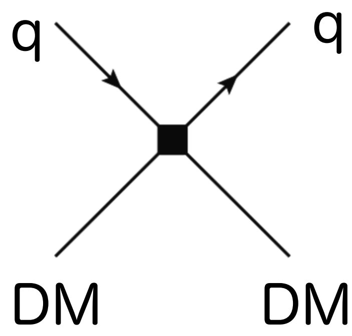
The tree-level diagram:




1-loop diagrams:




Matching

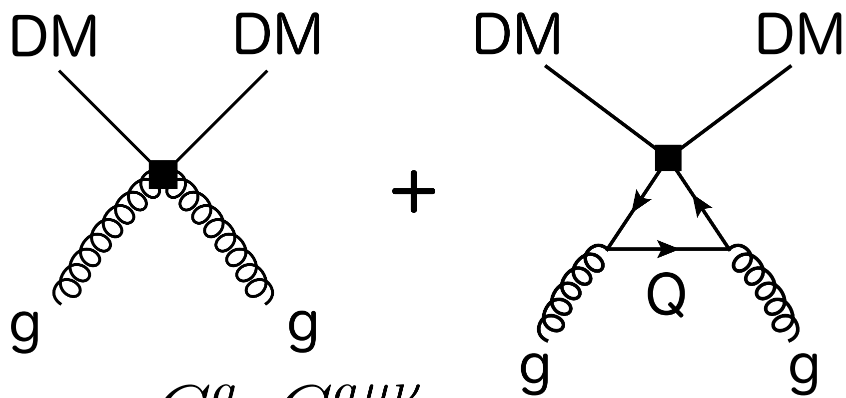



Scalar

$\propto \bar{q}q$

twist-2

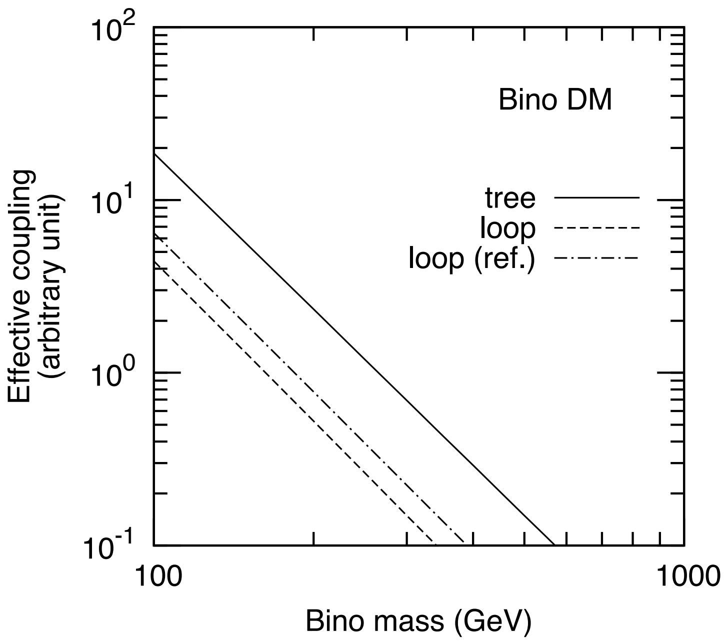
$\propto \mathcal{O}^q_{\mu\nu}$



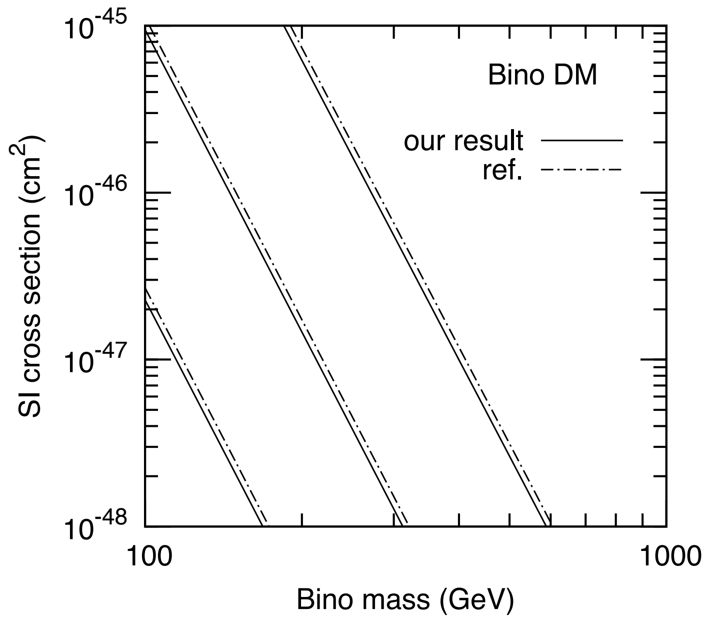

$\propto G^a_{\mu\nu} G^{a\mu\nu}$

Only the short-distance contribution should be included into the Wilson coefficients.

## Pure Bino DM




Each contribution to effective coupling $f_p$




Bino DM-proton SI cross sections

ref.) M. Drees and M. Nojiri, Phys. Rev. **D48** (1993) 3483.

We found O(10)% alternations in the SI cross sections

Due to a lack of matching in the previous calculation…

J. Hisano, K. Ishiwata, and NN, Phys. Rev. **D82** (2010) 115007.

## High-scale SUSY

High-scale SUSY scenario has a lot of fascinating aspects from a phenomenological point of view.

- 126 GeV Higgs boson can be achieved

  (sufficient radiative corrections)

- SUSY CP/flavor problems are relaxed

  (suppressed by sfermion masses)

- Gravitino problem is avoided

  (heavy gravitino)

- Gauge coupling unification

  (sfermions form SU(5) multiplets)

This scenario also accommodates the existence of Dark Matter (DM) .

w/ light gauginos

(chiral symmetries)

## Mass spectrum

On the assumption of a generic Kahler potential and no singlet field in the SUSY breaking sector

Scalar Particles Gravitino Higgsinos

Higgsinos can be light

(additional symmetries)

Gauginos (Loop suppressed)

Gluino

Bino

Wino

Anomaly mediation

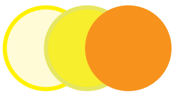
L. Randall and R. Sundrum (1998)

G.F. Giudice, M.A. Luty, H. Murayama, R. Rattazzi (1998)

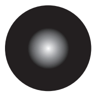
# Mass spectrum

On the assumption of a generic Kahler potential and no singlet field in the SUSY breaking sector

