

## ***Some recommended exercises***

- 1 Look at the classical derivation of the the Bethe-Bloch formula**
- 2 Kinematics of Compton scattering and (e+e-)-pair creation**
- 3 Cerenkov threshold for electrons in water**
- 4 Estimate the nuclear interaction length in Iron**  
(Fe,  $A=56$ ;  $\rho=7.8 \text{ g/cm}^3$ )
  - 1 The number of particles in a elm shower is proportional to the Energy. If we can measure the number of particles in a shower, how will the energy resolution scale with energy ?**
  - 2 Movement of a charged particle in a magnetic field. If the curvature is measured, how well can we measure the momentum of the charged particle ?**

# ***Bibliographie***

## **Text books :**

- C. Grupen, **Particle Detectors**, Cambridge University Press, 1996
- **G. Knoll, Radiation Detection and Measurement, 3rd ed. Wiley, 2000**
- **W. R. Leo, Techniques for Nuclear and Particle Physics Experiments, Springer, 1994**
- K. Kleinknecht, **Detectors for particle radiation** , 2nd edition, Cambridge Univ. Press, 1998
- D. Green, **The physics of <<<particle Detectors**, Cambridge Univ. Press 2000
- S. Tavernier, **Experimental Techniques in Nuclear and particle Physics**, Springer 2010
- G. Lutz, **Semiconductor Radiation Detectors**, Springer, 1999
- W. Blum, L. Rolandi, **Particle Detection with Drift Chambers**, Springer, 1994
- R. Wigmans, **Calorimetry**, Oxford Science Publications, 2000

## **Review Articles**

- **Experimental techniques in high energy physics**, T. Ferbel (editor), World Scientific, 1991.
- **Instrumentation in High Energy Physics**, F. Sauli (editor), World Scientific, 1992.
- **Many excellent articles can be found in Ann. Rev. Nucl. Part. Sci.**

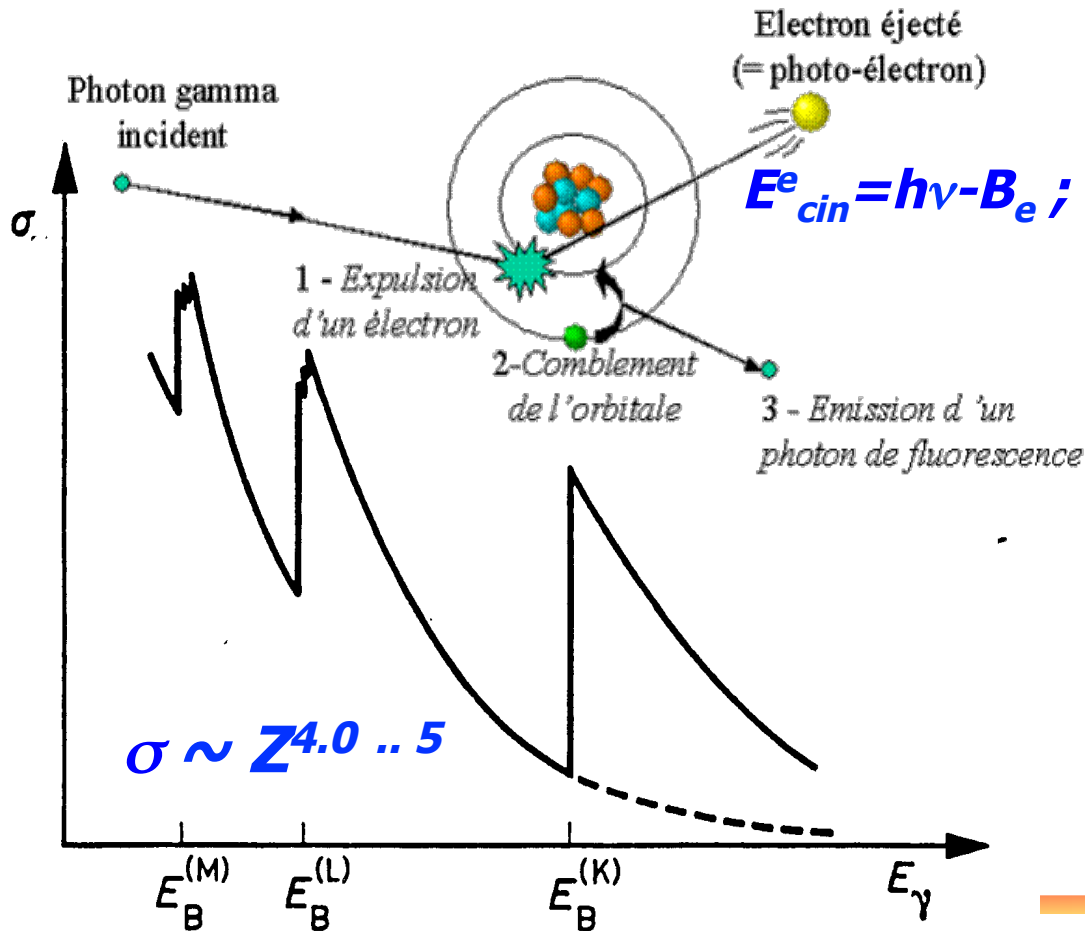
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## ***Summer student lectures and academic training***

- **Particle Detectors - Principles and Techniques: C. D'Ambrosio, T. Gys, C. Joram, M. Moll and L. Ropelewski, CERN Academic Training Programme 2004/2005 .....**
- **Summer Student Lectures 2010, Werner Riegler, CERN,**
- **Summer Student Lectures 2012, Detectors for Particle Physics, D. Bortoletto, Purdue University**
- **Particle detection and reconstruction at the LHC (I), African School of Physics, Stellenbosch, South Africa, August 2010 (D. Froidevaux, CERN)**
- **Particle detectors and large HEP experiments, L. Serin LAL/Orsay & IN2P3/CNRS, lecture at the European Summer Campus 2011, Strasbourg France**
- **Physics of Particle Detection, ICFA, Instrumental school, South Africa 2001, Claus Grupen, University of Siegen**
- **.....**

# Photo-electric effect

$$\sigma_{p.e.}^K |_{atom} = \sqrt{\frac{32}{\left(E_\gamma / m_e c^2\right)^7}} \cdot Z^5 \alpha^4 \times \underbrace{\left(\frac{8}{3} \pi r_e^2\right)}_{\text{corrections}}$$



At high Z, the hole in the K-shell is filled by an electron under the emission of a fluorescence x-ray of energy  $E_\gamma = E_K - E_{L,M,N}$

At low Z, Auger electrons occur: electrons of higher shells (L) are ejected with energy

$$E_{Auger} = E_K - 2E_L$$

# Compton-effect

## Scattering of a gamma on a "free" electron

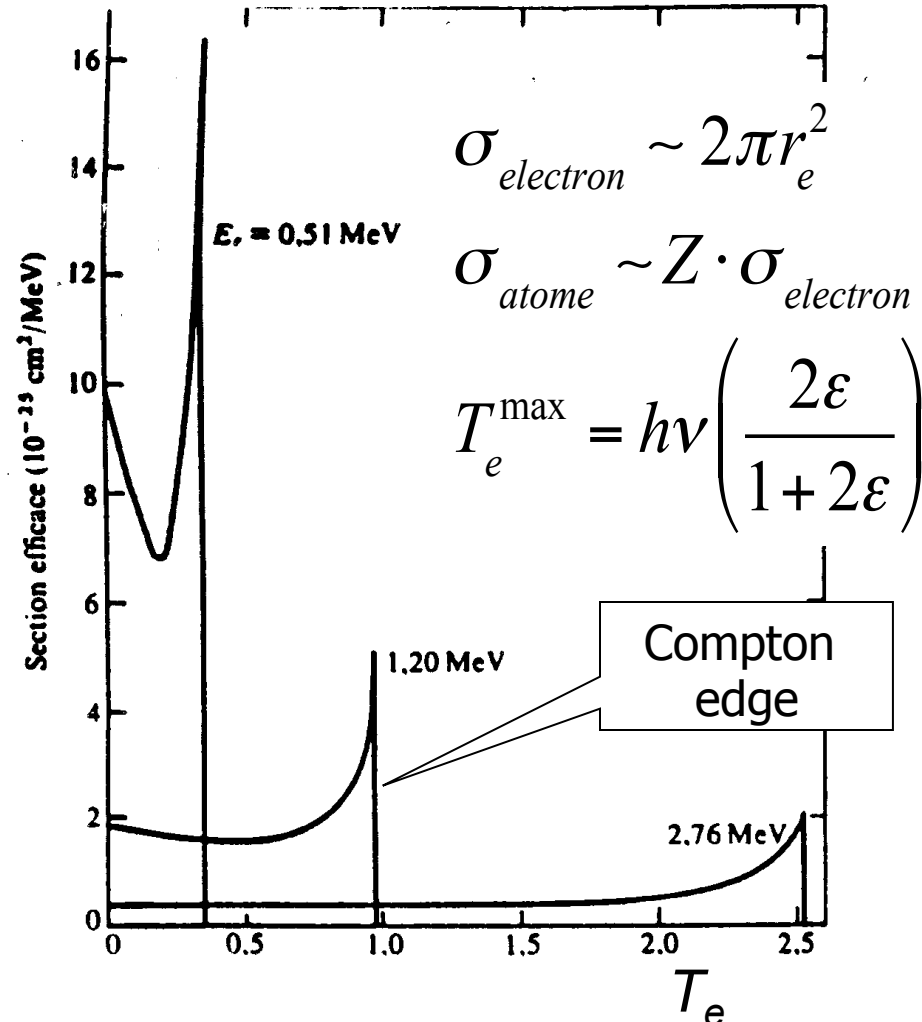
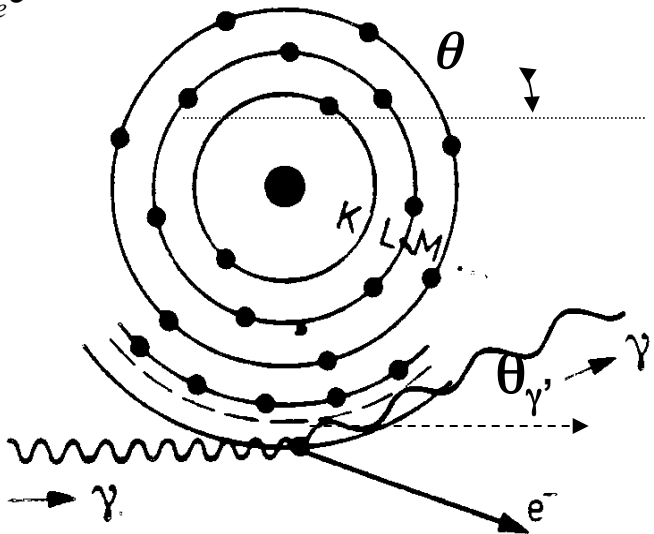
$$h\nu' = \frac{h\nu}{1 + \varepsilon(1 - \cos\theta_{\gamma'})}$$

