$B_s \to \mu^+\mu^-$
The Standard Model Prediction

Emmanuel Stamou

emmanuel.stamou@weizmann.ac.il

Weizmann Institute of Science

Aspen Center for Physics
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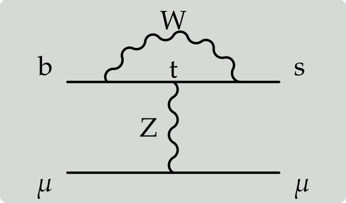
In collaboration with:

C. Bobeth, M. Gorbahn — arXiv:1311.1348

C. Bobeth, M. Gorbahn, T. Hermann, M. Misiak, M. Steinhauser — arXiv:1311.0903

$B_s \to \mu^+\mu^-$ within and beyond the SM



Within the SM

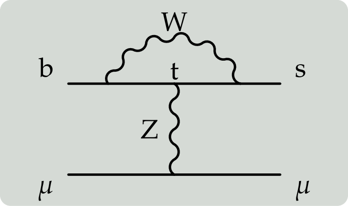
- FCNC ➜ loop-induced
- helicity suppressed ➜ BR $\propto \frac{m_\mu^2}{M_{B_s}^2}$
- CKM suppressed ➜ BR $\propto |V_{tb}^* V_{td}|^2 \propto \sin^4\theta_c \sim 0.0016$

rare decay BR ~ 10^{-9}

- single dim-6 vectorial operator $Q_{10} = [\bar{s}_L \gamma_\nu b_L][\bar{\mu}\gamma^\nu\gamma_5\mu]$
  (B_s pseudo-scalar, no γ-penguin, scalar operators suppressed)
- single hadronic quantity, decay constant f_{B_s} at the 2% level

theoretically clean decay

$B_s \to \mu^+\mu^-$ within and beyond the SM



Beyond the SM

- **sensitive** to scalar and pseudo-scalar operators in models with **extended Higgs sector** (2HDM, MSSM)

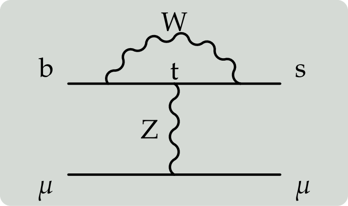
  i.e. in MSSM BR $\propto \tan^6 \beta$.

- constraints on effective Z couplings to quarks **comparable** with **electroweak precision** tests

  i.e. MFV, models with partial compositeness

[Haisch, Weiler '07, Guadagnoli, Isidori '13]

$B_s \to \mu^+\mu^-$ within and beyond the SM



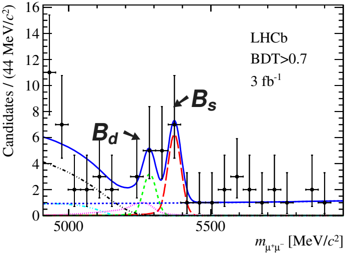
Experimental status

- LHCb and CMS measure the time-integrated BR

$$\mathbf{BR}(B_s \to \mu^+\mu^-) = (2.9 \pm 0.7) \times 10^{-9}$$
$$\mathbf{BR}(B_d \to \mu^+\mu^-) = (3.6^{+1.6}_{-1.4}) \times 10^{-10}$$

[CMS@(25fb) arXiv:1307.5025]

[LHCb@(3/fb) arXiv:1307.5024]



- in SM related to instantaneous $\mathrm{BR}^{[t=0]}$:

$$\mathrm{BR} = \frac{1}{1 - \tau_{B_s} \Delta\Gamma_s/2} \mathrm{BR}^{[t=0]}$$

[De Bruyn, Fleischer, Knegjens, Koppenburg, Merk ’12, arXiv:1204.1737]

Outline

- Recent development in SM prediction of $B_s \to \mu^+\mu^-$
  - ➜ **NNLO QCD corrections** **($1.8\% \to 0.2\%$)**

    [Hermann, Misiak, Steinhauser ’13, arXiv:1311.1347]
  - ➜ **NLO electroweak corrections** **($8\% \to 0.6\%$)**

