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SUSY COHERENT STATES AND CLASSICAL TRAJECTORIES

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1. The supersymétric-algébraic Method

This method requires the factorisation of the hamiltonian in the resolution of the Schrodinger equation. One introduces the superpotentiel $x(q)$ which is related to the physical potential by :

$$V(q) - E_0 = \frac{1}{2}\left[x^2(q) + \frac{dx(q)}{dq}\right] \ (1.01).$$

The hamiltonian can be writen as a product of the annihilation and creation operators:

$$\hat{H} = \hat{A}^+\hat{A} - E_0 \ (1.02)$$

these ones being given by :

$$\hat{A}^+ = \frac{1}{\sqrt{2}}\left[\frac{d}{dq} - x(q)\right] \ et \ \hat{A} = \frac{1}{\sqrt{2}}\left[-\frac{d}{dq} - x(q)\right](1.03).$$

The cohérent states are the eigenvectors of the annihilation operator:

$$\hat{A}|\alpha\rangle = \alpha\ |\alpha\rangle(1.04),$$

so that the corresponding wave functions take the form:

$$\psi_\alpha(q) = N\ exp\left[\sqrt{2}\alpha q + \int_0^q x(\xi)d\xi\right](1.05).$$

2.1 The classical approach

In classical mechanics ,we can calculate the particle's position in terms of quadratures:

$$t - t_0 = \sqrt{\frac{m}{2}}\int_{x_0}^{x}\frac{dx}{\sqrt{E-V(x)}}(2.01).$$

This can be inverted for the Morse potential, for example.

2.2 The quantum approach

According to the Ehrenfest theorem, the classical position can in principle be related to the mean value of the position operator

$$\langle\hat{Q}\rangle = Q_0\frac{{}_S\langle\psi|\hat{q}|\psi\rangle_S}{{}_S\langle\psi|\psi\rangle_S} \ (2.02).$$

From the relation between the Schrödinger and the Heisenberg pictures, we can write the wave functions as the following serie :

$$\psi_S(q,t,\alpha) = \sum_{m=0}^{\infty}\frac{(-i\omega t)^m}{m!}\left[(\hat{A}^+\hat{A})^m\ \psi_H(q)\right]\exp(-\frac{i}{\hbar}E_0 t)(2.03).$$

With reference to the preceding lines, the numérator of (2,02) is equal to :

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^n i^{m+n}\frac{(\omega t)^{m+n}}{m!\,n!}\int_{-\infty}^{+\infty} dq\ q\ f_m^*(q,\alpha)f_n(q,\alpha)\psi_H^*(q,\alpha)\psi_H(q,\alpha)(2.04)$$

The functions f_n can be calculated from the reccurence relation :

$$f_{n+1} = -\frac{1}{2}\,\partial^2 f_n - \left[\sqrt{2}\alpha + x(q)\right]\partial f_n - \left[\sqrt{2}\alpha x(q) + 2\alpha^2 - 2E_0\right]f_n \ (2.05)$$

With $f_0 = 1$.

For the Morse potential, one has :

$$x(q) = \frac{1}{C_0}\left[-C_0 + \exp(-C_1 q)\right] \ (2.06).$$

The wave function of the coherent state takes the form:

$$\psi_\alpha(q) = exp\left[-\frac{1}{C_1^2}\exp(-C_1 q) - \frac{C_0}{C_1}q + \sqrt{2}\alpha\ q\right](2.07)$$

Which is not square integrable. The integrals in (2.02) diverge, so that, one can not make a link with classical trajectories in this case.

3. Our project

To go forward,we consider the superpotential :

$$x(q) = x_0 + x_1 q + x_2 q^2 + x_3 q^3 \ (2.08)$$

The associated physical potential is a 6th degree polynomial. With a good choice of the parameters, this potential has only one minimum and goes to infinity for big values of the position. The Schrodinger equation will give an infinite discete spectrum. Moreover, the coherent states of equation (1,05) are normalisable.

We are now in a position such that we can compute and compare classical and quantum trajectories using the formulas (2.01) and

4. Bibliography

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