Model Building for Lepton Mixing and Neutrino Masses

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Outline

- introduction: lepton mixing
- origin of mixing
- different approaches
  - finite, discrete, non-abelian symmetry G_f
  - G_f and CP
  - G_f and $m_{\rm lightest} = 0$
  - $G_f = SU(3)_L \times SU(3)_E \times O(3)_N$
- comments on models
- conclusions & outlook

Parametrization of lepton mixing

Parametrization ***(PDG)***

$$U_{PMNS} = \tilde{U}\ \mathrm{diag}(1, e^{i\alpha/2}, e^{i(\beta/2+\delta)})$$

with

$$\tilde{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

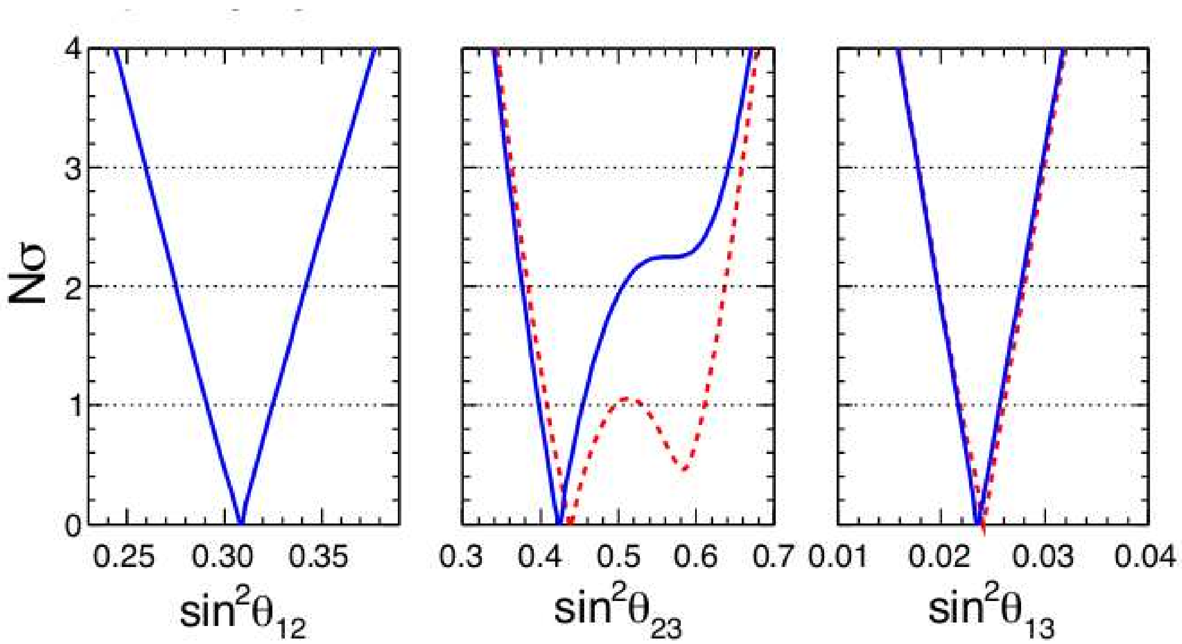
and $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$

Jarlskog invariant J_{CP}

$$\begin{aligned} J_{CP} &= \mathrm{Im}\left[U_{PMNS,11}U^*_{PMNS,13}U^*_{PMNS,31}U_{PMNS,33}\right] \\ &= \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta \end{aligned}$$