    [Bobeth, Gorbahn, ES ’13, arXiv:1311.1348]

- **New** BR prediction in the SM
  - ➜ error budget

    [Bobeth, Gorbahn, Hermann, Misiak, ES, Steinhauser ’13, arXiv:1311.0903]

The multi-scales of B Decays

Λ

u,d,s,c,b,t
g,γ,W,Z,H

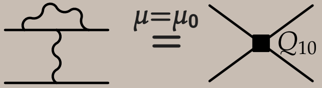
μ_0 — $\Lambda_{\mathbf{EW}} \simeq \mathcal{O}(100\mathrm{GeV})$

u,d,s,c,b
g,γ

μ_b — $M_{\mathbf{B}} \simeq \mathcal{O}(5\mathrm{GeV})$

$\mathcal{L}_{\mathrm{SM}}$

Matching



$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{QCD}} + \mathcal{L}_{\mathrm{QED}} + \sum_i \boldsymbol{C_i(\mu)} \underbrace{Q_i}_{\text{non-renorm.}}$

RGE

- simple for QCD, no mixing
  (Q_{10} conserved current)
- operator mixing under QED
  (mixing of Q_9, Q_2 & QCD penguins)

$\langle 0|\bar{s}\gamma_\nu\gamma_5 b|\overline{\mathrm{B}}_{\mathrm{s}}(\mathrm{p})\rangle = i f_{\mathbf{B_s}} p_\nu$ computed on the lattice

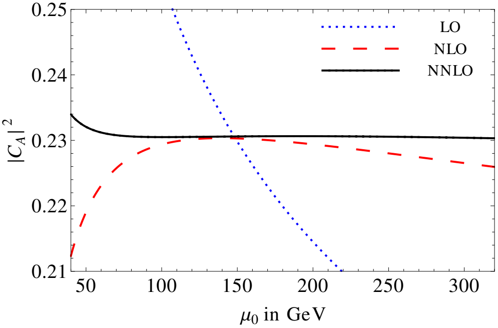
formally μ_0, μ_b independent
→ estimate of theo. uncertainty

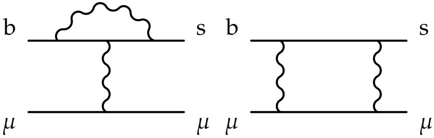
$$\mathbf{BR} \propto f^2_{\mathrm{B_s}} |C_{10}(\mu_b)|^2$$

NNLO QCD

$B_s \to \mu^+\mu^-$ at LO, NLO & NNLO QCD

$$C_{10} = G_F \frac{\alpha_{em}}{4\pi s_w^2}\left(c_{10}^{\mathrm{LO}} + \dots\right)$$





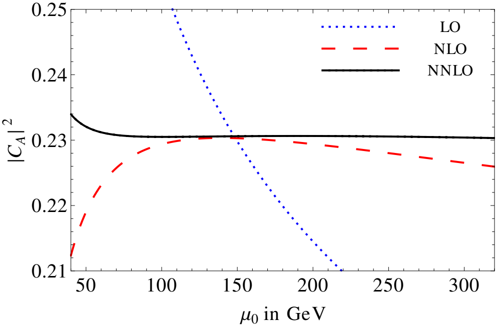
[Inami, Lim '81]

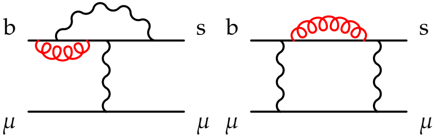
- neglect EW scale/scheme dependence for now

- LO has large μ_0 dependence from $m_t(\mu_0)$
- $m_t(\mu_0)$ $\overline{\mathrm{MS}}$ mass w.r.t. QCD and on-shell w.r.t. EW interactions
  ($m_t(50\mathrm{GeV}) = 180.8$ GeV, $m_t(163.5) = 163.5$ GeV, $m_t(300\mathrm{GeV}) = 156.2$ GeV)

