

$B \rightarrow K^* l^+ l^-$: SM or beyond?

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Aspen Winter 2014

22 January 2014

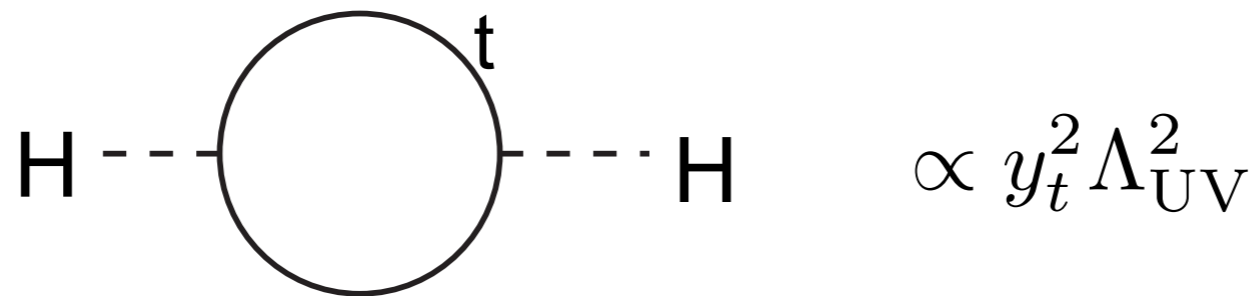
largely based on work with J Martin Camalich: arXiv:1212.2263, JHEP,
and work to appear

Content

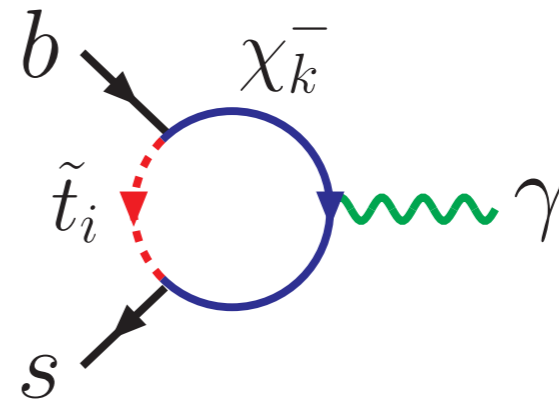
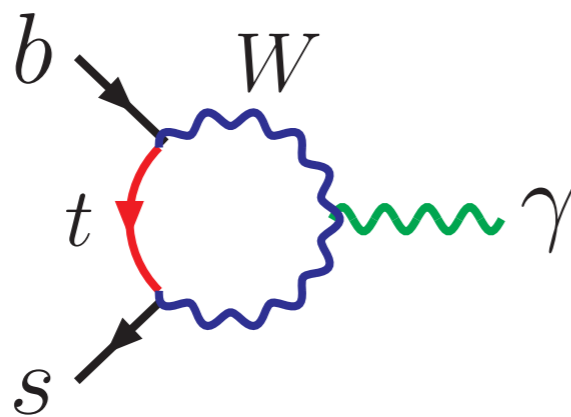
- Rare semileptonic B decays: BSM sensitivity and SM bugbears
- QCD anatomy of the decay amplitude
- BSM sensitivity at (very) low q^2
- LHCb anomaly: SM or beyond?

Why rare B decays

Solutions to the hierarchy problem must bring in particles to cut off the top contribution to the weak scale (Higgs mass parameter).



The new particles' couplings tend to break flavour (they do in all the major proposals for TeV physics)



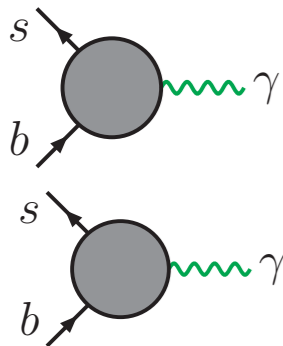
At least they will have CKM-like flavour violations (minimal flavour violation), so will always affect rare decays

weak $\Delta B=\Delta S=1$ Hamiltonian

= EFT for $\Delta B=\Delta S=1$ transitions (up to dimension six)

$$\mathcal{H}_{\text{eff}}^{\text{had}} = \frac{4G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3\dots 6} C_i P_i + C_{8g} Q_{8g} \right] \quad C_i \sim g_{\text{NP}} \frac{m_W^2}{M_{\text{NP}}^2}$$

$$\mathcal{H}_{\text{eff}}^{\text{sl}} = -\frac{4G_F}{\sqrt{2}} \lambda_t \left[C_7 Q_{7\gamma} + C'_7 Q'_{7\gamma} + C_9 Q_{9V} + C'_9 Q'_{9V} + C_{10} Q_{10A} + C'_{10} Q'_{10A} + C_S Q_S + C'_S Q'_S + C_P Q_P + C'_P Q'_P + C_T Q_T + C'_T Q'_T \right].$$

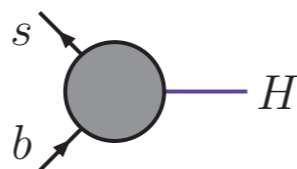


$$\mathcal{O}_7 = \frac{e}{16\pi^2} \hat{m}_b \bar{s} \sigma_{\mu\nu} P_R F^{\mu\nu} b,$$

$$\mathcal{O}_V = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s} \gamma_\mu P_L b) (\bar{l} \gamma^\mu l),$$

$$\mathcal{O}_S = \frac{\alpha_{\text{em}}}{4\pi} \hat{m}_b (\bar{s} P_R b) (\bar{l} l),$$

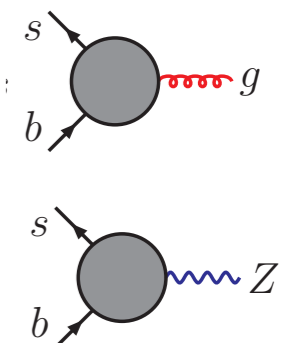
$$\mathcal{O}_T = \frac{\alpha_{\text{em}}}{4\pi} \hat{m}_b (\bar{s} \sigma_{\mu\nu} P_R b) (\bar{l} \sigma^{\mu\nu} P_R l),$$



$$\mathcal{O}_8 = \frac{g_s}{16\pi^2} \hat{m}_b \bar{s} \sigma_{\mu\nu} P_R G^{\mu\nu} b,$$

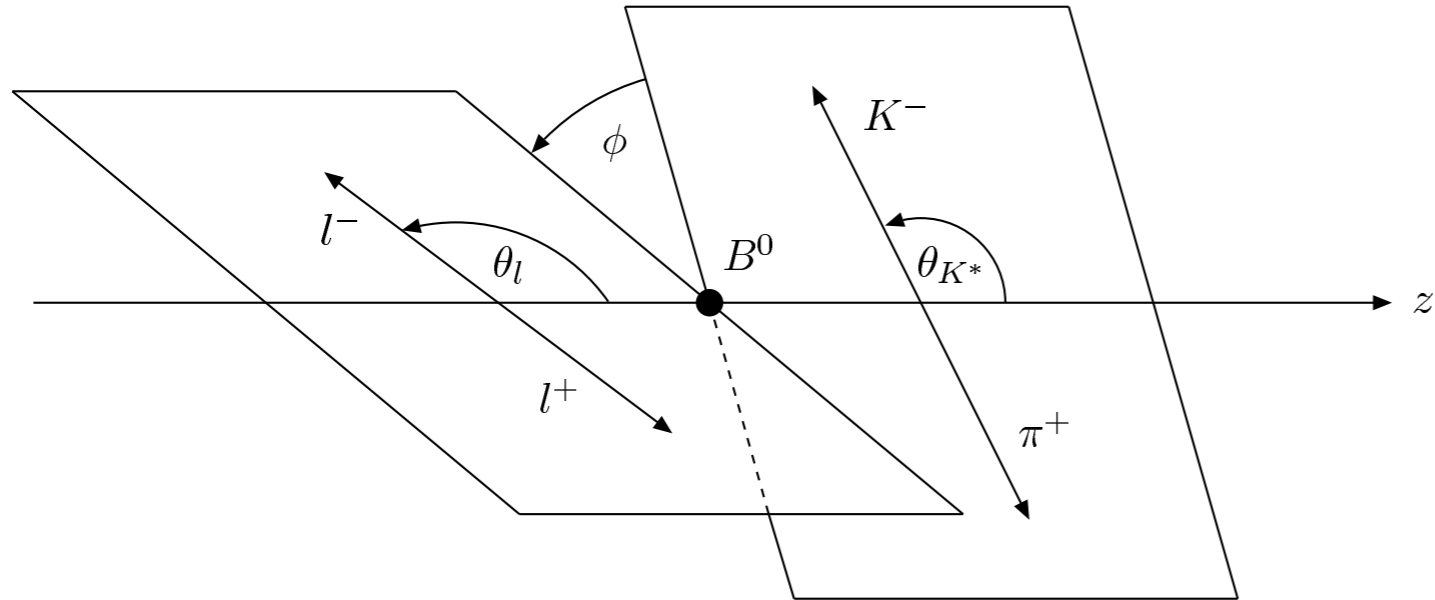
$$\mathcal{O}_A = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s} \gamma_\mu P_L b) (\bar{l} \gamma^\mu \gamma^5 l)_A$$

$$\mathcal{O}_P = \frac{\alpha_{\text{em}}}{4\pi} \hat{m}_b (\bar{s} P_R b) (\bar{l} \gamma^5 l),$$



look for observables sensitive to C_i 's, specifically those that are suppressed in the SM

$B \rightarrow K^* l \bar{l}$: angular distribution



θ_K in K^* rest frame

θ_l in dilepton cm frame

ϕ boost-invariant (w.r.t. z axis)

fig. Krueger, Matias 2002

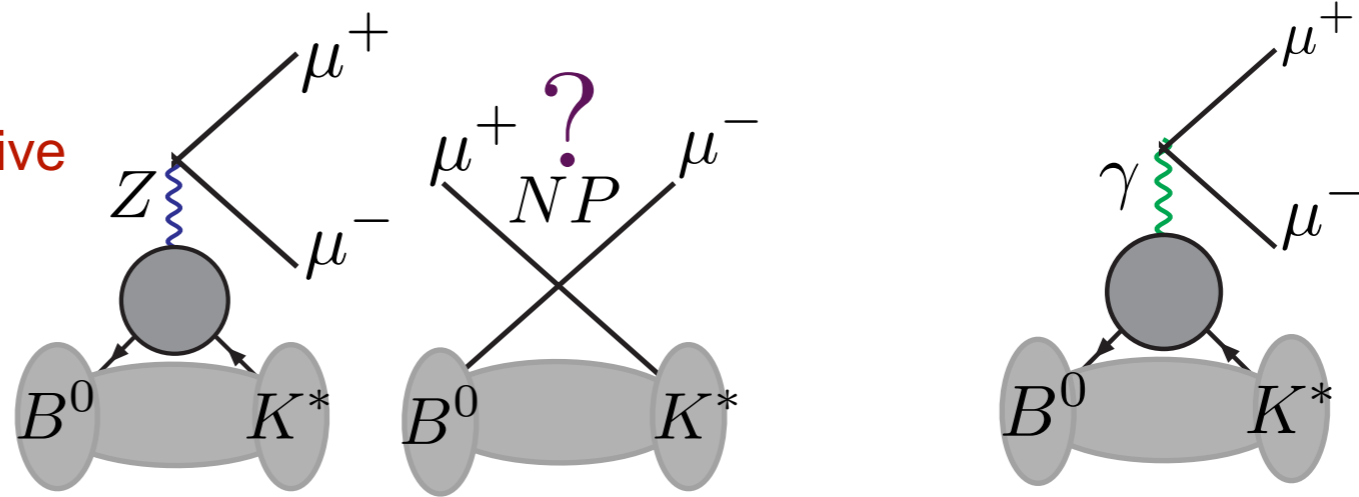
$$\frac{d^{(4)}\Gamma}{dq^2 d(\cos \theta_l) d(\cos \theta_k) d\phi} = \frac{9}{32\pi}$$

$$\begin{aligned} & \times \left(I_1^s \sin^2 \theta_k + I_1^c \cos^2 \theta_k + (I_2^s \sin^2 \theta_k + I_2^c \cos^2 \theta_k) \cos 2\theta_l \right. \\ & + I_3 \sin^2 \theta_k \sin^2 \theta_l \cos 2\phi + I_4 \sin 2\theta_k \sin 2\theta_l \cos \phi \\ & + I_5 \sin 2\theta_k \sin \theta_l \cos \phi + (I_6^s \sin^2 \theta_k + I_6^c \cos^2 \theta_K) \cos \theta_l \\ & \left. + I_7 \sin 2\theta_k \sin \theta_l \sin \phi + I_8 \sin 2\theta_k \sin 2\theta_l \sin \phi + I_9 \sin^2 \theta_k \sin^2 \theta_l \sin 2\phi \right) \end{aligned}$$

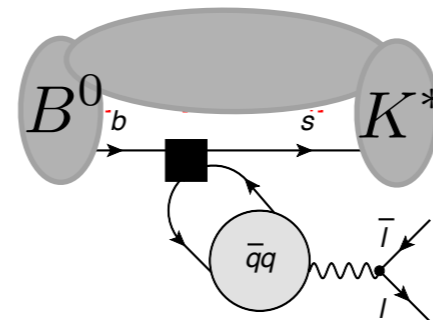
The angular coefficients are functions of the Wilson coefficients, and can be used to probe for new physics