Experimental results on lepton mixing

Lepton mixing parameters as of end 2013 ***(Capozzi et al. ('13))***



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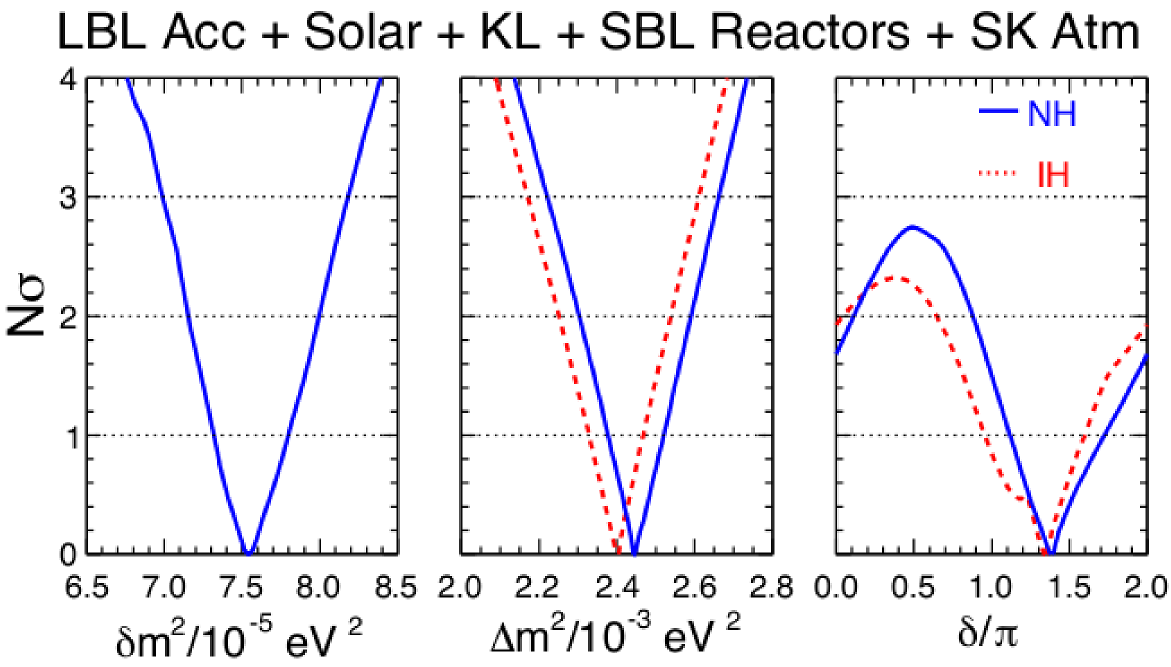

Experimental results on lepton mixing

Latest global fits NH [IH] ***(Capozzi et al. ('13))***

best fit and $1\,\sigma$ error	$3\,\sigma$ range
$\sin^2\theta_{13} = 0.0234[9]^{+0.0022[1]}_{-0.0018[21]}$	$0.0177[8] \leq \sin^2\theta_{13} \leq 0.0297[300]$
$\sin^2\theta_{12} = 0.308^{+0.017}_{-0.017}$	$0.259 \leq \sin^2\theta_{12} \leq 0.359$
$\sin^2\theta_{23} = \begin{cases} 0.425[37]^{+0.029[59]}_{-0.027[9]} \\ [0.531 \leq \sin^2\theta_{23} \leq 0.610] \end{cases}$	$0.357[63] \leq \sin^2\theta_{23} \leq 0.641[59]$
$\delta = 1.39[5]\,\pi^{+0.33[24]\,\pi}_{-0.27[39]\,\pi}$	$0 \leq \delta \leq 2\,\pi$
$\alpha\ ,\ \ \beta$	unconstrained

Experimental results on lepton mixing

Latest global fits NH [IH] ***(Capozzi et al. ('13))***

$$||U_{PMNS}|| \approx \begin{pmatrix} 0.82 & 0.55 & 0.15 \\ 0.40[39] & 0.65 & 0.64[5] \\ 0.40[2] & 0.52 & 0.75[4] \end{pmatrix}$$

and no information on Majorana phases

$$\Downarrow$$

Mismatch in lepton flavor space is large!

Origin of lepton mixing

- interpret this mismatch in lepton flavor space as mismatch of residual symmetries G_ν and G_e
- if we want to predict lepton mixing, we have to derive this mismatch
- let us assume that there is a symmetry, broken to G_ν and G_e
- this symmetry is in the following a

finite, discrete, non-abelian symmetry G_f

(Blum et al. ('07), Lam ('07,'08), de Adelhart Toorop et al. ('11))

[Masses do not play a role in this approach.]

Non-trivial breaking of G_f

Idea:

Derivation of the lepton mixing from how G_f is broken

Interpretation as mismatch of embedding of different subgroups G_ν and G_e into G_f

$$
\begin{array}{ccc}
 & G_f & \\
\swarrow & & \searrow \\
\text{neutrinos} & & \text{charged leptons} \\
G_\nu & & G_e
\end{array}
$$

Non-trivial breaking of G_f

Idea:

Derivation of the lepton mixing from how G_f is broken
Interpretation as mismatch of embedding of different subgroups G_ν and G_e into G_f

$$G_f$$

$$\swarrow \qquad \searrow$$

neutrinos

$G_\nu = Z_2 \times Z_2$ (Majorana)

$G_\nu = Z_M$ with $M \geq 3$ (Dirac)

charged leptons

$G_e = Z_N$ with $N \geq 3$

Non-trivial breaking of G_f

$$G_f$$

$$\swarrow \qquad \searrow$$

neutrinos

$G_\nu = Z_2 \times Z_2$ (Majorana)

$G_\nu = Z_M$ with $M \geq 3$ (Dirac)

charged leptons

$G_e = Z_N$ with $N \geq 3$

Further requirements

- two/three non-trivial angles $\Rightarrow$ irred. 3-dim. rep. of G_f
- fix angles through G_ν, G_e $\Rightarrow$ 3 families transform diff. under G_ν, G_e