$$\varepsilon = h\nu / m_e c^2$$

$$T_e = h\nu - h\nu'$$

$$\Delta\lambda = \lambda' - \lambda = \frac{hc}{m_e c^2} (1 - \cos\theta_{\gamma'})$$

$$\tilde{\lambda}_c = \frac{hc}{m_e c^2} \text{ Compton wave length of an electron}$$

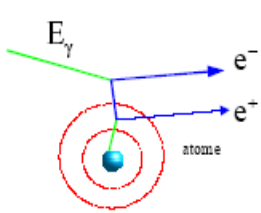


$$\sigma_{\text{electron}} \sim 2\pi r_e^2$$

$$\sigma_{\text{atome}} \sim Z \cdot \sigma_{\text{electron}}$$

$$T_e^{\text{max}} = h\nu \left( \frac{2\varepsilon}{1 + 2\varepsilon} \right)$$

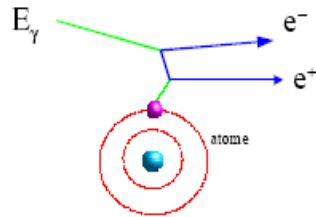
# Creation of electron positron pairs



Dans le champ du noyau

$$E_\gamma \geq 2m_e + \frac{2m_e^2}{m_N}$$

$$m_N \gg m_e \Rightarrow E_\gamma \geq 2m_e$$

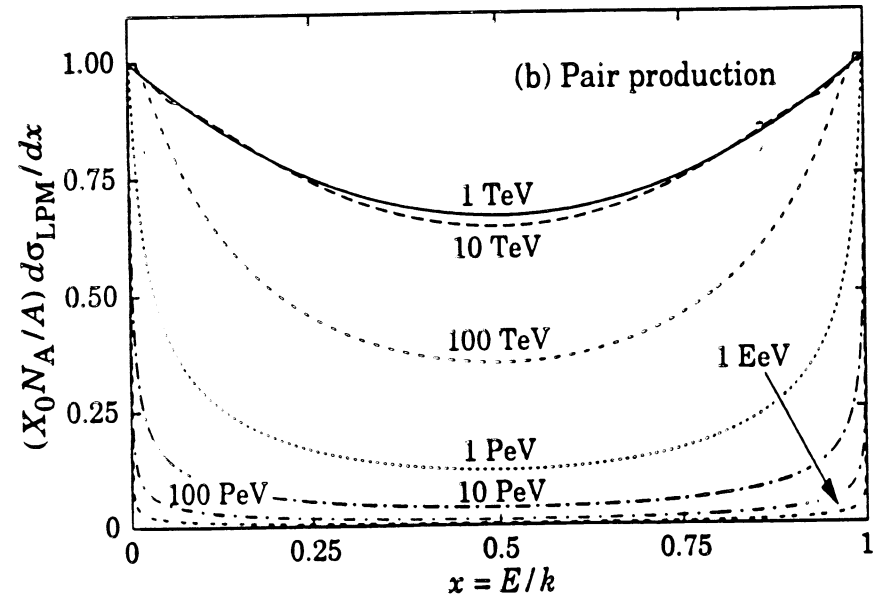
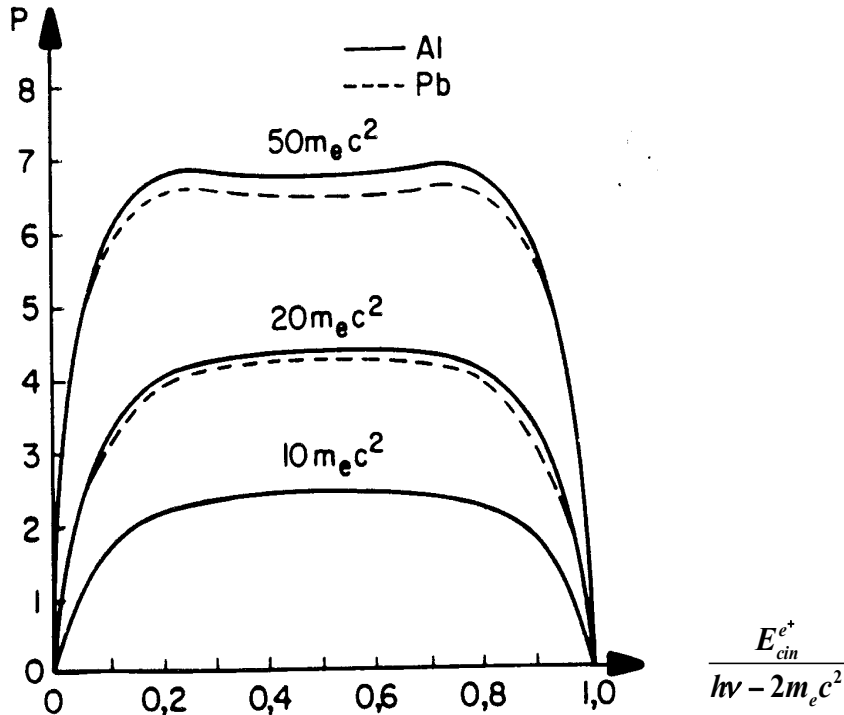