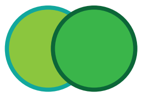
Scalar Particles



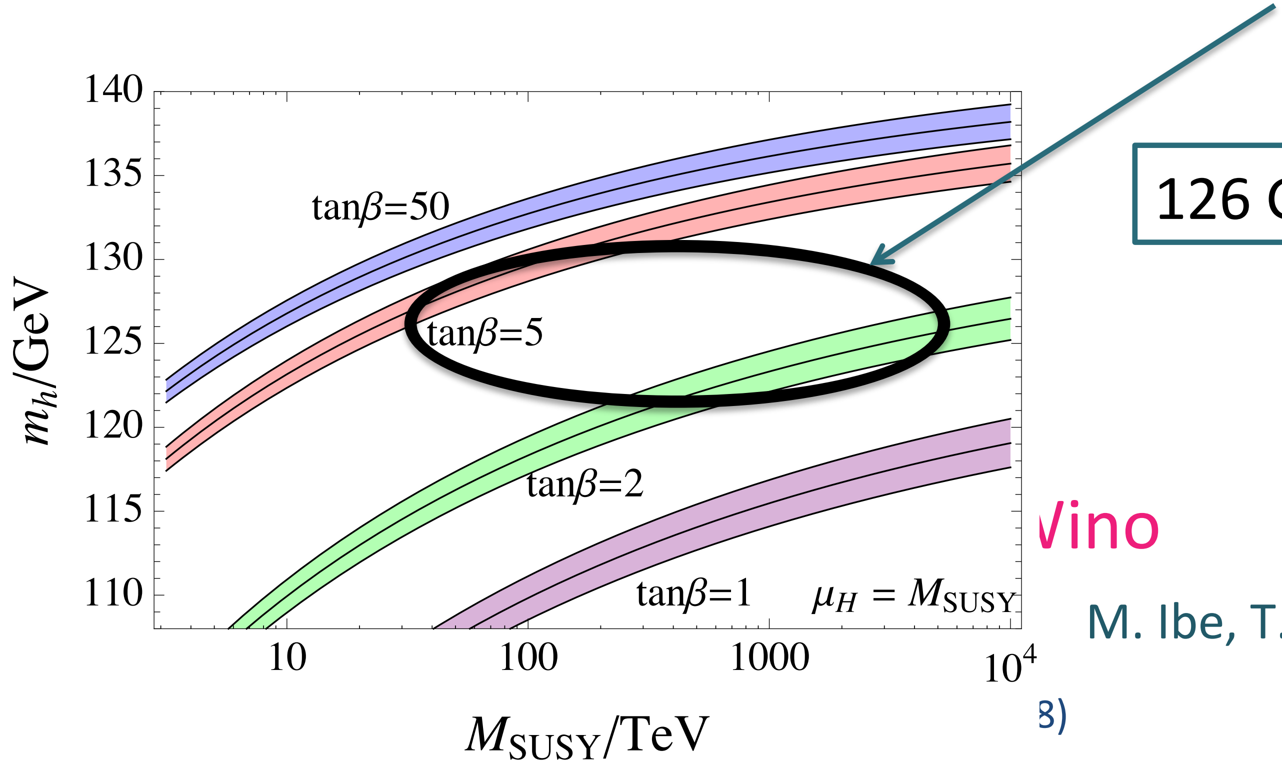
Gravitino



Higgsinos



$M_S = 10^{(2\text{-}3)}$ TeV




126 GeV Higgs boson

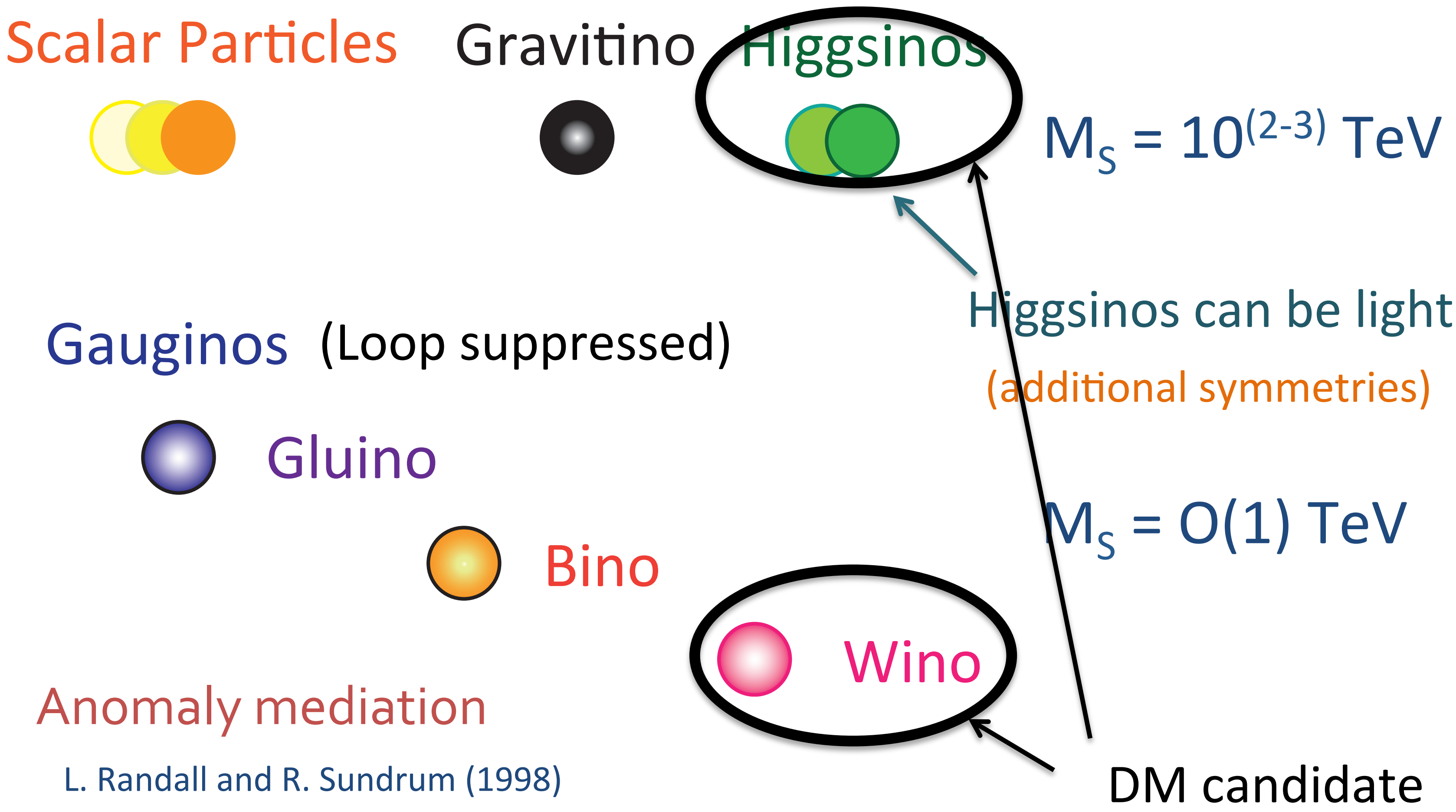
Vino

M. Ibe, T.T. Yanagida (2012).

3)

## Mass spectrum

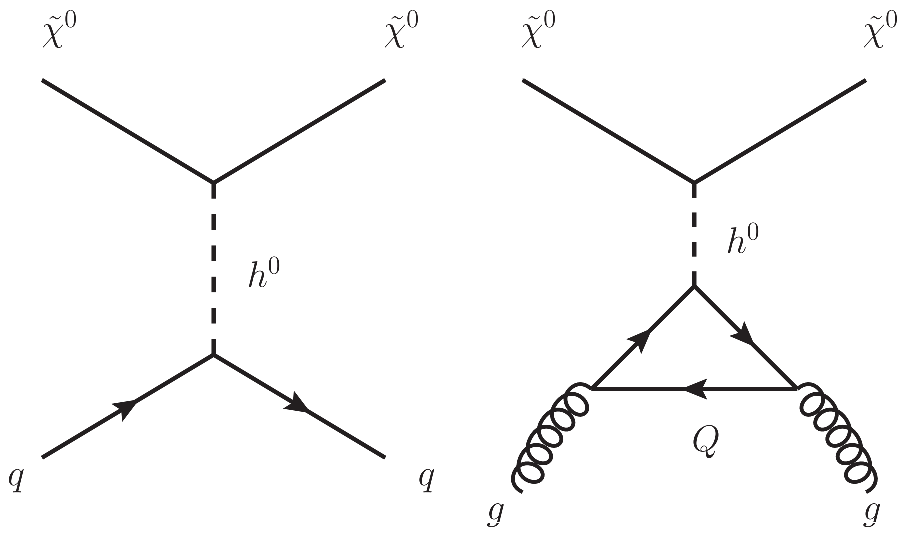
On the assumption of a generic Kahler potential and no singlet field in the SUSY breaking sector