Non-trivial breaking of G_f

- neutrino sector: $Z_2 \times Z_2$ or Z_M, $M \geq 3$, preserved and generated by

$$\Omega_\nu^\dagger \, Z_i \, \Omega_\nu = Z_i^{diag} \; , \quad i = 1, 2$$

or $$\Omega_\nu^\dagger \, Z \, \Omega_\nu = Z^{diag} \quad \text{with} \quad \Omega_\nu \quad \text{unitary}$$

- charged lepton sector: Z_N, $N \geq 3$, preserved and generated by

$$\Omega_e^\dagger \, Q_e \, \Omega_e = Q_e^{diag} \quad \text{with} \quad \Omega_e \quad \text{unitary}$$

Non-trivial breaking of G_f

- neutrino sector: $Z_2 \times Z_2$ or Z_M, $M \geq 3$, preserved

$$Z_i^T \, m_\nu \, Z_i = m_\nu \ , \quad i = 1, 2$$

or

$$Z^\dagger \, m_\nu^\dagger m_\nu \, Z = m_\nu^\dagger m_\nu$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

$$Q_e^\dagger \, m_e^\dagger m_e \, Q_e = m_e^\dagger m_e$$

Non-trivial breaking of G_f

- neutrino sector: $Z_2 \times Z_2$ or Z_M, $M \geq 3$, preserved

  $\rightarrow$ neutrino mass matrix m_ν fulfills

  $\Omega_\nu^T \, m_\nu \, \Omega_\nu$ is diagonal

  or $\Omega_\nu^\dagger \, m_\nu^\dagger m_\nu \, \Omega_\nu$ is diagonal

- charged lepton sector: Z_N, $N \geq 3$, preserved

  $\rightarrow$ charged lepton mass matrix m_e fulfills

  $\Omega_e^\dagger \, m_e^\dagger m_e \, \Omega_e$ is diagonal

Non-trivial breaking of G_f

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu$$

- 3 unphysical phases are removed by $\Omega_e \to \Omega_e K_e$
- neutrino masses are made real and positive through $\Omega_\nu \to \Omega_\nu K_\nu$
- permutations of columns of Ω_e, Ω_ν are possible: $\Omega_{e,\nu} \to \Omega_{e,\nu} P_{e,\nu}$

$$\Downarrow$$

Predictions:

Mixing angles up to exchange of rows/columns

J_{CP} up to sign

Majorana phases undetermined

Some examples

- tri-bimaximal (TB) mixing from S_4 ***(Lam ('07,'08))***

$$||U_{PMNS}|| = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Some examples

- tri-bimaximal (TB) mixing from S_4 ***(Lam ('07,'08))***
- generators S, T and U of S_4

$$S^2 = 1 \ , \ \ T^3 = 1 \ , \ \ U^2 = 1 \ ,$$
$$(ST)^3 = 1 \ , \ \ (SU)^2 = 1 \ , \ \ (TU)^2 = 1 \ , \ \ (STU)^4 = 1$$

Some examples

- tri-bimaximal (TB) mixing from S_4 ***(Lam ('07,'08))***
- generators S, T and U of S_4

$$S^2 = 1 \; , \;\; T^3 = 1 \; , \;\; U^2 = 1 \; ,$$
$$(ST)^3 = 1 \; , \;\; (SU)^2 = 1 \; , \;\; (TU)^2 = 1 \; , \;\; (STU)^4 = 1$$

- subgroups $G_e = Z_3$ and $G_\nu = Z_2 \times Z_2$

Some examples

- tri-bimaximal (TB) mixing from S_4 ***(Lam ('07,'08))***
- subgroups $G_e = Z_3$ and $G_\nu = Z_2 \times Z_2$
- subgroup $G_\nu = Z_2 \times Z_2$ generated by $Z_1 = S$ and $Z_2 = U$, diagonalized by Ω_ν

$$Z_1 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \; , \quad Z_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Some examples

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- subgroup $G_\nu = Z_2 \times Z_2$ generated by $Z_1 = S$ and $Z_2 = U$, diagonalized by Ω_ν

$$\Omega_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Some examples

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- subgroup $G_\nu = Z_2 \times Z_2$ generated by $Z_1 = S$ and $Z_2 = U$, diagonalized by Ω_ν
- subgroup $G_e = Z_3$ generated by $Q_e = T$, diagonalized by Ω_e

$$Q_e = \frac{1}{2} \begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ -1 & \sqrt{2} & -1 \end{pmatrix}$$