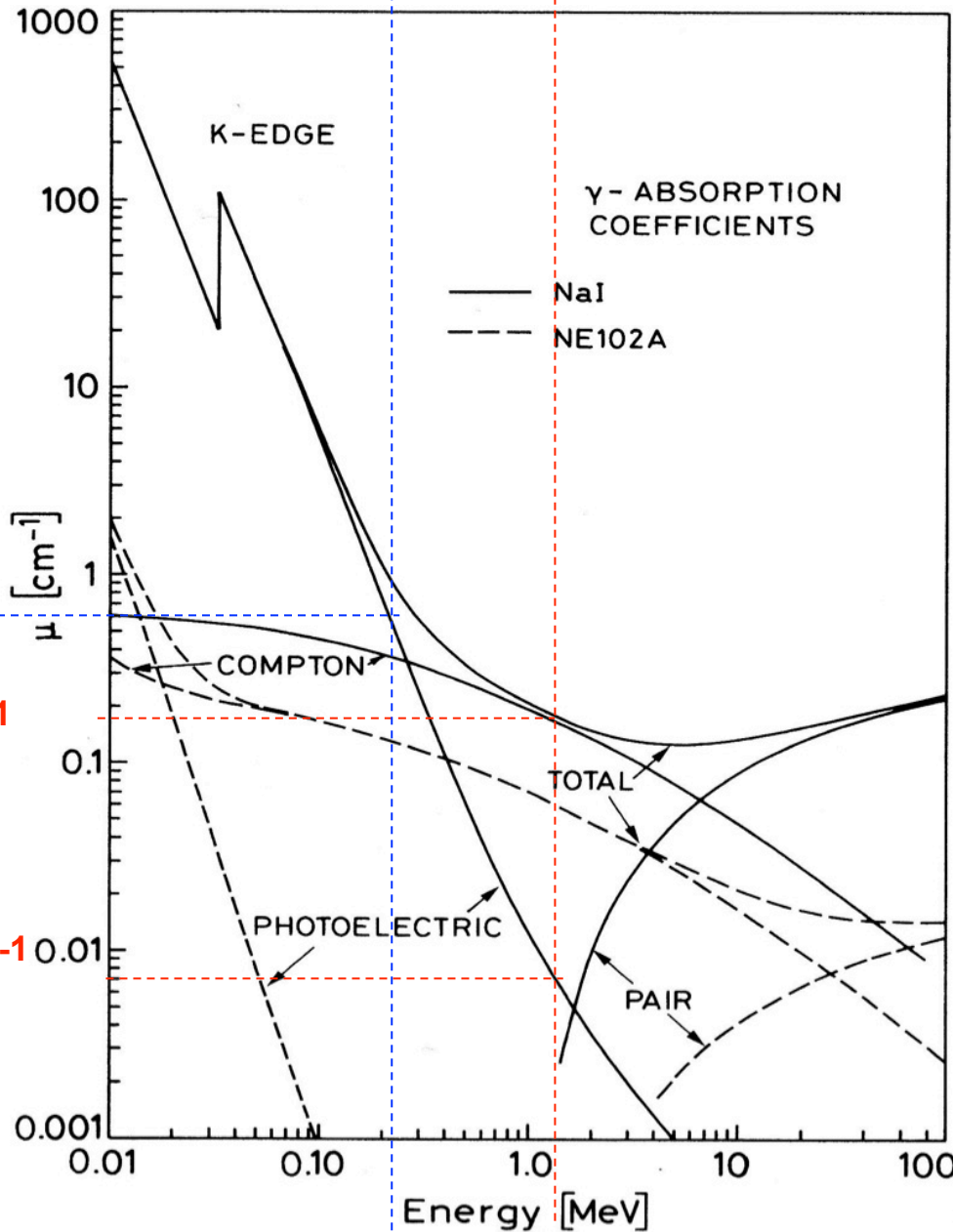
Dans le champ d'un électron

$$E_\gamma \geq 4m_e$$

$$\sigma_{pair} \approx \frac{7}{9} \frac{A(g)}{N_A} \cdot \frac{1}{X_0} \sim Z(Z+1)$$

$$\mu_{pair} = \frac{N_A}{A} \sigma_{pair} \approx \frac{7}{9} \frac{1}{X_0} ; \lambda_{pair} = \frac{1}{\mu_{pair}} = \frac{9}{7} X_0$$





**Photo-electric effect**

**Absorption of  $\gamma$**

**Compton scattering**

**scattering  $\gamma \rightarrow \gamma'$**

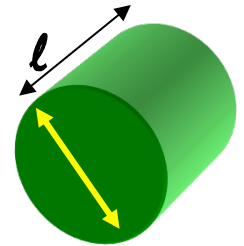
**Creation of  $(e^+e^-)$**

**pairs**

**Absorption of  $\gamma$**

## ***NaI (TI)***

- Reference/standard of efficiency:  $\varepsilon = 1,22 \times 10^{-3}$ 
  - Cylindrical detector NaI(Tl),  $7,62(\varnothing) \times 7,62(\ell)$  cm<sup>3</sup>
  - Source of <sup>60</sup>Co (1,33 MeV) at 25 cm



### **Properties of NaI:**

- $Z = 53$  high  $\Rightarrow$  good efficiency
- Relatively short decay time (230 ns)
- intense signal
- Relative good energy resolution
- But NaI is very hygroscopic!!

**Exercise : verify efficiency!**



# Efficiency of a detector

(valid in general!!)

## Absolute or total efficiency

$$\varepsilon_{tot} = \frac{\text{(particles or gammas) registered}}{\text{(particles or gammas) emitted}}$$

- This depends on the geometry between the source and the detector (its distance and opening, its solid angle)

$$\varepsilon_{tot} = \underbrace{\left[ 1 - \exp\left(\frac{-S_p}{\lambda}\right) \right]}_{\text{probability of an interaction}} \times \underbrace{\frac{\Delta\Omega}{4\pi}}_{\text{probability of an emission in the solid angle of the detector}}$$

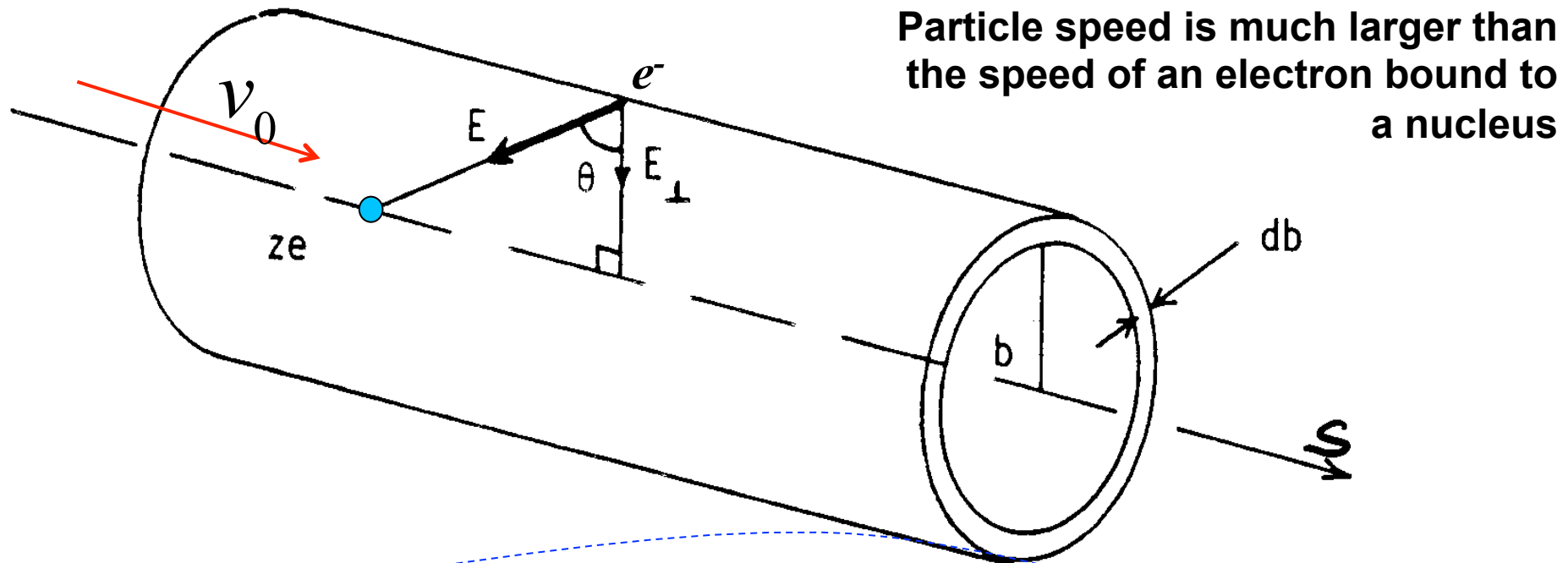
$$\varepsilon_{tot} \cong \varepsilon_{int} \times \varepsilon_{geom}$$

$$\lambda = \text{attenuation length; } \left\{ \frac{1}{\lambda} = \sigma \cdot n_b \right\}; S_p = \text{Depth of the detector}$$

## Intrinsic efficiency

$$\varepsilon_{int} = \frac{\text{(particles or gammas) "registered"}}{\text{(particles or gammas) in the acceptance of the detector}}$$

# Interaction of charged “heavy” particles with the electrons of matter



$$-\frac{dE}{ds} = -\int_0^{\infty} \frac{dE}{db} db = \frac{4\pi z^2 e^4 k^2}{m_e v_0^2} n_e \ln \frac{b_{\max}}{b_{\min}}$$

$$\Delta p_e = \int_{-\infty}^{\infty} F dt = e \int_{-\infty}^{\infty} \mathcal{E}_{\perp} dt = \frac{e}{v_0} \int_{-\infty}^{\infty} \mathcal{E}_{\perp} ds; \quad \mathcal{E}_{\perp} = \text{electric field}$$

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**GAUSS** :  $\iiint_V \text{div } \vec{\psi} dx dy dz = \oint_A \vec{\psi} d\vec{a}; \quad \vec{\psi} = \text{vector field}$

$$\iint_A \mathcal{E}_{\perp} da = \iiint_V \text{div } \vec{\mathcal{E}} dx dy dz = \frac{1}{\epsilon_0} \iiint_V \rho dx dy dz = \frac{ze}{\epsilon_0}; \quad \text{div } \vec{\mathcal{E}} = \frac{\rho}{\epsilon_0}$$


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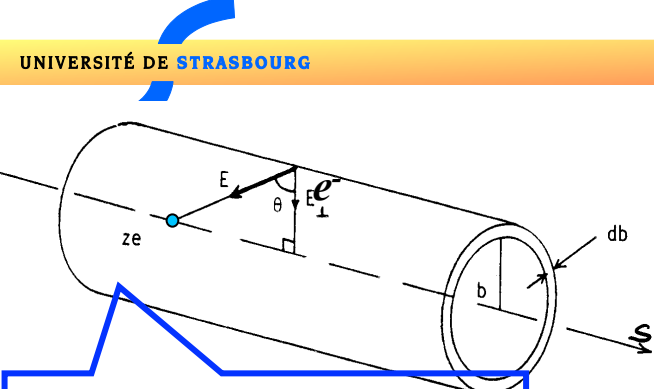
$$da = 2\pi b ds; \quad 2\pi b \int_{-\infty}^{\infty} \mathcal{E}_{\perp} ds = \frac{ze}{\epsilon_0}$$

$$\Delta p_e = \frac{2}{4\pi\epsilon_0} \frac{ze^2}{bv_0} = 2k \frac{ze^2}{bv_0}; \quad k = \frac{1}{4\pi\epsilon_0}$$

$$\Delta E = -\Delta E_e = -\frac{(\Delta p_e)^2}{2m_e} = -2 \frac{z^2 e^4}{b^2 m_e} \left( \frac{k}{v_0} \right)^2$$

$$-dE(b) = \Delta E(b) n_e dV = 4\pi n_e \frac{z^2 e^4}{m_e} \left( \frac{k}{v_0} \right)^2 \frac{db}{b} ds; \quad (dV = 2\pi b db ds)$$

$$-\frac{dE}{ds} = -\int_0^{\infty} \frac{dE}{db} db = 4\pi n_e \frac{z^2 e^4}{m_e} \left( \frac{k}{v_0} \right)^2 \ln \left( \frac{b_{\max}}{b_{\min}} \right)$$