L. Randall and R. Sundrum (1998)

G.F. Giudice, M.A. Luty, H. Murayama, R. Rattazzi (1998)

## Diagrams Tree-level




``Higgs" contribution

$$f_q^H \; \overline{\tilde{\chi}^0}\tilde{\chi}^0\bar{q}q$$

$$-\frac{\alpha_s}{12\pi} f_Q^H \; \overline{\tilde{\chi}^0}\tilde{\chi}^0 G^a_{\mu\nu}G^{a\mu\nu}$$

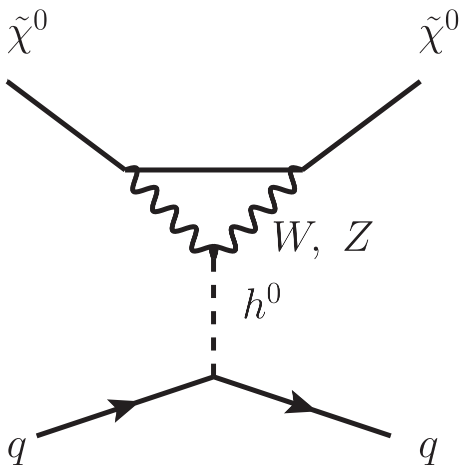
Effective coupling

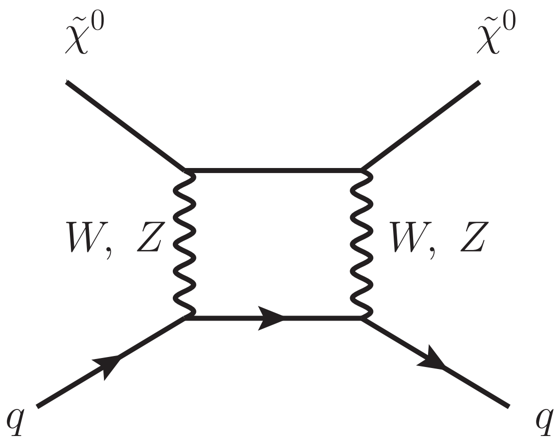
$$f_q^H = \frac{g_2^2}{4m_W m_h^2}(Z_{12} - Z_{11}\tan\theta_W)(Z_{13}\cos\beta - Z_{14}\sin\beta)$$

($Z_{ij}$: Neutralino mixing matrix)

$$f_q^H \simeq \frac{g_2^2(M_2 + \mu\sin 2\beta)}{4m_h^2(M_2^2 - \mu^2)} \qquad (|\mu \pm M_2| \gg m_Z)$$

## Diagrams 1-loop




``Scalar”

$\propto \bar{q}q$

``twist-2”

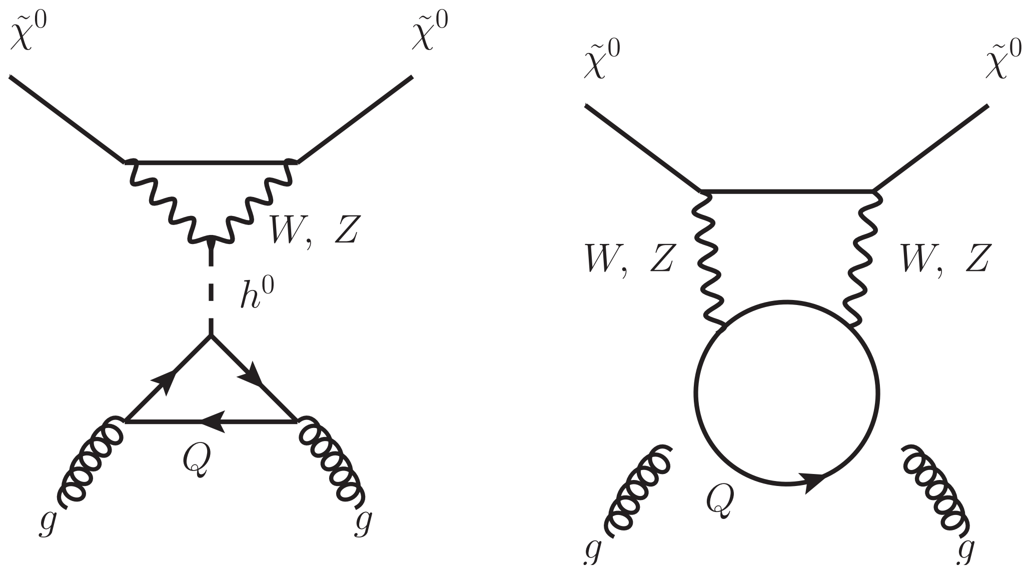
$\propto \mathcal{O}^q_{\mu\nu}$

These interactions are not suppressed even if the DM mass is much larger than the W/Z boson mass.