$B \rightarrow K^* l^+ l^-$ decay amplitude

matrix elements of semileptonic/radiative Hamiltonian factorize “naively”



$$\mathcal{A}(\bar{B} \rightarrow V l^- l^+) = \sum_i C_i \langle l^- l^+ | \bar{l} \Gamma_i l | 0 \rangle \langle V | \bar{s} \Gamma'_i b | \bar{B} \rangle + C_7^{(\prime)} \frac{e^2}{q^2} \langle l^+ l^- | \bar{l} \gamma^\mu l | 0 \rangle \langle V | \bar{s} \sigma_{\mu\nu} P_{R(L)} b | \bar{B} \rangle + \frac{e^2}{q^2} \langle l^- l^+ | \bar{l} \gamma^\mu l | 0 \rangle F.T. \langle V | T(j_{\mu,em}^{\text{had}}(x) \mathcal{H}_W^{\text{had}}(0)) | \bar{B} \rangle$$



nonlocal “quark loops”
do **not** factorize naively

correct to lowest order in electromagnetism
exact in QCD - no assumptions (yet)

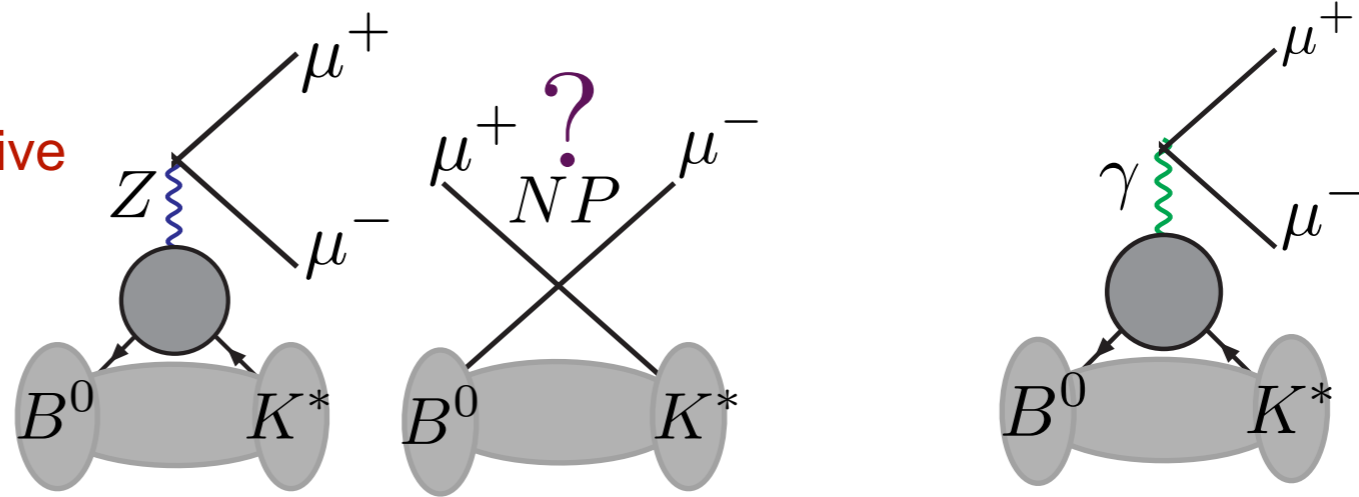
three helicity states for $V=K^*$
dilepton can have $J=0$ or $J=1$
several leptonic currents

} 7 (14) complex helicity amplitudes
in SM (BSM)

photon couples only to **vector** leptonic current. At $q^2 = 0$ photon pole

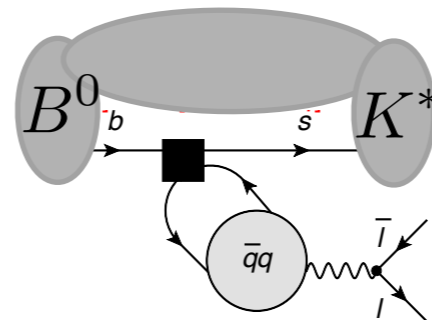
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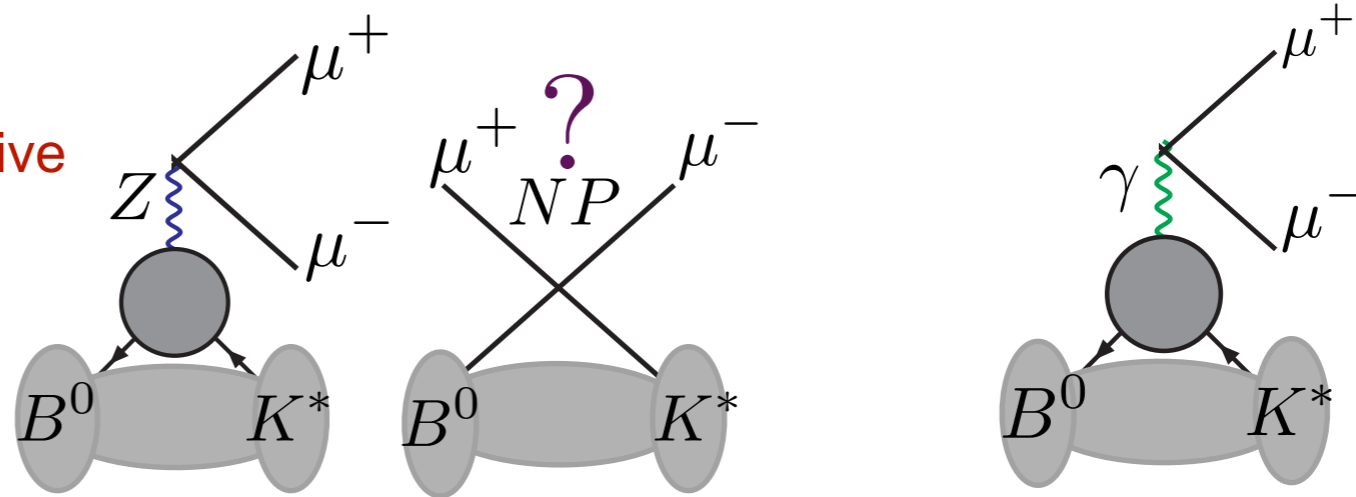
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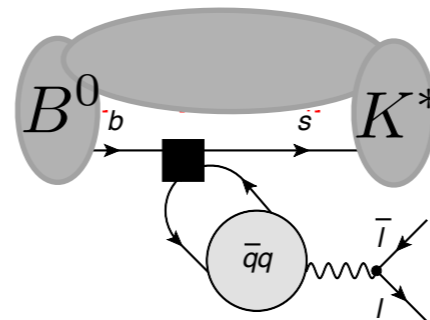
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 & + \frac{e^2}{q^2} \langle l^- l^+ | \bar{l} \gamma^\mu l | 0 \rangle F.T. \langle V | T(j_{\mu,em}^{\text{had}}(x) \mathcal{H}_W^{\text{had}}(0)) | \bar{B} \rangle
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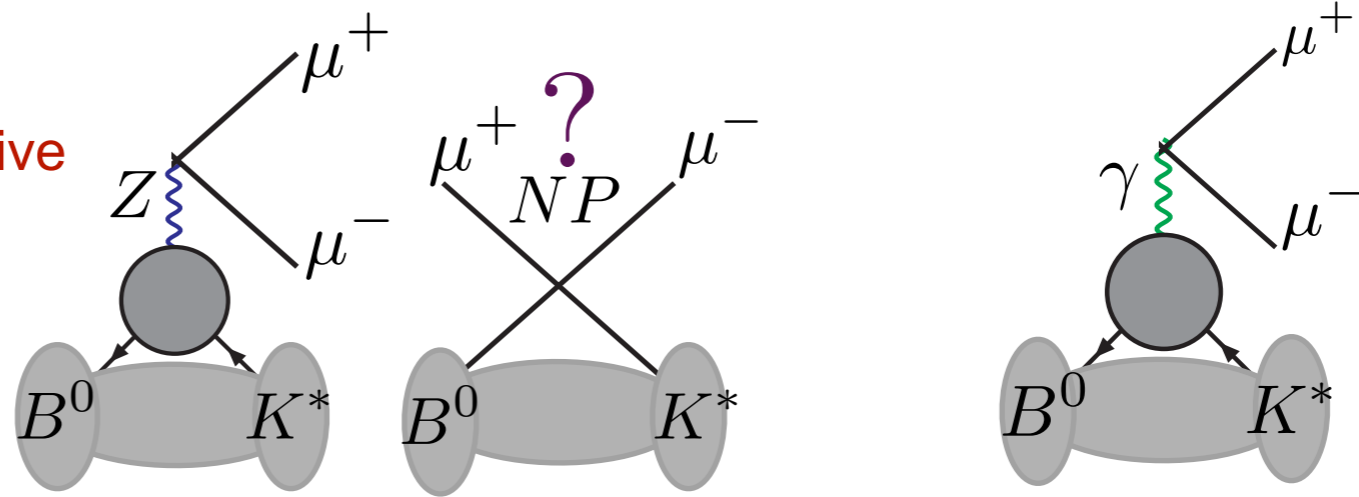
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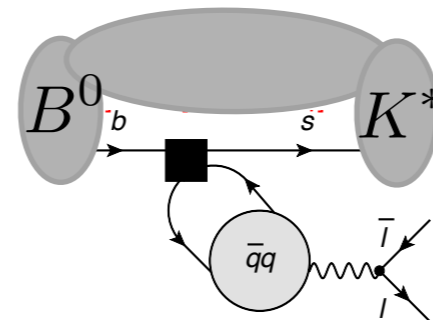
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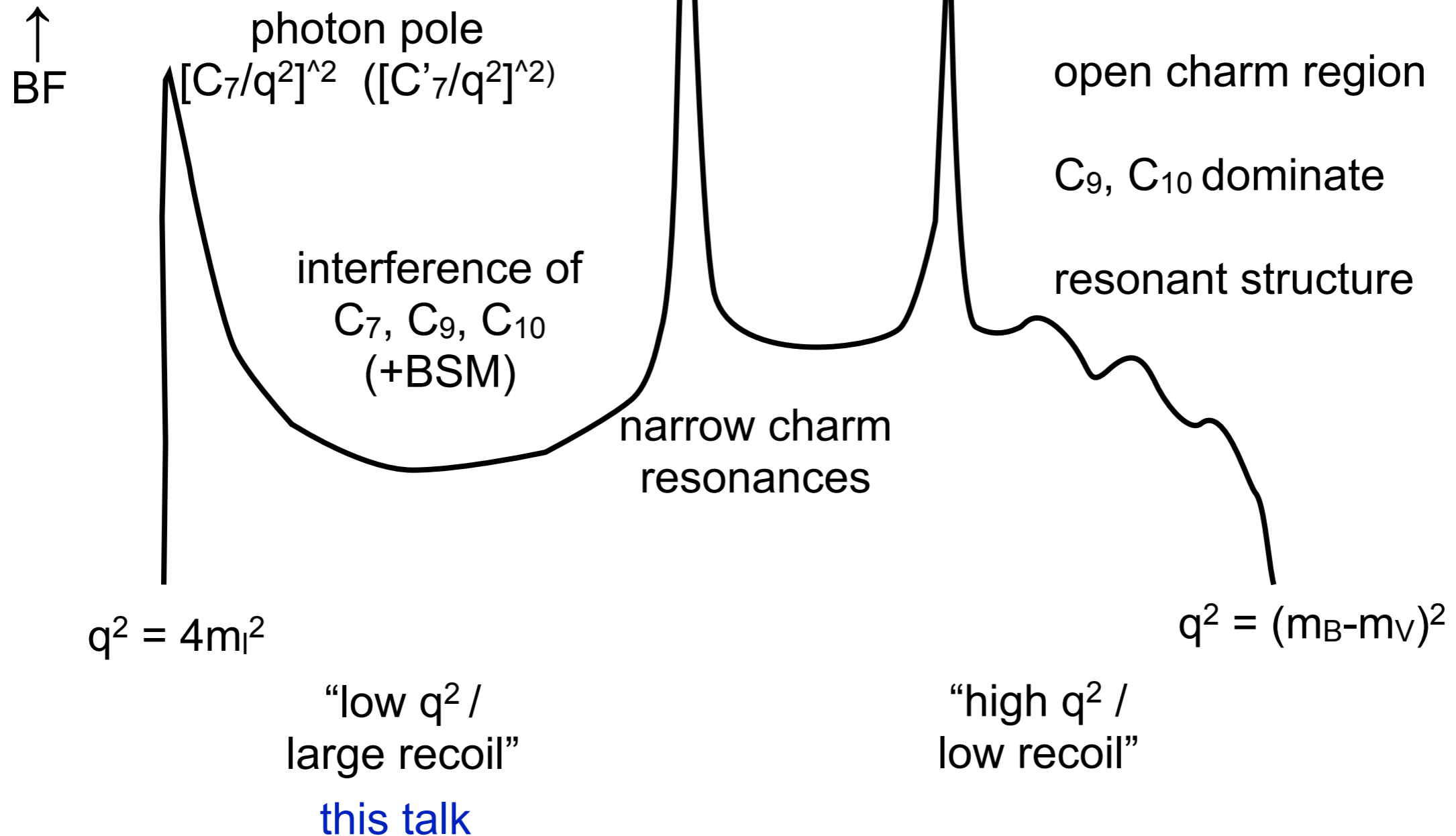
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Rate: q^2 dependence (qualitative)

