A closer look into the CMB anisotropies

A simple test of cosmological principle

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- Introduction
  - CMB anisotropies in harmonic space
  - Angular power spectrum
  - Statistical anomalies?
- tests for the primordial fluctuations
  - A test of the zero-mean hypothesis
  - A test of the power law spectrum
    - small scale features in the spectrum?
- summary

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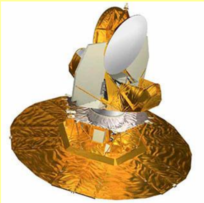
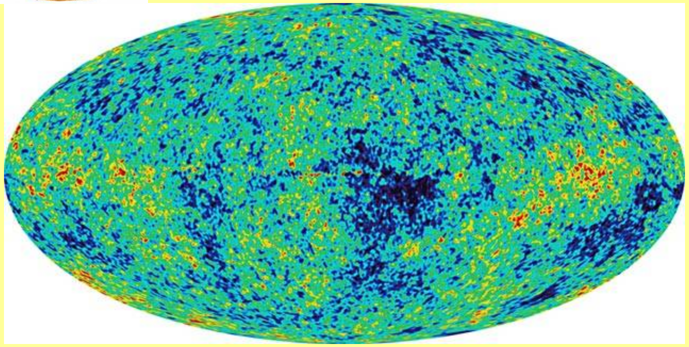
CMB anisotropies



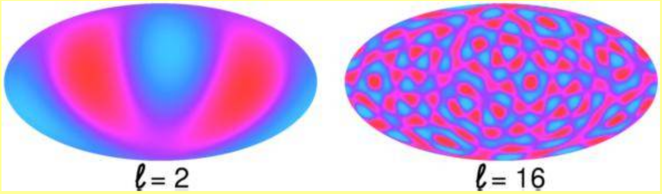
Microwave sky: $\frac{\Delta T(\hat{n})}{T_0}$



CMB: Density fluctuations
380,000 yr after the big-bang

Expand with the spherical harmonics

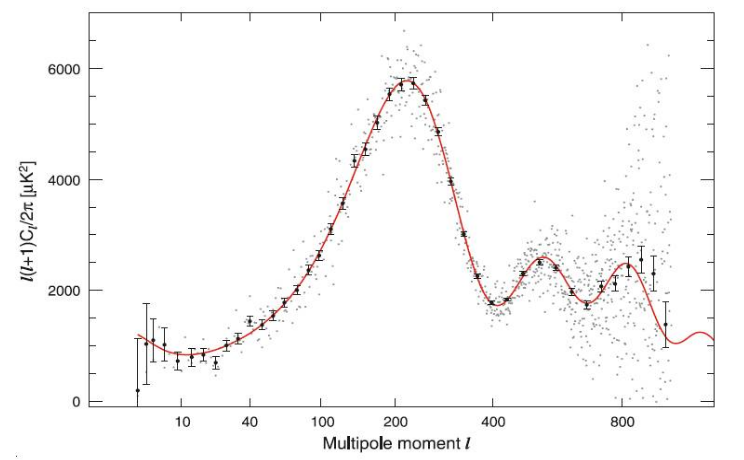
$$\frac{\Delta T(\hat{n})}{T_0} = \sum_{\ell m} a_{\ell m} Y_{\ell m}$$



Power spectrum: Calc the variance of the expanded coefficients

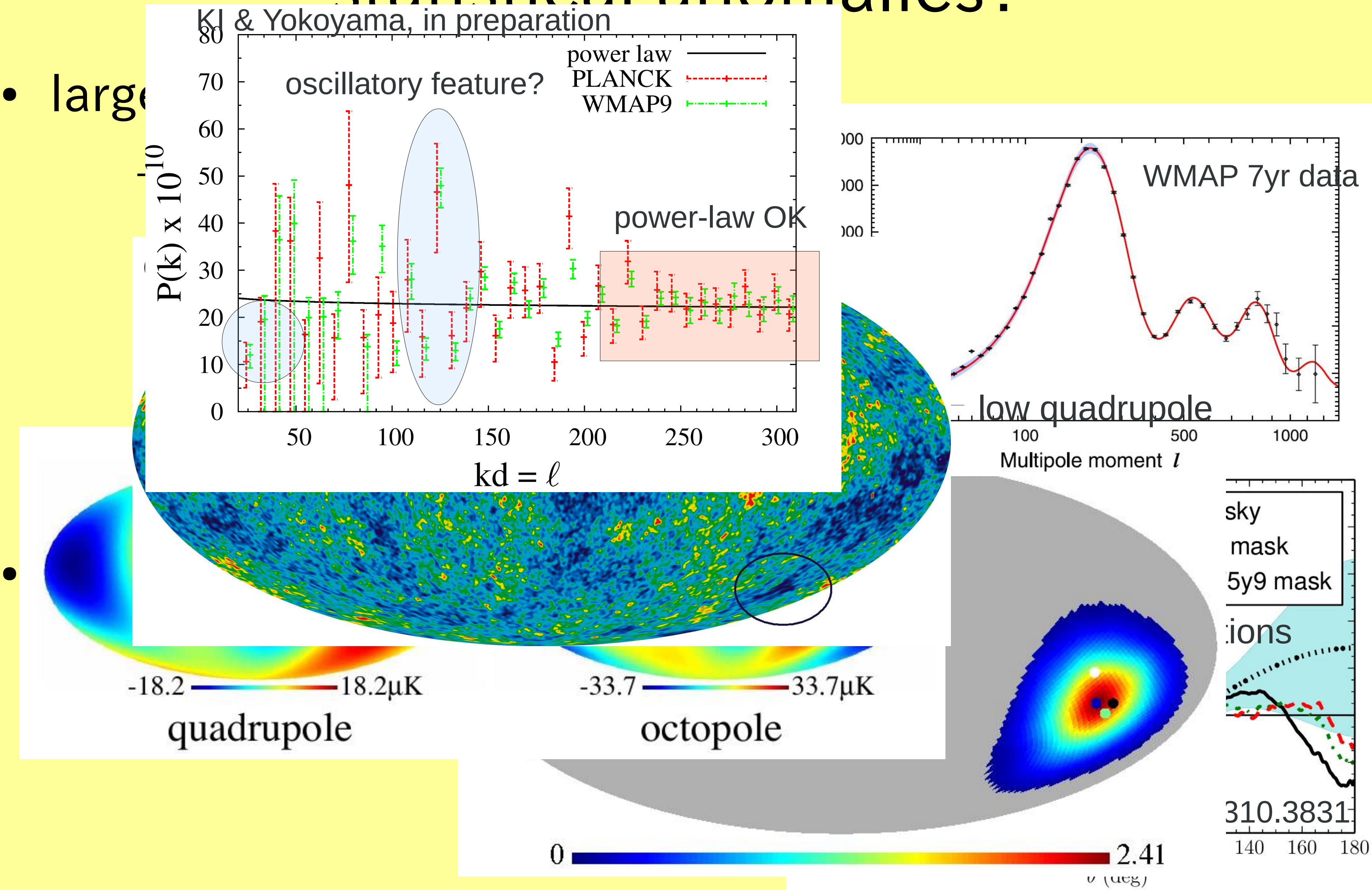
$$C_\ell = \langle a^*_{\ell m} a_{\ell m} \rangle$$
$$= \frac{1}{2\ell + 1} \sum_m |a_{\ell m}|^2$$

WMAP power spectrum



statistical anomalies?

