Tachyonic tensor pertubation in AdS/CFT correspondance

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Abstract

We study inflation parameters in the slow roll regime and perturbations in a universe filled by tachyon field in the AdS/CFT correspondence . By analyzing the parameters space of the model and by using a quadratic potential. The viability of the model in confrontation with recent observational data is considered.

Introduction

The AdS/CFT correspondence implies that a gravitational theory on (d + 1) dimensional Anti-de Sitter (AdS) space admits a dual description in terms of a conformal field theory (CFT) propagating on the d - dimensional boundary [2].
On the other hand, it is well known that inflation is to date the most compelling solution to many long-standing problems of the big bang cosmology (horizon, flatness, monopoles, etc.). One of the success of the inflationary universe model is that it provides a causal interpretation of the origin of the observed anisotropy of the cosmic microwave background (CMB) radiation, and also provides a mechanism for production of density perturbations required to seed the formation of structures in the universe. In a simple inflationary model, the universe is dominated by a scalar field called inflaton whose potential energy dominates over the kinetic term (the slow-roll conditions), followed by a reheating period. Focusing on late-time dynamics, recent cosmological observations show that our universe is undergoing an accelerating phase of expansion and transition to the accelerated phase has been occurred in the recent cosmological past.
The scalar field we are going to explore its cosmological dynamics is the tachyon field . This field can be responsible for early time inflation in the history of the universe and also can be considered as a dark energy in the late time. In the case that the tachyon condensate starts to roll down slowly the potential, a universe dominated by this field evolves smoothly from a phase of accelerated expansion to an era dominated by a non relativistic fluid [1]. These features show that tachyon fields may provide suitable candidates to realize initial inflation and late time cosmic speed-up.

Modified Friedmann equation and the tachyon field

- The Friedmann equation in the AdS/CFT correspondence reads [2] :

$$H^2 = \frac{\hat{m}_p^2}{4c}[1+\varepsilon\sqrt{1-\frac{8c}{3\hat{m}_p^4}\rho}] \tag{1}$$

where branch $\epsilon = -1$ reduces to the correct form of the Friedmann equation at low-energy limit, $\hat{m}_p^2 = \frac{m_p^2}{8\pi}$ is the reduced Planck mass and c the coefficient of the conformal anomaly.

- The energy density of the tachyon field reads [1].

$$\rho = \frac{V}{\sqrt{1-\dot{\phi}^2}} \tag{2}$$

where $V(\phi)$ is its scalar potential.

- The cosmological dynamics of the tachyon is described by the following equation

$$\frac{\ddot{\phi}}{1-\dot{\phi}} + 3H\dot{\phi} + \frac{V'}{V} = 0 \tag{3}$$

where $V' = \frac{\partial V(\phi)}{\partial \phi}$

- During the inflationary epoch the energy density associated to the tachyon field is of the order of the potential, $\rho \approx V$. Assuming the set of slow-roll conditions, $\dot{\phi}^2 \ll 1$ and $\ddot{\phi} \ll 3H\dot{\phi}$. the Friedmann equation (1) reduces to

$$H^2 = \frac{\hat{m}_p^2}{4c}[1-\sqrt{1-\frac{8c}{3\hat{m}_p^4}V}] \tag{4}$$

and Eq. (3) becomes

$$3H\dot{\phi} \approx -\frac{V'}{V}. \tag{5}$$

- Introducing the dimensionless slow-roll parameters [1], we write

$$\varepsilon = -\frac{\dot{H}}{H^2} \simeq \frac{\hat{m}_P^2}{2}\frac{1}{(1-\frac{4c}{3\hat{m}_P^4}V)}\frac{V'^2}{V^3} \tag{6}$$

$$\eta = -\frac{\ddot{\phi}}{H\dot{\phi}} \simeq -\hat{m}_P^2[(\frac{V'^2}{V^3})(1+\frac{1}{2(1-\frac{4c}{3\hat{m}_P^4}V)}) - \frac{V''}{V^2}] \tag{7}$$

$$\gamma = -\frac{V'\dot{\phi}}{2HV} \tag{8}$$

- The number of e-folds

$$N = \int_{thc}^{tf} H dt \tag{9}$$

or equivalently

$$N \simeq -\int_{\phi_{hc}}^{\phi_f} 3H^2\frac{V}{V'}d\phi \tag{10}$$

where ϕ_{hc} denotes the value of ϕ when the universe scale observed today crosses the Hubble horizon during inflation and ϕ_f is the value of ϕ when the universe exits the inflationary phase.