**Cylinder of surface  $A$  and volume  $V$**

**Classical calculation by Bohr:**

**Momentum transfer  $\Delta p$  to the electron;**

**Energy loss of particle = - energy transfer to electron  $\Delta E$ ;**

$n_e$  = electron density

## Classical calculation by Bohr, $b_{\min}$ and $b_{\max}$

$b_{\min}$  : Maximal energy transfer to electron

$$T_e^{\max} = 2m_e v_0^2 \gamma^2 = 2 \frac{z^2 e^4}{b_{\min}^2 m_e} \left( \frac{k}{v_0} \right)^2$$

$$b_{\min} = \frac{z \cdot e^2 k^2}{\gamma m_e v_0^2}; \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}; \quad \beta = \frac{v_0}{c}; \quad v_0 = \text{particle speed !}$$

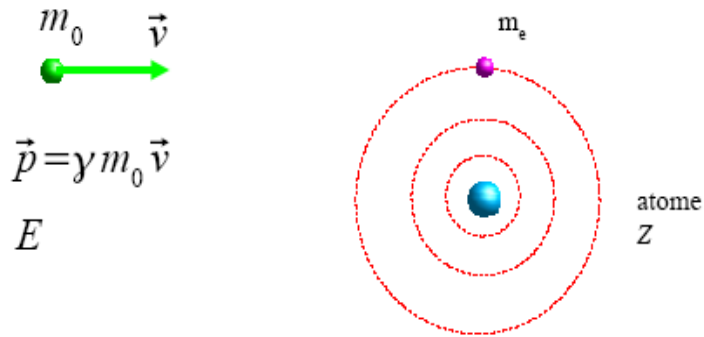
$b_{\max}$  : interaction time  $\ll$  Orbit time  $\bar{T}$

$$\frac{b_{\max}}{\gamma v_0} \ll \bar{T}$$

$$b_{\max} = \gamma v_0 \bar{T}$$

$$-\frac{dE}{ds} = -\int_0^{\infty} \frac{dE}{db} db = \frac{4\pi z^2 e^4 k^2}{m_e v_0^2} n_e \ln \frac{\gamma^2 m_e v_0^3 \bar{T}}{z^2 e^2 k^2}$$

# Maximal energy transfer of charged “heavy” particles to the electrons of matter



$$T_e^{\max} = E_e^{\max} - m_e c^2 = \frac{2m_e^2 c^2 \beta^2 \gamma^2}{\left(E_{CM} / m_0 c^2\right)^2}$$

$$v \gg v_e \approx Z\alpha c$$

$$E_{CM} = \left(m_0^2 c^4 + m_e^2 c^4 + 2m_e c^2 E\right)^{\frac{1}{2}}$$

$$p_e^{CM} = p \frac{m_e c^2}{E_{CM}}$$

$$E_e^{CM} = (E + m_e c^2) \frac{m_e c^2}{E_{CM}}$$

$$\gamma^{CM} = \frac{E + m_e c^2}{E_{CM}}; \beta^{CM} = \frac{pc}{E + m_e c^2}$$

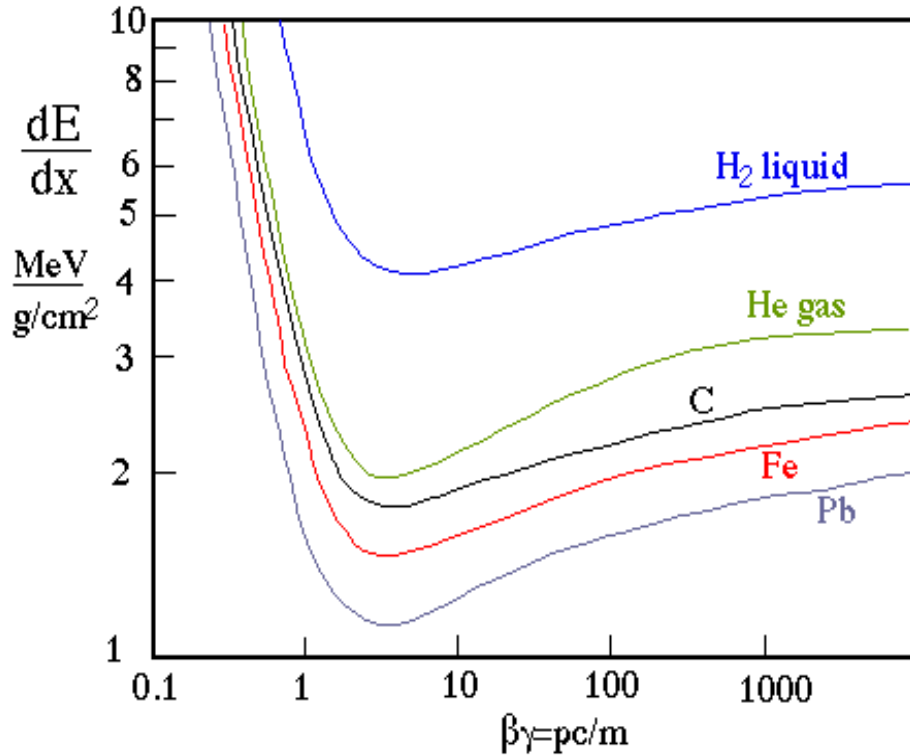
$$m_0 \gg m_e; 2\gamma m_e / m_0 \ll 1$$

$$T_e^{\max} = 2m_e c^2 \beta^2 \gamma^2$$

$$m_0 = m_e$$

$$T_e^{\max} = \frac{E^2 - m_e^2 c^4}{m_e c^2 + E} = E - m_e c^2 = T_e = T_0$$

# Bethe – Bloch formula



$$-\frac{dE}{dx} = -\frac{1}{\rho} \frac{dE}{ds}$$

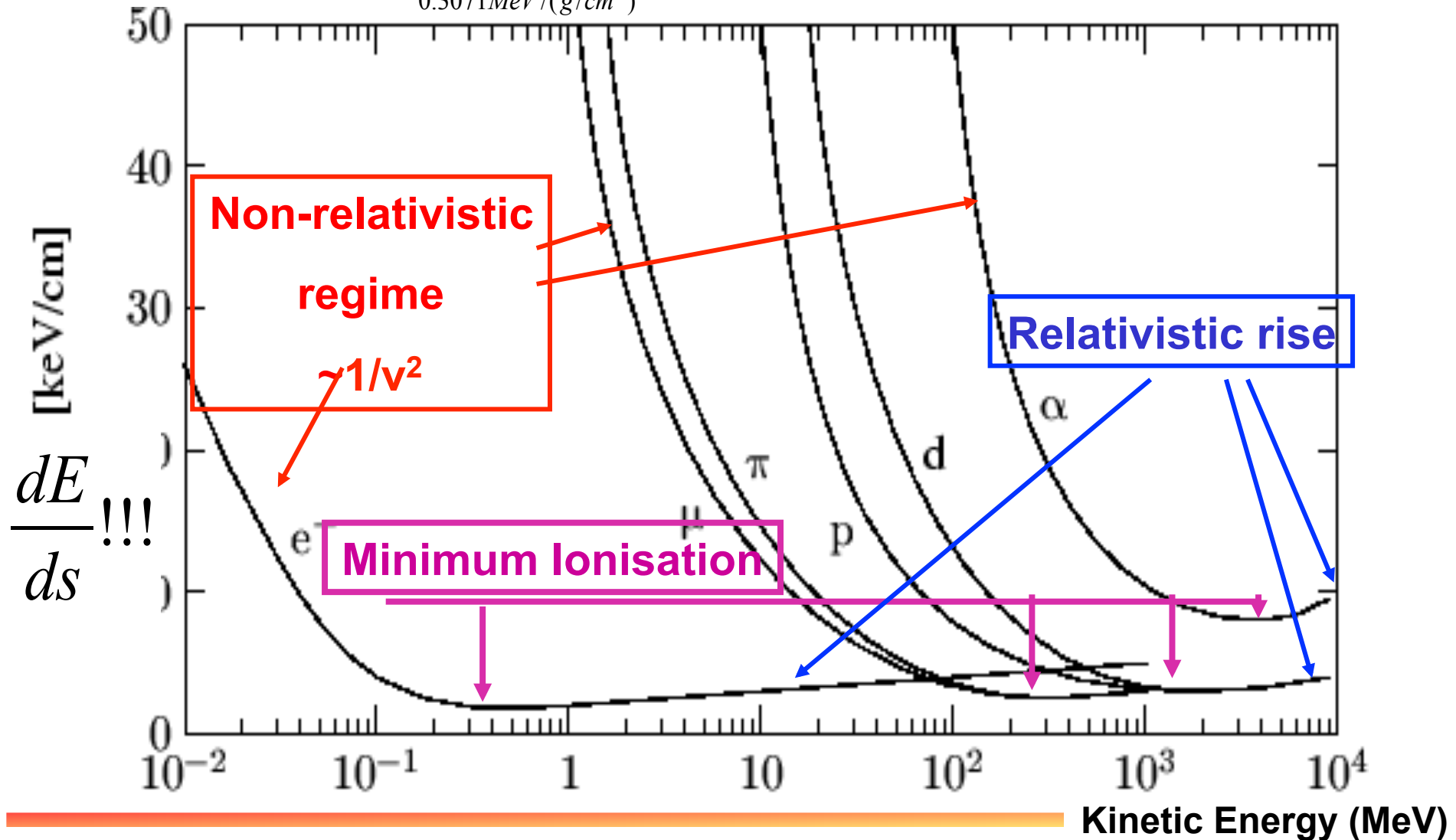
$$n_e = N_A \cdot \rho \cdot \frac{Z}{A}$$