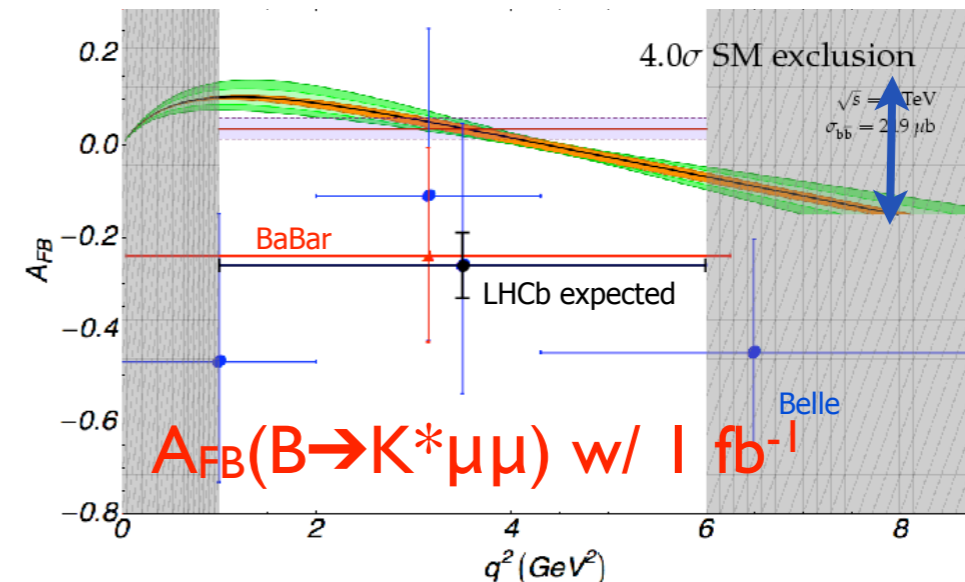
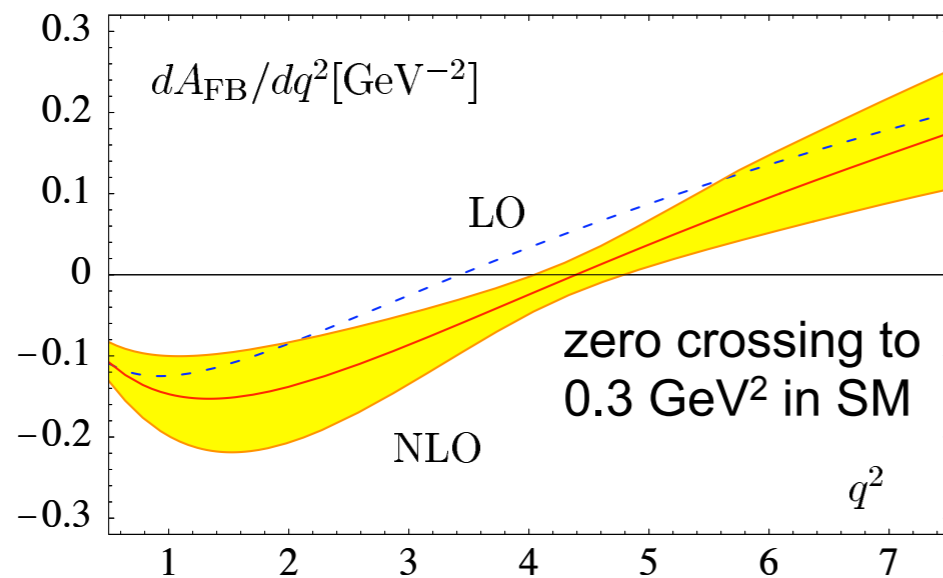


Note - artist's impression only.

LHCb has not yet published sufficiently fine binning to show the resonant features [open charm resonances are however visible in published $B \rightarrow K \ell \ell$ data]

Phenomenology issues

Examples of theory predictions - intentionally dated ones!
Here, forward-backward asymmetry



$$q_0^2[K^{*0}] = 4.36^{+0.33}_{-0.31} \text{ GeV}^2, \quad q_0^2[K^{*+}] = 4.15^{+0.27}_{-0.27} \text{ GeV}^2$$

Beneke et al Eur Phys J C 41 (2005) 173

F Muheim @ FPCP2010

our original motivation

- critically (re)examine **all** theory uncertainties, specifically power corrections: **separate parameterisation from estimation**

- Should one cut at low q^2 end? Costs sensitivity to C_7' , C_7
What is the residual error with a given set of cuts?

This is also (very) relevant to current "anomalies" in data (P_5') !

Angular coefficients

$$I_1^c = F \left\{ \frac{1}{2} (|H_V^0|^2 + |H_A^0|^2) + 4|H_P|^2 + \frac{2m_\ell^2}{q^2} (|H_V^0|^2 - |H_A^0|^2) + 4\beta^2 |H_S|^2 \right\},$$

$$I_1^s = F \left\{ \frac{\beta^2 + 2}{8} (|H_V^+|^2 + |H_V^-|^2 + (V \rightarrow A)) + \frac{m_\ell^2}{q^2} (|H_V^+|^2 + |H_V^-|^2 - (V \rightarrow A)) \right\}$$

$$I_2^c = -F \frac{\beta^2}{2} (|H_V^0|^2 + |H_A^0|^2),$$

$$I_2^s = F \frac{\beta^2}{8} (|H_V^+|^2 + |H_V^-|^2) + (V \rightarrow A),$$

$$I_3 = -\frac{F}{2} \text{Re} [H_V^+ (H_V^-)^*] + (V \rightarrow A),$$

$$I_4 = F \frac{\beta^2}{4} \text{Re} [(H_V^- + H_V^+) (H_V^0)^*] + (V \rightarrow A),$$

$$I_5 = F \left\{ \frac{\beta}{2} \text{Re} [(H_V^- - H_V^+) (H_A^0)^*] + (V \leftrightarrow A) - \frac{2\beta m_\ell}{\sqrt{q^2}} \text{Re} [H_S^* (H_V^+ + H_V^-)] \right\}$$

$$I_6^s = F\beta \text{Re} [H_V^- (H_A^-)^* - H_V^+ (H_A^+)^*],$$

$$I_6^c = 8F \frac{\beta m_\ell}{\sqrt{q^2}} \text{Re} [H_S^* H_V^0],$$

$$I_7 = F \left\{ \frac{\beta}{2} \text{Im} [(H_A^+ + H_A^-) (H_V^0)^*] + (V \leftrightarrow A) - \frac{2\beta m_\ell}{\sqrt{q^2}} \text{Im} [H_S^* (H_V^- - H_V^+)] \right\},$$

$$I_8 = F \frac{\beta^2}{4} \text{Im} [(H_V^- - H_V^+) (H_V^0)^*] + (V \rightarrow A),$$

$$I_9 = F \frac{\beta^2}{2} \text{Im} [H_V^+ (H_V^-)^*] + (V \rightarrow A),$$

strongly suppressed in SM
good sensitivity to NP with
different chirality structure
("right-handed currents")

Melikhov 1998
Krueger, Matias 2002
Lunghi, Matias 2006

suppression of I_3, I_9 due to
suppression of +-amplitudes
must quantify corrections

Heavy-quark limit and corrections

$$F(q^2) = F^\infty(q^2) + a_F + b_F q^2/m_B^2 + \mathcal{O}([q^2/m_B^2]^2)$$

heavy quark limit

Power corrections - parameterise

At most 1-2%
over entire 0..6
GeV² range ->
ignore

$$F^\infty(q^2) = F^\infty(0)/(1 - q^2/m_B^2)^p + \Delta_F(\alpha_s; q^2)$$

(Charles et al)

(Beneke, Feldmann)

For $\alpha_s=0$, q^2 dependence
fixed in heavy-quark limit (argument relies
on properties of vector light-cone DA)

$$\begin{aligned} V_+^\infty(0) = 0 & \quad T_+^\infty(0) = 0 & \text{from heavy-quark/} \\ V_-^\infty(0) = T_-^\infty(0) & & \text{large energy} \\ V_0^\infty(0) = T_0^\infty(0) & & \text{symmetry alone} \end{aligned}$$

Corrections are **unambiguously**
calculable in terms of perturbation
theory, decay constants, light cone
distribution amplitudes

$$V_+^\infty(q^2) = 0 \quad T_+^\infty(q^2) = 0$$

hence

$$\begin{aligned} T_+(q^2) &= \mathcal{O}(q^2) \times \mathcal{O}(\Lambda/m_b) \\ V_+(q^2) &= \mathcal{O}(\Lambda/m_b). \end{aligned}$$

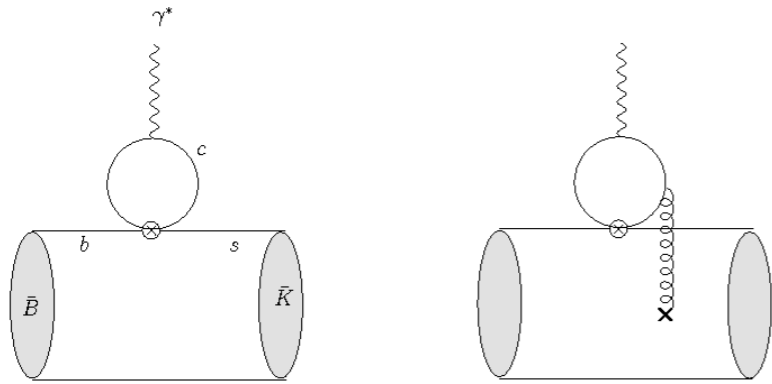
- “naively factorizing” part of the helicity amplitudes H_{V,A^+} strongly
suppressed as a consequence of chiral SM weak interactions

(quark picture: Burdman,
Hiller 1999)

- We see the suppression is **particularly strong** near low- q^2 endpoint

- Form factor relations imply reduced uncertainties in suitable observables

Nonfactorizable contributions



no known way to treat charm resonance region to the necessary precision (would need $\ll 1\%$ to see short-distance contribution)

“solution”: cut out $6 \text{ GeV}^2 < q^2 < 14 \text{ GeV}^2$

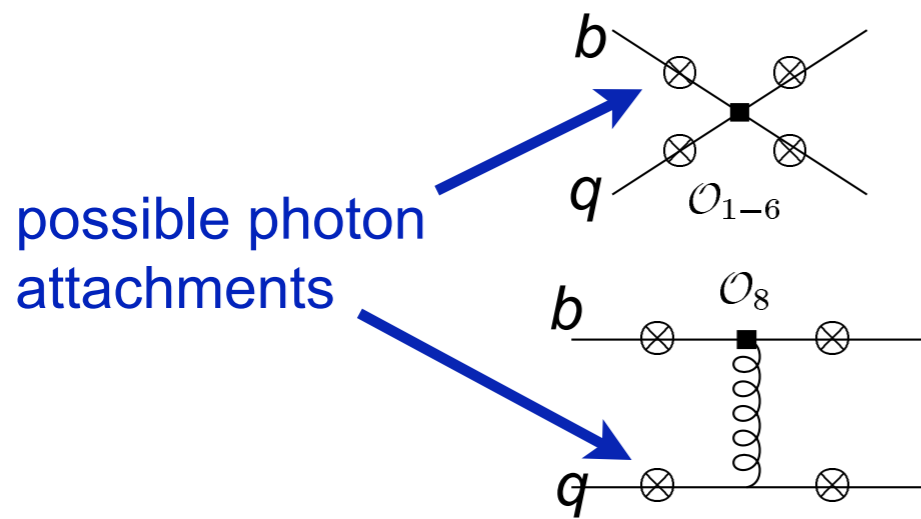
above (high- q^2) charm loops calculable in OPE

Grinstein, Pirjol 2004; Beylich, Buchalla, Feldmann 2011
phenomenological applications: Bobeth et al 2008-2013

at low q^2 , long-distance charm effects also suppressed, but photon can now be emitted from *spectator* without power suppression

systematic framework (QCD factorisation) based on $1/m_b$ expansion

Beneke, Feldmann, Seidel 01



small Wilson coefficients

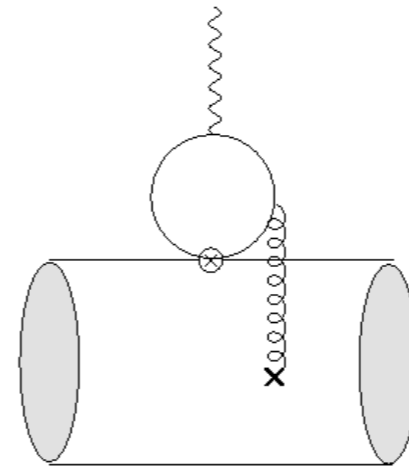
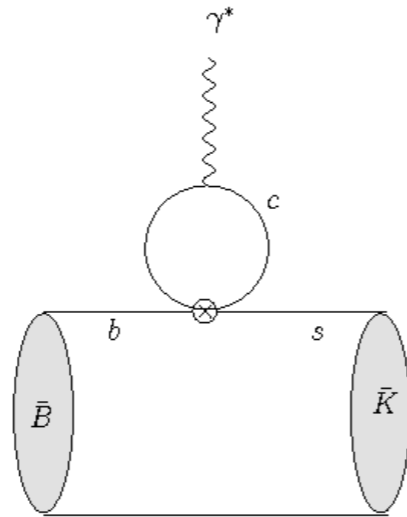
more significant for $b \rightarrow s$ transitions

$$\frac{\pi^2}{N_c} \frac{f_B f_{K^*,a}}{M_B} \Xi_a \sum_{\pm} \int \frac{d\omega}{\omega} \Phi_{B,\pm}(\omega) \int_0^1 du \Phi_{K^*,a}(u) T_{a,\pm}(u, \omega)$$

light-cone wave functions

calculable

“Charm loop” (operators with charm)



leading-power: factorises into perturbative kernels, form factors, LCDA's (including hard/hard-collinear gluon corrections to all orders)

$$\alpha_s^0 : C_7 \rightarrow C_7^{\text{eff}}$$

$$C_9 \rightarrow C_9^{\text{eff}}(q^2)$$

+ 1 annihilation diagram

α_s^1 : (convergent) convolutions of hard-scattering kernels with meson light cone-distribution amplitudes

Beneke, Feldmann, Seidel 2001

state-of-the-art in phenomenology

unambiguous (save for parametric uncertainties)

at subleading powers:
breakdown of factorisation

partial estimates as end-point divergent convolutions with a cut-off [Feldmann, Matias](#)

can perform light-cone OPE of charm loop & estimate resulting (nonlocal) operator matrix elements by light-cone QCD sum rules

[Khodjamirian et al 2010](#)

one can show that the helicity suppression of H_V^+ **survives** long-distance corrections

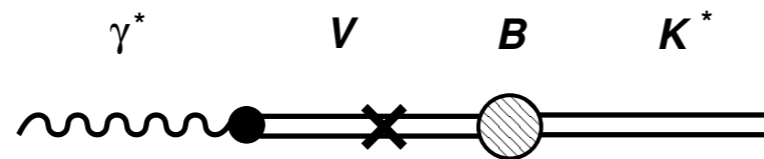
[SJ, J Martin Camalich 2012](#)

Light-quark contributions

Operators without charm have strong charm or CKM suppression; power corrections should be negligible.

