Introduction to Accelerators
Tutorial

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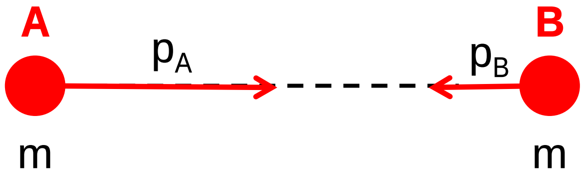
1. Answers to questions raised after the course

Fixed-target vs head-on: identical particles



- Relativistic invariant $(\Sigma m)^2 c^4 = (\Sigma U)^2 - (\Sigma p)^2 c^2$
- In the laboratory frame $4m^2c^4 = (U_A + U_B)^2 - (\overrightarrow{p_A} + \overrightarrow{p_B})^2 c^2$
- Let U^* be the total energy available in the collision
- In the center-of-mass frame $\overrightarrow{p^*} = \overrightarrow{p_A}* + \overrightarrow{p_B}* \equiv 0$

$$4m^2c^4 = {U^*}^2$$

$$ {U^*}^2 = (U_A + U_B)^2 - (\overrightarrow{p_A} + \overrightarrow{p_B})^2 c^2$$

- Fixed-target

$$p_B = 0\ ;\ U_B = mc^2$$

$$ {U^*}^2 = {U_A}^2 - {p_A}^2c^2 + m^2c^4 + 2U_Amc^2$$

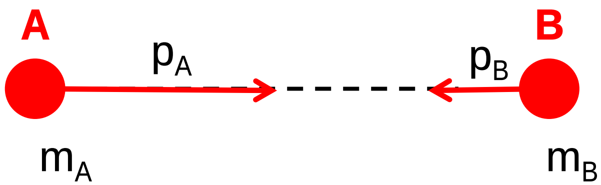
$$ {U^*}^2 = 2m^2c^4 + 2U_Amc^2 \approx 2U_Amc^2$$

$$U^* \approx \sqrt{2U_Amc^2}$$

- Head-on collision

$$U^* = U_A + U_B$$

Fixed-target vs head-on: general case



- Relativistic invariant $(\Sigma m)^2c^4 = (\Sigma U)^2 - (\Sigma p)^2c^2$
- In the laboratory frame $(m_A + m_B)^2c^4 = (U_A + U_B)^2 - (\overrightarrow{p_A} + \overrightarrow{p_B})^2c^2$
- Let U^* be the total energy available in the collision
- In the center-of-mass frame $\overrightarrow{p^*} = \overrightarrow{p_A} * + \overrightarrow{p_B} * \equiv 0$

$$(m_A + m_B)^2c^4 = U^{*2}$$

$$\boxed{U^{*2} = (U_A + U_B)^2 - (\overrightarrow{p_A} + \overrightarrow{p_B})^2c^2}$$

- Fixed-target

$$p_B = 0 \; ; \; U_B = m_Bc^2$$

$$U^{*2} = U_A{}^2 - p_A{}^2c^2 + m_B{}^2c^4 + 2U_Am_Bc^2$$

$$U^{*2} = (m_A{}^2 + m_B{}^2)c^4 + 2U_Am_Bc^2 \approx 2U_Am_Bc^2$$

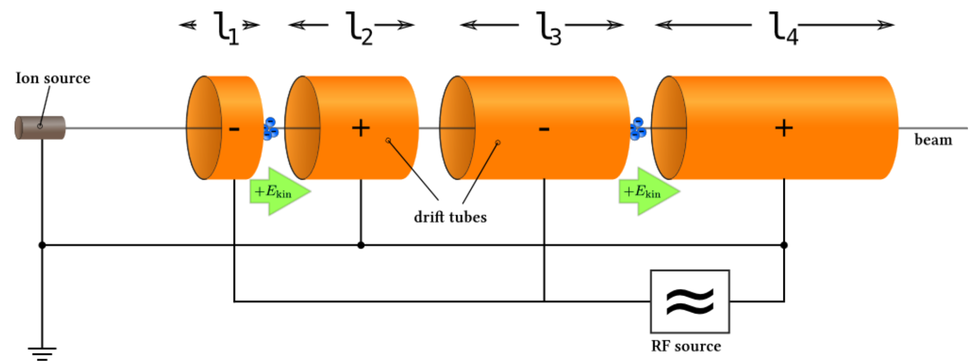
$$\boxed{U^* \approx \sqrt{2U_Am_Bc^2}}$$