Some examples

- tri-bimaximal (TB) mixing from S_4 ***(Lam ('07,'08))***
- subgroups $G_e = Z_3$ and $G_\nu = Z_2 \times Z_2$
- subgroup $G_\nu = Z_2 \times Z_2$ generated by $Z_1 = S$ and $Z_2 = U$, diagonalized by Ω_ν
- subgroup $G_e = Z_3$ generated by $Q_e = T$, diagonalized by Ω_e

$$\Omega_e = \begin{pmatrix} \sqrt{2/3} & -1/\sqrt{6} & -1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 0 & i/\sqrt{2} & -i/\sqrt{2} \end{pmatrix}$$

Some examples

- tri-bimaximal (TB) mixing from S_4 ***(Lam ('07,'08))***
- subgroups $G_e = Z_3$ and $G_\nu = Z_2 \times Z_2$
- subgroup $G_\nu = Z_2 \times Z_2$ generated by $Z_1 = S$ and $Z_2 = U$, diagonalized by Ω_ν
- subgroup $G_e = Z_3$ generated by $Q_e = T$, diagonalized by Ω_e
- PMNS mixing matrix

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -i/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & i/\sqrt{2} \end{pmatrix}$$

Some examples

- series $\Delta(6n^2)$ of subgroups of $SU(3)$ with faithful irred. 3-dim. reps.
- isomorphic to $(Z_n \times Z_n) \rtimes S_3$; described with four generators
- generic form of mixing patterns for Majorana neutrinos

$$U_{PMNS} = U_{TB} R_{13}(\theta)$$

and θ depends on n ***(King et al. ('13))***, i.e. mixing angles are of the form

$$\sin^2\theta_{12} = \frac{1}{2+\cos 2\theta} \; , \quad \sin^2\theta_{23} = \frac{1}{2}\left(1 - \frac{\sqrt{3}\,\sin 2\theta}{2+\cos 2\theta}\right) \quad \text{and} \quad \sin^2\theta_{13} = \frac{2}{3}\sin^2\theta$$

$$\delta = 0 \, , \; \pi$$

Some examples

- series $\Delta(6n^2)$ of subgroups of $SU(3)$ with faithful irred. 3-dim. reps.
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- generic form of mixing patterns for Majorana neutrinos

$$U_{PMNS} = U_{TB} R_{13}(\theta)$$

  and θ depends on n ***(King et al. ('13))***

- we conjectured ***(de Adelhart Toorop et al. ('11))***

$$\theta = \frac{\pi}{n} \quad \text{for} \quad G_e = Z_3 \;\; , \; G_\nu = Z_2 \times Z_2$$
$$\theta = \frac{\pi}{3n} \quad \text{for} \quad G_e = Z_3 \;\; , \; G_\nu = Z_2 \times Z_2$$

Some examples

- series $\Delta(6n^2)$ of subgroups of $SU(3)$ with faithful irred. 3-dim. reps.
- isomorphic to $(Z_n \times Z_n) \rtimes S_3$; described with four generators
- generic form of mixing patterns for Majorana neutrinos

$$U_{PMNS} = U_{TB} R_{13}(\theta)$$

and θ depends on n ***(King et al. ('13))***

- we conjectured ***(de Adelhart Toorop et al. ('11))*** [for Dirac neutrinos]

$$\theta = \frac{\pi}{2n} \quad \text{for} \quad G_e = Z_3 \;\; , \; G_\nu = Z_{2n}$$

$$\theta = \frac{\pi}{6n} \quad \text{for} \quad G_e = Z_3 \;\; , \; G_\nu = Z_{2n}$$

Variants of non-trivial breaking of G_f

Reduce residual symmetry in charged lepton or neutrino sector

(Ge et al. ('11), Hernandez/Smirnov ('12,'13), Lavoura/Ludl ('14))

$$G_f$$

$$\swarrow \qquad\qquad \searrow$$

neutrinos | charged leptons

$G_\nu = Z_2$ | $G_e = Z_N$ with $N \geq 3$

$$\Downarrow$$

- G_ν cannot distinguish all three generations anymore
- only one column of PMNS mixing matrix is fixed;
  rest depends on free parameter

Origin of lepton mixing

- interpret this mismatch in lepton flavor space as mismatch of residual symmetries G_ν and G_e
- if we want to predict lepton mixing, we have to derive this mismatch
- let us assume that there is a symmetry, broken to G_ν and G_e
- this symmetry is in the following a combination of a

finite, discrete, non-abelian symmetry G_f and CP

(Feruglio et al. ('12,'13), Holthausen et al. ('12), Grimus/Rebelo ('95))

[Masses do not play a role in this approach.]