- large



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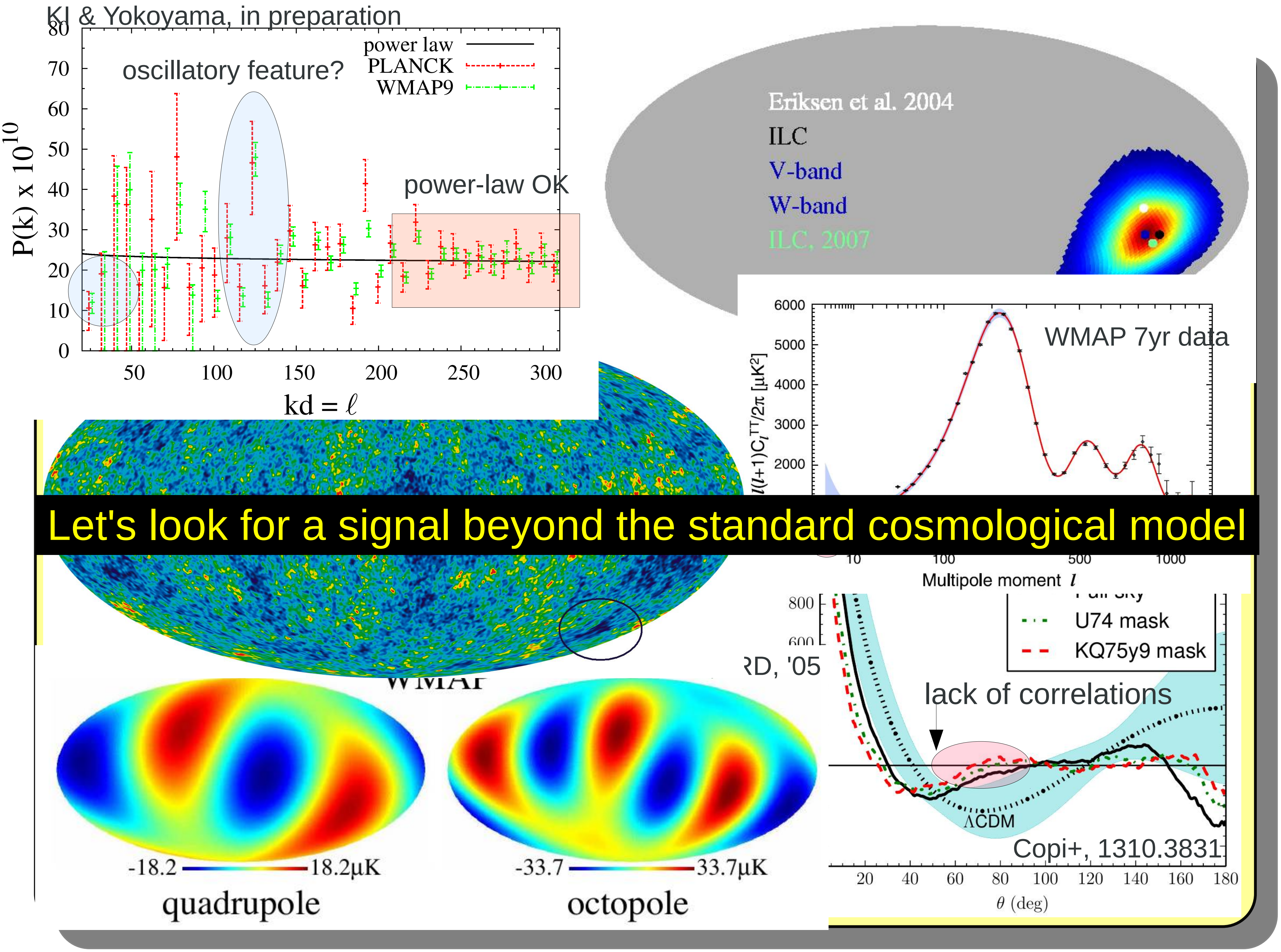
KI & Yokoyama, in preparation
oscillatory feature?
power law
PLANCK
WMAP9
power-law OK
P(k) x 10^10
kd = ℓ
Eriksen et al. 2004
ILC
V-band
W-band
ILC, 2007
WMAP 7yr data
Multipole moment l
Let's look for a signal beyond the standard cosmological model
U74 mask
KQ75y9 mask
lack of correlations
ΛCDM
Copi+, 1310.3831
θ (deg)
-18.2
18.2μK
quadrupole
-33.7
33.7μK
octopole

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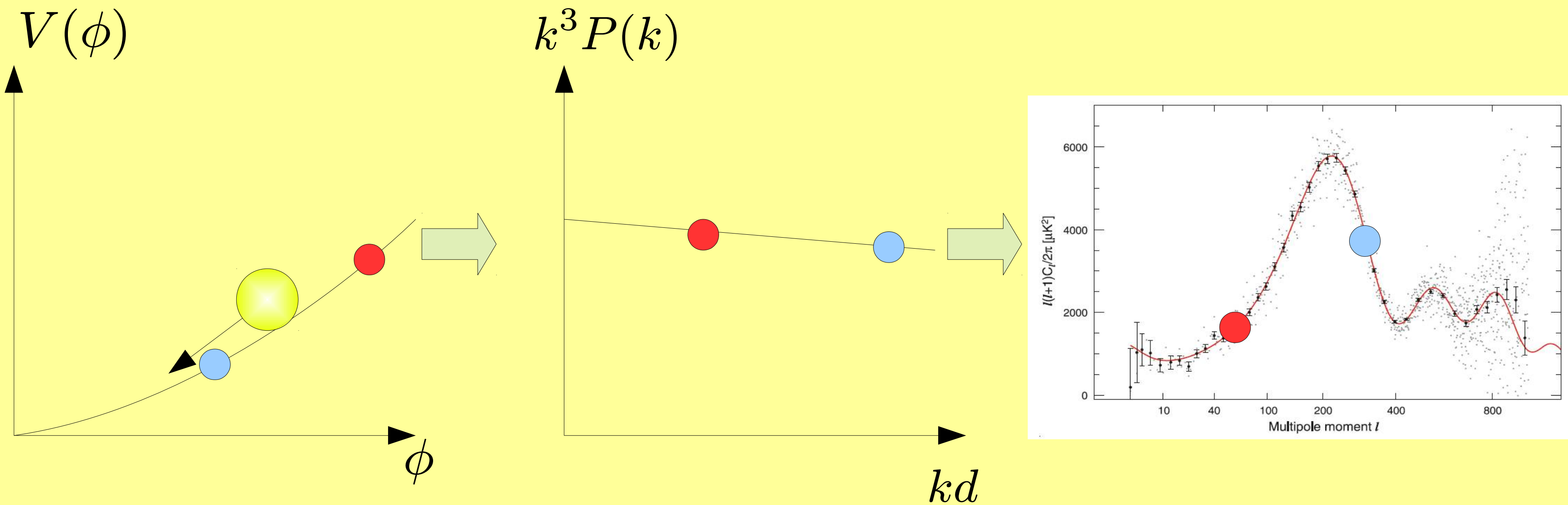
Primordial fluctuations to the CMB



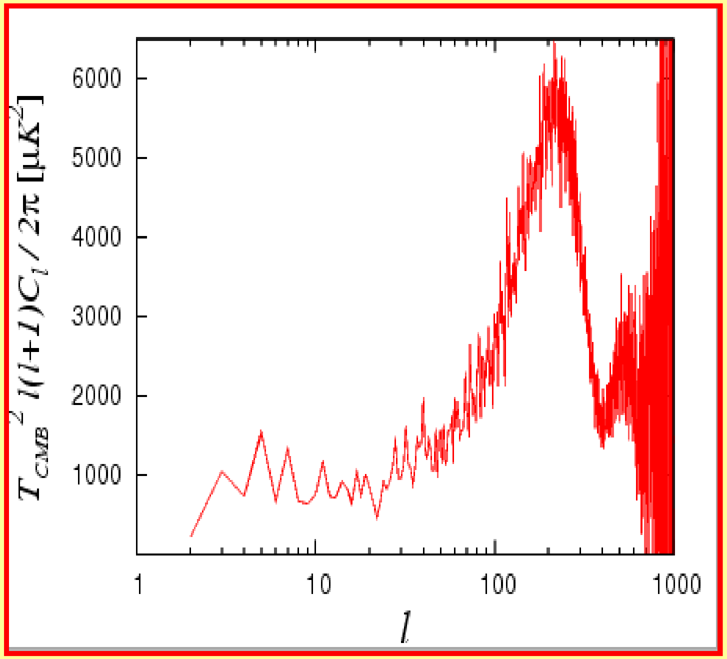
$V(\phi)$ → $P(k)$ → C_ℓ

Inflation will produce... primordial fluctuation that will be transfered to... the CMB anisotropy