$$r_e = \frac{e^2}{4\pi\epsilon_0 m_e c^2} = \frac{\alpha\hbar c}{m_e c^2}$$

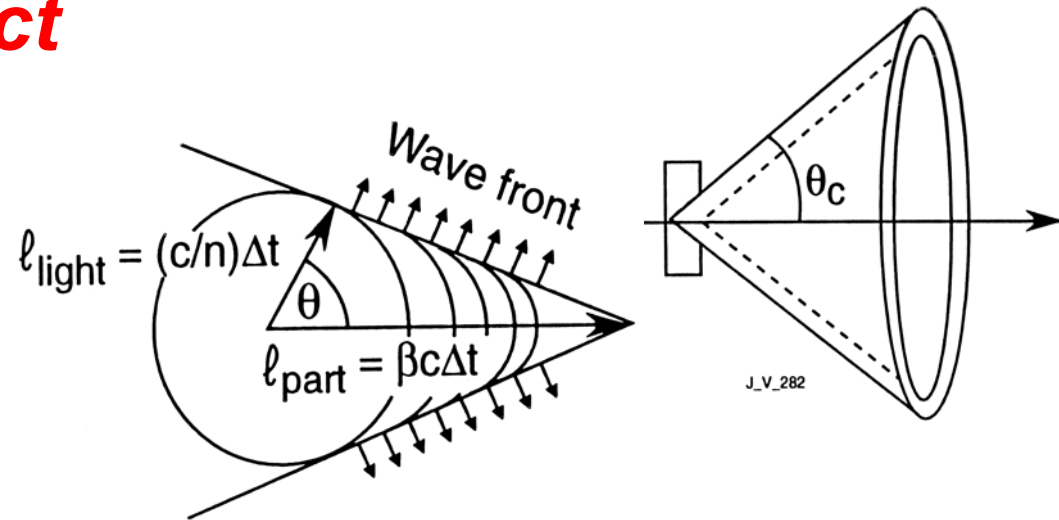
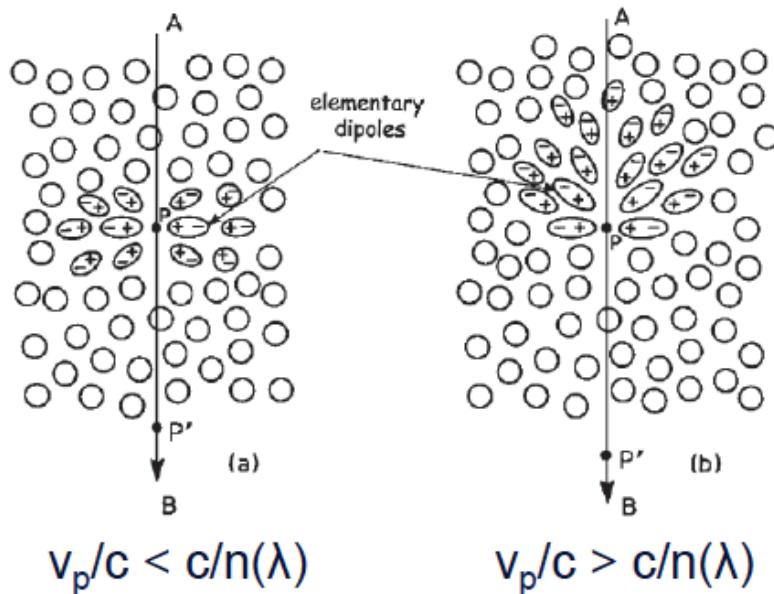
$$-\frac{1}{\rho} \frac{dE}{ds} = -\frac{dE}{dx} = \underbrace{4\pi N_A r_e^2 m_e c^2}_{0.3071 \text{ MeV}/(\text{g}/\text{cm}^2)} \frac{Z}{A} z^2 \frac{1}{\beta^2} \left[ \frac{1}{2} \ln \frac{2m_e c^2 \gamma^2 \beta^2 \cdot T_e^{\max}}{I^2} - \beta^2 \frac{\delta}{2} - \frac{C}{Z} \right]$$

Density- shell correction

$$-\frac{1}{\rho} \frac{dE}{ds} = -\frac{dE}{dx} = \underbrace{4\pi N_{Av} r_e^2 m_e c^2}_{0.3071 \text{ MeV}/(\text{g/cm}^2)} \frac{Z}{A} z^2 \frac{1}{\beta^2} \left[ \frac{1}{2} \ln \frac{2m_e c^2 \gamma^2 \beta^2 \cdot T_e^{\max}}{I^2} - \beta^2 - \frac{\delta}{2} - \frac{C}{Z} \right]$$



# Cerenkov effect



- **Coherent superposition of the radiation of the atoms**
- **Mainly blue light**
- **Very few photons**
- **Very small energy loss**
- **Identification of particles!**

$$v = \beta c > c/n$$

$$\cos \theta_c = \frac{c \cdot \Delta t / n}{\beta c \cdot \Delta t} = \frac{1}{\beta n}$$

$$\Rightarrow \beta > \frac{1}{n}; \cos \theta_c^{\text{max}} = \frac{1}{n}$$

$$\lambda_{\text{photons}} \approx 200 - 700 \text{ nm}$$

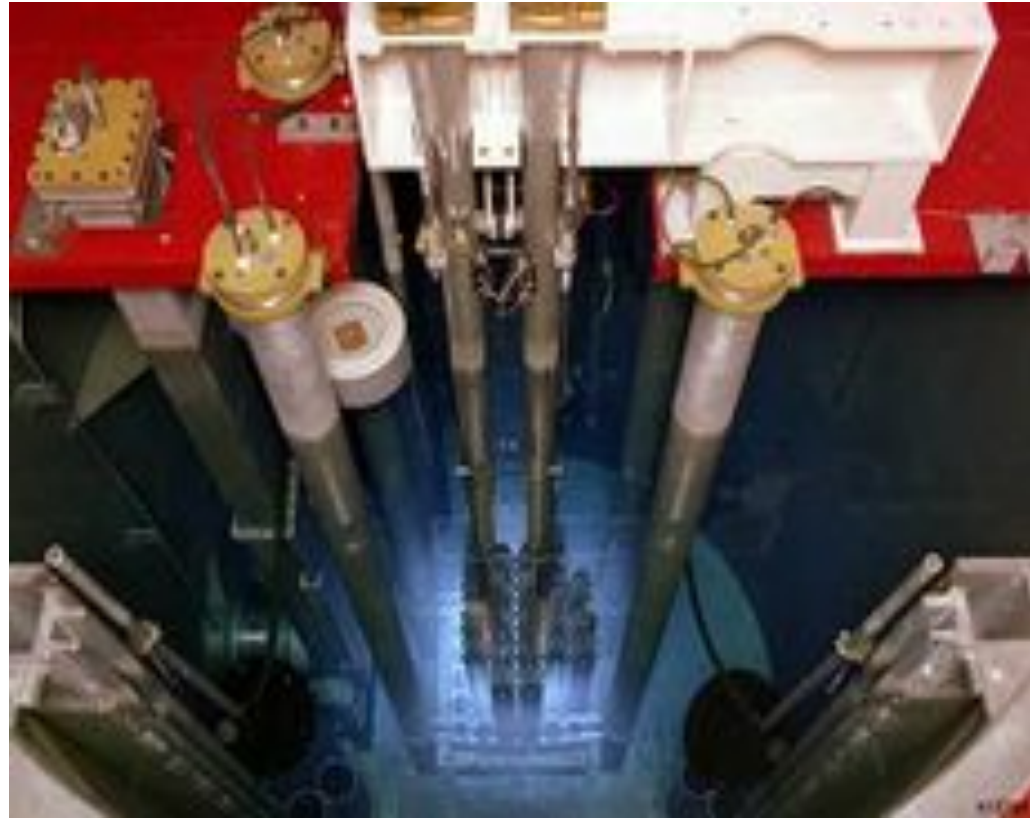
$$\frac{d^2 N_{h\nu}}{dE_{h\nu} dx} \approx 370 \sin^2 \theta_c \text{ eV}^{-1} \text{ cm}^{-1}$$