Non-trivial breaking of G_f and CP

Idea:

Relate lepton mixing to how G_f and CP are broken
Interpretation as mismatch of embedding of different subgroups G_ν and G_e into G_f and CP

G_f & CP

$\swarrow \qquad \searrow$

neutrinos — charged leptons

G_ν — G_e

Non-trivial breaking of G_f and CP

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Relate lepton mixing to how G_f and CP are broken
Interpretation as mismatch of embedding of different subgroups G_ν and G_e into G_f and CP

G_f & CP

$\swarrow$ $\searrow$

neutrinos
assume 3 generations
of Majorana neutrinos

charged leptons
distinguish 3 generations

Non-trivial breaking of G_f and CP

Idea:

Relate lepton mixing to how G_f and CP are broken
Interpretation as mismatch of embedding of different subgroups G_ν and G_e into G_f and CP

G_f & CP

$\swarrow \qquad \searrow$

neutrinos: $G_\nu = Z_2 \times$ CP

charged leptons: $G_e = Z_N$ with $N \geq 3$

An example: $\mu\tau$ reflection symmetry ***(Harrison/Scott ('02,'04), Grimus/Lavoura ('03))***

Non-trivial breaking of G_f and CP

G_f & CP

$\swarrow$ $\searrow$

neutrinos	charged leptons
$G_\nu = Z_2 \times$ CP	$G_e = Z_N$ with $N \geq 3$

Further requirements

- two/three non-trivial mixing angles $\Rightarrow$ irred 3-dim rep of G_f
- "maximize" predictability of approach

Non-trivial breaking of G_f and CP

Consistency conditions have to be fulfilled:

- definition of generalized CP transformation ***(see e.g. Branco et al. ('11))***

$$\phi_i \xrightarrow{\text{CP}} X_{ij}\phi_j^\star \quad \text{with} \quad XX^\dagger = XX^\star = \mathbb{1}$$

- "closure" relations

$$(X^\star A X)^\star = A' \quad \text{with in general} \quad A \neq A' \quad \text{and} \quad A,\, A' \in G_f$$

- realize direct product of $Z_2 \subset G_f$ and CP; Z generates Z_2

$$XZ^\star - ZX = 0$$

Non-trivial breaking of G_f and CP

- neutrino sector: $Z_2 \times$ CP preserved and generated by

$$X = \Omega_\nu \Omega_\nu^T \quad \text{and} \quad Z = \Omega_\nu Z^{diag} \Omega_\nu^\dagger$$
$$Z^{diag} = \text{diag}\,(-1, 1, -1) \quad \text{and} \quad \Omega_\nu \quad \text{unitary}$$

- charged lepton sector: Z_N, $N \geq 3$, preserved and generated by

$$\Omega_e^\dagger \, Q_e \, \Omega_e = Q_e^{diag} \quad \text{with} \quad \Omega_e \quad \text{unitary}$$

Non-trivial breaking of G_f and CP

- neutrino sector: $Z_2 \times$ CP preserved

$$Z^T m_\nu Z = m_\nu \quad \text{and} \quad X m_\nu X = m_\nu^\star$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

$$Q_e^\dagger \, m_e^\dagger m_e \, Q_e = m_e^\dagger m_e$$

Non-trivial breaking of G_f and CP

- neutrino sector: $Z_2 \times$ CP preserved

  $\rightarrow$ neutrino mass matrix m_ν fulfills

$$Z^{diag}[\Omega_\nu^T m_\nu \Omega_\nu] Z^{diag} = [\Omega_\nu^T m_\nu \Omega_\nu] \quad \text{and} \quad [\Omega_\nu^T m_\nu \Omega_\nu] = [\Omega_\nu^T m_\nu \Omega_\nu]^\star$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

  $\rightarrow$ charged lepton mass matrix m_e fulfills

$$\Omega_e^\dagger \, m_e^\dagger m_e \, \Omega_e \quad \text{is diagonal}$$

Non-trivial breaking of G_f and CP

- neutrino sector: $Z_2 \times$ CP preserved

  $\rightarrow$ neutrino mass matrix m_ν is diagonalized by

  $$\Omega_\nu(X, Z) R(\theta) K_\nu$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

  $\rightarrow$ charged lepton mass matrix m_e fulfills

  $$\Omega_e^\dagger \, m_e^\dagger m_e \, \Omega_e \quad \text{is diagonal}$$

Non-trivial breaking of G_f and CP

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu R(\theta) K_\nu$$

- 3 unphysical phases are removed by $\Omega_e \to \Omega_e K_e$
- U_{PMNS} contains one parameter θ
- permutations of rows and columns of U_{PMNS} possible

$$\Downarrow$$

Predictions:

Mixing angles and CP phases are predicted
in terms of one parameter θ only,
up to permutations of rows/columns