$B_s \to \mu^+\mu^-$ at LO, NLO & NNLO QCD

$$C_{10} = G_F \frac{\alpha_{em}}{4\pi s_w^2}\left(c_{10}^{\mathrm{LO}} + \frac{\alpha_s}{4\pi} c_{10}^{\mathrm{NLO\,QCD}} + \ldots\right)$$





[Buchalla, Buras '93; Misiak, Urban '99]

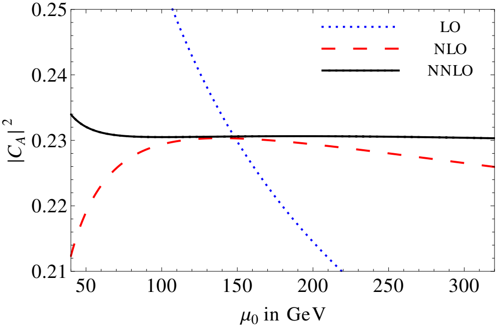
- neglect EW scale/scheme dependence for now

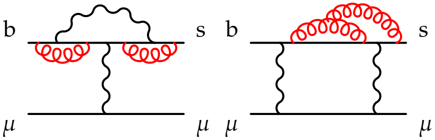
- @NLO strong reduction of scale uncertainty on BR (1.8%)
- @NLO good convergence for $\mu \simeq m_t$, $\sim +2.2\%$ on BR
- **1.8%NLO → 0.2%NNLO scale uncertainty on BR**

[Hermann, Misiak, Steinhauser '13, arXiv:1311.1347]

$B_s \to \mu^+\mu^-$ at LO, NLO & NNLO QCD

$$C_{10} = G_F \frac{\alpha_{em}}{4\pi s_w^2}\left(c_{10}^{\mathrm{LO}} + \frac{\alpha_s}{4\pi} c_{10}^{\mathrm{NLO\,QCD}} + \frac{\alpha_s^2}{(4\pi)^2} c_{10}^{\mathrm{NNLO\,QCD}} + \ldots \right)$$





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NLO EW

Turning on Electroweak Interactions

- so far we treated EW parameters as numbers

Questions

- which numbers?
  EW parameters G_F, M_Z^{pole}, M_W^{pole}, M_H^{pole}, M_t^{pole}, $\alpha_{em}^{\overline{\mathrm{MS}}}$, $s_w^{\overline{\mathrm{MS}}}$ are not all independent
- how does the BR depend on this choice (EW renormalisation scheme)?
  i.e. $s_w^{2\ \overline{\mathrm{MS}}} = 0.231$ VS $s_w^{2\ \mathrm{on-shell}} = 0.223$
- what about scale dependence?

➜ 8% uncertainty on the BR

- uncertainty was not included in SM prediction so far
- ambiguities removed at NLO in EW interactions

Electroweak Corrections I/II

Step 1: effective Lagrangian normalisation

$$\mathcal{L} = \frac{4G_F}{\sqrt{2}} \frac{\alpha_{em}}{4\pi s_w^2} \left(c_{10}^{\mathrm{LO}}\left(\frac{m_t^2}{M_W^2}\right) + \frac{\alpha_{em}}{4\pi} c_{10}^{\mathrm{EW}}(m_t, M_Z, s_w, \ldots) + \ldots \right)$$

$$\tilde{\mathcal{L}} = \frac{G_F^2 M_W^2}{\pi^2} \left(\tilde{c}_{10}^{\mathrm{LO}}\left(\frac{m_t^2}{M_W^2}\right) + \frac{\alpha_{em}}{4\pi} \tilde{c}_{10}^{\mathrm{EW}}(m_t, M_Z, s_w, \ldots) + \ldots \right)$$

- $\tilde{\mathcal{L}} \rightarrow$ no EW ambiguity at LO [Misiak '11, arXiv:1112.5978]
- only the product invariant under EW scheme

Step 2: choice of numerical input

$$G_F, \quad \alpha_{em}^{\overline{\mathrm{MS}}}, \quad M_Z^{\mathrm{pole}}, \quad M_H^{\mathrm{pole}}, \quad M_t^{\mathrm{pole}}$$