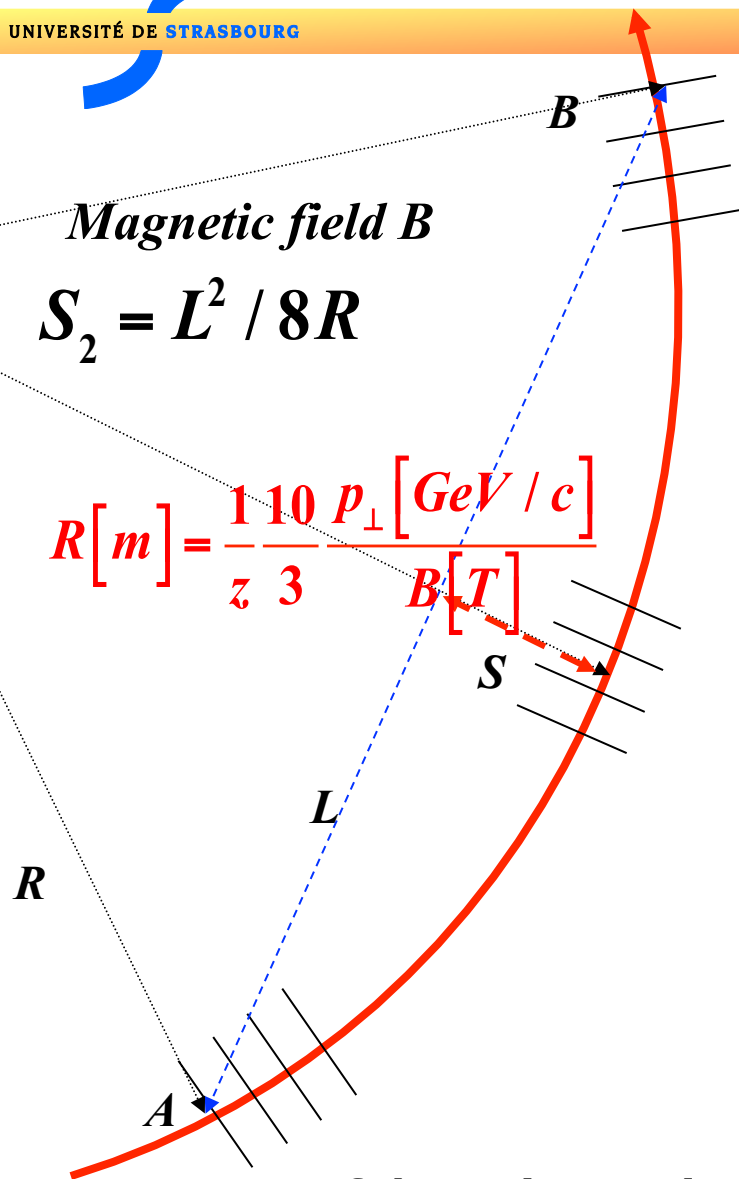


# Exercise

## Blue light in a reactor

1. What produces the light?
2. Water  $n=1.333$ . calculate the minimal energy of an electron to produce Cerenkov light





## Reconstruction of transverse momentum in a magnetic field

**Exercise !!!**

$$S_2 = L^2 / 8R$$

$$R[m] = \frac{110 p_{\perp} [GeV/c]}{z \cdot 3 B[T]}$$

- Movement of a charge  $z$  in a uniform magnetic field
- Momentum resolution  $dp/p$
- Spatial resolution of the sagitta  $dS/S$

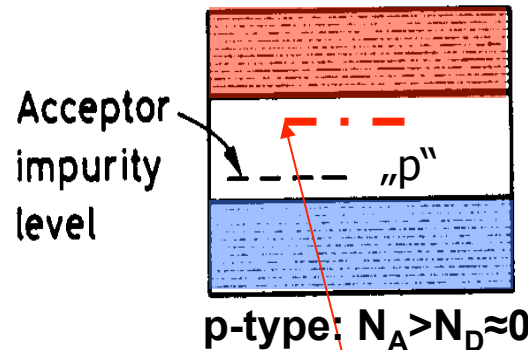
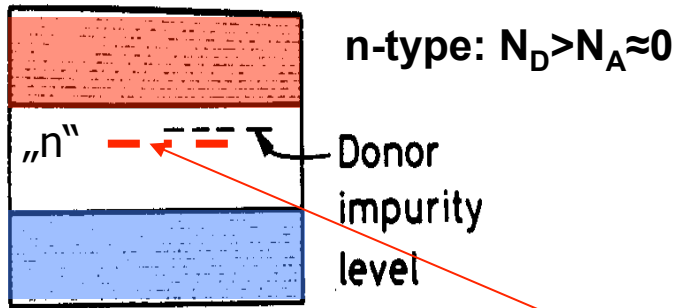
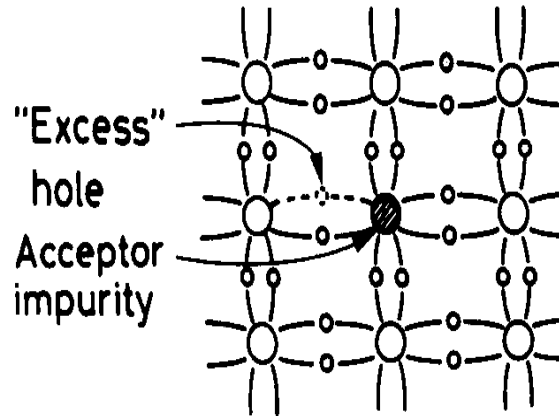
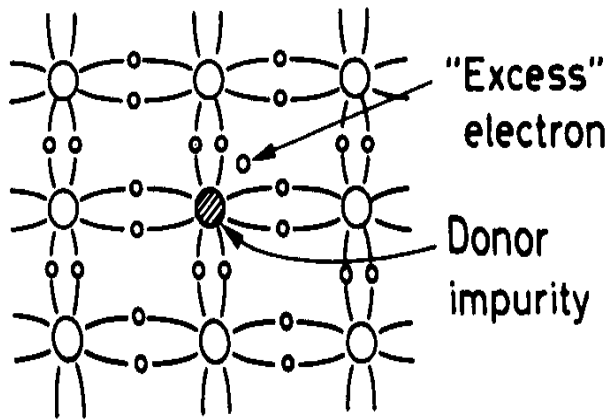
$$\frac{dS}{S} = \frac{dp_{\perp}}{p_{\perp}} = \frac{80}{3 \cdot z} \frac{1}{BL^2} p_{\perp} dS$$

$$[B] = \text{Tesla}; [L] = m; [p_{\perp}] = GeV/c$$

If the trajectory is measured with  $N$  points:

$$\left. \frac{\sigma(p_T)}{p_T} \right|^{meas.} = \frac{\sigma(x) \cdot p_T}{0.3 \cdot BL^2} \sqrt{720/(N+4)} \quad (\text{for } N \geq \sim 10)$$

# „Doped“ Semi-conductors



	Li	Sb	P	As	Bi
Donor levels (eV)	0.033	0.039	0.044	0.049	0.069
Acceptor levels (eV)	0.045	0.057	0.065	0.16	0.26

Silicon band gap 1.1eV

Intrinsic:  $1.5 \cdot 10^{10}/\text{cm}^3$  ( $N_A = 6.022 \cdot 10^{23}/\text{cm}^3$  !!)  
 $n, p : 10^{13}/\text{cm}^3$   
 $n^+, p^+ : 10^{20}/\text{cm}^3$

- Impurities**
- traps
  - recombination

Energy levels within the band gap corresponding to various n- and p-type dopants [6]

# Semi-conductor detectors

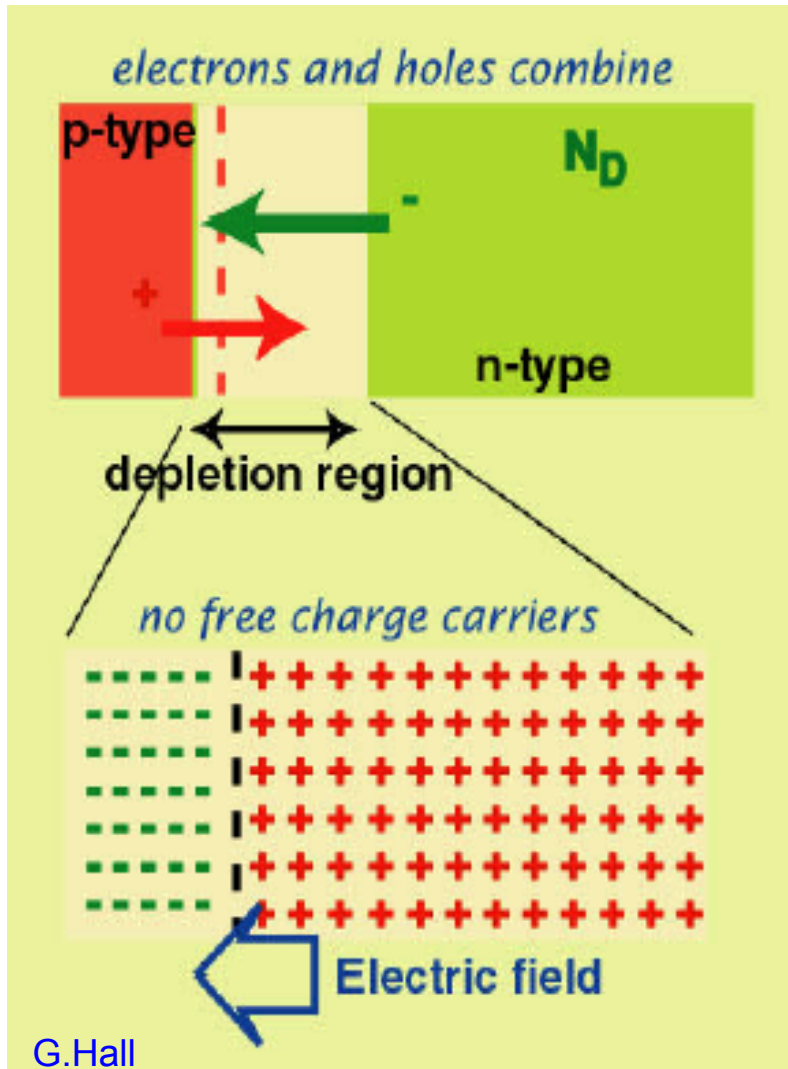
Material	$E_g$ [eV]	$w$ [eV]	Mobility (velocity/E)		$\tau_e$ [s]	$\tau_h$ [s]	density g/cm <sup>3</sup>	Z [a.m.u]
			$\mu_e$ [cm <sup>2</sup> /Vs]	$\mu_h$ [cm <sup>2</sup> /Vs]				
<b>C</b> (diamond)	5.5	13	1800	1200	$2 \cdot 10^{-9}$	$2 \cdot 10^{-9}$	3.515	6
<b>Si</b>	1.12	3.61	1350	480	$5 \cdot 10^{-3}$	$5 \cdot 10^{-3}$	2.33	14
<b>Ge</b>	0.67	2.98	3900	1900	$2 \cdot 10^{-5}$	$2 \cdot 10^{-5}$	5.32	32
<b>GaAs</b>	1.42	4.70	8500	450	$5 \cdot 10^{-8}$	$5 \cdot 10^{-8}$	5.32	31,33
<b>CdTe</b>	1.56	4.43	1050	100	$1 \cdot 10^{-6}$	$1 \cdot 10^{-6}$		48,52
<b>HgI<sub>2</sub></b>	2.13	4.20	100	–	$1 \cdot 10^{-6}$	$2 \cdot 10^{-6}$		53,80