Some example: $\boldsymbol{S_4}$ and $\boldsymbol{CP}$

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and certain X

(Harrison/Scott ('02,'04), Grimus/Lavoura ('03), Feruglio et al. ('12,'13))

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2\cos\theta & \sqrt{2} & 2\sin\theta \\ -\cos\theta + i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta - i\sqrt{3}\cos\theta \\ -\cos\theta - i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta + i\sqrt{3}\cos\theta \end{pmatrix} K_\nu$$

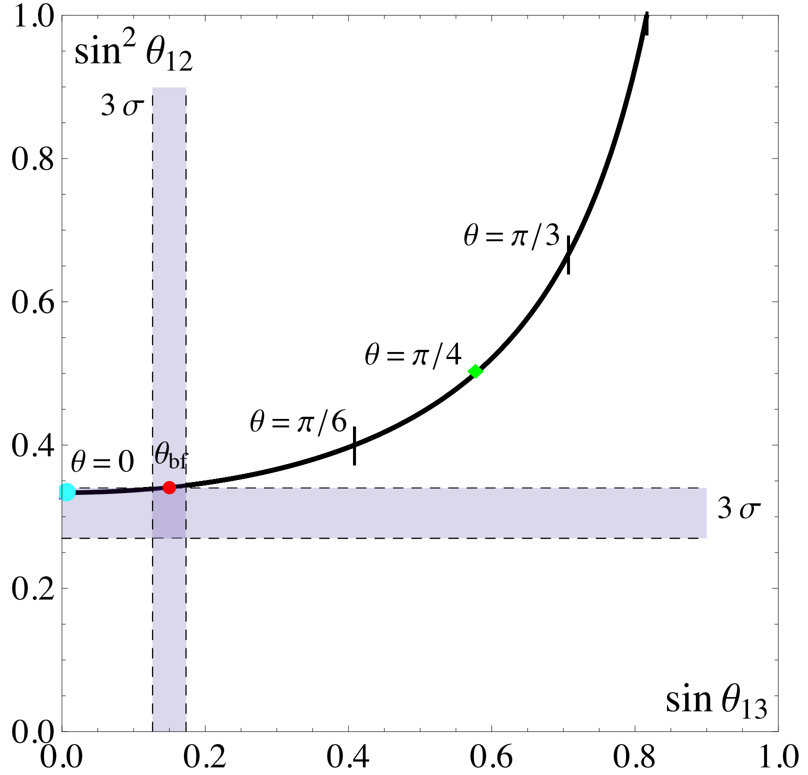
$$\sin^2\theta_{13} = \frac{2}{3}\sin^2\theta \; , \quad \sin^2\theta_{12} = \frac{1}{2+\cos 2\theta} \; , \quad \sin^2\theta_{23} = \frac{1}{2}$$

and

$$|\sin\delta| = 1 \;\; , \quad |J_{CP}| = \frac{|\sin 2\theta|}{6\sqrt{3}} \;\; , \quad \sin\alpha = 0 \;\; , \quad \sin\beta = 0$$

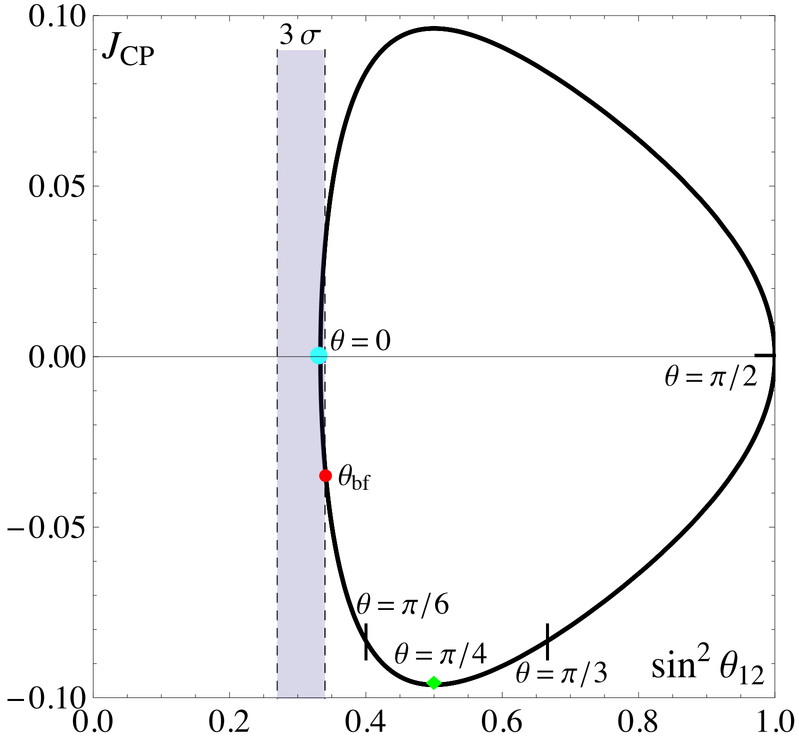
Some example: S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and certain X