However, they generate (mild) resonance structure even below the charm threshold, ie “duality violation”

Presumably ρ, ω, ϕ most important; use vector meson dominance supplemented by heavy-quark limit $B \rightarrow VK^*$ amplitudes



$$\tilde{a}_\mu^{\text{had, lq}} = \int d^4x e^{-iq \cdot x} \sum_{P, P'} \langle 0 | j_\mu^{\text{em, lq}}(x) | P' \rangle \langle P'(x) | P(0) \rangle \langle \bar{K}^* P | \mathcal{H}_{\text{eff}}^{\text{had}}(0) | \bar{B} \rangle$$

estimate **uncertainty** from difference between VMD model and the subset of heavy-quark limit diagrams corresponding to intermediate V states.

Helicity hierarchies in **hadronic** B decays prevent large uncertainties in H_V^+ from this source, too.

Phenomenology

Useful to consider functions of the angular coefficients for which form factors drop out in the heavy quark limit (ie neglecting power corrections) if perturbative QCD corrections are also neglected.

Becirevic, Schneider 2011
Matias, Mescia, Ramon, Virto 2012
Descotes-Genon et al 2012
(also Krueger, Matias 2005; Egede et al 2008, Altmannshofer et al 2008)

$$P_1 = \frac{\Sigma_3}{2\Sigma_{2s}},$$

$$P_2 = \frac{\Sigma_6}{8\Sigma_{2s}},$$

$$P_3 = -\frac{\Sigma_9}{4\Sigma_{2s}},$$

$$P'_4 = \frac{\Sigma_4}{\sqrt{-\Sigma_{2s}\Sigma_{2c}}},$$

$$P'_5 = \frac{\Sigma_5}{2\sqrt{-\Sigma_{2s}\Sigma_{2c}}},$$

$$P'_6 = -\frac{\Sigma_7}{2\sqrt{-\Sigma_{2s}\Sigma_{2c}}}$$

$$\Sigma_i = \frac{I_i + \bar{I}_i}{2},$$

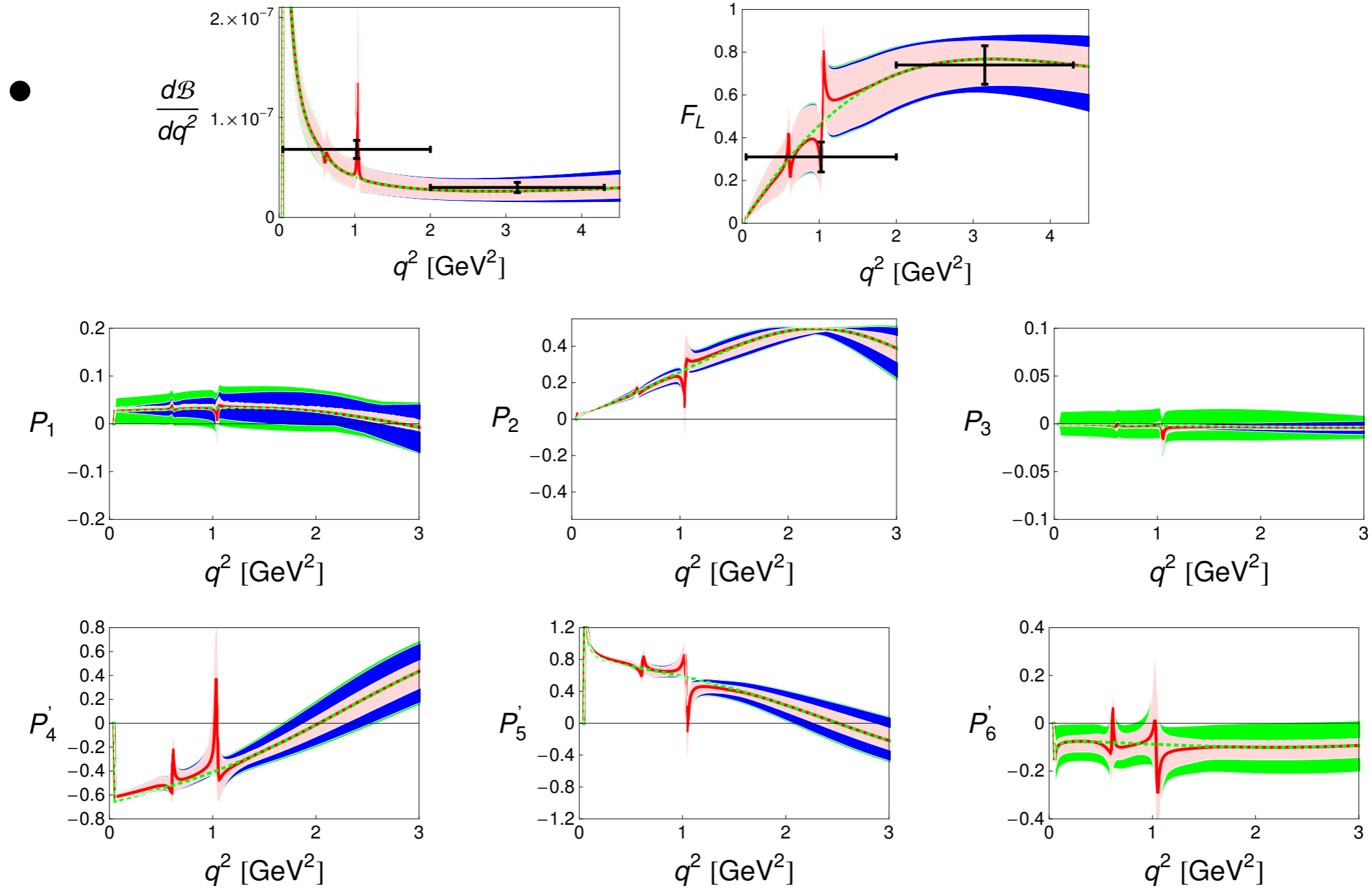
$$\Delta_i = \frac{I_i - \bar{I}_i}{2}$$

Matias, Mescia, Ramon, Virto 2012

(similar sets suitable at high q^2 : Bobeth, Hiller, Van Dyk 2010, 2012; Matias et al 2012)

Observables with these properties are *defined* to be “clean” (Matias et al) or “*form-factor independent*” (LHCb title). Terms not used in their usual meaning!
How do the observables fare in reality?

Predictions



SJ, Martin Camalich 2012

- As expected, P_1 and P_3 remain very cleanly zero in the SM. The other “clean” observables are more sensitive to long-distance effects (power corrections / duality violations)

Error budget

Obs.	$[q_{min}^2, q_{max}^2]$	Result	Hadronic	Fact.	c -quark	Light-quark
$10^7 \times \langle \frac{d\mathcal{B}}{dq^2} \rangle$	[0.1, 1]	$0.81^{+0.23}_{-0.20}$	+0.20 -0.17	+0.03 -0.03	+0.10 -0.10	± 0.00
	[0.1, 2]	$1.13^{+0.39}_{-0.38}$	+0.36 -0.24	+0.08 -0.07	+0.13 -0.12	± 0.02
	[2, 4.3]	$0.62^{+0.33}_{-0.26}$	+0.27 -0.21	+0.19 -0.15	+0.02 -0.01	± 0.00
	[1, 6]	$1.5^{+0.8}_{-0.6}$	+0.6 -0.5	+0.46 -0.37	+0.05 -0.05	± 0.02
$\langle F_L \rangle$	[0.1, 1]	$0.20^{+0.11}_{-0.10}$	+0.10 -0.09	+0.02 -0.02	+0.03 -0.02	± 0.01
	[0.1, 2]	$0.31^{+0.16}_{-0.12}$	+0.15 -0.11	+0.04 -0.04	+0.04 -0.03	± 0.01
	[2, 4.3]	$0.75^{+0.11}_{-0.16}$	+0.09 -0.13	+0.07 -0.9	+0.02 -0.02	± 0.00
	[1, 6]	$0.70^{+0.14}_{-0.17}$	+0.11 -0.13	+0.09 -0.11	+0.02 -0.02	± 0.00
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	[0.1, 2]	$1.57^{+0.19}_{-0.26}$	+0.08 -0.20	+0.13 -0.13	+0.11 -0.10	± 0.04
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SJ, Martin Camalich 2012

Error budget

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$10^7 \times \langle \frac{dB}{dq^2} \rangle$	[0.1, 1]	$0.81^{+0.23}_{-0.20}$	+0.20 -0.17	+0.03 -0.03	+0.10 -0.10	± 0.00
	[0.1, 2]	$1.13^{+0.39}_{-0.38}$	+0.36 -0.24	+0.08 -0.07	+0.13 -0.12	± 0.02
	[2, 4.3]	$0.62^{+0.33}_{-0.26}$	+0.27 -0.21	+0.19 -0.15	+0.02 -0.01	± 0.00
	[1, 6]	$1.5^{+0.8}_{-0.6}$	+0.6 -0.5	+0.46 -0.37	+0.05 -0.05	± 0.02
$\langle F_L \rangle$	[0.1, 1]	$0.20^{+0.11}_{-0.10}$	+0.10 -0.09	+0.02 -0.02	+0.03 -0.02	± 0.01
	[0.1, 2]	$0.31^{+0.16}_{-0.12}$	+0.15 -0.11	+0.04 -0.04	+0.04 -0.03	± 0.01
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SJ, Martin Camalich 2012

(P_1, P_3 helicity-suppressed in SM)

Error budget

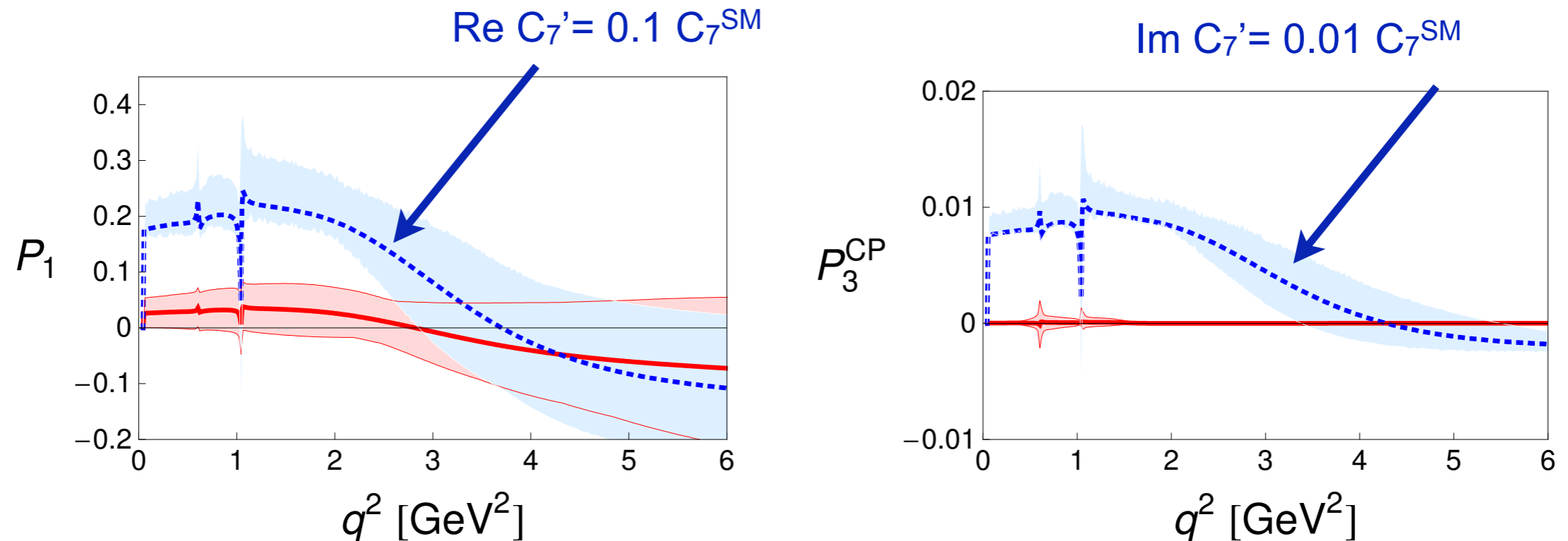
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SJ, Martin Camalich 2012

(P_1, P_3 helicity-suppressed in SM)

Long-distance effects (charm loop) dominate remaining (very small) error on P_1, P_3 and are important in all observables.