Step 3: EW renormalisation schemes (α_{em} always $\overline{\mathrm{MS}}$)

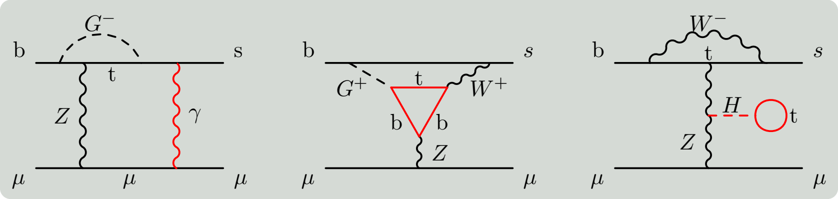
OS masses, $s_w^2 \equiv 1 - M_W^2/M_Z^2$

$\overline{\textbf{MS}}$ everything, "unbroken" parameters $v, g_1, g_2, y_t, \lambda$

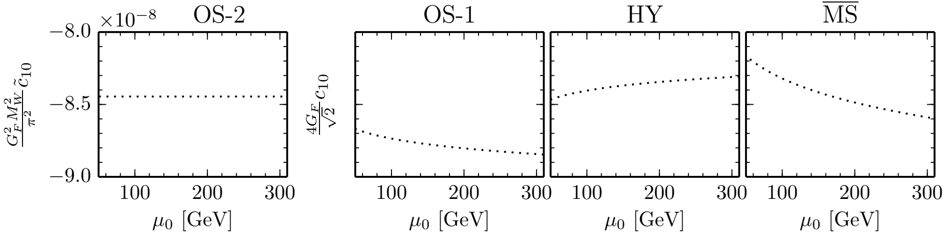
HY masses on-shell, $s_w^2\ \overline{\mathrm{MS}}$

Electroweak Corrections II/II

- 2-loop matching calculation in $R_\xi = 1$ gauge, divergences cancel,...



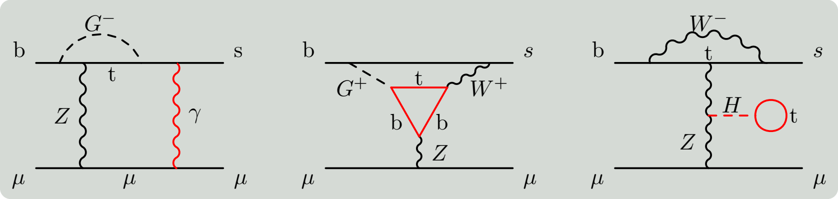
- here, switch off QCD and neglect mixing [Bobeth, Gorbahn, ES '13, arXiv:1311.1348]



- @LO large shifts → ±8% on BR
- @LO considerable μ_0-dependence in single G_F normalisation

Electroweak Corrections II/II

- 2-loop matching calculation in $R_\xi = 1$ gauge, divergences cancel,...



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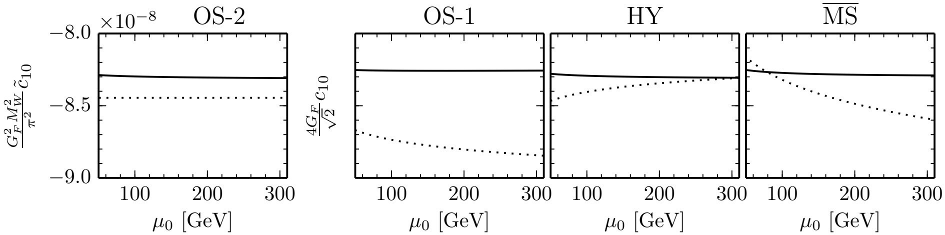
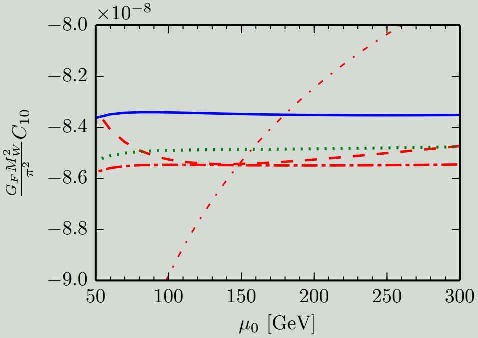