Non-decoupling effects


J. Hisano, S. Matsumoto, M. Nojiri, O. Saito, Phys. Rev. D **71** (2005) 015007.

## Diagrams 2-loop



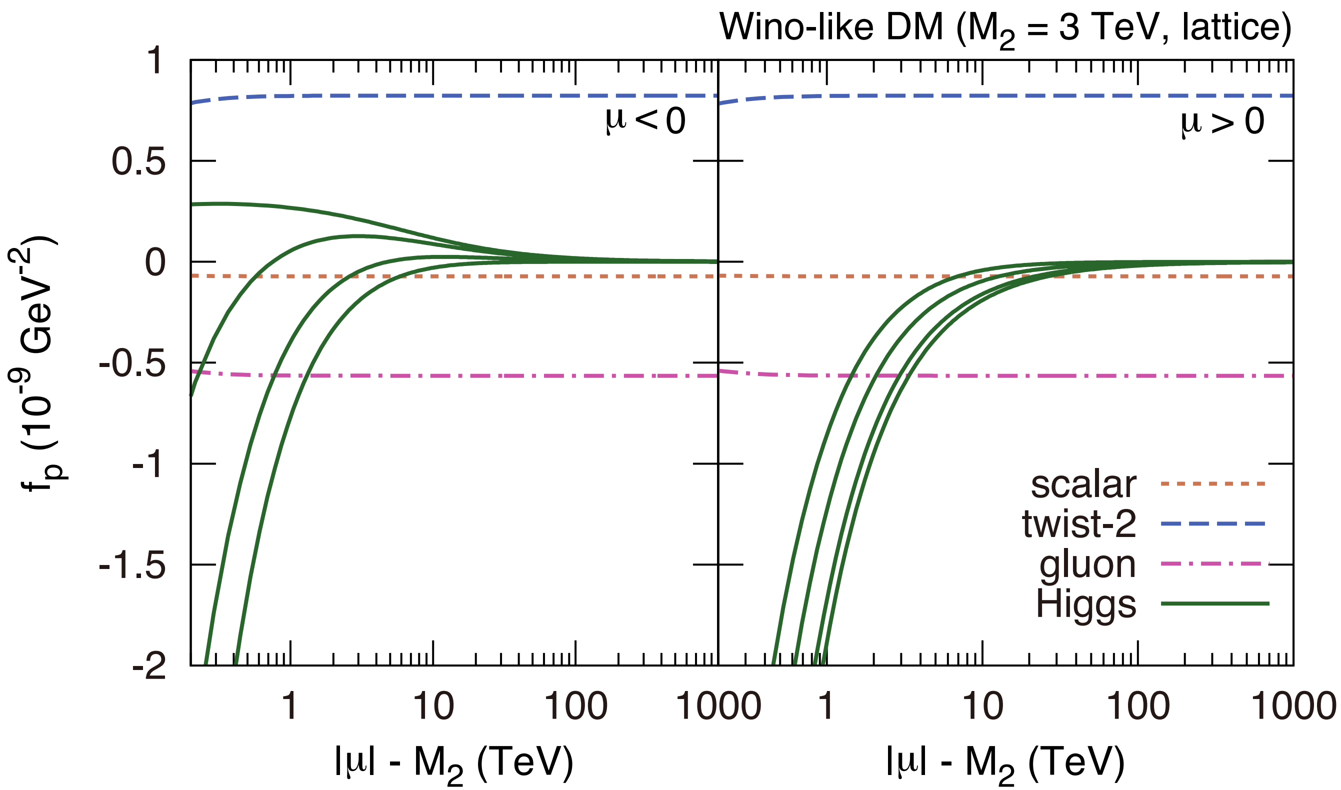

Gluon contribution

$$\propto G^{a}_{\mu\nu} G^{a\mu\nu}$$

- Neglected in previous calculations
- 2-loop gluon contribution can be comparable to 1-loop quark contribution
- non-decoupling

J. Hisano, K. Ishiwata, and NN, Phys. Lett. B **690** (2010) 311.

# Wino-like DM | Effective coupling with a proton




$\tan\beta$ = 1, 2, 5, 50

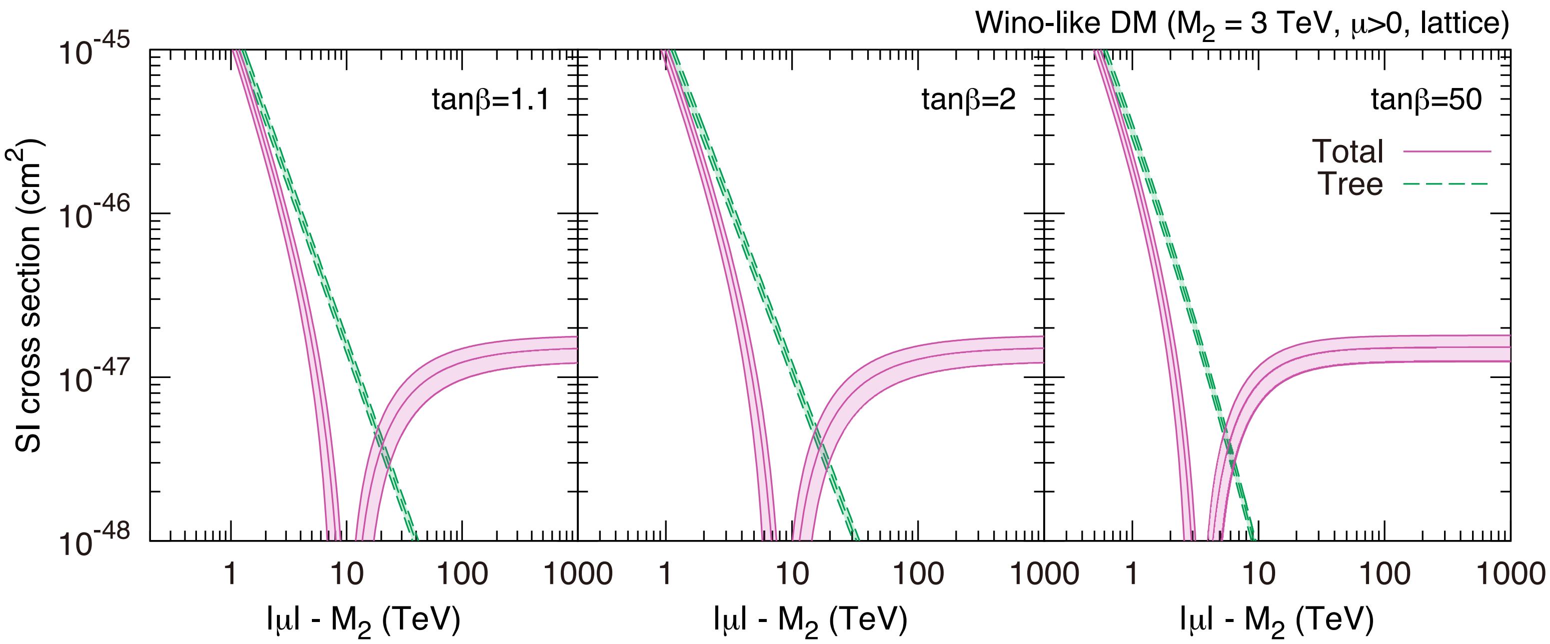
(from top to bottom)

$\tan\beta$ = 1, 2, 5, 50

(from bottom to top)

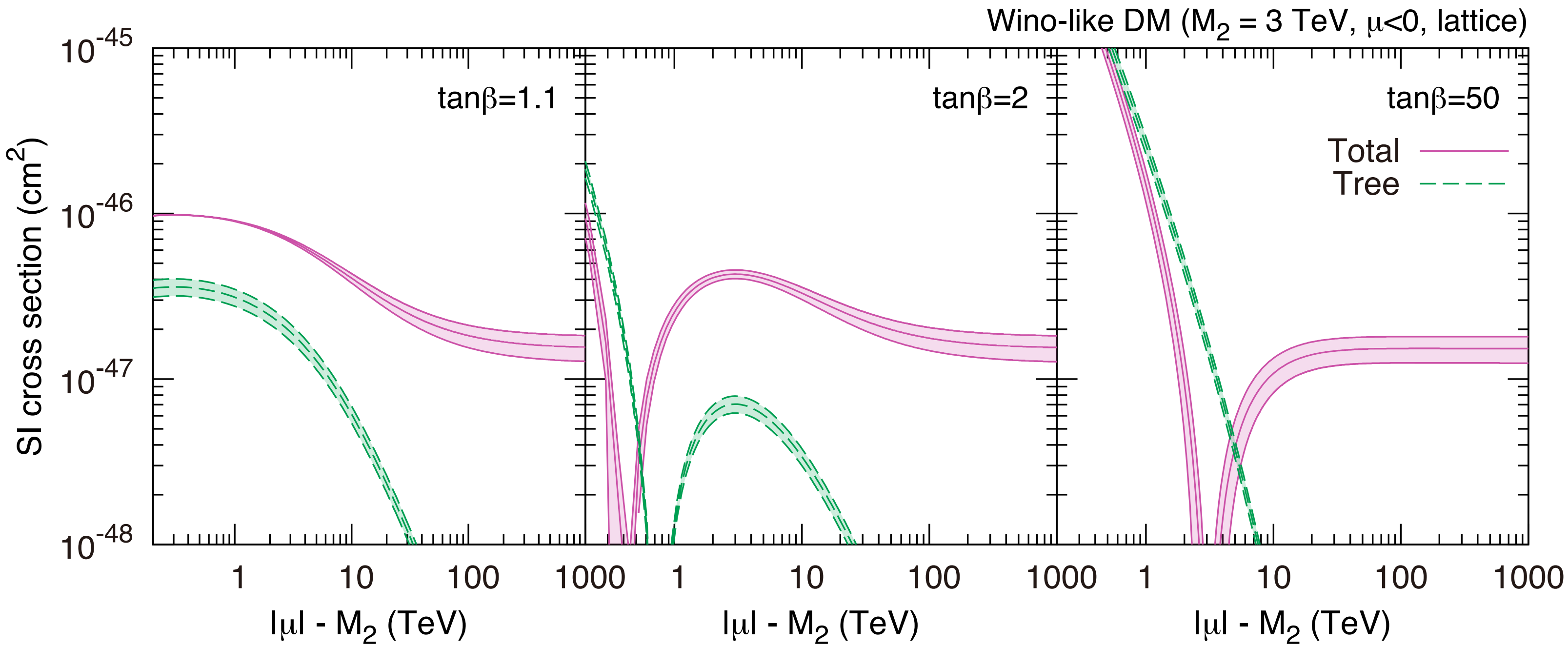
There is a cancellation among these contributions