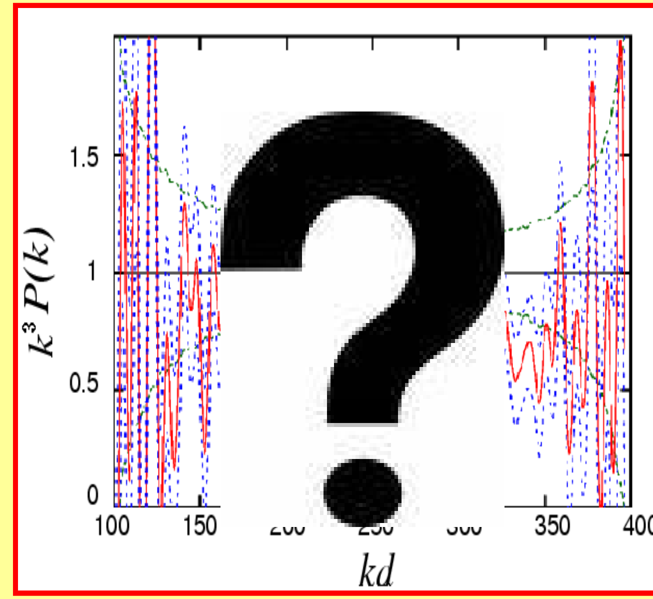
$$\frac{\ell(\ell+1)C_\ell}{2\pi} = \int d\ln k\, k^3 P(k) |T_\ell(\eta_0, k)|^2$$

CMB angular power spectrum   Primordial fluc.   transfer function

Reconstruction of primordial fluctuation from data

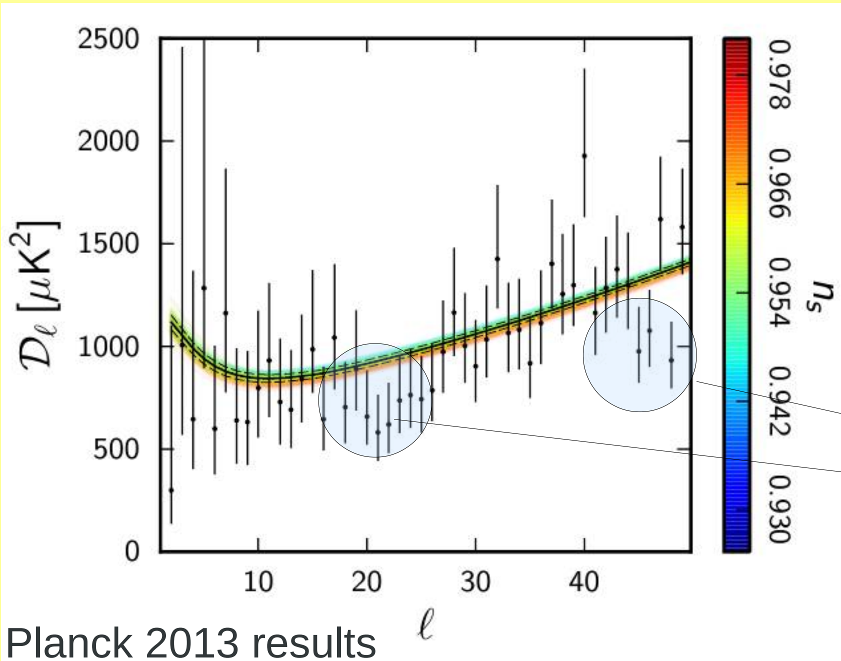


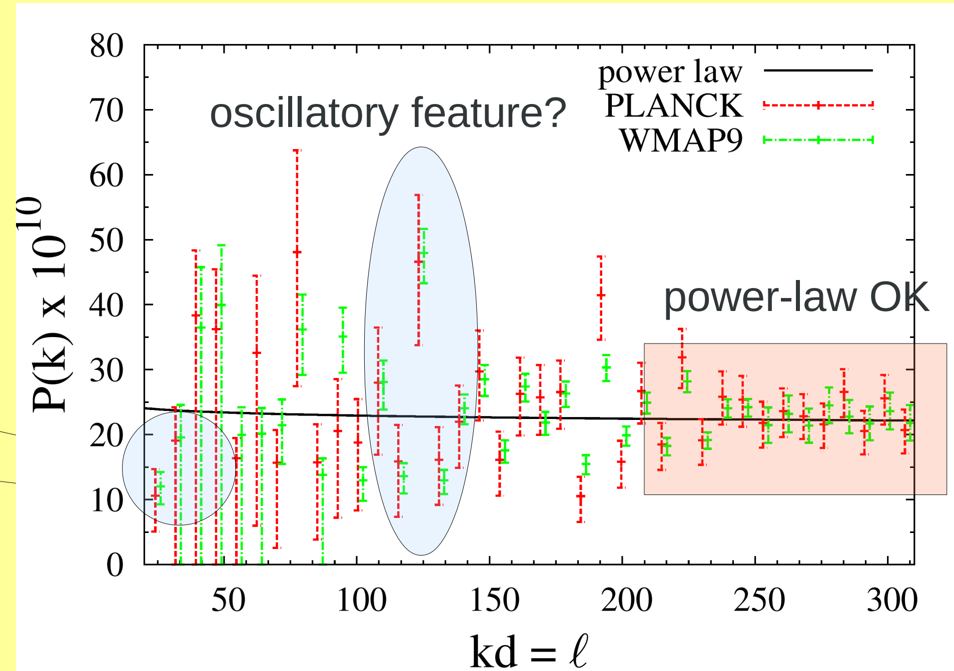
C_ℓ $\quad P(k)$



- Let's find the initial condition of our own Universe
- Simplest inflation models predict a smooth, power law P(k)
  - search for fine and global structures beyond sample variance