- @NLO prediction aligns in diff. schemes
- large shift in OS-1, better convergence in HY

- **EW scheme uncert.**
  ±8% → ±0.8%

LO + NLO QCD + NNLO QCD + Log QED + NLO EW

- Choose OS-2 as default scheme
- RGE evolution $C_{10}(\mu_b) = \sum_i U(\mu_b, \mu_0) C_i(\mu_0)$
- Log-enhanced QED corrections known [Bobeth, Gambino, Gorbahn, Haisch '03 hep-ph/0312090, Huber, Lunghi, Misiak, Wyler '05, hep-ph/0512066]
- **NLO EW reduce BR by 4% w.r.t. NNLO QCD**



for $\mu_b = 5$ GeV

Estimate of higher-order uncertainties

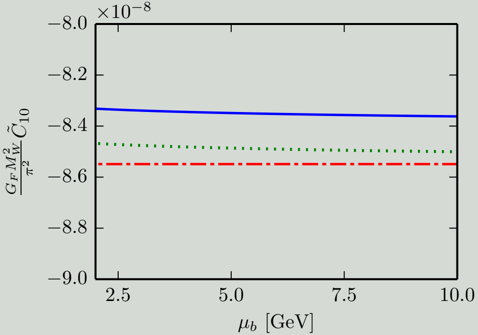
- μ_0 variation between $[m_t/2, 2m_t]$ ➜ **±0.2% (QCD) ±0.2% (EW)**
- residual scheme dependence from OS-2 & HY ➜ **±0.2%**

Residual μ_b Dependence

LO + NLO QCD + NNLO QCD + Log QED + NLO EW

- the Log enhanced QED corrections further reduce the BR
- the residual μ_b dependence cancels by yet **unknown** virtual QED corrections at μ_b.
- variation of μ_b gives a measure of uncertainty

$\pm$0.3% residual μ_b dependence

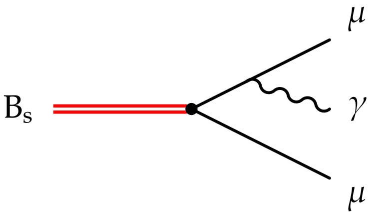


for $\mu_0 = 160$ GeV

Soft and Hard Photons

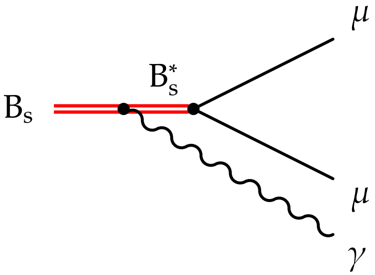
Soft photons from muons
Theoretical prediction is fully inclusive in bremsstrahlung. Otherwise sizeable corrections from phase-space cut.

[Buras, Girrbach, Guadagnoli, Isidori '12, arXiv:1208.0934]



Direct emission
Is phase-space supressed for invariant mass $m_{\mu\mu}$ close to M_{B_s}.

[Aditya, Healey, Petrov '12, arXiv:1212.4166]



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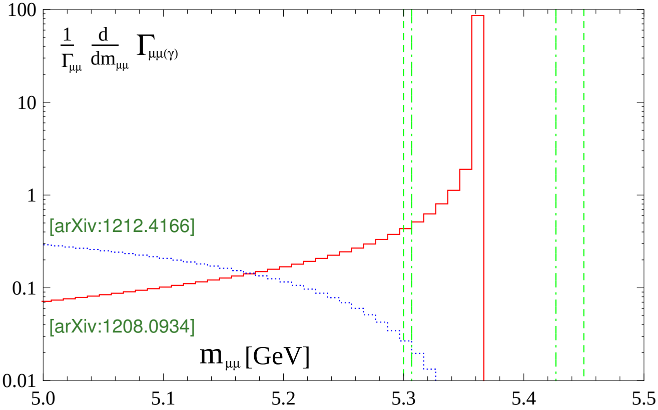