# Wino-like DM | Scattering cross sections with a proton




- Cancellations between tree- and loop-level contributions occur at a certain value of μ
- Loop contribution is dominant in a wide range of parameter region

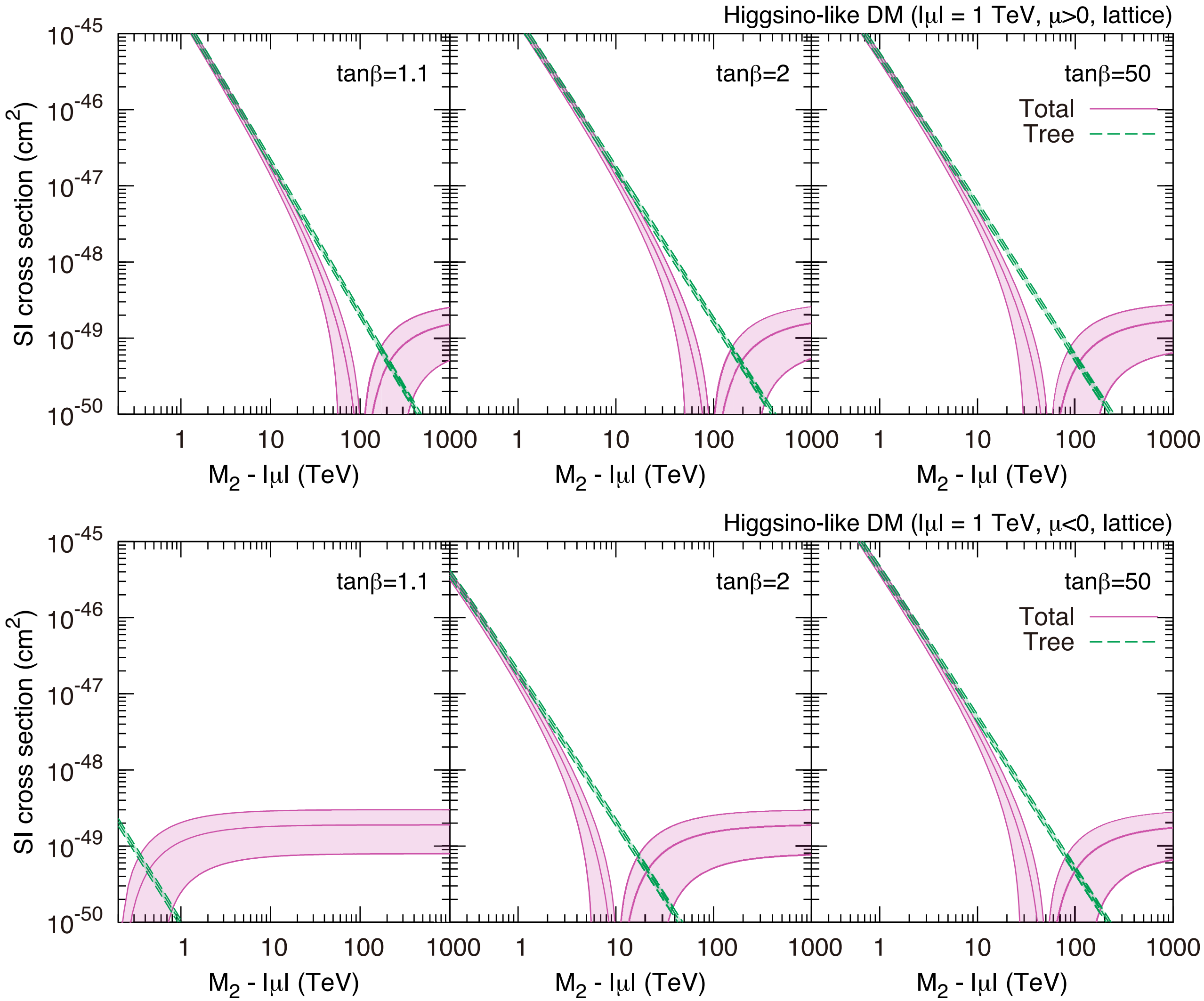
# Wino-like DM Scattering cross sections with a proton




Tree-level contribution interferes constructively to the loop contribution in the case of low tanβ

The cross sections are within a reach of future experiments in a wide range of parameter regions