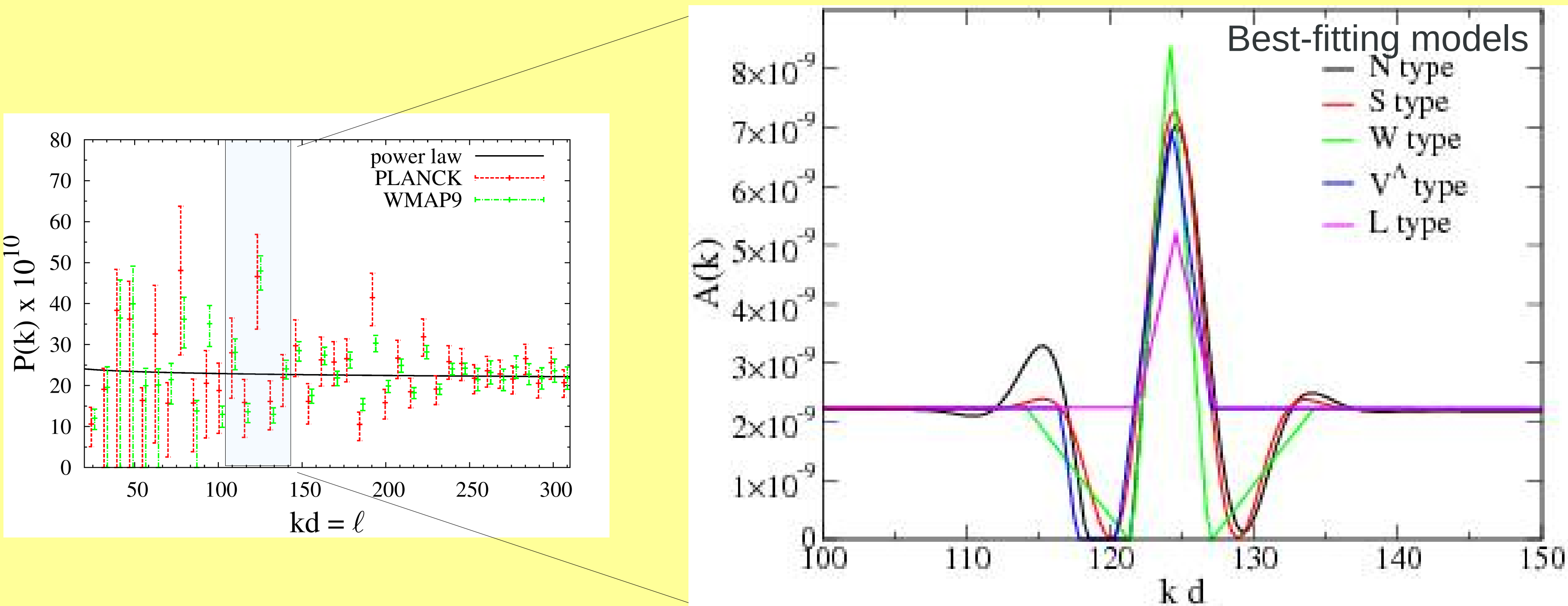
A brute force reconstruction





- Planck reports a power deficit at $\ell \approx 20$
- divide a range of wavenumber into many bins
- we find hints for the departure from the power law at $\ell \approx 120$
- constraints are weaker for PLANCK

A close look at around l=120



・A simple 3 parameter feature model: (S-type in the figure)

$$k^3 P(k) = \underbrace{A\left(\frac{k}{k_0}\right)^{n-1}}_{\text{power-law}} + \underbrace{B\left(\frac{k}{k_0}\right)^{n-1} \exp\left(-\frac{(k-k_*)^2}{\kappa^2}\right) \cos\left(\pi \frac{k-k_*}{\kappa}\right)}_{\text{oscillatory feature on top of the power-law}}$$

power-law　　oscillatory feature on top of the power-law

・Posterior probability to find B=0 is 8×10^{-4} including looking-elsewhere effect

Implications

- It is interesting that our featured spectrum improved the fit not only the TT spectrum, but also the TE (WMAP)
  - $\Delta\chi_{\rm eff} = -21 \quad \text{breakdown:} -12.5\ (\text{TT}) \quad -8.5\ (\text{TE})$
- Standard cosmological parameters, esp. $\Omega_b h^2,\ n_s$ can shift comparable to the statistical fluctuation depending on whether we incorporate this feature or not (KI and Yokoyama, in preparation)
- E-mode polarization will be more suitable for the P(k) reconstruction (Mortonson+, PRD, '09)
  - Let's wait for the PLANCK's polarization data next year

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Today's question

- The variance (C_ℓ) contains most of the cosmological information

$$C_\ell = \frac{1}{2\ell+1}\sum_m |a_{\ell m}|^2$$

- Why should we divide the squares by $(2\ell+1)$, and not by 2ℓ? (textbooks say we should divide by (d.o.f -1) to get an unbiased estimate)
- This is because we have *implicitly* assumed that the mean of $a_{\ell m}$ is zero.

condition for the zero-mean

- We believe that, according to the cosmological principle, we can write any perturbation variables as

$$\phi(t,x) = \phi_0(t) + \delta\phi(t,x)$$

$$\langle \delta\phi(t,x) \rangle = 0$$ Independent of position (x)

- This is possible when $\phi(t,x)$ is statistically homogeneous:

$$\langle \phi(\vec{x}) \rangle = \left\langle \phi(\vec{x} + \vec{T}) \right\rangle$$ $\vec{T}$: arbitrary vector

- The condition should be tested by observations!

Another motivation

- In the analysis of CMB anisotropies, **zero mean is usually assumed implicitly.**
  - Any higher order statistics, such as variance, skewness, kurtosis etc... are affected by this assumption.
  - Non-zero mean have been indicated by LSS (e.g., Labini, arXiv:1103.5974)
    - However, LSS suffers from bias, selection rules, galaxy evolution,...

↓

Let's look for in the CMB anisotropies !

Mean of CMB anisotrpies (1)

- CMB fluctuations $\delta T(x,\hat{n})$ are related with the primordial fluctuations (random variable) $\phi(\vec{k})$ through transfer function $\mathcal{T}(\vec{k},\hat{n})$ as

$$\delta T(x,\hat{n}) = \int \frac{d^3k}{(2\pi)^3}\mathcal{T}(\vec{k},\hat{n})\phi(\vec{k})e^{i\vec{k}\cdot\vec{x}}$$

- Expanded coefficients of CMB fluctuations:

$$a_{\ell m} = \int d^2\hat{n}\,\delta T(\hat{n})Y_{\ell m}(\hat{n})$$
$$= 4\pi(-i)^\ell \int \frac{d^3k}{(2\pi)^3}\mathcal{T}_\ell(k)\phi(\vec{k})Y_{\ell m}(\hat{k})$$

Legendre coefficients of the transfer function

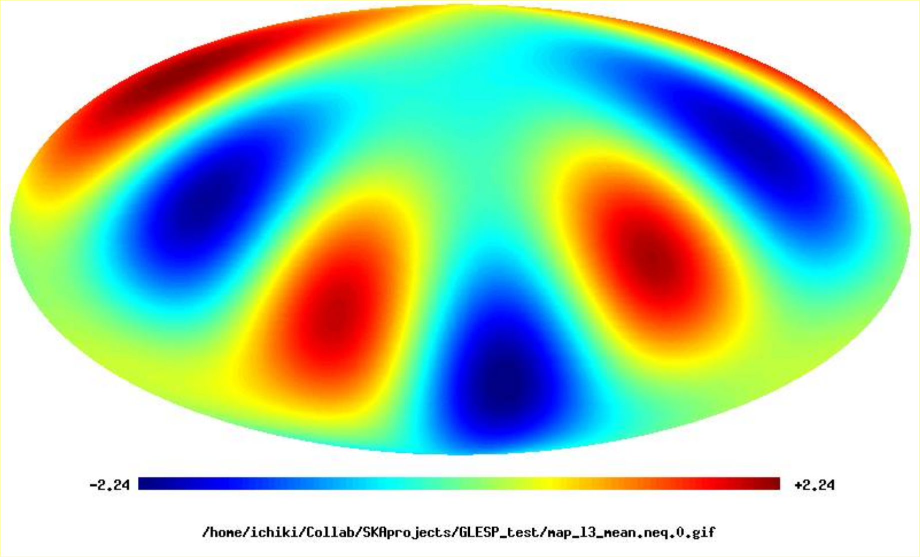
- Therefore, $\langle\phi\rangle = 0 \rightarrow \langle a_{\ell m}\rangle = 0$