$$\frac{dN}{N} = \frac{1}{\sqrt{N}} ; E \sim N ; N = \text{numb. of (e,h)}$$

Parameters Values for Materials Used in Fabricating Semiconductor Radiation Sensors

# Junction p-n

## Formation of a depletion zone

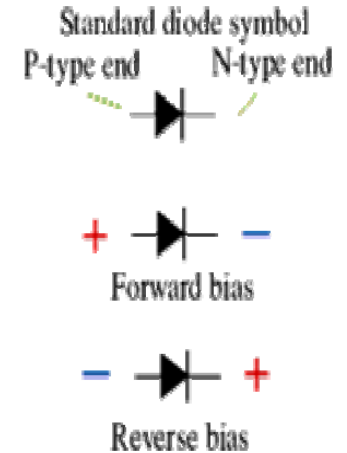
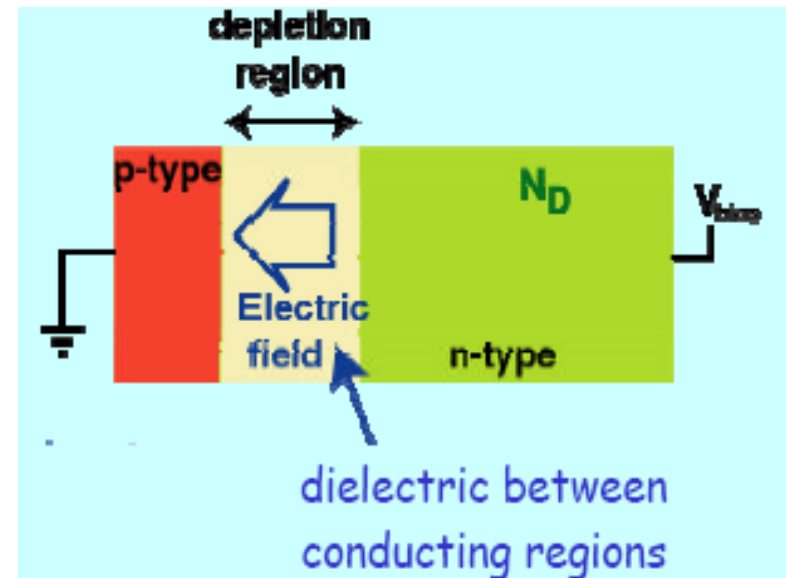


## Direct Polarisation

- conduction
- $I \sim I_0[\exp(qV/kT) - 1]$

## Inverse Polarisation

- increase of depletion zone
- reduction of capacitance



# Inverse Polarisation

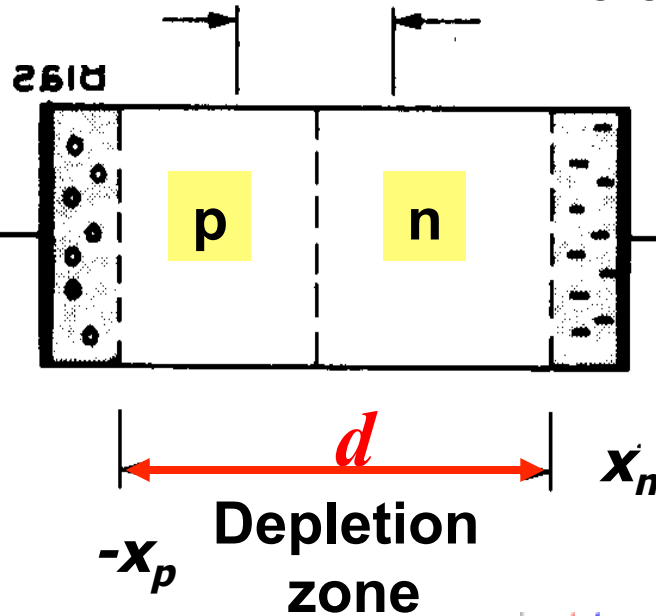
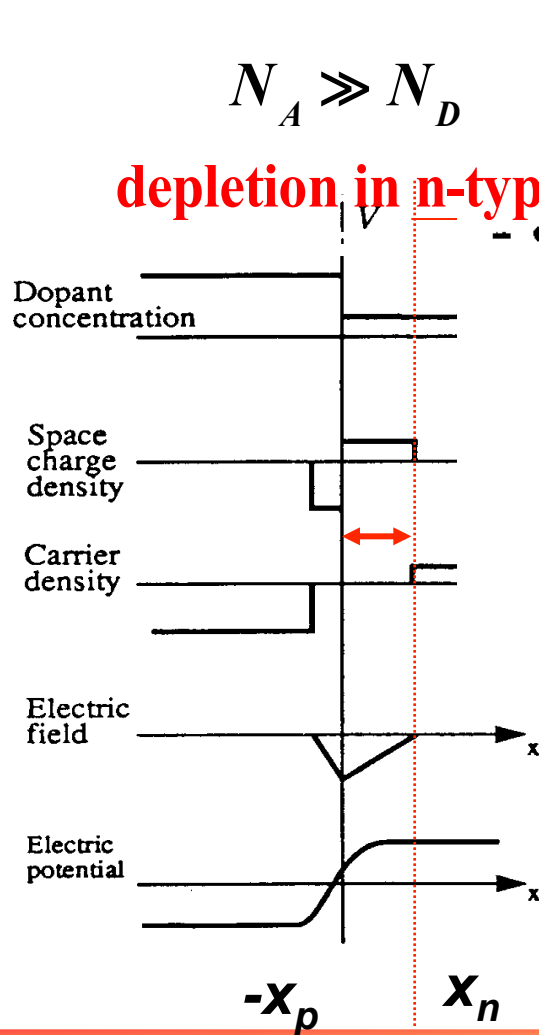
Holes move to « - »

$$d|_{V_{bias}} = x_n + x_p = \sqrt{\frac{2\epsilon(\phi_0 + V_{bias})(N_A + N_D)}{e N_A N_D}}$$

electrons move to contact "+"

$$N_A \gg N_D$$

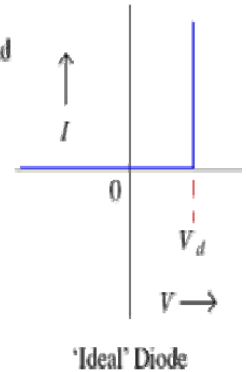
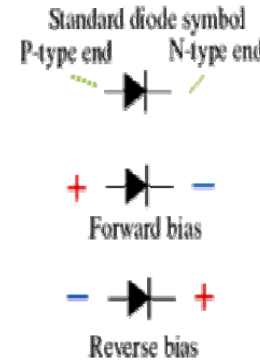
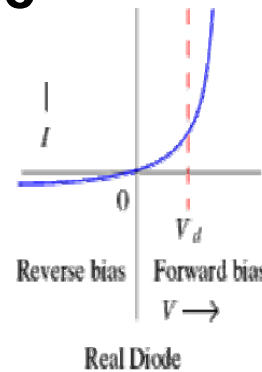
depletion in n-type