# Results Higgsino-like DM



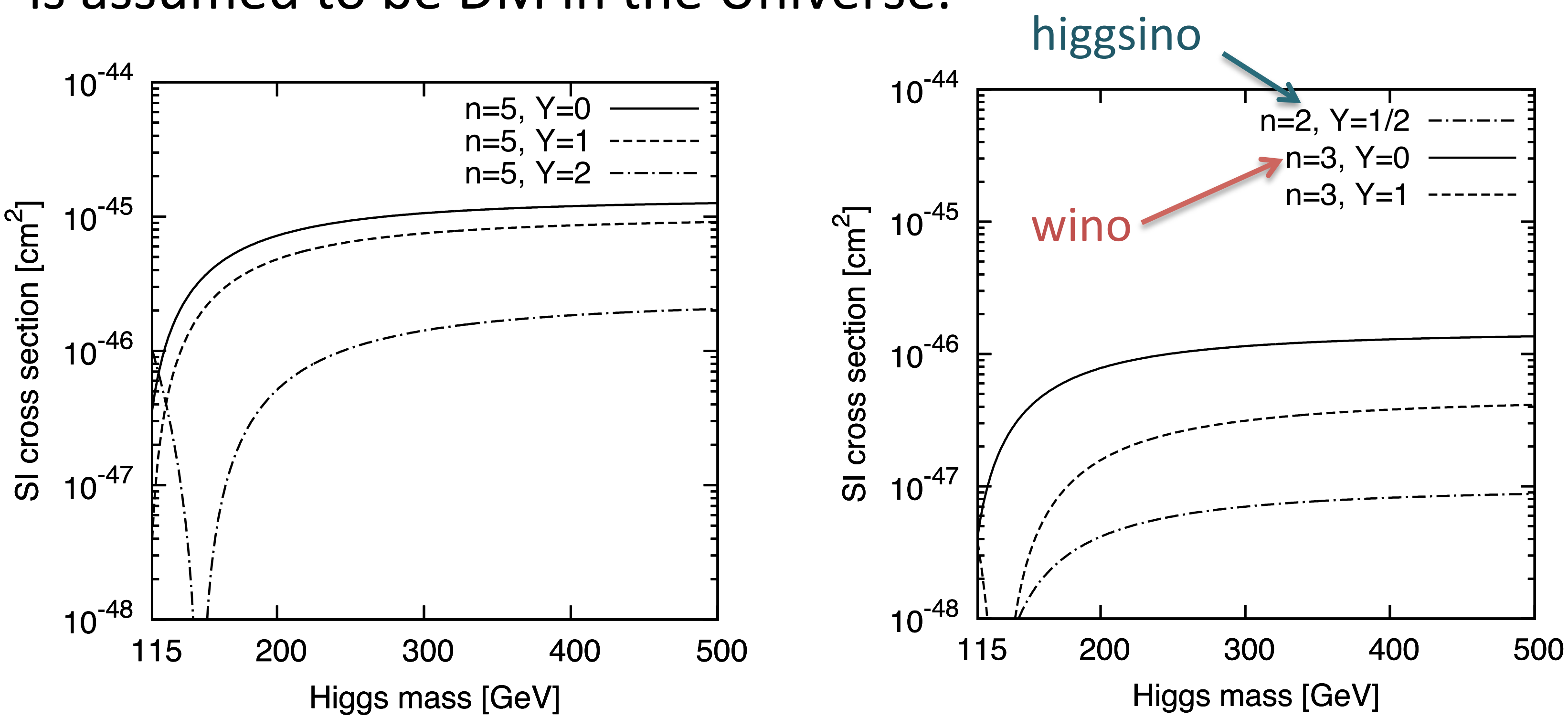
EWIMP DM

Minimal dark matter

## Electroweak interacting massive particles (EWIMPs)

Ibe-san's talk

The neutral component of an $SU(2)_L$ $n$-tuplet (hypercharge $Y$) is assumed to be DM in the Universe.




We again find cancellations among the contributions.

J. Hisano, K. Ishiwata, N. Nagata, and T. Takesako, JHEP **1107** (2011) 005.

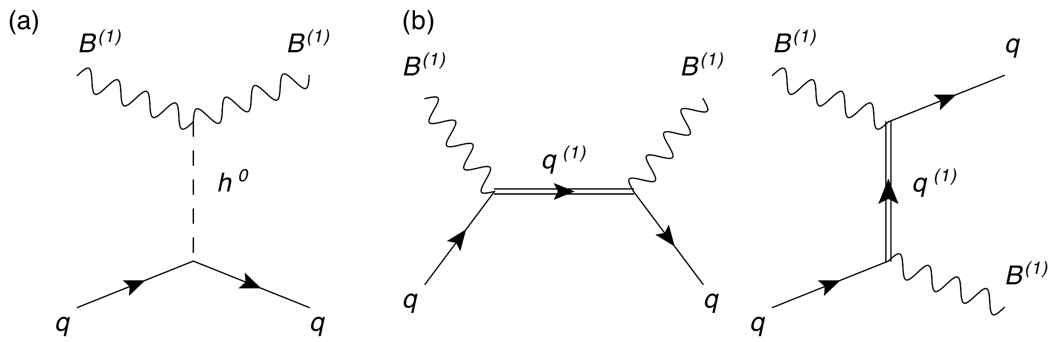
## Vector DM | KK photon DM (MUED)

### Minimal Universal Extra Dimension (MUED) model

The first KK photon becomes the lightest Kaluza-Klein particle (LKP).

