Applications of Quantum Mechanics to Complex Physical Systems: Alpha decay

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Introduction

The main objectives of this project are to use Quantum Mechanics and describe complex physical systems. In this poster we have explained the process of alpha particle decay from inside the nucleus of an atom after a fusion reaction. Due to the fact that there is a non zero probability for a particle of energy **E** to cross a barrier **V** even if **E** < **V**, called tunnelling effect in quantum mechanics, we are able to understand how the alpha particle decay from inside the daughter nucleus after the fusion reaction in the sun. Our work is based on a mathematical modelling of the alpha decay process and the process of generation of energy in the sun.

Assumptions and Settings

- We assume that the alpha particle is firstly formed inside the daughter nucleus (which is said to be spherical symmetric).
- The equation of motion of the particle is described by Schroedinger's equation in 3D, but reduced to 1D form for the radial solution.
- The form of the potential inside the nucleus is a constant and outside the nucleus is a Coulomb potential in $\frac{1}{r}$.
  - We neglected the angular momentum of the alpha particle and the screening effect due to the electronic cloud

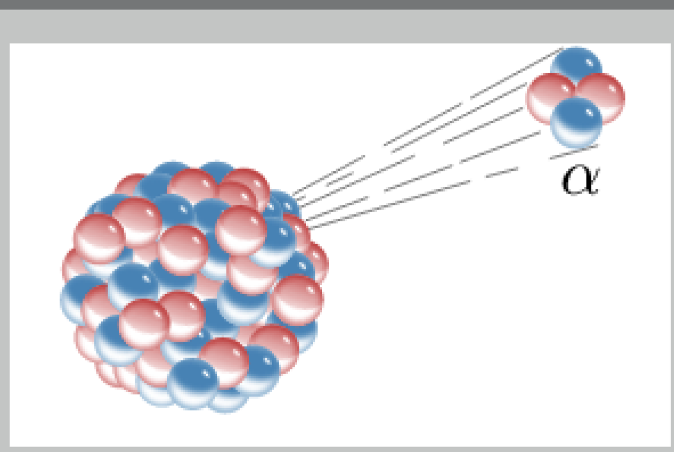

Figure 1: Alpha decay

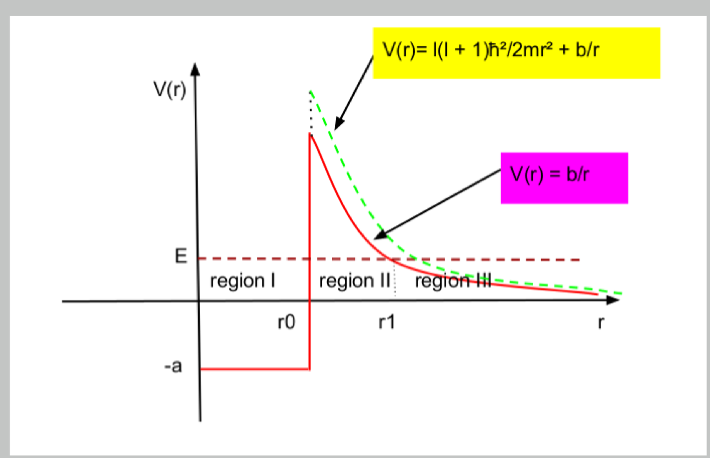

Figure 2: Potential

Methods

Two methods are used to describe the process of alpha decay:

- A numerical approach using Sage and Python.
- An asymptotic approach with WKB approximation.
- Application to alpha decay (fusion in the sun).
  - The transmission coefficient is given by the relation $\mathbf{T = 4e^{-2\Theta/\hbar}}$ with

$$\theta = \pi \mathbf{b}\sqrt{\frac{\mathbf{m}}{\mathbf{2E}}} - \mathbf{2}\sqrt{\mathbf{2mr_0b}}$$

  - We have also computed the decay rate.

$$\mathbf{P_f} = \sqrt{\left(\frac{\mathbf{m}}{\mathbf{2k_BT}}\right)^3} \mathbf{16}\pi \mathbf{v^2 e^{-\frac{mv^2}{2k_BT} - \frac{2\Theta}{\hbar}}}$$

  - We evaluated the influence of parameters as the energy, the temperature and the mass of the particle on the decay rate.

Radial wave function

- There is an exponential decay of the wave function in the region where the particle is tunnelling
- Inside the nucleus and far away from the tunnelling region we have the sinusoidal behaviour of the wave function, with different frequencies and amplitudes due to the difference in the kinetic energy of the particle on each side.