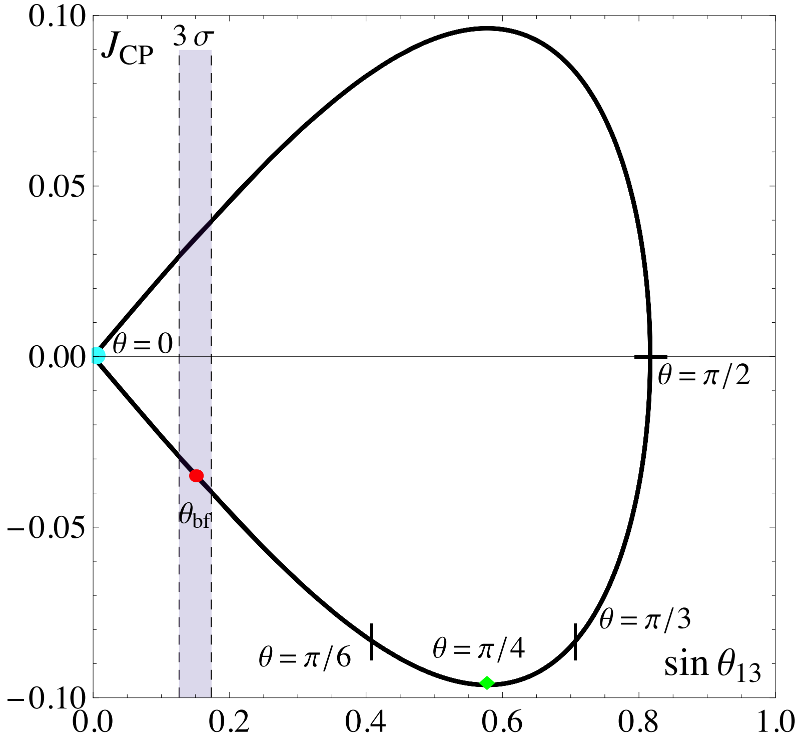
Some example: $\boldsymbol{S_4}$ and $\boldsymbol{CP}$

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and certain X



Some example: $\boldsymbol{S_4}$ and $\boldsymbol{CP}$

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and certain X



Some example: S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and certain X

$$\theta_{\text{bf}} \approx 0.185 \ , \quad \chi^2_{\text{min}} \approx 18.4 \quad \text{for} \quad \theta_{23} < \pi/4$$

$$\sin^2\theta_{13}(\theta_{\text{bf}}) \approx 0.023 \ , \quad \sin^2\theta_{12}(\theta_{\text{bf}}) \approx 0.341 \ ,$$
$$|J_{CP}(\theta_{\text{bf}})| \approx 0.0348$$

Origin of lepton mixing

- interpret this mismatch in lepton flavor space as mismatch of residual symmetries G_ν and G_e
- if we want to predict lepton mixing, we have to derive this mismatch
- let us assume that there is a symmetry, broken to G_ν and G_e
- this symmetry is in the following a

finite, discrete, non-abelian symmetry $G_f \subset U(3)$
leading to $m_{\text{lightest}} = 0$

(Joshipura/Patel ('13))

Non-trivial breaking of G_f and $m_{\text{lightest}} = 0$

$$G_f$$

$$\swarrow \qquad\qquad \searrow$$

Majorana neutrinos

$G_\nu = Z_2 \times Z_M$ with $M \geq 3$

$G_\nu = Z_M$ with $M \geq 3$ even

charged leptons

$G_e = Z_N$ with $N \geq 3$

Further requirements

- G_ν and G_e distinguish three generations
- $Z_M \subset G_\nu$, $M \geq 3$ forbids one of the three neutrino masses
  crucial: ordering of neutrino masses is not arbitrary anymore!