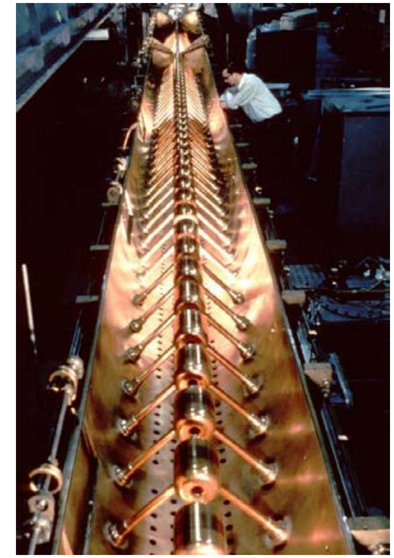
- Head-on collision

$$\boxed{U^* = U_A + U_B}$$

Alvarez linac

L. Alvarez





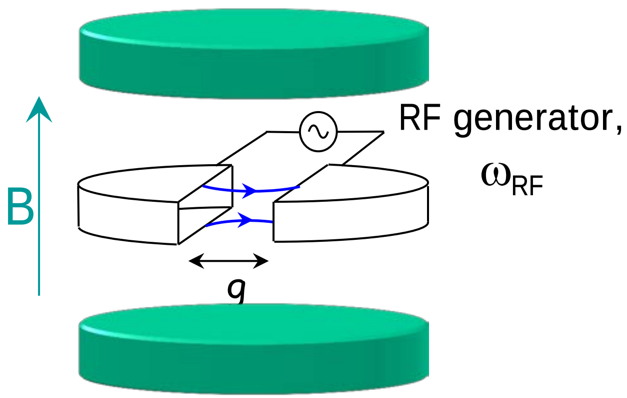
- Synchronism condition $$L = v\frac{T_{RF}}{2} = \frac{v}{2f_{RF}}$$

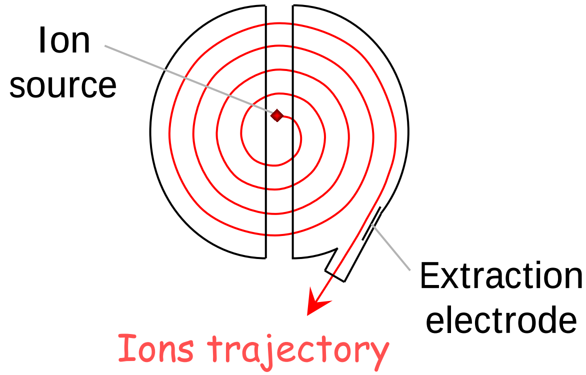
- Acceleration occurs in the gaps between the drift tubes
- First practical linac (200 MHz, 32 MeV) built by L. Alvarez at Berkeley in 1946
- As particle velocity increases, the drift tubes get longer $\Rightarrow$ lost length
- This can be contained by increasing $f_{RF} \Rightarrow$ increased power loss
- To limit power loss, enclose the system into a resonant cavity