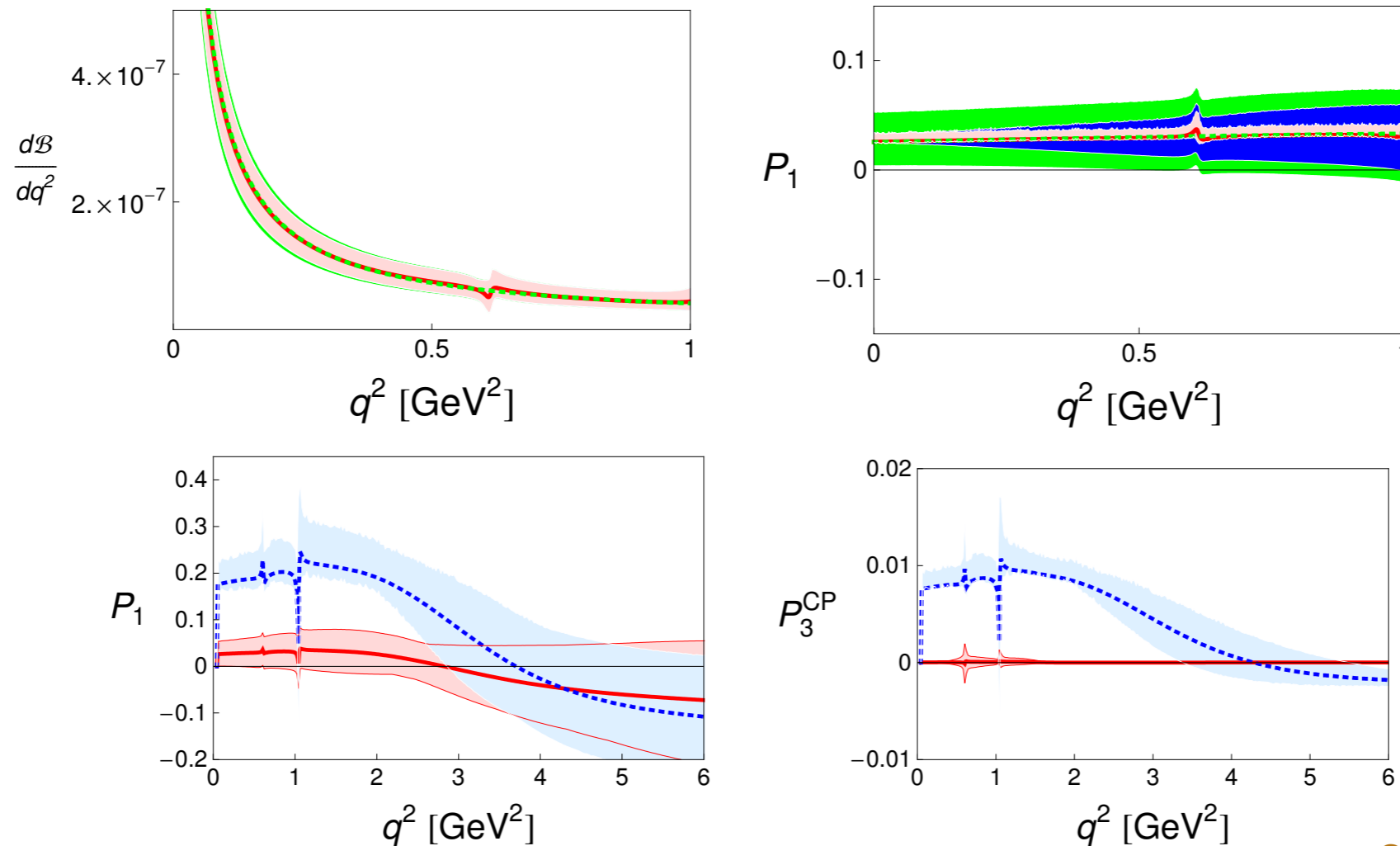
Sensitivity to C_7' (muonic mode)



SJ, Martin Camalich 2012

- (theoretical limit on) sensitivity to $\text{Re } C_7'$ at $<10\%$ (C_7^{SM}) level, to $\text{Im } C_7'$ at $<1\%$
- sensitivity stems from $q^2 \in [0.1, 2]$ GeV². There is no need to discard the data below 1 GeV², theory is perfectly fine!
- other observables' cleanness reduced by LD effects

Electronic mode



SJ, Martin Camalich 2012

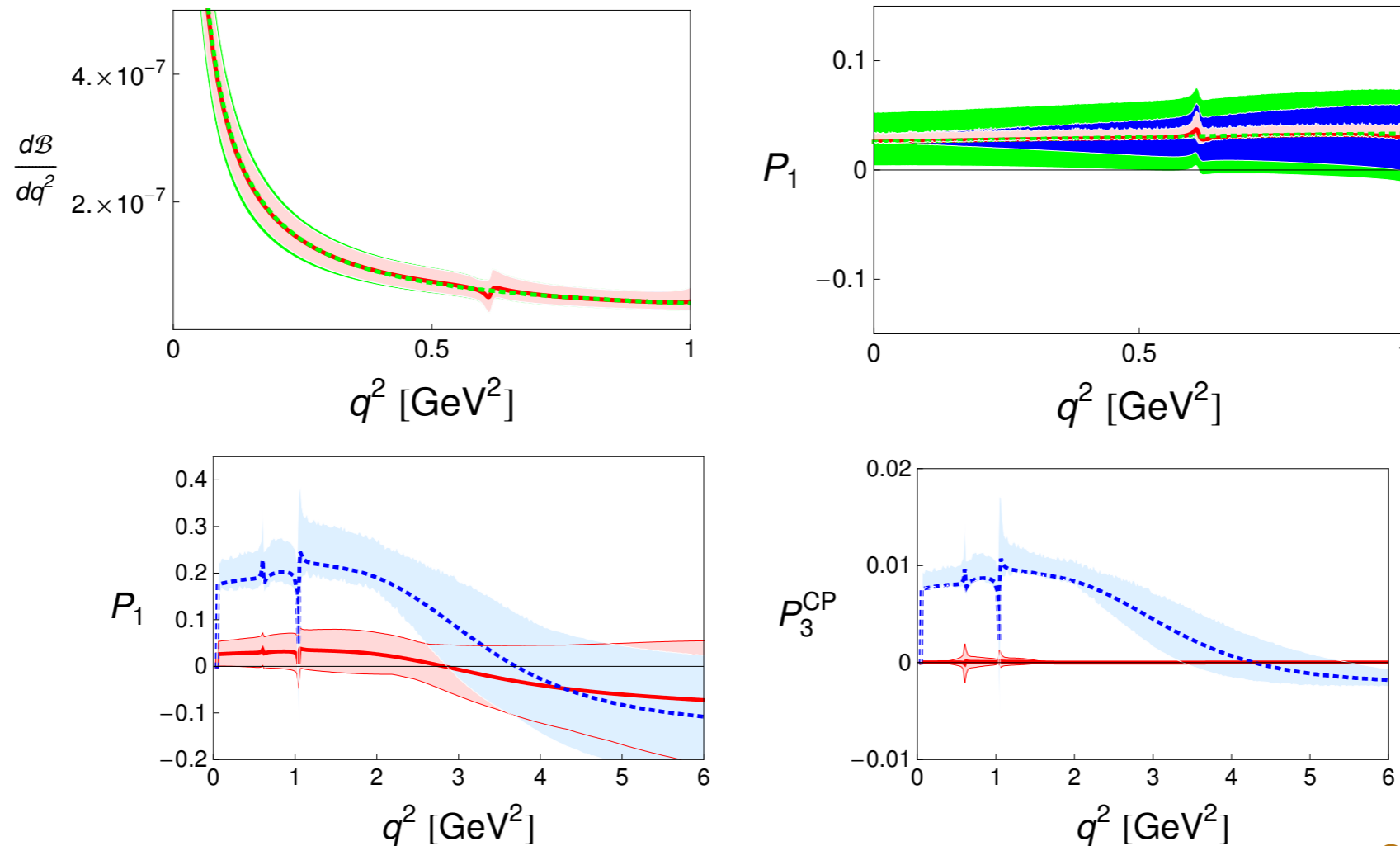
- P_1 and P_3^{CP} clean null test of SM down to end point

Obs.	Result	Hadronic	Fact.	c -quark	Light-quark
$10^7 \times \langle \frac{d\mathcal{B}}{dq^2} \rangle$	$2.43^{+0.66}_{-0.47}$	$+0.50$ -0.39	$+0.10$ -0.05	$+0.42$ -0.25	± 0.03
$10^2 \times \langle P_1 \rangle$	$2.7^{+3.0}_{-2.7}$	$+0.8$ -0.1	$+1.0$ -1.2	$+2.7$ -2.3	± 0.0

$(0.03 \text{ GeV})^2 < q^2 < 1 \text{ GeV}^2$

- Statistical advantage over muonic mode from closeness to photon pole - offsets in part experimental difficulty

Electronic mode



SJ, Martin Camalich 2012

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$(0.03 \text{ GeV})^2 < q^2 < 1 \text{ GeV}^2$

$$\mathcal{B}(B^0 \rightarrow K^{*0} e^+ e^-)_{30-1000 \text{ MeV}/c^2} = (3.1^{+0.9}_{-0.8} \text{ } ^{+0.2}_{-0.3} \pm 0.2) \times 10^{-7}$$

LHCb, Moriond 2013

- Statistical advantage over muonic mode from closeness to photon pole - offsets in part experimental difficulty

LHCb anomaly

●
PRL 111, 191801 (2013)

PHYSICAL REVIEW LETTERS

week ending
8 NOVEMBER 2013



Measurement of Form-Factor-Independent Observables in the Decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$

R. Aaij *et al.**

(LHCb Collaboration)

(Received 9 August 2013; published 4 November 2013)

We present a measurement of form-factor-independent angular observables in the decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$. The analysis is based on a data sample corresponding to an integrated luminosity of 1.0 fb^{-1} , collected by the LHCb experiment in pp collisions at a center-of-mass energy of 7 TeV. Four observables are measured in six bins of the dimuon invariant mass squared q^2 in the range $0.1 < q^2 < 19.0 \text{ GeV}^2/c^4$. Agreement with recent theoretical predictions of the standard model is found for 23 of the 24 measurements. A local discrepancy, corresponding to 3.7 Gaussian standard deviations is observed in one q^2 bin for one of the observables. Considering the 24 measurements as independent, the probability to observe such a discrepancy, or larger, in one is 0.5%.

DOI: [10.1103/PhysRevLett.111.191801](https://doi.org/10.1103/PhysRevLett.111.191801)

PACS numbers: 13.20.He, 11.30.Rd, 12.60.-i

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(see talk W Altmannshofer)

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BSM physics?

(see talk W Altmannshofer)

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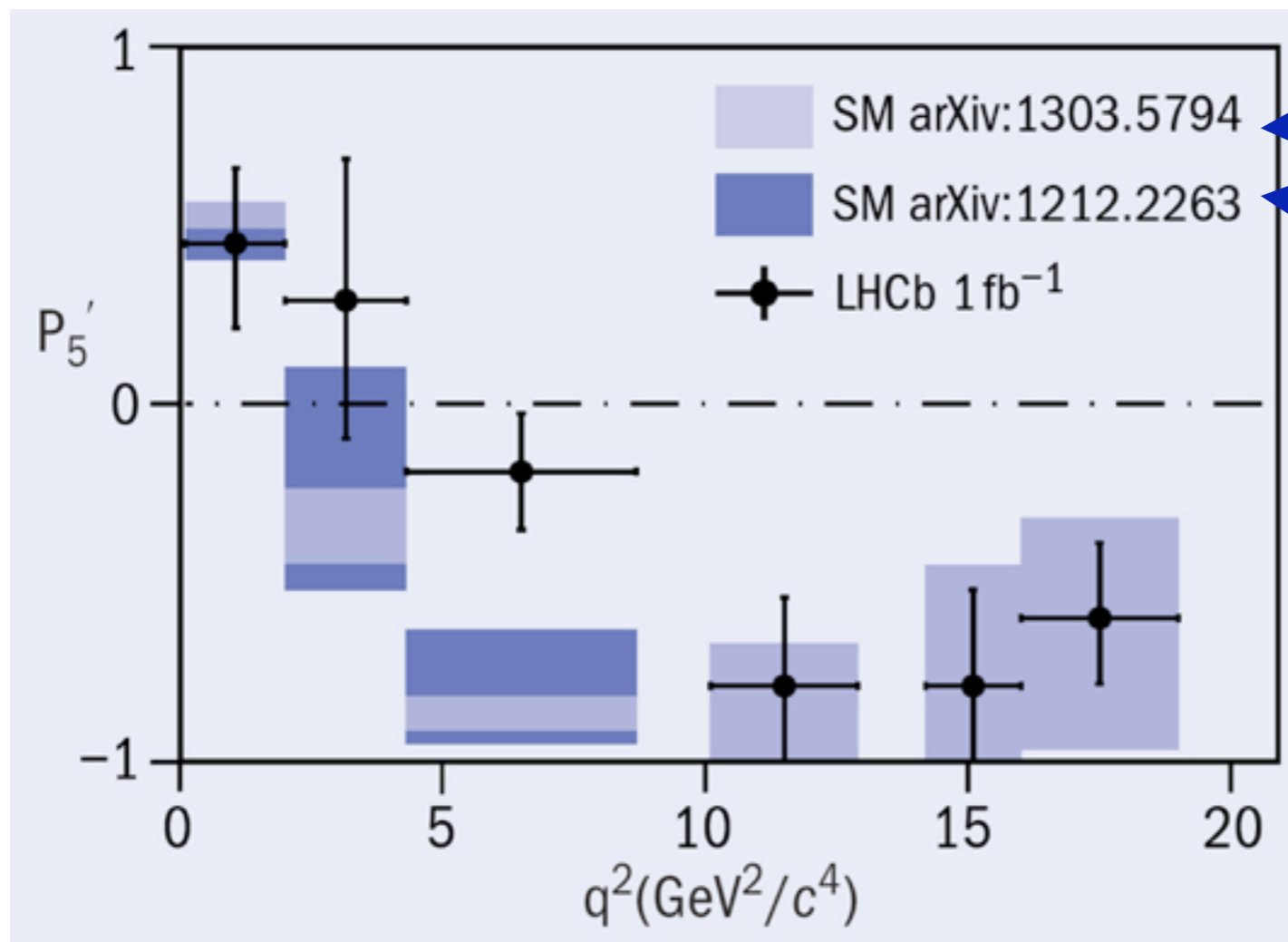
Let's check.