Suppose collaborations were using simple signal windows:

(they do not, analysis more involved)

- direct emission treated as background, tiny in signal window
- simulate signal fully inclusive in bremsstrahlung (PHOTOS)

$\mathbf{BR}(B_s \to \mu^+\mu^-)$

The new SM Prediction & Error Budget

$$\mathbf{BR}(B_s \to \mu^+\mu^-)^{\mathrm{exp}} = (2.9 \pm 0.7) \times 10^{-9}$$

[LHCb-CONF-2013-013,CMS-PAS-BPH-13-007]

$$\mathbf{BR}(B_s \to \mu^+\mu^-)^{\mathrm{SM}} = (3.65 \pm 0.23) \times 10^{-9}$$

[Bobeth, Gorbahn, Hermann, Misiak, ES, Steinhauser '13, arXiv:1311.0903]

Error Budget

f_{B_s}	CKM	τ_H^s	M_t	α_s	other param.	**non-param.**	$\sum$
4.0%	4.3%	1.3%	1.6%	0.1%	$< 0.1\%$	1.5%	6.4%

param.

- $f_{B_s} = 227.4(4.5)$ MeV

  [FLAG '13, arXiv:1310.8555]
- CKM from recent inclusive fit

  [Gambino, Schwanda '13, arXiv:1307.4551]

non param.

- 0.3% from $\mu_b \in [m_b/2, 2m_b]$
- $2 \times 0.3\%$ from $\mathcal{O}(\alpha_s^3, \alpha_{em}^2, \alpha_s\alpha_{em})$ for $\mu_0 \in [m_t/2, 2m_t]$
- 0.3% from top-mass conversion
- 0.5% additional uncertainties $(\mathcal{O}(m_b^2/M_W^2) + \ldots)$

Conclusions

We revisited and improved the SM prediction of the rare decay $B_s \to \mu^+\mu^-$

(actually of all $B_{s,d} \to \ell^+\ell^-$ decays)

- NNLO QCD corrections reduce μ_0 dependence from $m_t(\mu_0)$
  from 1.8%@NLO QCD $\to$ 0.2%@NNLO QCD
- NLO EW corrections reduce EW scheme dependence
  from 8%@LO $\to$ 0.6%@NLO EW
- the size of the NLO EW corrections is
  (3 – 5)% depending on μ_0 and scheme
- the theory uncertainty of the BR is
  $\leq$ 7% mainly from $f_{B_s}(4\%)$, $V_{cb}(4.3\%)$, non.-param.(1.5%)

The SM prediction is

$$\mathbf{BR}(B_s \to \mu^+\mu^-)^{\mathrm{SM}} = (3.65 \pm 0.23) \times 10^{-9}$$

in good agreement with the LHCb and CMS measurements.

Backup

The Branching Ratio of $B_s \to \mu^+\mu^-$

- LHCb, CMS report average time-integrated BR
- within the SM it is related to instantaneous $\mathrm{BR}^{[t=0]}$:

$$\mathrm{BR} = \frac{1}{1-\tau_{B_s}\Delta\Gamma_s/2}\mathrm{BR}^{[t=0]}$$

[De Bruyn, Fleischer, Knegjens, Koppenburg, Merk '12, arXiv:1204.1737]

$$\mathrm{BR}(B_s \to \mu^+\mu^-) = \frac{M_{B_s}^3 f_{B_s}^2}{32\pi\Gamma_H^s}\beta r^2 |V_{tb}^* V_{ts}|^2 \left|\frac{G_F^2 M_W^2}{\pi^2} C_{10}(\mu_b)\right|^2 + \mathcal{O}(\alpha_{em})$$

with $r = 2m_\mu/M_{B_s}$ and $\beta = \sqrt{1-r^2}$

r^2: helicity suppression
$1/\Gamma_H$: effect of $B_s^0 - \bar{B}_s^0$ mixing
f_{B_s}: decay constant from lattice input
$|\ldots|^2$: perturbative Wilson coefficient