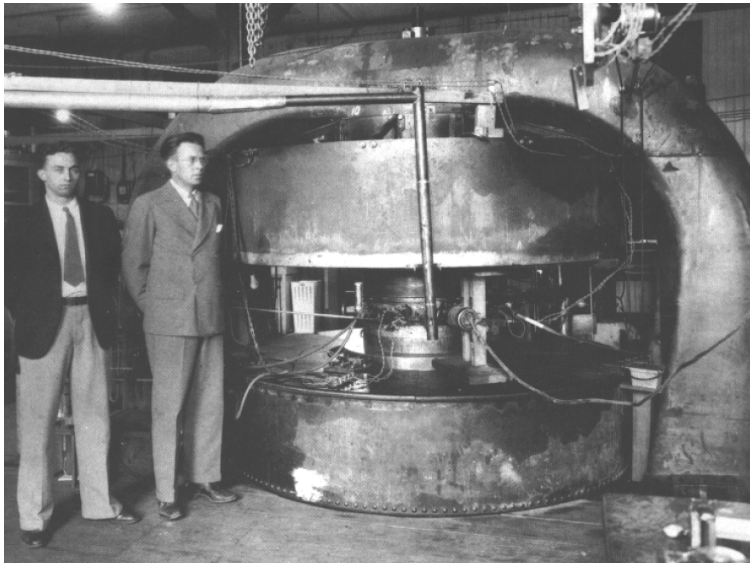
Length of drift tubes in linac

- $L = v\frac{T_{RF}}{2} = \frac{v}{2f_{RF}} = \frac{\beta c}{2f_{RF}}$
- Consider linac with $\beta = 0.1$
  - Calculate length of drift tubes for a) $f_{RF} = 1\text{MHz}$, b) $f_{RF} = 100\text{MHz}$
  - Answers: a) 15 m !!! b) 15 cm
- First proton linac (Alvarez, 1946) accelerated from 4 to 32 MeV, with RF frequency 200 MHz
  - Calculate length of drift tubes a) at beginning of linac, b) at end of linac
  - At beginning, $\gamma = \frac{938+4}{938} = 1.0043$, hence $\beta = 0.092$ and $L = 0.069\ m$
  - At end, $\gamma = \frac{938+32}{938} = 1.0342$, hence $\beta = 0.255$ and $L = 0.191\ m$
  - Remark: the formula gives the minimum length, i.e. this which covers the half-alternance when the field is retarding. In practice, the drift tubes are usually made longer to limit the span of accelerating field variation seen by the particles

Lawrence & Livingston's Cyclotron







S. Livingston & E.O. Lawrence (1933)

Synchronism $2\pi\rho = vT_{RF} = v/f_{RF}$

Cyclotron frequency $\omega_{RF} = \dfrac{eB}{m_0\gamma}$

- "Folded-in" linac: the electrodes are the "dees" immersed in magnetic field
- Orbits are spirals as particles gain energy at each gap crossing
- Constant magnetic field means constant frequency at $\gamma = 1$, no exact synchronism for relativistic particles

Cyclotron frequency

- $\omega_{RF} = \frac{eB}{m_0\gamma}$
- $f_{RF} = \frac{eB}{2\pi m_0\gamma}$
- The AMIT compact superconducting cyclotron is being developed by CIEMAT and CERN (protons, 8.5 MeV, 4 T) for production of medical radionuclides
- Calculate the cyclotron frequency
- Answer
  - The beam is non-relativistic $m \approx m_0$
  - $f_{RF} \approx 64\ MHz$

Synchrotron radiation

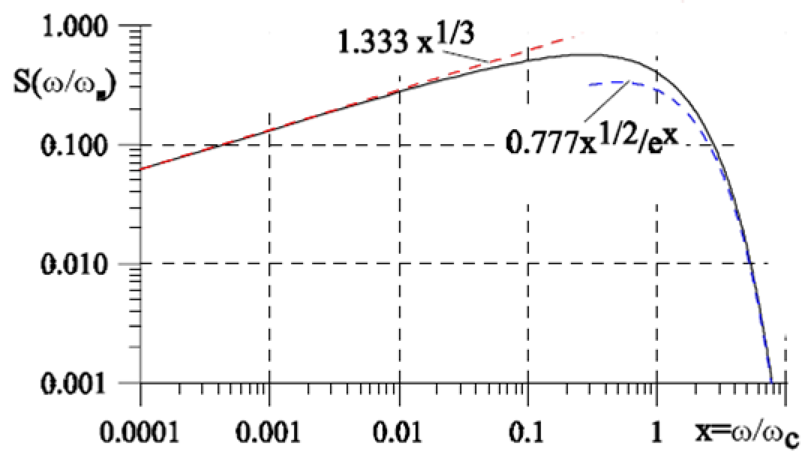
- Instantaneous power $P = \frac{cC_\gamma}{2\pi}\frac{U^4}{\rho^2}$

  with $C_\gamma = \frac{4\pi}{3}\frac{r_e}{(mc^2)^3}$ for electrons

- Energy loss per turn (constant B) $W_{turn} = C_\gamma \frac{U^4}{\rho}$

- From a bending magnet, emission in a continuous spectrum, the median of which is the "critical photon energy" $\mathcal{E}_c = \frac{3}{2}\hbar c \frac{\gamma^3}{\rho}$