(see talk W Altmannshofer)

P_5' “anomaly”

$$\langle P_5' \rangle = \frac{\langle \beta(\text{Re}[(H_V^- - H_V^+)H_A^{0*} + (H_A^- - H_A^+)H_V^{0*}]) \rangle}{\sqrt{\langle \beta^2 |H_V^0|^2 + |H_A^0|^2 \rangle \langle \beta^2 (|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2) \rangle}}$$

CERN Courier, December 2013



Descotes-Genon, Matias, Virto [DMV]

SJ, J Martin Camalich (4.3..8.68 bin is actually a private update, not stated in paper)

Whence the big difference in error estimate?

Differences in treatment

- DMV employ ad hoc 10% (multiplicative) power correction at amplitude level, not per form factor [less conservative]
- DMV (essentially) add in quadrature, we scan over theory ranges for 3 groups of inputs, final error in quadrature

The pros and cons of different statistical treatments are discussed elsewhere.

Instead, I will simply illustrate the strong sensitivity of P_5' to small power corrections, which is irrespective of a statistical treatment.

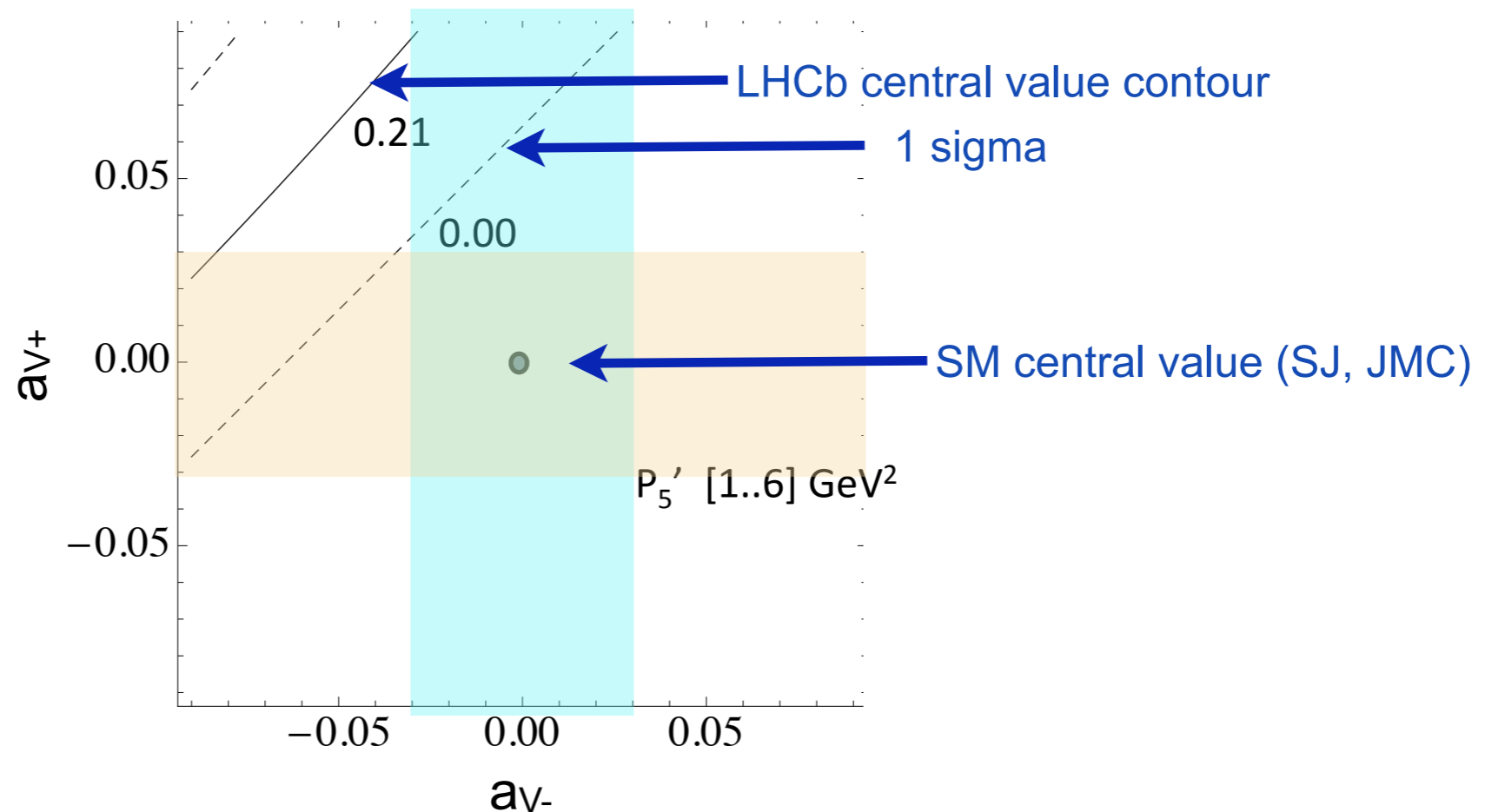
Moreover, I will **not** consider the [4.3 .. 8.68] GeV^2 bin in the following, which extends above the perturbative charm threshold into the resonance region. LD charm treatment is totally unreliable there. Consider published 1..6 GeV^2 data instead.

P_5' parametric dependence

plot in plane of two of the power correction parameters

relating to V_+ and V_- , respectively

(there are 10 power-correction parameters to order q^2/m_B^2)



$\sim \pm 0.03$ for either power correction parameter corresponds to a 10% power correction

Drawing conclusions based on this observable requires a high degree of trust in one's modelling of power corrections... or accuracy of widely employed LCSR estimates

Analytic approximations

IF

- neglecting power corrections
- neglecting perturbative QCD corrections in heavy-quark limit
- neglecting the helicity-+ amplitudes [given the other two assumptions, this just means neglecting m_s on top]

THEN

binned \rightarrow

$$\langle P'_5 \rangle = C_{10} \frac{\langle \beta f_1 [\tilde{C}_9^- + \tilde{C}_9^0] \rangle}{\sqrt{\langle \beta^2 f_2 ([\tilde{C}_9^0]^2 + C_{10}^2) \rangle \langle \tilde{\beta}^2 f_3 ([\tilde{C}_9^-]^2 + C_{10}^2) \rangle}}$$

$$f_1 = (1 - q^2/m_B^2)^{-5}, \quad f_2 = (1 - q^2/m_B^2)^{-6}, \quad f_3 = (1 - q^2/m_B^2)^{-4}$$

$$\tilde{C}_9^\pm = C_9^{\text{eff}}(q^2) + \frac{2m_b m_B}{q^2} C_7^{\text{eff}}, \quad \tilde{C}_9^0 = C_9^{\text{eff}}(q^2) + \frac{m_b \sqrt{\lambda(m_B^2, m_{K^*}^2, q^2)}}{m_B^2 q^2} C_7^{\text{eff}}$$

C_7 and C_9 have opposite sign

destructive interference enhances vulnerability to anything that violates the large-energy form factor relations

Conclusion

Rare semileptonic $B \rightarrow V \ell \ell$ is a rich probe of BSM physics.

It is also complex from a SM QCD point of view, involving many hierarchies and nonperturbative parameters

Excellent sensitivity to “magnetic” photon-induced effects remains upon taking into account all theory uncertainties if one goes to the very low end of the dilepton invariant mass distribution; there is no case for cutting at 1 GeV^2

The P_5' (and similar) SM predictions are very sensitive to even small-size power corrections.

Need to take into account properly in phenomenology, or in calculating p-values!

Helicity amplitudes

decompose amplitude in lepton currents & “dilepton helicity”

$$A = - \sum_{\lambda=\pm 1,0} \mathcal{L}_V(\lambda) H_V(\lambda) - \sum_{\lambda=\pm 1,0} \mathcal{L}_A(\lambda) H_A(\lambda) + L_S H_S + L_P H_P \\ - \sum_{\lambda=\pm 1,0} \mathcal{L}_{TL}(\lambda) H_{TL}(\lambda) - \sum_{\lambda=\pm 1,0} \mathcal{L}_{TR}(\lambda) H_{TR}(\lambda),$$

polarisation vectors for dilepton

$$\mathcal{L}_V(\lambda) = \epsilon_\mu(\lambda) L_V^\mu,$$

$$\mathcal{L}_A(\lambda) = \epsilon_\mu(\lambda) L_A^\mu,$$

$$\mathcal{L}_{TL}(\lambda) = \epsilon_\mu(\lambda) L_{TL}^\mu,$$

$$\mathcal{L}_{TR}(\lambda) = \epsilon_\mu(\lambda) L_{TR}^\mu,$$

$$\mathcal{L}_S = L_S$$

$$\mathcal{L}_P = L_P$$

$$L_V^\mu = \langle \ell^+ \ell^- | \bar{l} \gamma^\mu l | 0 \rangle,$$

$$L_S = \langle \ell^+ \ell^- | \bar{l} l | 0 \rangle,$$

$$L_{TL}^\mu = \frac{i}{\sqrt{q^2}} \langle \ell^+ \ell^- | q_\nu \bar{l} \sigma^{\mu\nu} P_L l | 0 \rangle,$$

$$L_A^\mu = \langle \ell^+ \ell^- | \bar{l} \gamma^\mu \gamma^5 l | 0 \rangle,$$

$$L_P = \langle \ell^+ \ell^- | \bar{l} \gamma^5 l | 0 \rangle,$$

$$L_{TR}^\mu = \frac{i}{\sqrt{q^2}} \langle \ell^+ \ell^- | q_\nu \bar{l} \sigma^{\mu\nu} P_R l | 0 \rangle$$

most of the literature employs transversity amplitudes

$$A_{\parallel L(R)} = \frac{1}{\sqrt{2}} (H_{+1,L(R)} + H_{-1,L(R)}), \quad A_{\perp L(R)} = \frac{1}{\sqrt{2}} (H_{+1,L(R)} - H_{-1,L(R)})$$

$$H_{\lambda L/R} = i \sqrt{f} \frac{1}{2} (H_V(\lambda) \mp H_A(\lambda)), \quad A_t = i \frac{\sqrt{q^2}}{2m_\ell} \sqrt{f} H_P, \quad A_S = -i \sqrt{f} H_S$$

Helicity amplitudes

express in terms of Wilson coefficients, form-factors and a nonlocal operator product

$$H_A(\lambda) = N(C_{10A}\tilde{V}_{L\lambda} + C'_{10A}\tilde{V}_{R\lambda}),$$

$$H_{TR}(\lambda) = N\frac{4\hat{m}_b m_B}{m_W\sqrt{q^2}} C_T\tilde{T}_{L\lambda},$$

$$H_{TL}(\lambda) = N\frac{4\hat{m}_b m_B}{m_W\sqrt{q^2}} C'_T\tilde{T}_{R\lambda},$$

$$H_S = -N\frac{\hat{m}_b}{m_W}(C_S\tilde{S}_L + C'_S\tilde{S}_R),$$

$$H_P = -N\left\{\frac{\hat{m}_b}{m_W}(C_P\tilde{S}_L + C'_P\tilde{S}_R) + \frac{2m_l\hat{m}_b}{q^2}\left[C_{10A}\left(\tilde{S}_L - \frac{m_s}{m_b}\tilde{S}_R\right) + C'_{10A}\left(\tilde{S}_R - \frac{m_s}{m_b}\tilde{S}_L\right)\right]\right\}$$

11 helicity amplitudes factorize naively (into form factors and Wilson coefficients)

(drop tensor amplitudes H_{TL} , H_{TR} in the following)

$$H_V(\lambda) = N\left\{C_{9V}\tilde{V}_{L\lambda} + C'_{9V}\tilde{V}_{R\lambda} - \frac{m_B^2}{q^2}\left[\frac{2\hat{m}_b}{m_B}(C_{7\gamma}\tilde{T}_{L\lambda} + C'_{7\gamma}\tilde{T}_{R\lambda}) - 16\pi^2 h_\lambda\right]\right\}$$

$$h_\lambda \equiv \frac{i}{m_B^2}\epsilon^{\mu*}(\lambda)a_\mu^{\text{had}}$$

only 3 helicity amplitudes are sensitive to non-(naively-)factorizing long-distance physics

$$\frac{e^2}{q^2}L_V^\mu a_\mu^{\text{had}} = -i\frac{e^2}{q^2}\int d^4x e^{-iq\cdot x}\langle\ell^+\ell^-|j_\mu^{\text{em,lept}}(x)|0\rangle\int d^4y e^{iq\cdot y}\langle M|j^{\text{em, had},\mu}(y)\mathcal{H}_{\text{eff}}^{\text{had}}(0)|\bar{B}\rangle$$

form factors and non-factorizable contributions control theory errors

nb - often transversity amplitudes are used, e.g. $H_V(+)\propto A_{||,L} + A_{||,R} + A_{T,L} + A_{T,R}$ (all LD sensitive)

Form factors

Helicity amplitudes naturally involve helicity form factors

$$-im_B \tilde{V}_{L(R)\lambda}(q^2) = \langle M(\lambda) | \bar{s} \not{\epsilon}^*(\lambda) P_{L(R)} b | \bar{B} \rangle,$$

$$m_B^2 \tilde{T}_{L(R)\lambda}(q^2) = \epsilon^{*\mu}(\lambda) q^\nu \langle M(\lambda) | \bar{s} \sigma_{\mu\nu} P_{R(L)} b | \bar{B} \rangle \quad \sim \text{Bharucha et al 2010}$$

$$im_B \tilde{S}_{L(R)}(q^2) = \langle M(\lambda = 0) | \bar{s} P_{R(L)} b | \bar{B} \rangle.$$

(& rescale $\lambda=0$ form factors by kinematic factor.)