### Tree-level diagrams:

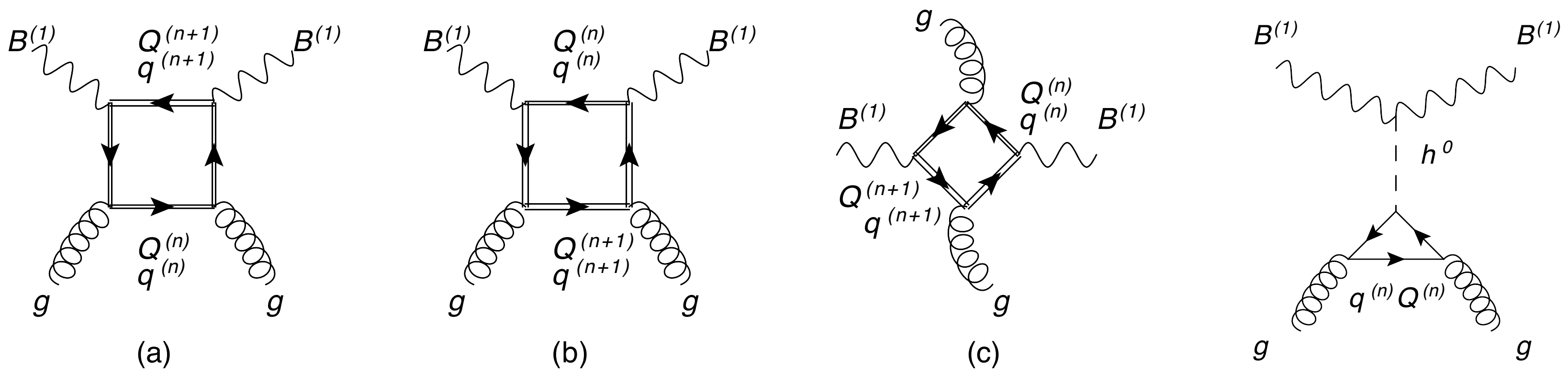
→ Vector DM



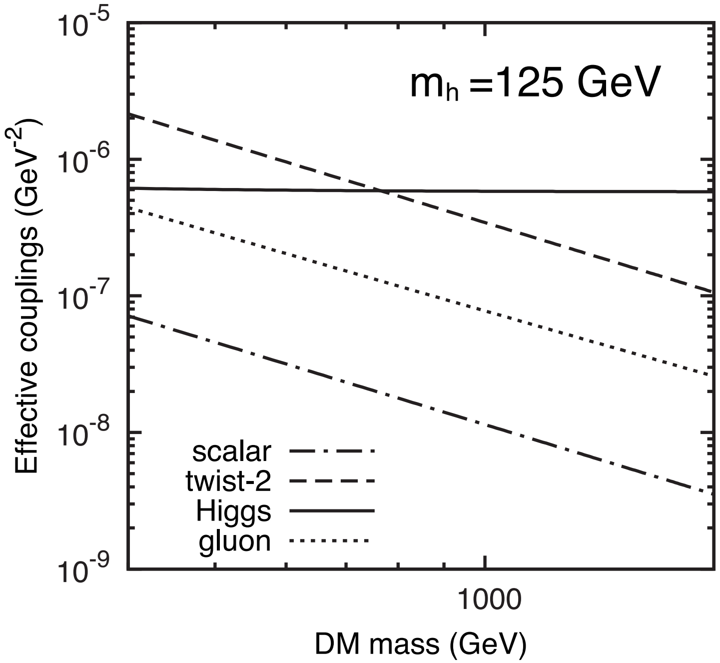

$B^{(1)}$ : KK photon DM  $q^{(1)}$ : the first KK quark

### 1-loop diagrams:

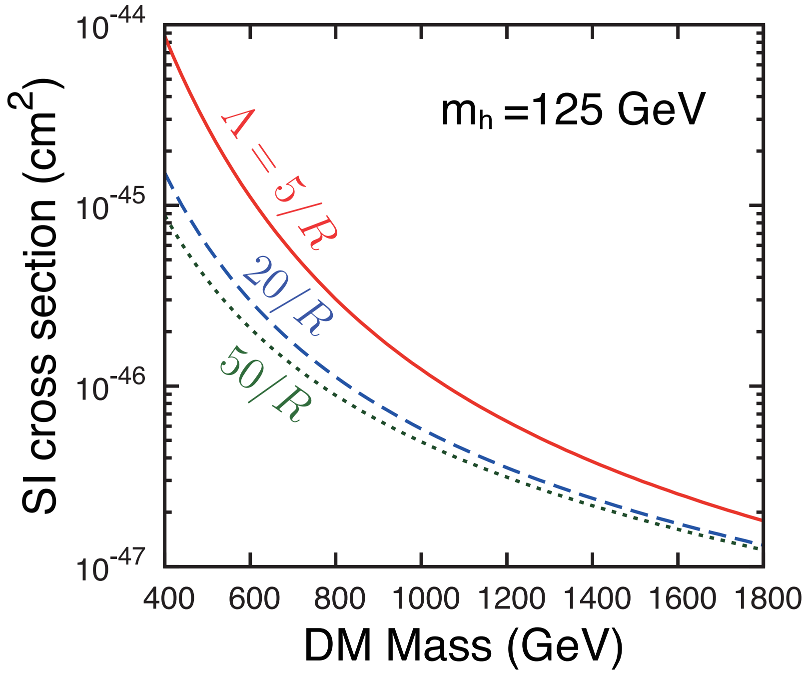
Neglected in previous calculations



## Vector DM | KK photon DM (MUED)




Each contribution in the effective coupling




The SI scattering cross sections

- All of the contributions have the same sign (constructive).
- Resultant scattering cross sections are larger than those in previous work by about an order of magnitude.

J. Hisano, K. Ishiwata, NN, and M. Yamanaka, Prog. Theor. Phys. Vol. 126, No. **3** (2011) 435.

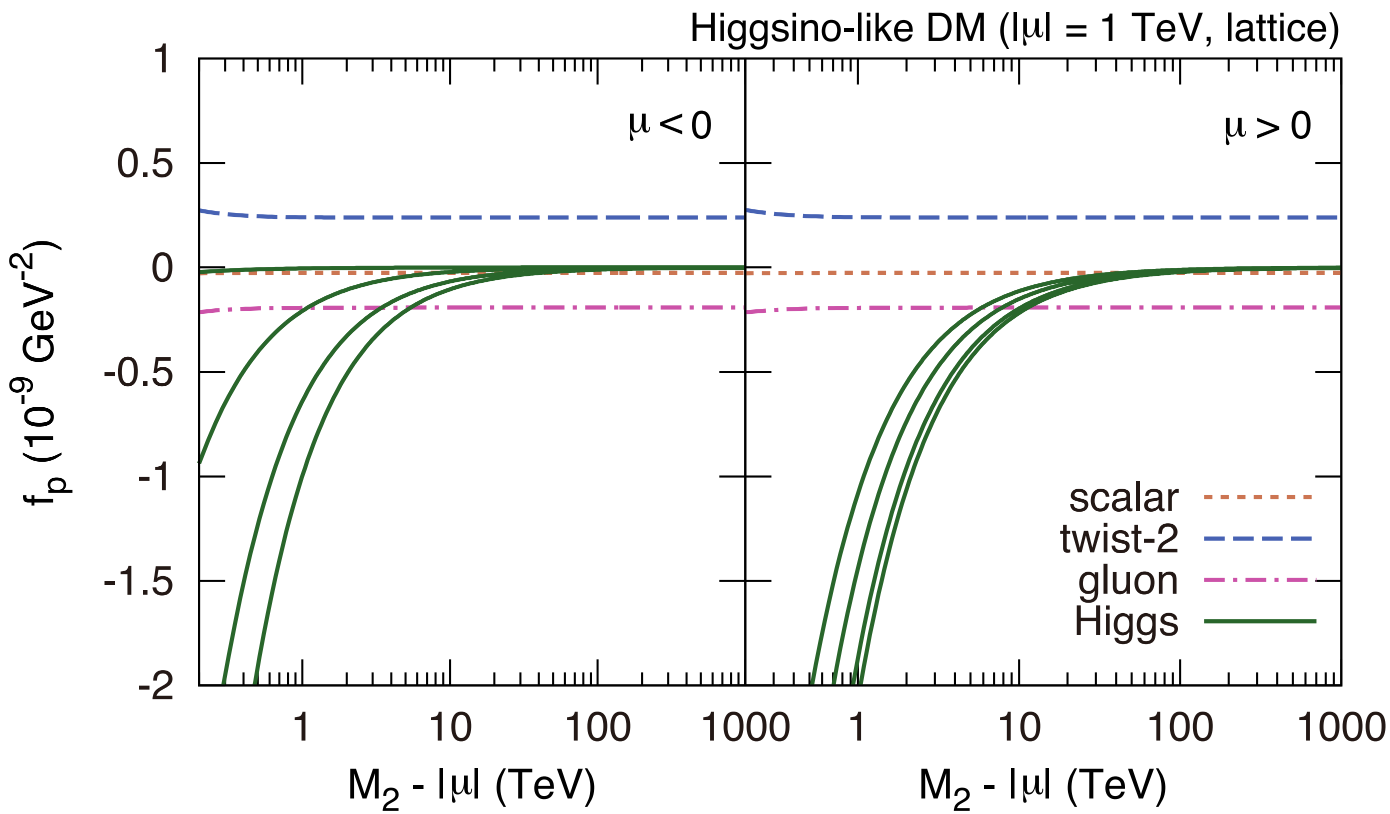
# 4. Summary

# Summary

- We evaluate the elastic scattering cross sections of WIMP DM with nucleon based on the method of effective theory.
- The interaction of DM with gluon as well as quarks yields sizable contribution to the cross section, though the gluon contribution is induced at loop level.
- In the wino dark matter scenario we find the cross section is smaller than the previous results by more than an order of magnitude
- The cross section of the first Kaluza-Klein photon dark matter turns out to be larger by up to a factor of ten.