$$d \approx x_n \approx 0.53 \sqrt{\rho_n \phi_0} \mu m$$

$$\rho \sim 2 \cdot 10^4 \Omega cm, \phi_0 \sim 1V$$

$$\Rightarrow d \sim 75 \mu m$$



# Surface barrier detectors



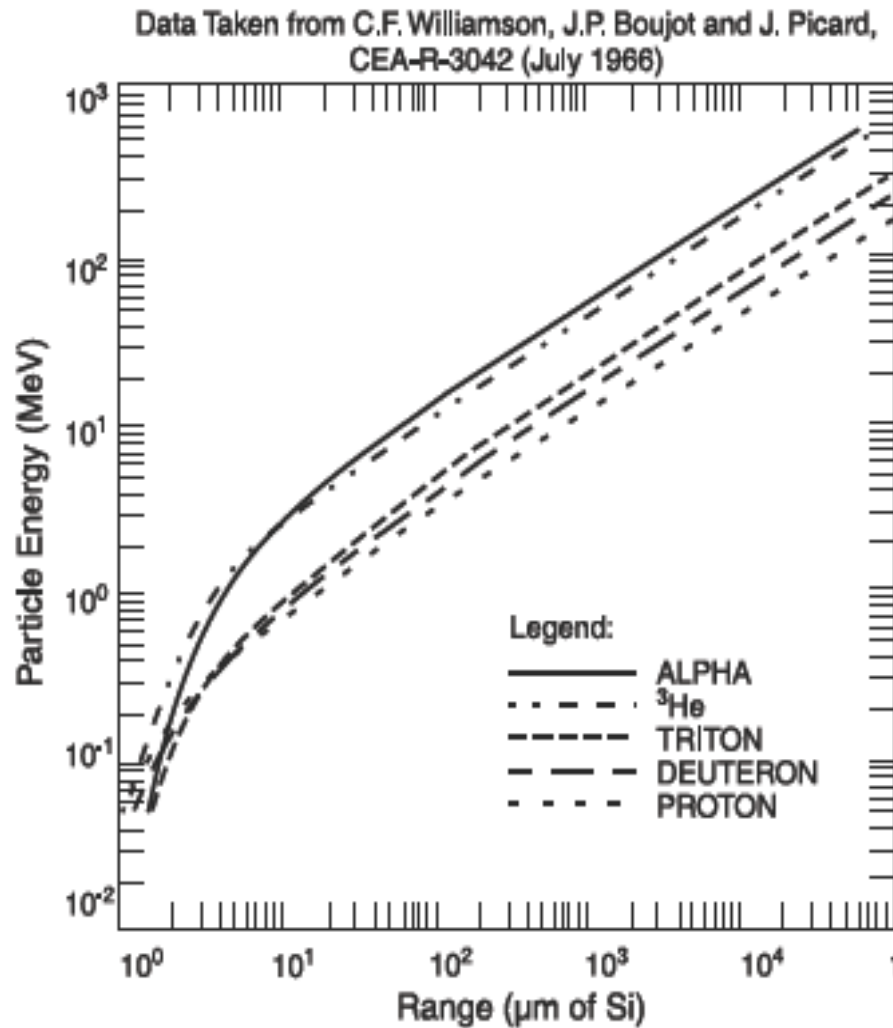
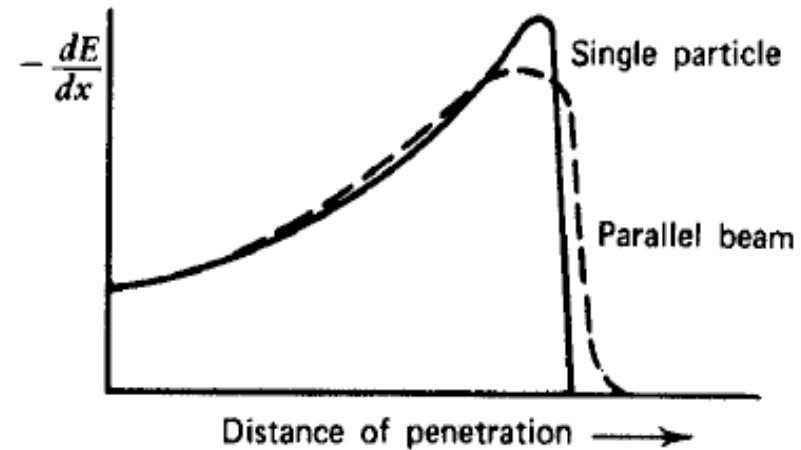


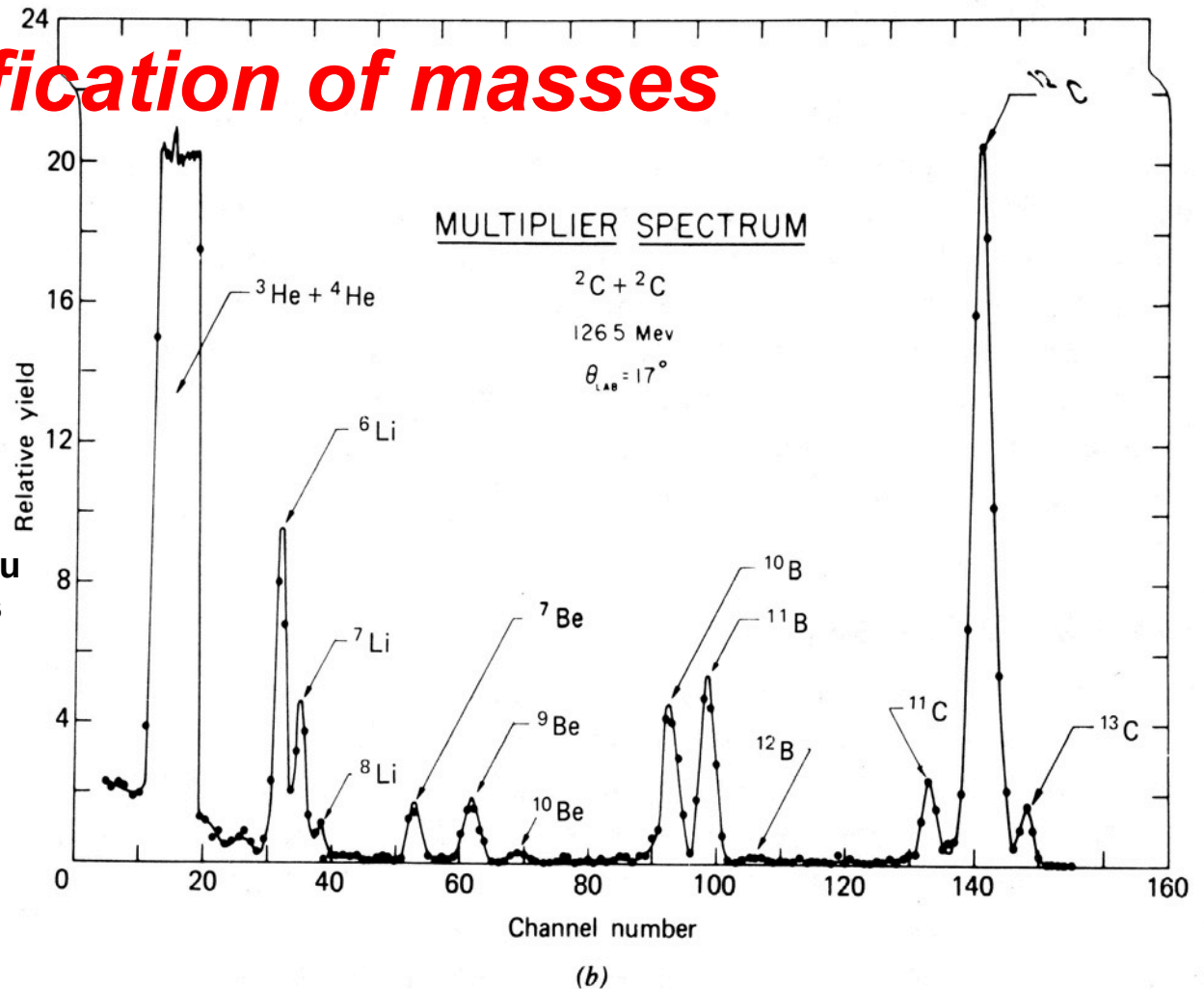
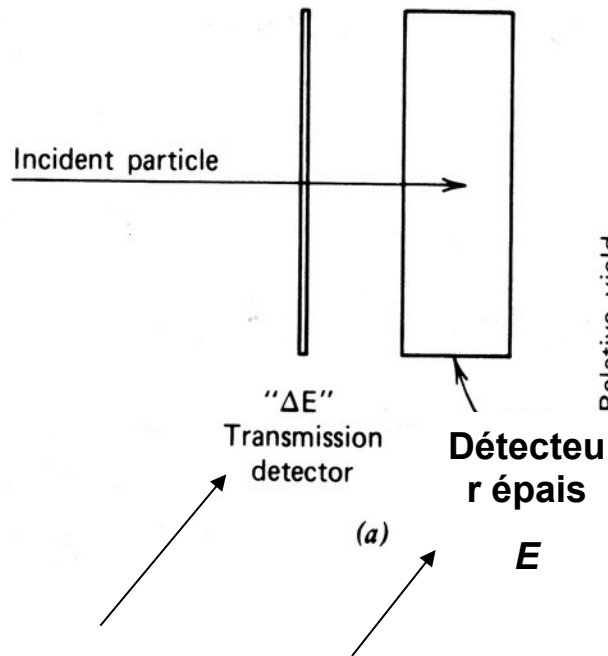
Figure 1.10 Range-Energy Curves in Silicon

## Interaction des particules chargées avec le silicium





# Identification of masses



**Figure 11-16** (a) A particle identifier arrangement consisting of tandem  $\Delta E$  and  $E$  detectors operated in coincidence. (b) Experimental spectrum obtained for the  $\Delta E \cdot E$  signal product for a mixture of different ions. (From Bromley.<sup>90</sup>)