Perturbations

- In this section we will study the scalar and the tensor perturbations for our model. For a tachyon field the power spectrum of the curvature perturbations is given by the following expression [1]

$$A_S^2 = \frac{1}{4\pi^2}\frac{H^4}{\dot{\phi}^2}\frac{1}{V} \tag{11}$$

The scalar spectral index n_s is given by [3] $n_s - 1 = \frac{d\ln A_s^2}{d\ln k}$. From Eq. (11), we obtain

$$n_S \approx 1 - 4\varepsilon + 2\eta + 2\gamma \tag{12}$$

or equivalently

$$n_S \approx 1 - 3\hat{m}_P^2[\frac{V'}{V^3}(1+\frac{1}{1-\frac{4c}{3\hat{m}_P^4}V}) - \frac{2}{3}\frac{V''}{V^2}] \tag{13}$$

The final WMAP9 year data gives [4] $n_S = 0.9608 \pm 0.0080$.

- On the other hand, the generation of tensor perturbations during inflation would produce gravitational waves and its amplitudes are given by [1]

$$A_T^2 = \frac{2}{\pi^2\hat{m}_P^2}H^2 \tag{14}$$

where the spectral index n_T is given by $n_T = \frac{d\ln A_T^2}{d\ln k}$.
We define a new variable θ such as $\rho = \frac{3\hat{m}_p^4}{8c}\cos^2\theta$ and $\theta \in [0, \frac{\Pi}{2}]$

$$n_T = -\frac{1}{2}(\frac{1+\sin\theta}{\sin\theta})\frac{25}{16\pi}\frac{A_T^2}{A_S^2} \tag{15}$$

We deduce that the magnitude of the spectral index is enhanced by a factor of $\frac{1}{2}(\frac{1+\sin\theta}{\sin\theta})$ relative to the standard scenario.
From expressions (11) and (14) we write the tensor-scalar ratio as [3]:

$$r = \frac{A_T^2}{A_S^2} = 8\hat{m}_P^2\frac{V'^2}{V^3} \tag{16}$$

The recent $Planck + WP + highL + BAO$ results [4] give the value for the scalar-tensor ratio $r < 0.111$.

Quadratic tachyon potential.

In this section, we consider the quadratic tachyon potential[3].

$$V = \frac{1}{2}\sigma\phi^2. \tag{17}$$

from Eq (10) the number of e-folds in the slow-roll approximation takes the following form

$$N = -\frac{\sigma}{16\hat{m}_p^2}(\phi_f^4 - \phi_{hc}^4) \tag{18}$$

By assuming $\phi_{hc} \gg \phi_f$, we find from Eq (18).

$$N = \frac{\sigma}{16\hat{m}_p^2}\phi_{hc}^4 \tag{19}$$

Substituting Eq (19) into Eq (13) the scalar spectral index becomes

$$n_S = 1 - \frac{3}{2N}(\frac{2}{3} + \frac{1}{1-\frac{8c}{3\hat{m}_P^3}\sqrt{\sigma N}}) \tag{20}$$

The running of scalar spectral index n_s is given by [1] $\alpha = \frac{dns}{dlnk} = -\frac{dns}{dN}$. From Eq (20) we get

$$\alpha = -\frac{3}{2N^2}[\frac{2}{3} + \frac{1}{(1-\frac{8c}{3\hat{m}_P^3}\sqrt{\sigma N})}(1 - \frac{\frac{4c}{3\hat{m}_P^3}\sqrt{\sigma N}}{(1-\frac{8c}{3\hat{m}_P^3}\sqrt{\sigma N})})] \tag{21}$$

From $Planck + WP + highL + BAO$ [4] $\frac{dns}{dlnk} = -0.014^{+0.016}_{-0.017}$.