Normalized frequency spectrum



Fraction of beam energy lost by synchrotron radiation

- $W_{turn} = C_\gamma \frac{U^4}{\rho}$
- Calculate fraction of particle energy lost per turn in LEP
  - $U = 100\ GeV,\ \rho = 3026\ m$
- $\frac{W_{turn}}{U} = C_\gamma \frac{U^3}{\rho} = \frac{4\pi}{3} \frac{r_e}{(mc^2)^3} \frac{U^3}{\rho} = \frac{4\pi}{3} \frac{r_e}{\rho} \gamma^3$
- With $r_e \approx 2.82E - 15\ m$ classical electron radius
- Answer
  - $\gamma = 1.96E5$
  - $\frac{W_{turn}}{U} \approx 2.9\%$
  - A powerful RF system is needed to compensate continuously for the synchrotron power radiated by the beams

2. Some transverse beam dynamics

(from slides by D. Brandt)

Intro to beam dynamics

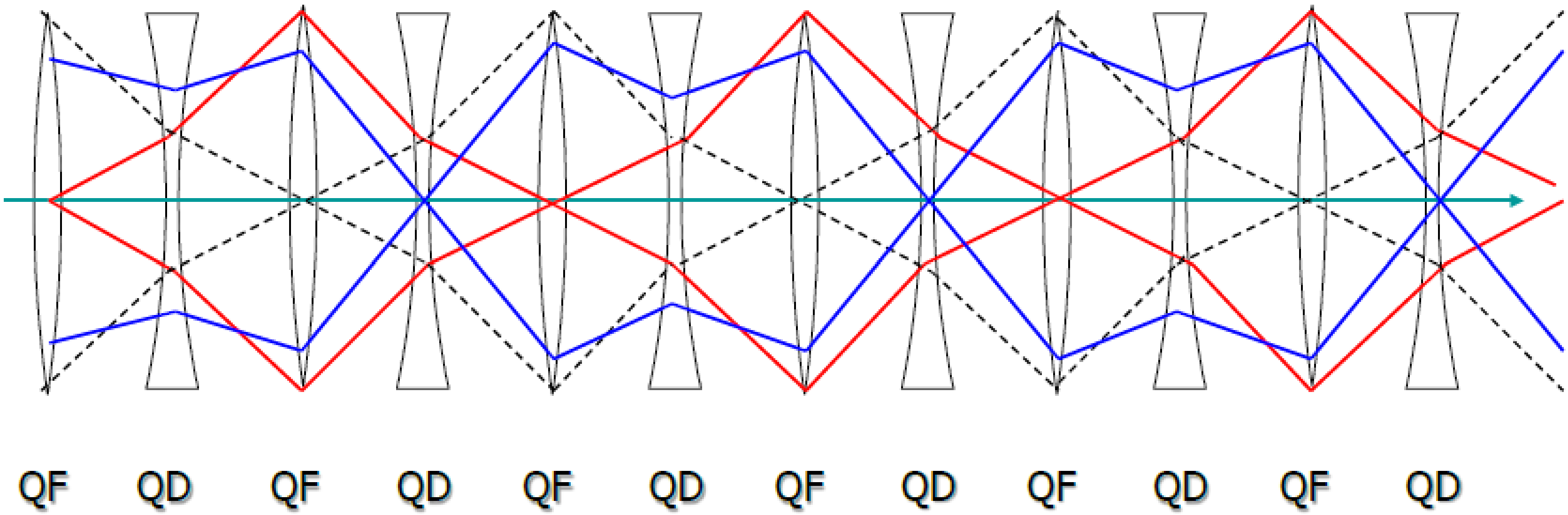
In order to describe the motion of the particles, each particle is characterised by:

- Its azimuthal position along the machine: s
- Its momentum: p (or Energy E)
- Its horizontal position: x
- Its horizontal slope: x'
- Its vertical position: y
- Its vertical slope: y'

i.e. a sixth dimensional vector

(s, p, x, x', y, y')

Alternating-gradient focussing: the FODO cell



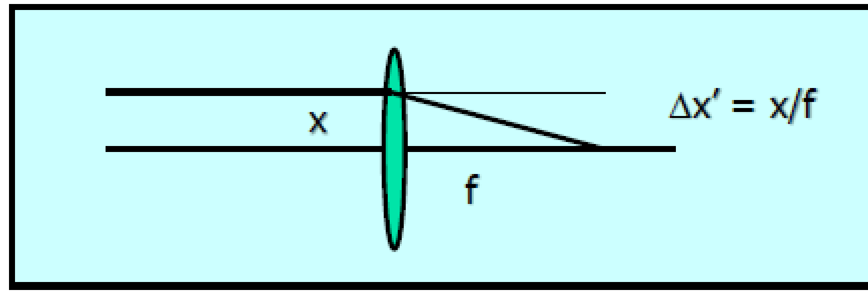
Particles for which $x, x', y, y' \neq 0$ thus oscillate around the ideal particle ...

but the trajectories remain inside the vacuum chamber !