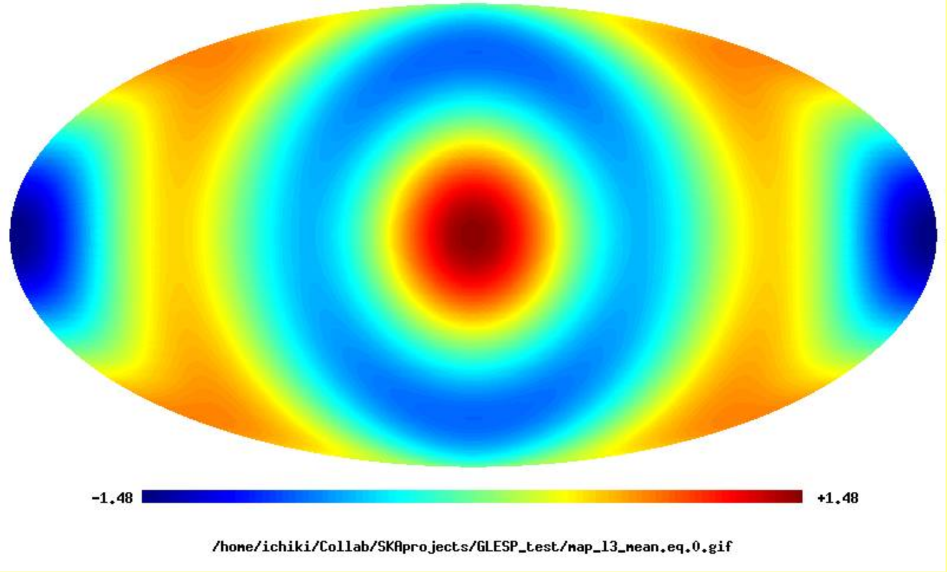
- Furthermore, if $\phi(\vec{k})$ are Gaussian, so are $a_{\ell m}$

Meaning of the zero mean

Fluctuations such that $\int d\hat{n}\frac{\Delta T(\hat{n})}{T} = \sum_{\ell m}\int d\hat{n} a_{\ell m} Y_{\ell m}(\hat{n}) = 0$

do not necessarily mean $\langle a_{\ell m}\rangle = 0$





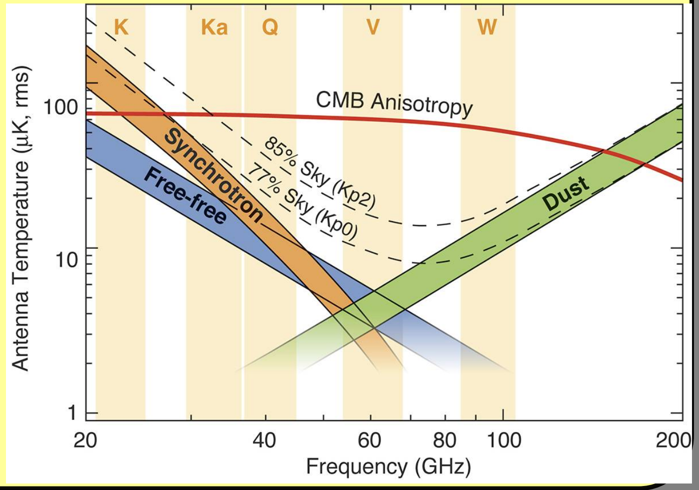
$a_{3m} = (1,1)$ for $m \geq 0$

$a_{31} = (1,0),\ a_{3,3} = (-1,0)$
$a_{30},\ a_{3,2} = (0,0)$

Difficulty…Foreground,Noise,Mask

- Some of the CMB photons are not primordial origin
  - Dust emission, synchrotron, free-free…
  - They have non-Gaussian dist., **non-zero mean**
- Cleaning should not be perfect
  - masking the galactic disk
  - **induces unwanted correlations**
- Instrumental noises
  - they have zero-mean

Bennett et al., 2003



CMB MAP in practice

- Putting the mask $M(\hat{n})$

$$\delta T(\hat{n})_{\rm obs} \equiv M(\hat{n})\delta T_{\rm CMB}(\hat{n})$$

Going to the spherical harmonic space

$$(\delta T_{\rm obs})_{\ell m} \equiv c_{\ell m} = \sum M_{\ell m;\ell' m'}(a_{\ell' m'} + N_{\ell' m'})$$

observed

signal

- Zero mean still holds true if $\langle N_{\ell m}\rangle = 0$, however $c_{\ell m}$ and $c_{\ell' m'}$ are *not independent* due to the mask coupling

We cannot use a simple statistical test
(such as the student's t-test)

Beating the mask

- Mask introduces unwanted correlations between the sample $a_{\ell m}$ s
- Simple statistical tests rely on the independence… what would you do?
  - Do a test including the correlations
    - Monte Carlo simulation (Kashino, KI, Takeuchi, PRD '12)
  - Construct a de-correlated variable
    - V-vector method (Armendariz-Picon, JCAP '11)
    - Principal component analysis

v-vector method (Armendariz-Picon, JCAP, '11)

- Goal: to remove the effect of the mask from the observed spherical harmonic coefficients

$$c_{\ell m} = \sum M_{\ell m;\ell' m'} a_{\ell' m'}$$

observed → $c_{\ell m}$; signal → $a_{\ell' m'}$

- Let us use a vector notation:

$$\vec{c}_m = M \cdot \vec{a}_m$$

- Find m-independent v-vectors that satisfy

$$\vec{v}^{\,\mathrm{t}} = \vec{v}^{\,t} M$$

$$v_{\ell m} = \begin{cases} v_\ell & \text{for } |m| \le m_{\max} \text{ and } m_{\max} \le \ell \le \ell_{\max} \\ 0 & \text{(otherwise)} \end{cases}$$

binning

- Construct d_m as a dot product of $\vec{v}$ and $\vec{c}_m$

$$d_m \equiv \vec{v} \cdot \vec{c}_m = \sum_\ell v_{\ell m} c_{\ell m} = \vec{v} \cdot (M \vec{a}_m) = \vec{v} \cdot \vec{a}_m$$

$$\text{for } \quad (|m| \le m_{\max})$$

v-vector method (summary)

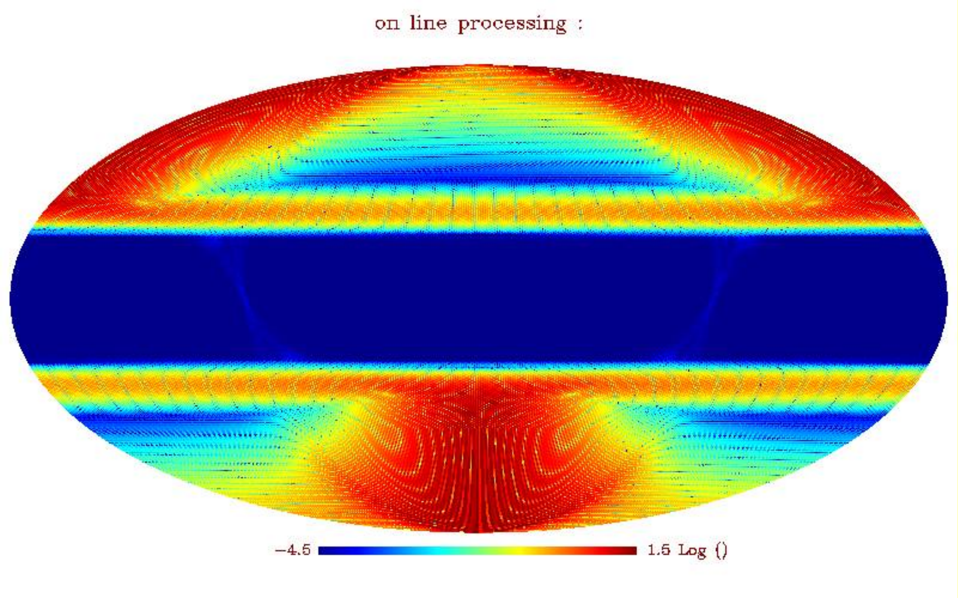
- Construct d_m as a dot product of $\vec{v}$ and $\vec{c}_m$

$$d_m \equiv \vec{v} \cdot \vec{c}_m = \vec{v} \cdot \vec{a}_m = \sum_{\ell=m_{\max}}^{\ell_{\max}} v_{\ell m} c_{\ell m} \quad (|m| \leq m_{\max})$$