# Backup

## Results | Higgsino LSP



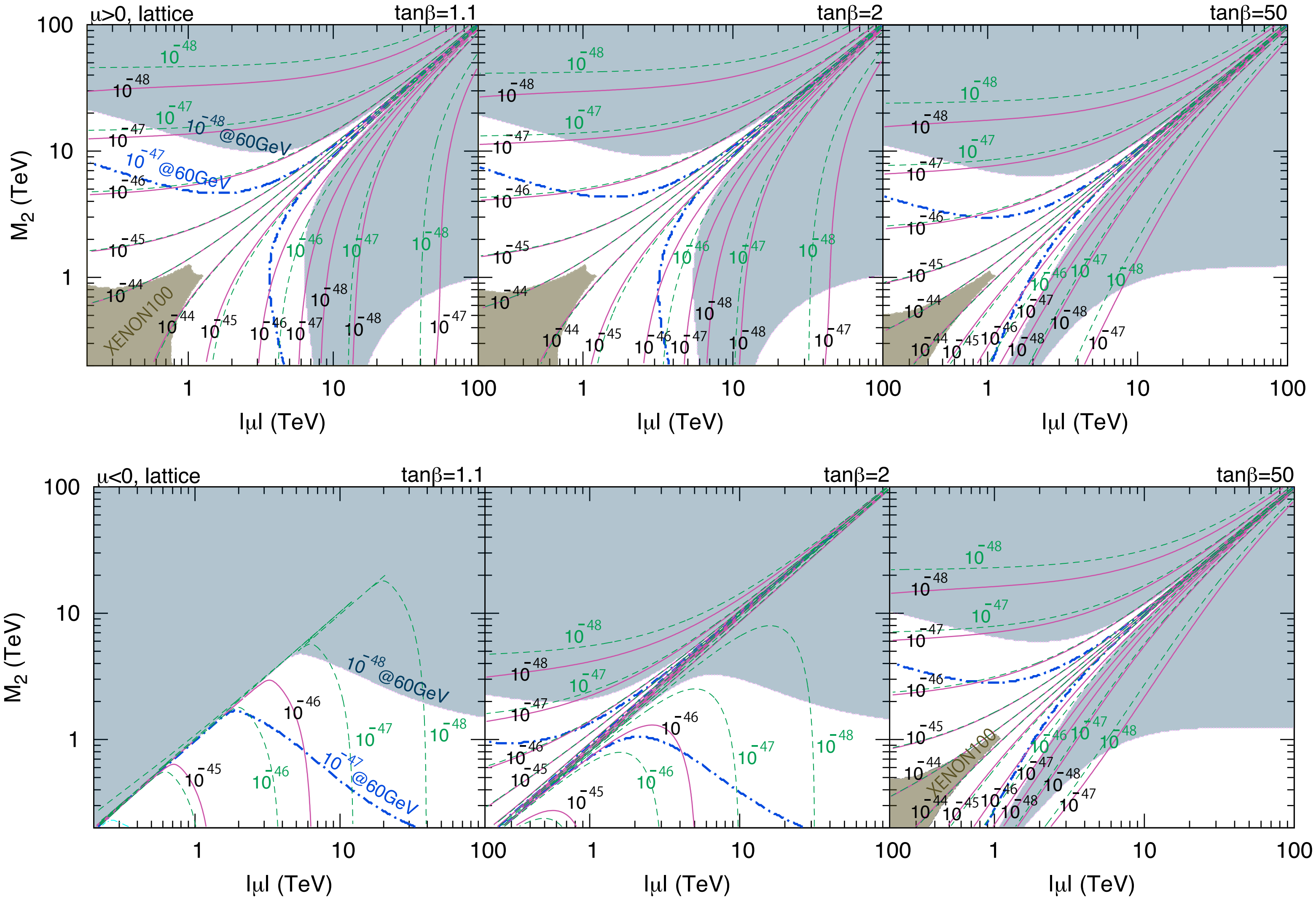

$\tan\beta$ = 1, 2, 5, 50

(from top to bottom)

$\tan\beta$ = 1, 2, 5, 50

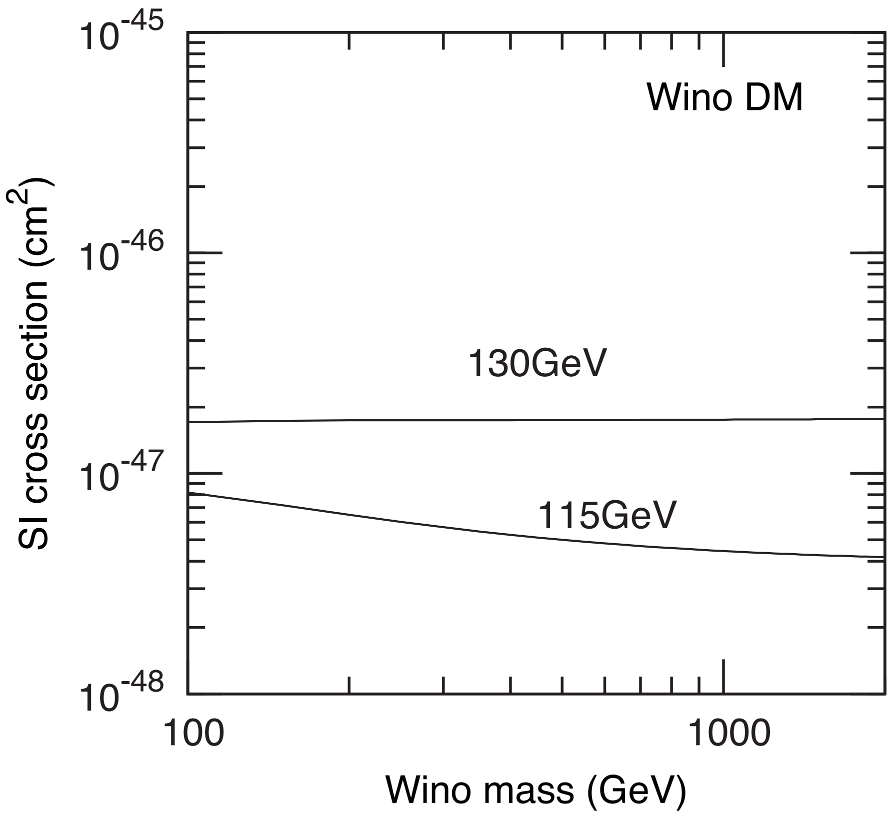
(from bottom to top)

# Results



μ>0, lattice
tanβ=1.1
tanβ=2
tanβ=50
μ<0, lattice
$M_2$ (TeV)
|μ| (TeV)
XENON100
$10^{-48}$ @60GeV
$10^{-47}$ @60GeV

# Loop contributions only




The SI cross section is almost independent of the wino mass.

J. Hisano, K. Ishiwata, and N. Nagata, Phys. Lett. B **690** (2010) 311.

## Higgs-nucleon coupling

$$\mathcal{L}_{NNh} = -g_{NNh}\bar{N}Nh$$

$$\begin{aligned} g_{NNh} &= \frac{\sqrt{2}}{v}\sum_q \langle N|m_q\bar{q}q|N\rangle \\ &= \frac{\sqrt{2}}{v}\left[m_N(f_{Tu}+f_{Td}+f_{Ts}) - \frac{\alpha_s}{4\pi}\langle N|G^a_{\mu\nu}G^{a\mu\nu}|N\rangle\right] \\ &= \frac{\sqrt{2}}{v}m_N\left[\frac{2}{9}+\frac{7}{9}(f_{Tu}+f_{Td}+f_{Ts})\right] \end{aligned}$$

Large mass fractions (f_Tq → large)

→ Higgs-nucleon couplings are enhanced

## Input parameters

$$\sigma_{\pi N} = \frac{m_u + m_d}{2} \langle p|\bar{u}u + \bar{d}d|p\rangle$$

$$\xi = \frac{\langle p|\bar{u}u - \bar{d}d|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle}$$

$$y = \frac{2\langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle}$$

$$\xi = 0.135 \pm 0.035$$

H. Y. Cheng (1989)

### Lattice results (ours)

$$\sigma_{\pi N} = 53 \pm 2(\mathrm{stat})^{+21}_{-7}(\mathrm{syst})\ \mathrm{MeV}$$

$$y = 0.030 \pm 0.016(\mathrm{stat})^{+0.006}_{-0.008}(\mathrm{syst})$$

H. Ohki et al. (2008)

### Chiral perturbation (traditional)

$$\sigma_{\pi N} = 64 \pm 7\ \mathrm{MeV}$$

M. M. Pavan et al. (2002)

$$y = 0.44 \pm 0.13$$

B. Borasoy and U. G. Meissner (1997)