Thin-lens analogy of AG focussing

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{out} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{in}$$

$$X_{out} = x_{in} + 0.x'_{in}$$
$$x'_{out} = (-1/f).x_{in} + x'_{in}$$

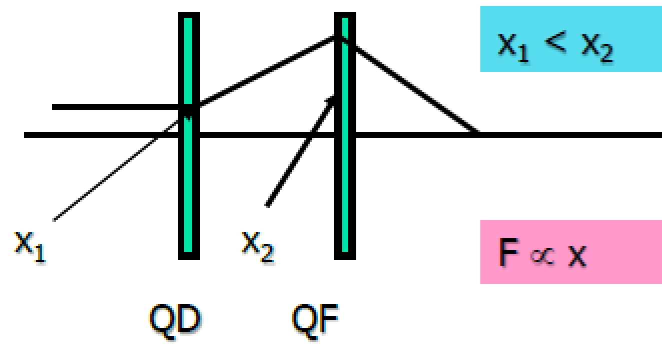


$$\text{Drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

$$\text{QF-Drift-QD} = \begin{pmatrix} 1-L/f & L \\ -L/f^2 & 1+L/f \end{pmatrix}$$

Initial: $x = x_0$ and $L < f$
$x' = 0$

More intuitively:

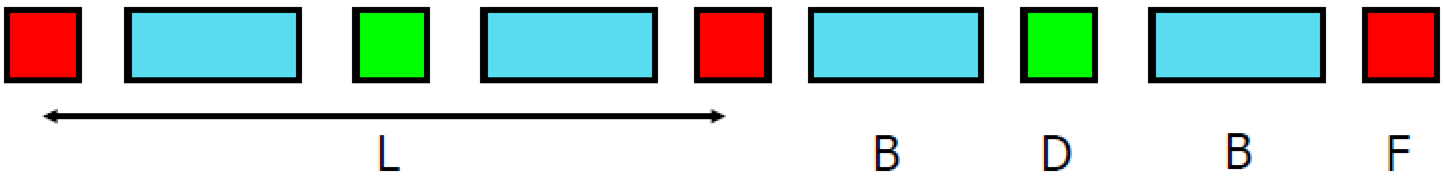


$x_1 < x_2$

$F \propto x$

Periodic lattice and tune

The accelerator is composed of a periodic repetition of cells:

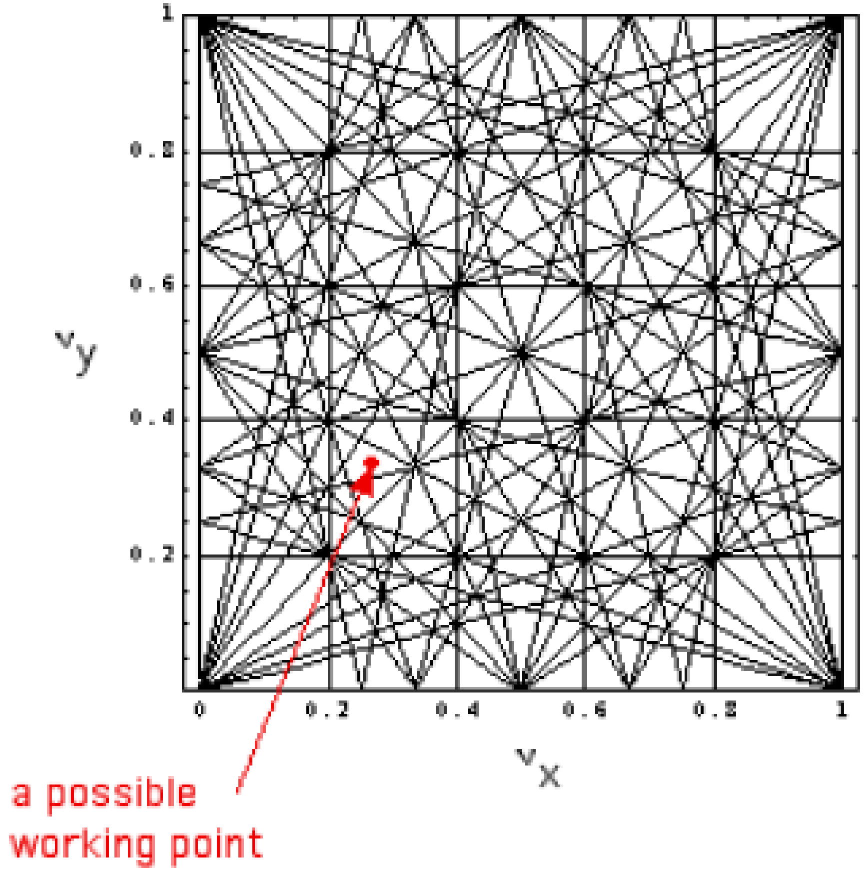


- The ideal particle will follow a particular trajectory, which closes on itself after one revolution: the closed orbit.

- The real particles will perform oscillations around the closed orbit.

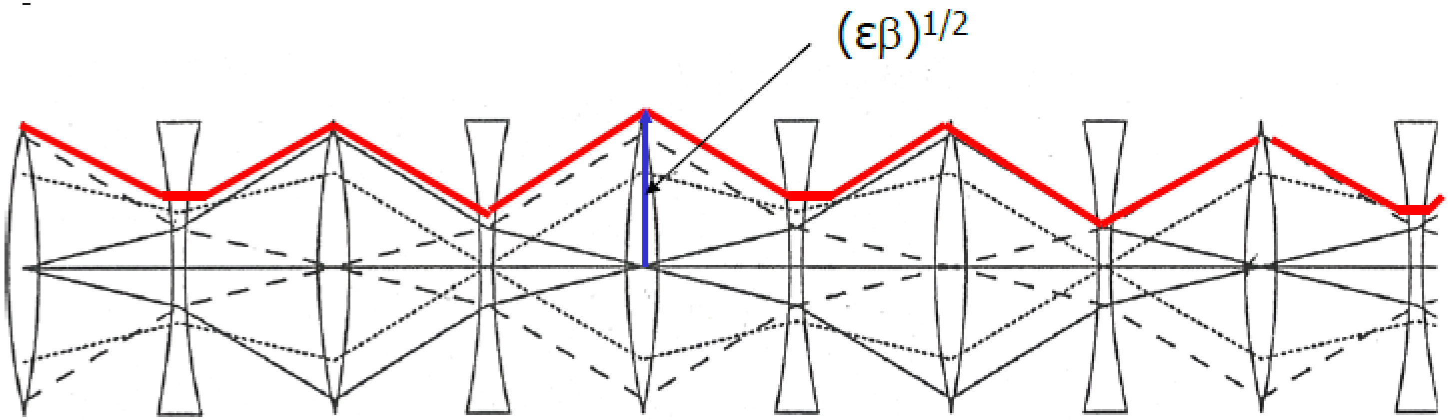
- The number of oscillations for a **complete revolution** is called the **Tune Q** of the machine (Qx and Qy).

Tune diagram and integer resonances



Resonance $\quad n\, \nu_x + m\, \nu_y = K$ (integer)

The beta function β(s)



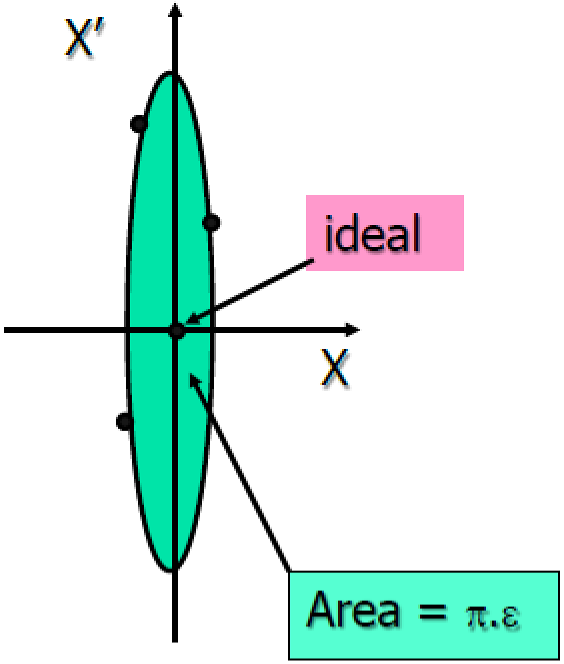
The β-function is the envelope around all the trajectories of the particles circulating in the machine.