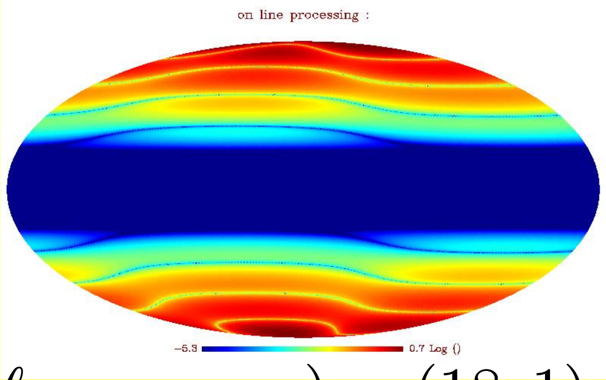
- The new stochastic variable d_m have following properties:
  - Foreground insensitive (because we work on $c_{\ell m}$)
  - Statistically independent samples (because $\vec{v}$ is constant)
  - zero-mean Gaussian if $a_{\ell m}$ are zero mean Gaussian
  - have m-independent variance

$$\sigma^2 = \sum_{\ell=m_{\max}}^{\ell_{\max}} K_\ell^2 (B_\ell^2 C_\ell + N_0) v_\ell^2$$

Visualization of v-vectors



$(\ell_{\max}, m_{\max}) = (212, 177)$