Some example

- inverted ordering and bi-maximal mixing from $G_f = S_4(2)$
- G_f is isomorphic to $A_4 \rtimes Z_4$
- group "similar" to S_4 also described with three generators
- take $G_e = Z_4$ and $G_\nu = Z_4$

$$Q_e = \begin{pmatrix} -i & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \;, \quad \tilde{Z} = \begin{pmatrix} 0 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & -i \end{pmatrix}$$

Some example

- inverted ordering and bi-maximal mixing from $G_f = S_4(2)$
- G_f is isomorphic to $A_4 \rtimes Z_4$
- group "similar" to S_4 also described with three generators
- take $G_e = Z_4$ and $G_\nu = Z_4$
- Ω_e is

$$\Omega_e = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & -i & i \\ 0 & 1 & 1 \end{pmatrix}$$

Some example

- inverted ordering and bi-maximal mixing from $G_f = S_4(2)$
- G_f is isomorphic to $A_4 \rtimes Z_4$
- group "similar" to S_4 also described with three generators
- take $G_e = Z_4$ and $G_\nu = Z_4$
- $\tilde{Z}$ has eigenvalues -1, +1, $-i$ and Ω_ν is

$$\Omega_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} -i & i & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}$$

Some example

- inverted ordering and bi-maximal mixing from $G_f = S_4(2)$
- G_f is isomorphic to $A_4 \rtimes Z_4$
- group "similar" to S_4 also described with three generators
- take $G_e = Z_4$ and $G_\nu = Z_4$
- PMNS mixing matrix

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu = \frac{1}{2} \begin{pmatrix} -\sqrt{2}\,i & \sqrt{2}\,i & 0 \\ i & i & \sqrt{2} \\ -i & -i & \sqrt{2} \end{pmatrix}$$

Origin of lepton mixing

- interpret this mismatch in lepton flavor space as mismatch of residual symmetries G_ν and G_e
- if we want to predict lepton mixing, we have to derive this mismatch
- let us assume that there is a symmetry, broken to G_ν and G_e
- this symmetry is in the following the

continuous symmetry $G_f = SU(3)_L \times SU(3)_E \times O(3)_N$

(Cirigliano et al. ('05), Alonso et al.('12), Alonso et al. ('13))

Non-trivial breaking of continuous G_f

- Lagrangian relevant for lepton masses, assuming type I seesaw mechanism

$$\bar{l}\, Y_E\, H\, E_R + \bar{l}\, Y_\nu\, \tilde{H}\, N_R + \frac{1}{2}\, N_R^t M N_R$$

- $Y_E \sim (3, \bar{3}, 1)$, $Y_\nu \sim (3, 1, 3)$ under G_f
- $M \propto \mathbb{1}$ leaves $O(3)_N$ invariant

Non-trivial breaking of continuous G_f

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- study of potential shows ("natural solutions")

$$\langle Y_E \rangle \propto \text{diag}\,(1,1,1)$$

or

$$\langle Y_E \rangle \propto \text{diag}\,(0,0,1)$$

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$$SU(3)_L \times SU(3)_E \quad \xrightarrow{\langle Y_E \rangle} \quad SU(3)_{L+E}$$

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"chiral" solution

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and

$$\langle Y_\nu \rangle \propto \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & -i \\ 0 & 1 & i \end{pmatrix}$$

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$$SU(3)_L \times SU(3)_E \times O(3)_N \quad \xrightarrow{\langle Y_E \rangle, \langle Y_\nu \rangle} \quad SU(2)_E \times U(1)_{L+N}$$

Non-trivial breaking of continuous G_f

- results
  - heavy tau lepton
  - three degenerate light neutrinos $\rightarrow$ large $0\nu\beta\beta$ signal
  - maximal mixing $\theta_{23} = \pi/4$, $\theta_{13} = 0$, θ_{12} free
  - maximal Majorana phase

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  - heavy tau lepton
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  - maximal mixing $\theta_{23} = \pi/4$, $\theta_{13} = 0$, θ_{12} free
  - maximal Majorana phase
- challenge: break the residual symmetry in controlled way in order to generate electron and muon mass, $\Delta m^2_{\rm atm}$, $\Delta m^2_{\rm sol}$, θ_{13} small
- heuristic: perturbations ***(Alonso et al. ('13))***
- probably additional spurions necessary (see quark sector ***(Espinosa et al. ('12), Fong/Nardi ('13))***)

Comments on models

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- construction of potential ("natural solutions"), stability under higher-order corrections (if G_f is spontaneously broken)
- non-canonical kinetic terms
- corrections from RG running
- after all, additional predictions can be possible in concrete models

Conclusions

- until now no "standard" theory for fermion masses and mixing, especially in the lepton sector
- flavor symmetries might be the key
  - finite, discrete, non-abelian G_f
  - G_f and CP
  - $G_f \subset U(3)$ and $m_{\text{lightest}} = 0$
  - $G_f = SU(3)_L \times SU(3)_E \times O(3)_N$

Outlook

- complete study of finite, discrete $SU(3)$ and $U(3)$ subgroups
- more examples and models with G_f and CP
- more on Dirac neutrinos?
- quarks and leptons described with same G_f?
- sterile neutrinos?

Thank you for your attention.