We write the tensor-scalar ratio as

$$r_{T-S} = \frac{4}{N} \tag{22}$$

In the following we perform numerical analysis in our setup, by considering [4] $\sigma = 1$, $c = 5.7\ 10^8$, $\hat{m}_p = 0.77\ 10^{16}$.

Figure 1: Evolution of the tensor-scalar ratio versus the number of e-folds

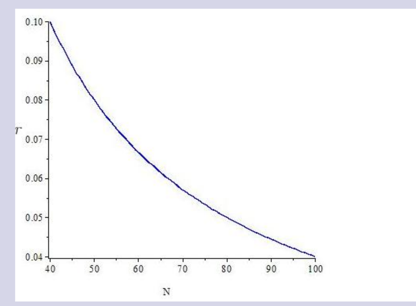


Figure2: Evolution of the running of scalar spectral index versus the number of e-folds

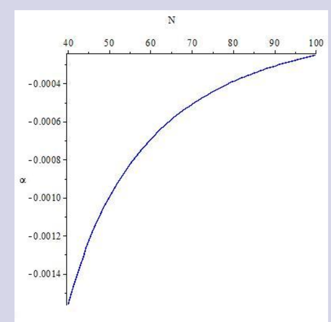


Figure 3:Evolution of the scalar spectral index versus the number of e-folds

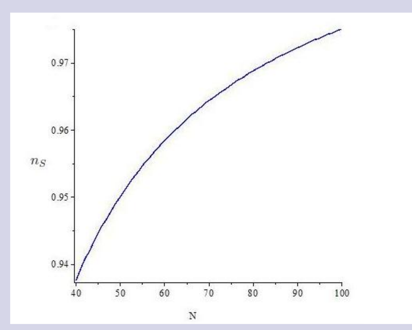


Figure 4:Evolution of the running scalar spectral index versus the scalar spectral index

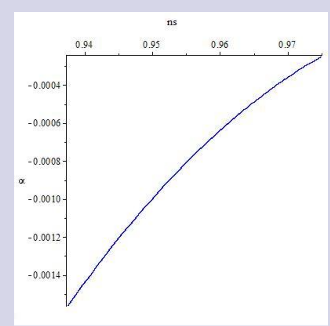


Figure5: Evolution of the running scalar spectral index versus the tensor-scalar ratio

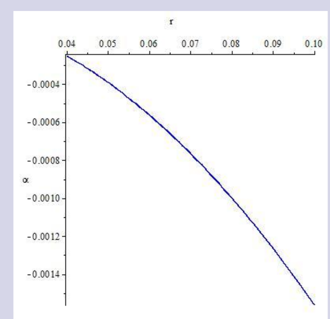
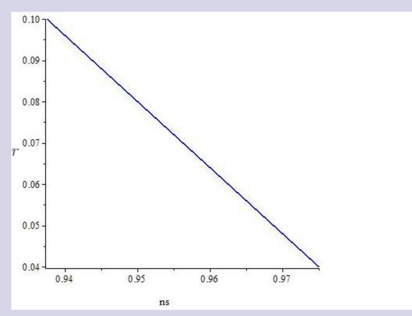


Figure6:Evolution of the tensor-scalar ratio versus the scalar spectral index.



The figure 1 shows that the number of e-folds increases as the tensor-scalar ratio decreases, whereas the figure 2 shows that the scalar spectral index increases as the number of e-folds increases too.The figure 3 shows that the number of e-folds increases as the running of scalar spectral index increases too and takes negative and very small value .
To compare with observational data, we plot the evolution of the running of scalar spectral index versus the tensor-scalar ratio in figure 4, the evolution of the running of scalar spectral index versus the scalar spectral index in figure 5 and the evolution of the tensor to scalar ratio versus the scalar spectral index in figure 6 for $40 < N < 100$.
For $60 < N < 70$ the values of r and n_s are compatible with observational data.

Conclusions

We have studied cosmological dynamics of a tachyon field as a field responsible for initial inflation. We have calculated the inflation parameters and perturbations in the AdS/CFT correspondence . By adopting a quadratic tachyon potential, we have performed a numerical analysis of the modelfs parameter space and the results have been shown in figures. Also, we have compared the tachyon model with observational data for $c = 5.7\ 10^8$ by plotting the evolution of the different parametre.

References

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