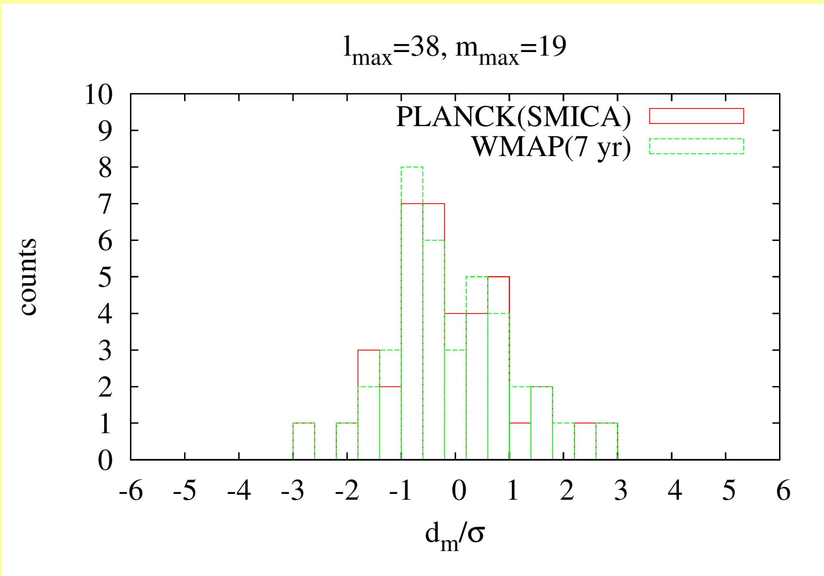


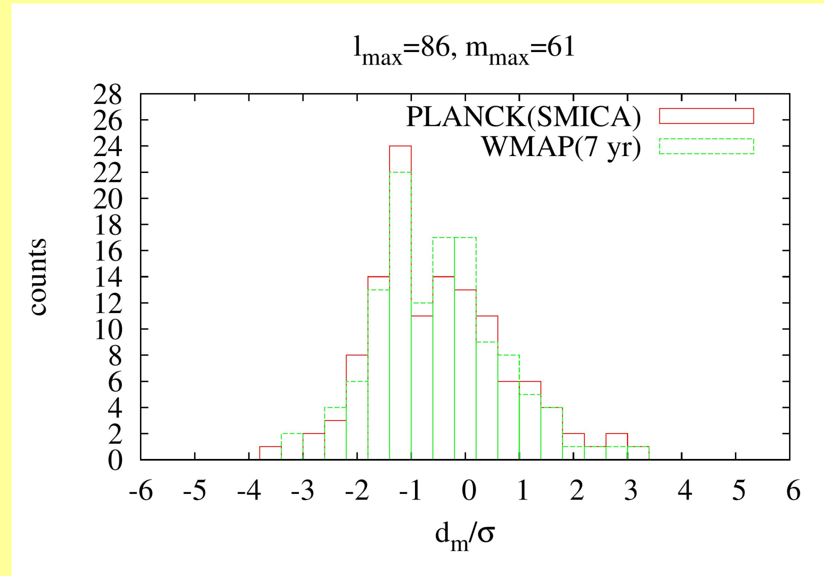
$(\ell_{\max}, m_{\max}) = (18, 1)$

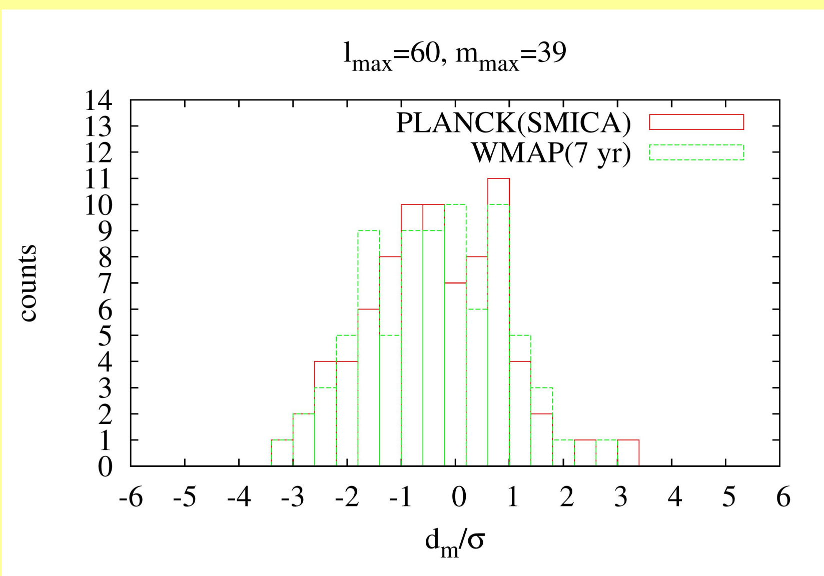
銀河面が除かれた重み付けがなされる。

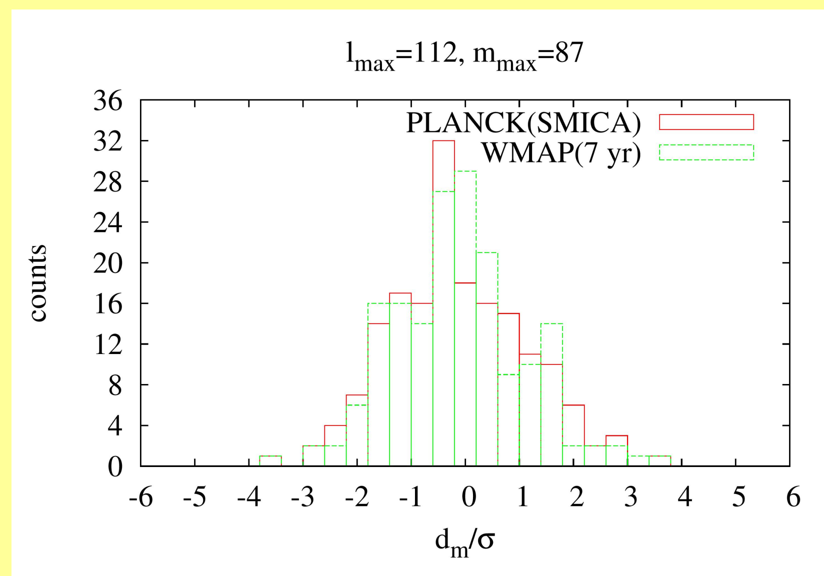
RESULTS

Distribution of the stochastic variable from PLANCK and CMB









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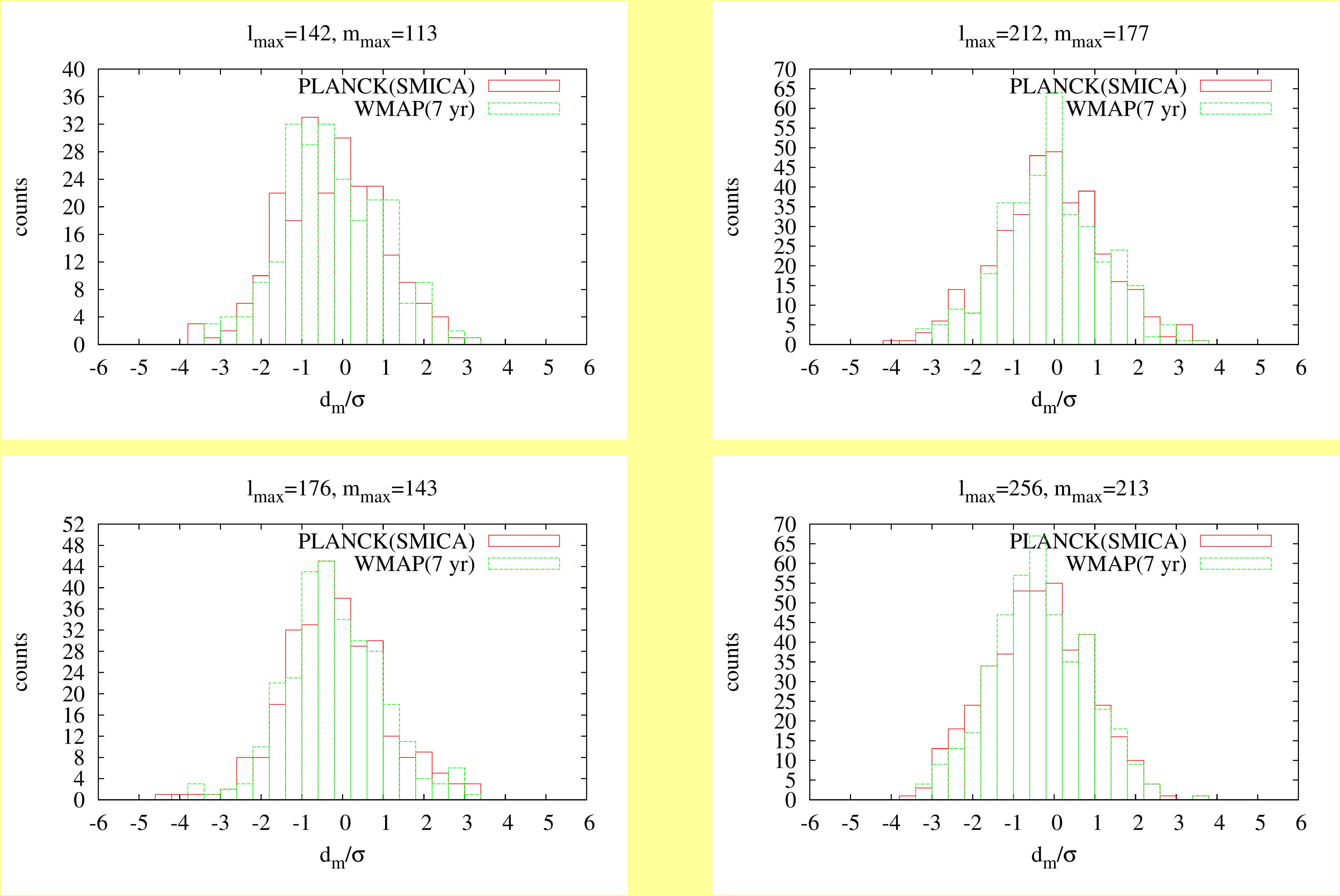
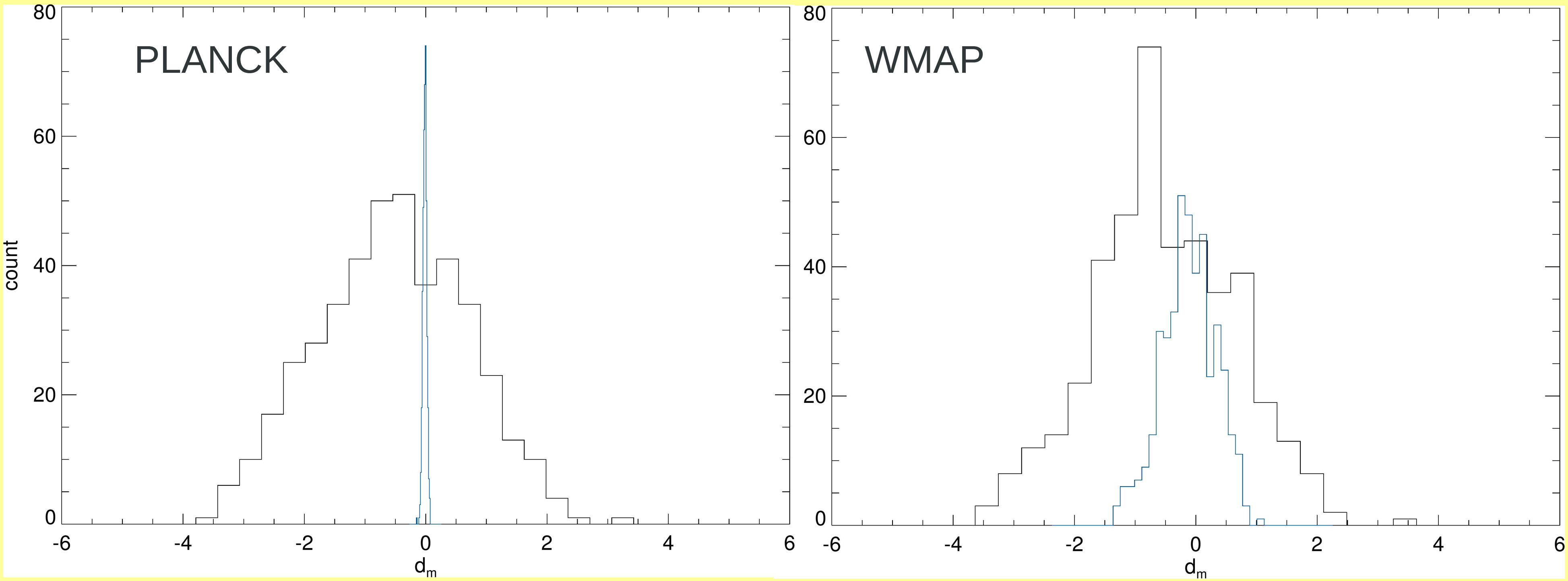
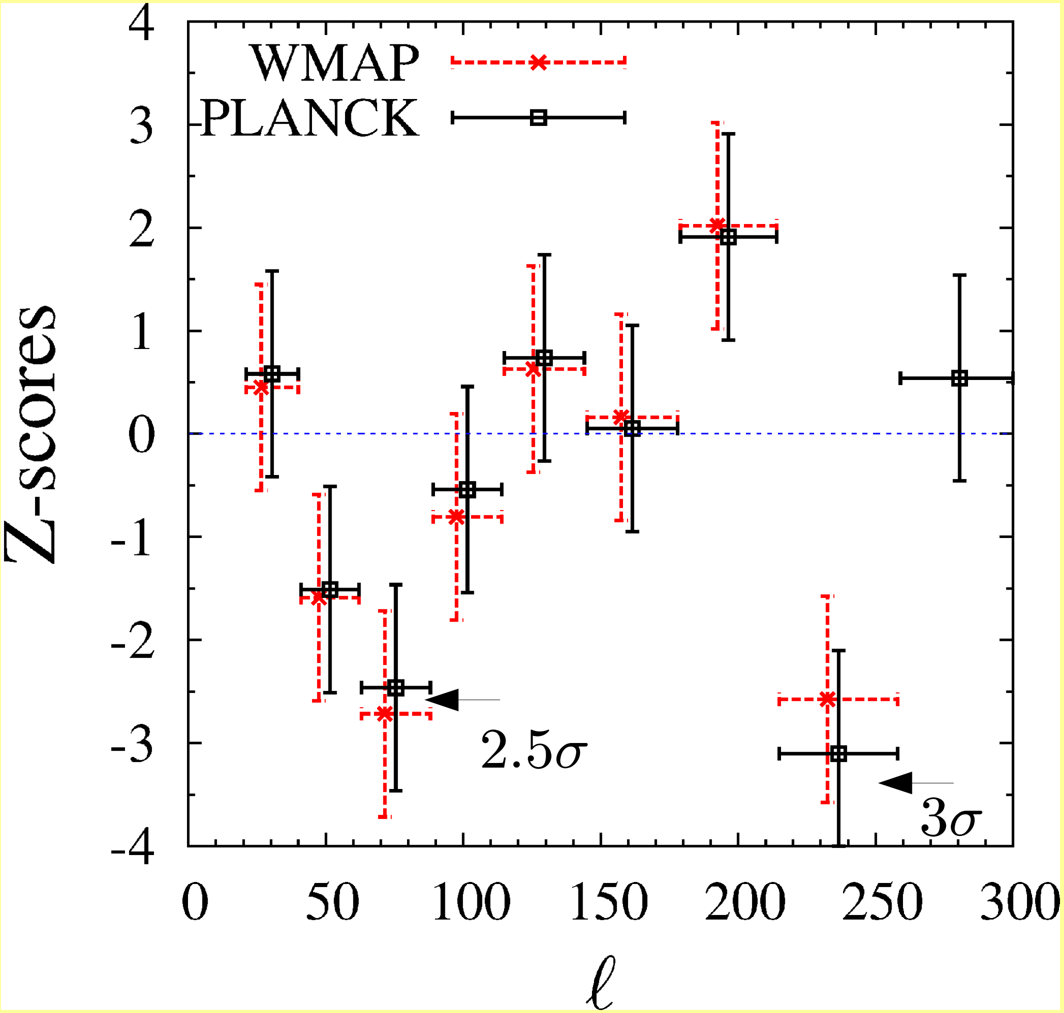