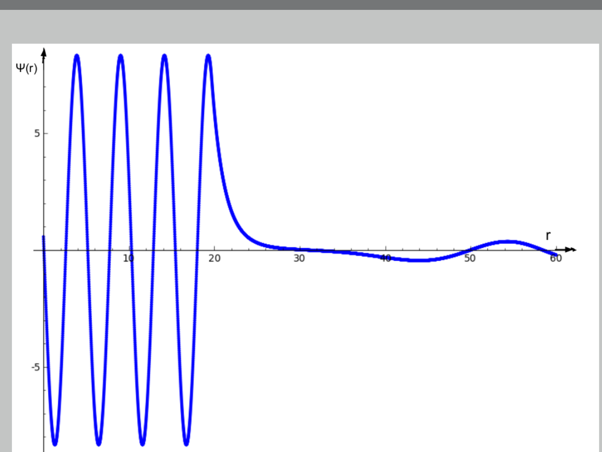

Figure 3: Numerical solution

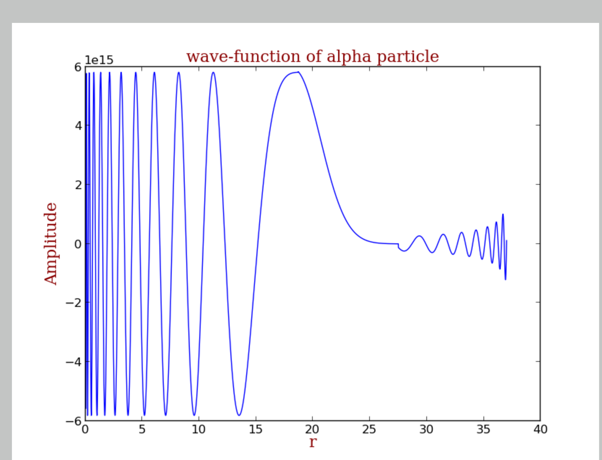

Figure 4: Asymptotic solution

Gamow peak

- This is the plot of the decay rate against the energy carried by the alpha particle called Gamow peak.
- The Gamow peak shows that there is a range of energies values for which the decay process is likely to happen. That is the values of energies under the window. As the temperature increases the window become wither and higher. The inverse effect is observed when increasing the mass of the particle.

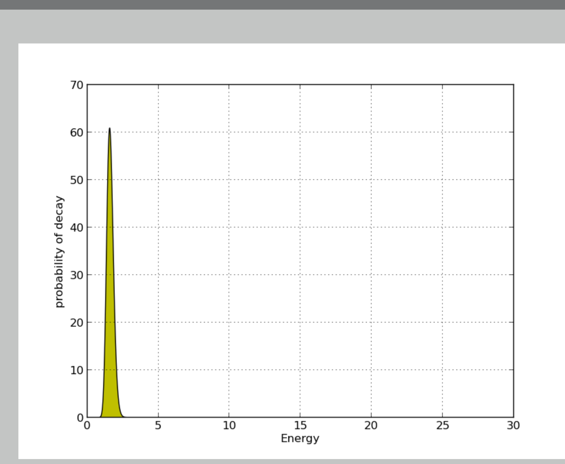

Figure 5: Gamow peak

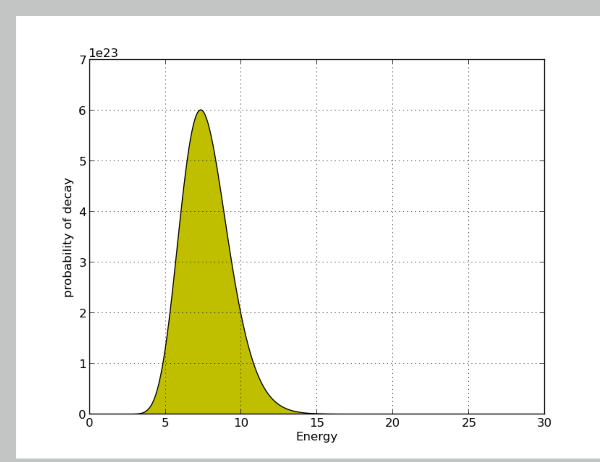

Figure 6: Gamow peak

Conclusion

This project lead us to the following conclusions:

- We are convinced that there is a non zero probability of decay for a certain range of energy values given by the Gamow peak. The highest decay rate is obtained for a value of energy $\mathbf{E_0}$ which is the maximum of the Gamow peak.
- The production of energy with fusion reactions remain a challenge for the scientific world, and if it have to be done, light elements are much advisable and high temperatures are required.
- The main issue in the production of energy from fusion reaction is to have is to have a control over the parameters influencing the decay rate and this work come as a contribution for further understanding.
- Further works can be done as part of this project taking into account the angular momentum of the particle and the screening effect.

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