Can be expressed in terms of traditional “transversity” FFs

$$V_{\pm}(q^2) = \frac{1}{2} \left[\left(1 + \frac{m_V}{m_B}\right) A_1(q^2) \mp \frac{\lambda^{1/2}}{m_B(m_B + m_V)} V(q^2) \right],$$

$$V_0(q^2) = \frac{1}{2m_V \lambda^{1/2} (m_B + m_V)} \left[(m_B + m_V)^2 (m_B^2 - q^2 - m_V^2) A_1(q^2) - \lambda A_2(q^2) \right]$$

$$T_{\pm}(q^2) = \frac{m_B^2 - m_V^2}{2m_B^2} T_2(q^2) \mp \frac{\lambda^{1/2}}{2m_B^2} T_1(q^2),$$

$$T_0(q^2) = \frac{m_B}{2m_V \lambda^{1/2}} \left[(m_B^2 + 3m_V^2 - q^2) T_2(q^2) - \frac{\lambda}{(m_B^2 - m_V^2)} T_3(q^2) \right],$$

$$S(q^2) = A_0(q^2),$$

The form factors satisfy two exact relations:

$$T_+(q^2 = 0) = 0,$$

$$S(q^2 = 0) = V_0(0)$$

note - M can be multiparticle state. Eg for a two-pseudoscalar state

$$\tilde{V}_{L\lambda} = -\eta(-1)^L \tilde{V}_{R,-\lambda} \equiv \tilde{V}_\lambda,$$

$$\tilde{T}_{L\lambda} = -\eta(-1)^L \tilde{T}_{R,-\lambda} \equiv \tilde{T}_\lambda,$$

$$\tilde{S}_L = -\eta(-1)^L \tilde{S}_R \equiv \tilde{S},$$

L = angular momentum

η = intrinsic parity

+ invariant mass dependence

SJ, J Martin Camalich 2012

Large-energy relations

J Charles et al 1999
Beneke, Feldmann 2000
Beneke, Yang 2004

- At small q^2 (energetic hadronic final state) one has, up to corrections $O(1/m_b)$, the relations

$$T_- = \frac{2E}{m_B} \xi_\perp,$$

$$T_+ = 0,$$

$$T_0 = \frac{E}{m_{K^*}} \xi_\parallel \left(1 + \frac{\alpha_s C_F}{4\pi} \left[\ln \frac{m_b^2}{\mu^2} - 2 + 4L \right] \right) + \frac{\alpha_s C_F}{4\pi} \Delta T_0,$$

$$V_- = \frac{2E}{m_B} \xi_\perp \left(1 + \frac{\alpha_s C_F}{4\pi} \left[\ln \frac{\mu^2}{m_b^2} + L \right] \right) + \frac{2E}{m_B + m_{K^*}} \frac{\alpha_s C_F}{4\pi} \Delta V_-,$$

$$V_+ = 0,$$

$$V_0 = \frac{E}{m_{K^*}} \xi_\parallel \left(1 + \frac{\alpha_s C_F}{4\pi} [-2 + 2L] \right) + \frac{\alpha_s C_F}{4\pi} \Delta V_0,$$

- The “soft” form factors ξ_\perp ξ_\parallel are ambiguous at $O(1/m_b)$

We define ξ_\perp such that the first equation holds exactly, and ξ_\parallel in terms of the “full-QCD” form factor A_0 .

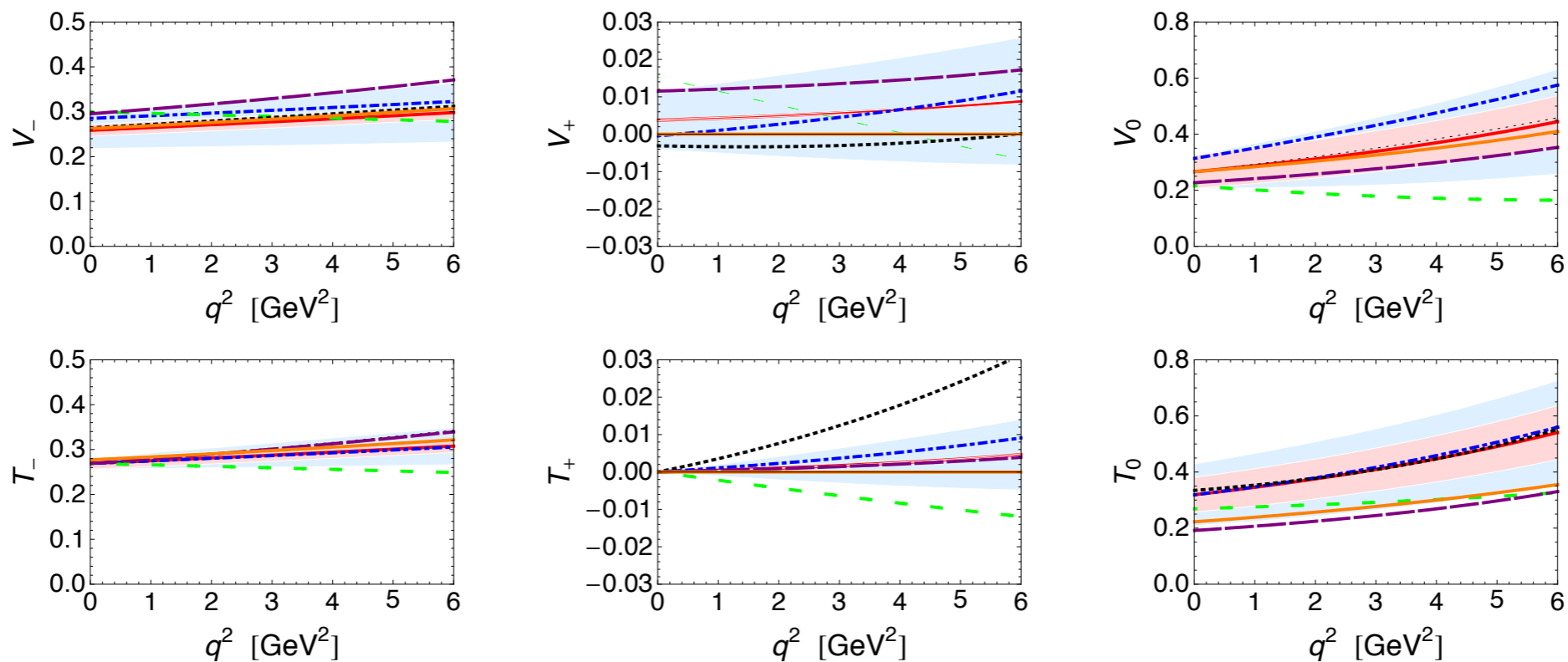
- $T_+ = V_+ = 0$ at leading power, to all orders (V-A structure)
- Calculable higher-order corrections to eqns 3, 4, and 6

Comparison of FF predictions

- parameterize form factor power corrections as

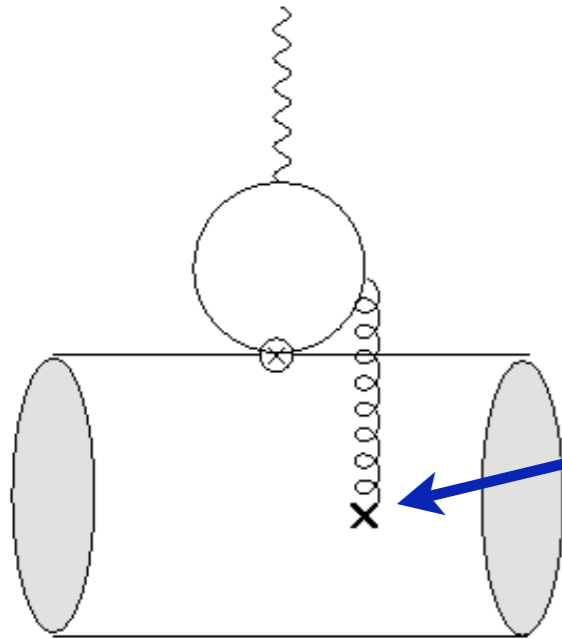
$$F^{\text{p.c.}} = a_F + b_F \frac{q^2}{m_B^2} + \mathcal{O}\left(\left(\frac{q^2}{m_B^2}\right)^2; \Lambda^2/m_b^2\right)$$

for phenomenology, will take $a_F, b_F =$ spread of th. predictions
(in absence of dedicated calculations!)



observed behaviour consistent with expectations

Charm loop



$$h_\lambda|_{c\bar{c}} = \frac{1}{m_B^2} \frac{2}{3} \epsilon^{\mu*}(\lambda) \int d^4y e^{iq \cdot y} \langle M | T [(\bar{c} \gamma^\mu c)(y) (C_1^c Q_1^c + C_2^c Q_2^c)(0)] | \bar{B} \rangle$$

consider soft gluon (in B rest frame)

From collinear factorisation viewpoint
this is part of the endpoint region

perform a “light-cone OPE” [Khodjamirian et al 2010](#)
(This is equivalent to expanding the charm
loop, treating $\Lambda^2/(4 m_c^2) \sim \Lambda/m_b$)

obtain

$$h_\lambda|_{c\bar{c},\text{LD}} = \epsilon^{\mu*}(\lambda) \langle M(k, \lambda) | \tilde{\mathcal{O}}_\mu | \bar{B} \rangle$$

$$\tilde{\mathcal{O}}_\mu = \int d\omega I_{\mu\rho\alpha\beta}(q, \omega) \bar{s}_L \gamma^\rho \delta\left(\omega - \frac{in_+ \cdot D}{2}\right) \tilde{G}^{\alpha\beta} b_L$$

(a nonlocal, light-cone operator)

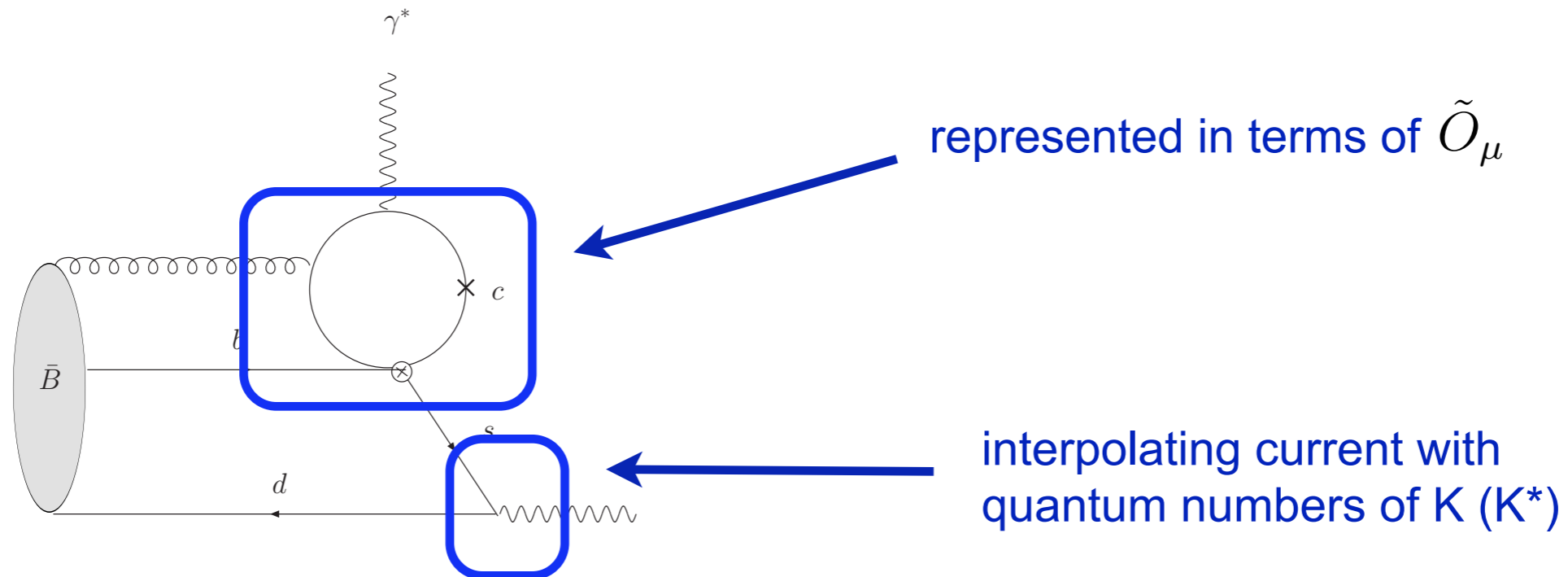
need estimate of $\langle M(k, \lambda) | \tilde{\mathcal{O}}_\mu | \bar{B} \rangle$