Noise levels



- black – signal, blue – noise
- noise becomes significant on smaller angular scales
- noise contributes upto 40 % for WMAP @ lmax=256

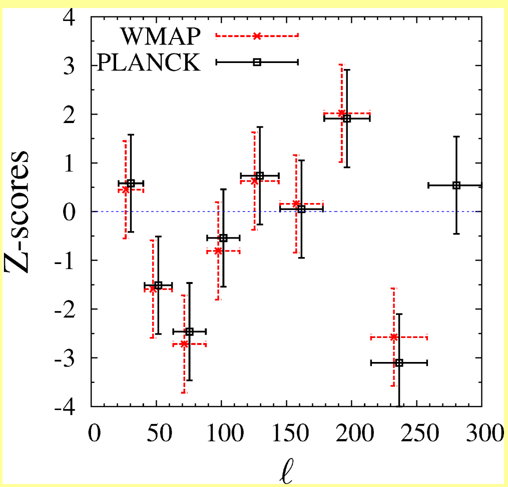
Result

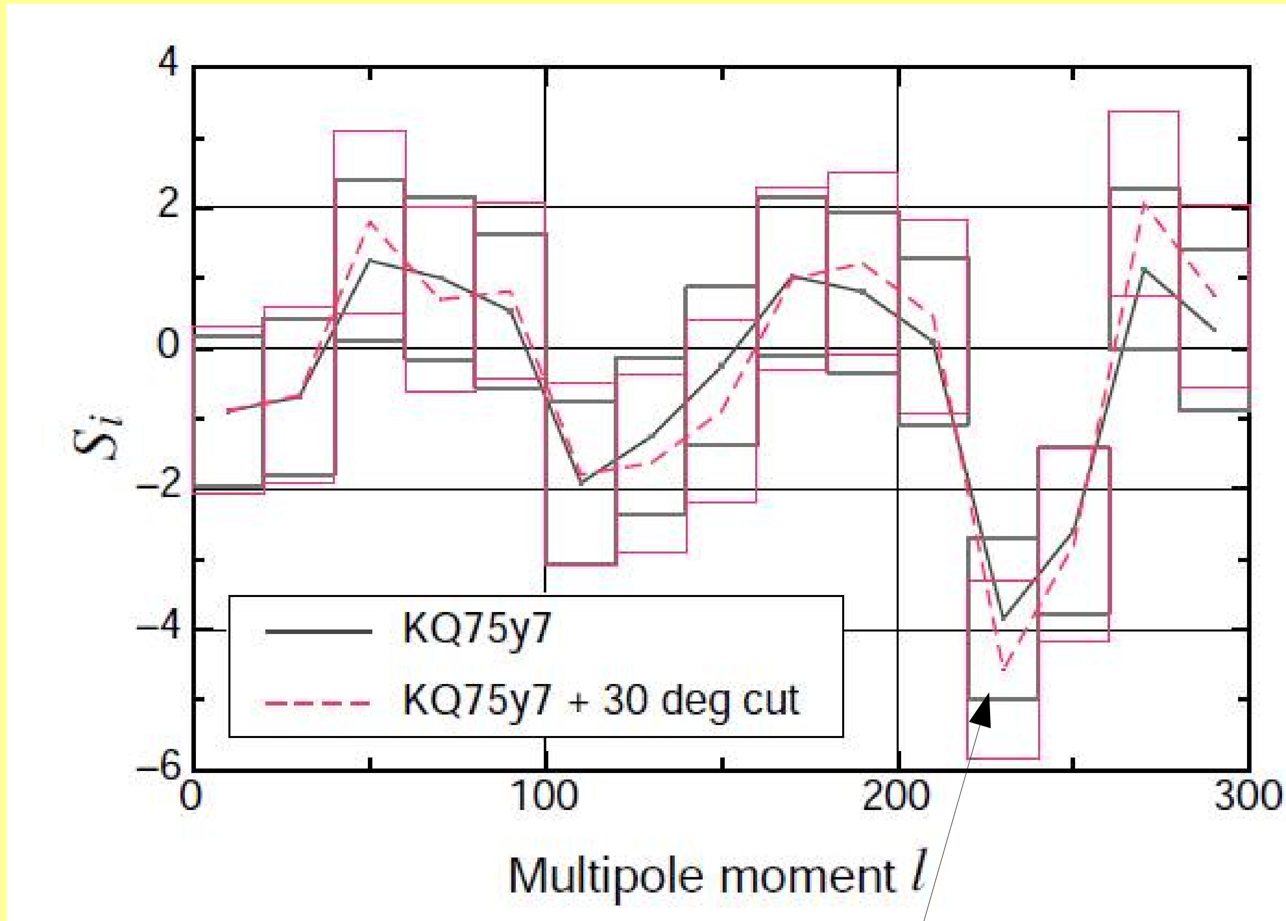


KI, in preparation

Monte-Carlo simulation results

Kashino, KI, Takeuchi, PRD, '12





We found the same tendency!

99.93% anomaly

summary

- In the analysis of CMB anisotropies, "zero mean" has been assumed implicitly (or by the Cosmo. Principle)
  - Zero mean should be confirmed by observation data themselves!
- We test this hypothesis using recent WMAP and PLANCK temperature anisotropies maps
- We find a hint of deviation from the zero-mean hypothesis at $\ell \approx 230$ in both WMAP and PLANCK