(which goes into h_λ)

LCSR for $\langle M(k, \lambda) | \tilde{\mathcal{O}}_\mu | \bar{B} \rangle$

to estimate the matrix element, use light-cone QCD sum rules

Khodjamirian et al 2010



$$\mathcal{F}_{\nu\mu}^{(B \rightarrow K)}(p, q) = i \int d^4 y e^{ip \cdot y} \langle 0 | T \{ j_\nu^K(y) \tilde{\mathcal{O}}_\mu(q) \} | B(p+q) \rangle$$

evaluate perturbatively as
light-cone expansion
($p^2 \sim -1 \text{ GeV}^2$ Euclidean,
far below K^* threshold)

insert \downarrow complete set of hadronic states

$$\mathcal{F}_{\nu\mu}^{(B \rightarrow K)}(p, q) = \frac{if_K p_\nu}{m_K^2 - p^2} [(p \cdot q) q_\mu - q^2 p_\mu] \tilde{\mathcal{A}}(q^2) + \int_{s_h}^{\infty} ds \frac{\tilde{\rho}_{\nu\mu}(s, q^2)}{s - p^2}$$

Lorentz expansion coefficient
containing matrix element

LCSR for $\langle M(k, \lambda) | \tilde{\mathcal{O}}_\mu | \bar{B} \rangle$

- Only numerical results given in [Khodjamirian et al 2010](#) expressed in terms of effective shift of C_9

$$\Delta C_9^{(\bar{c}c, B \rightarrow K^*, \mathcal{M}_i)}(q^2) = (C_1 + 3C_2) g(m_c^2, q^2) + 2C_1 \tilde{g}^{(\bar{c}c, B \rightarrow K^*, \mathcal{M}_i)}(q^2)$$

numerical results contribution to H^+ , H^- at O(8-10%) of leading-power contribution to H^- , significantly contaminating “clean” observables.

$\tilde{g}^{(\bar{c}c, B \rightarrow K^*, \mathcal{M}_1)}$	$\tilde{g}^{(\bar{c}c, B \rightarrow K^*, \mathcal{M}_2)}$	$\tilde{g}^{(\bar{c}c, B \rightarrow K^*, \mathcal{M}_3)}$
0.26	0.27	0.46
-0.08	-0.09	-0.15
-0.04 +0.07	-0.04 +0.08	-0.07 +0.12
+0.30 -0.17	+0.36 -0.18	+0.75 -0.33
+0.31 -0.19	+0.37 -0.21	+0.76 -0.37

However, coincidence of central values and error ranges suggest possibility of cancellations

contributing to transversity amplitudes ($H^+ \pm H^-$)

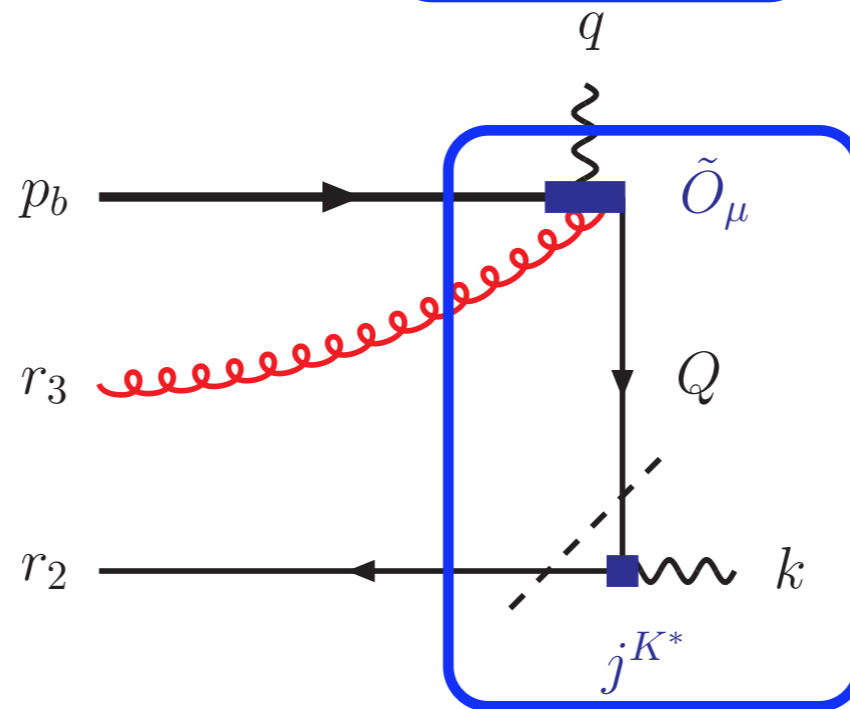
dedicated consideration of helicity amplitudes needed

LCSR for $\langle M(k, \lambda) | \tilde{O}_\mu | \bar{B} \rangle$

- obtain LCSR directly for helicity amplitudes

SJ, Martin Camalich 2012
(also for helicity-+ form factors!)

$$G_{h\lambda}(q^2; k^2) = -i \int d^4y e^{iky} \langle 0 | T \{ \epsilon^{\nu*}(\hat{z}; \lambda) j_\nu^{K*}(y) \epsilon^{\mu*}(-\hat{z}; \lambda) \tilde{O}_\mu(0) \} | B \rangle$$



light-cone OPE



key: project out helicities
through interpolating current

operator defining 3-particle
B-meson LCDA

cf Khodjamirian et al 2006,
2007, 2010

Prior art - $B \rightarrow K^* \gamma$

$$\begin{aligned} \mathcal{A}(\bar{B} \rightarrow V(\lambda) \gamma(\lambda)) &= \lim_{q^2 \rightarrow 0} \frac{q^2}{e} H_V(q^2 = 0; \lambda) \\ &= \frac{i N m_B^2}{e} \left[\frac{2 \hat{m}_b}{m_B} (C_7 \tilde{T}_\lambda(0) - C_7' \tilde{T}_{-\lambda}(0)) - 16 \pi^2 h_\lambda(q^2 = 0) \right] \end{aligned}$$

(only $\lambda = \pm 1$)

earlier estimates of $h_\lambda(0)$

Ball, Jones, Zwicky 2006

numerically small effect for both helicities

also Muheim, Xie, Zwicky 2008

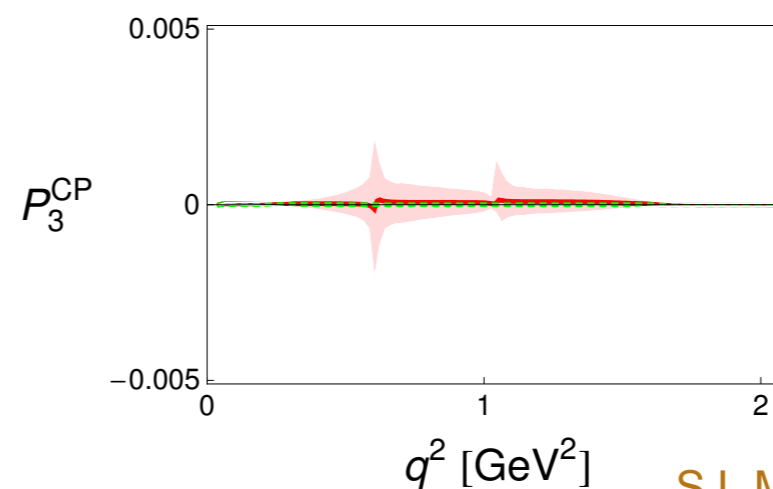
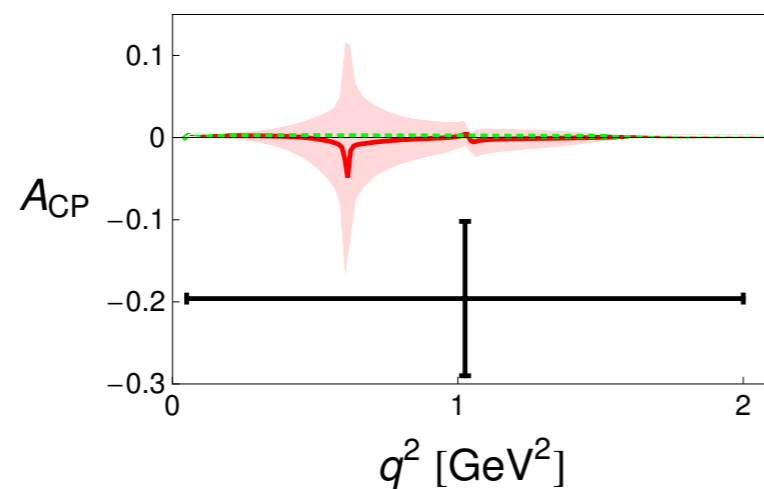
First ref employs expansion of \tilde{O}_μ in local operators, truncated after leading term.

However, neglected higher-dimensional operator matrix elements scale like $m_B^2 / (4 m_c^2)$. This is different from a somewhat analogous expansion in $B \rightarrow X_s \gamma$ where the scaling is like $m_B \Lambda / (4 m_c^2)$ giving a reasonable expansion parameter

Second ref only gives numerical result, which relies on unpublished result - cannot assess.

CP asymmetries

- LHCb has reported a large value of the (angular-integrated) CP asymmetry, particularly in the $[0.1, 2]$ GeV^2 bin



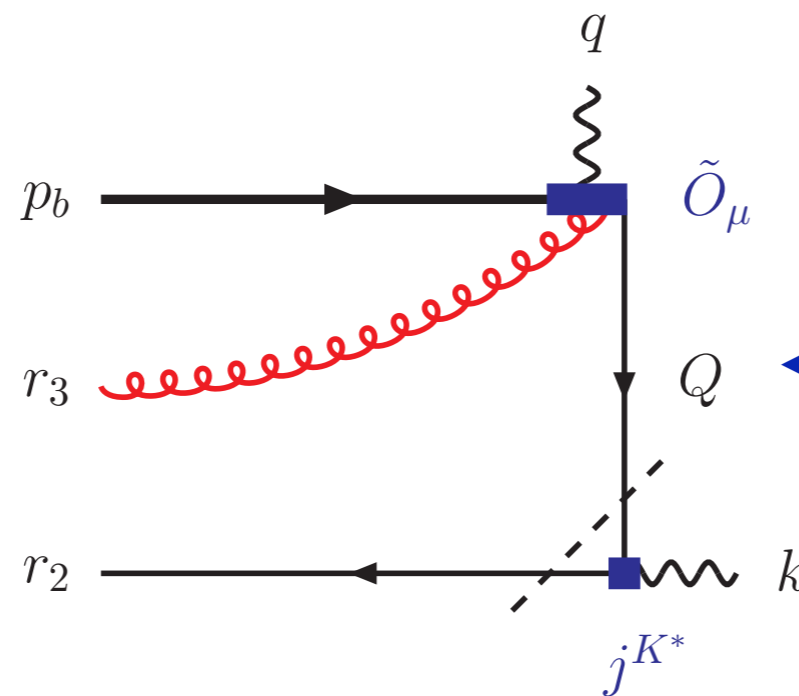
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- Large direct CP asymmetries cannot arise in a partonic description (small strong phases, strong CKM hierarchy)
- Resonance model provides large strong phases. Cannot explain the central value, but shows A_{CP} long-distance sensitive. Improved models? Eg [Khodjamirian et al 2012](#)
- Conversely the CP-asymmetric angular observable P_3^{CP} is another clean null test of the SM.

LCSR for $\langle M(k, \lambda) | \tilde{\mathcal{O}}_\mu | \bar{B} \rangle$

- obtain LCSR directly for helicity amplitudes

SJ, Martin Camalich 2012
(also for helicity-+ form factors!)



hard-collinear line

$$q = m_b \frac{n_-}{2} + \frac{q^2}{m_b + l} \frac{n_+}{2}$$

$$k = m_b \frac{n_+}{2} + \mathcal{O}(\Lambda),$$

$$Q = m_b \frac{n_+}{2} + \mathcal{O}(\Lambda),$$

$$\epsilon_\nu^*(\hat{z}, \lambda) j^\nu(k) \frac{Q + m_s}{Q^2 - m_s^2} P_R \epsilon^{\mu*}(-\hat{z}; \lambda) \tilde{\mathcal{O}}_\mu(q)$$

vanishes for + helicity to leading power

reduce estimate for long-distance charm-loop 10% -> 2%
in H_V^+

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phenomenological implementation via shifts of $C_9^{\text{eff}}(q)^2$ [helicity-dependent]