The β-function has a minimum at the QD and a maximum at the QF, ensuring the net focusing effect of the lattice.

It is a periodic function (repetition of cells). The oscillations of the particles are called betatron motion or **betatron oscillations**.

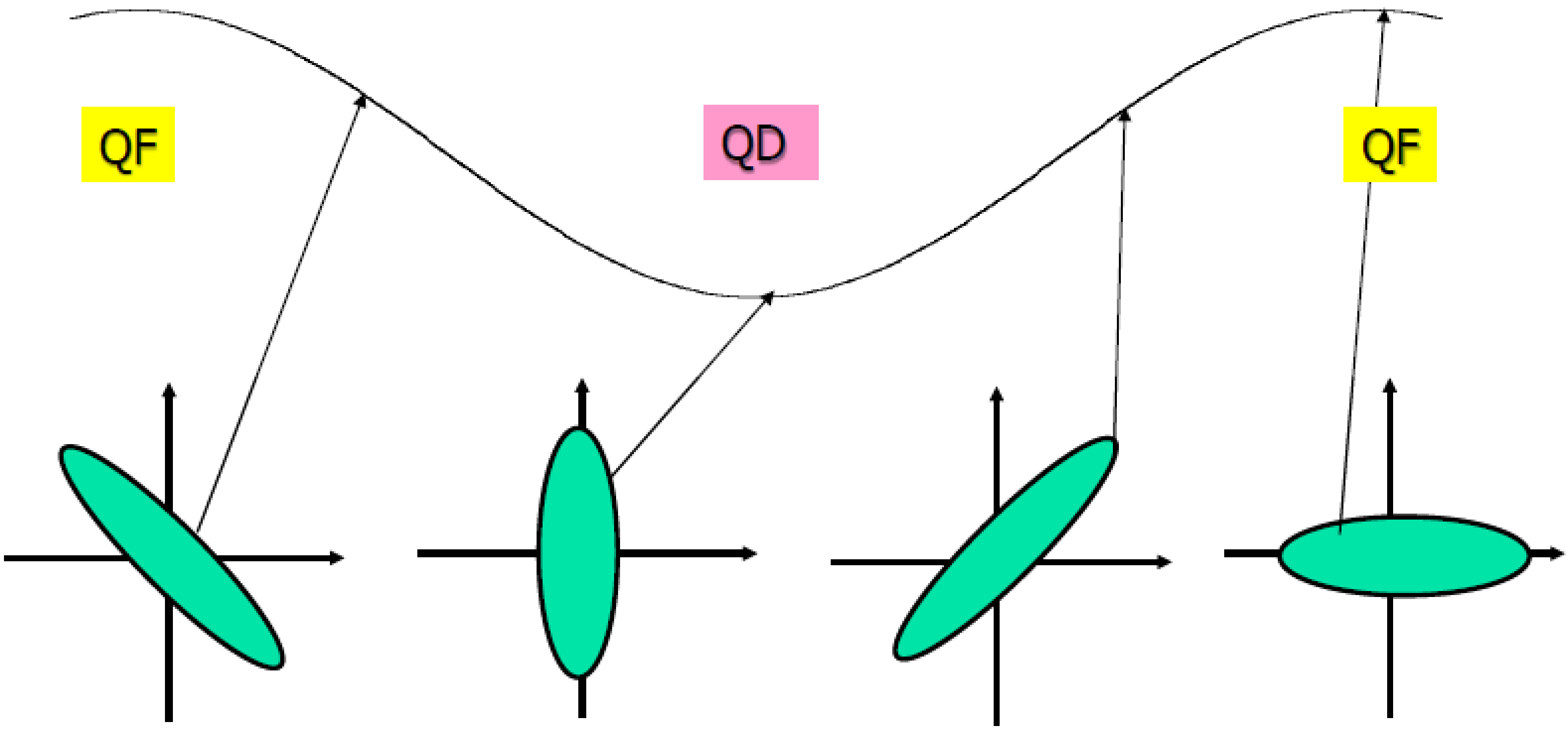
Phase space and emittance

- Select a particle in the beam being at 1 sigma (68%) of the distribution and plot its position vs. its phase (x vs. x') at some location in the machine for many turns.



- ε Is the emittance of the beam [mm mrad]
- ε describes the quality of the beam
- Measure of how much particle depart from ideal trajectory.
- β is a property of the machine (quadrupoles).

Emittance conservation



The shape of the ellipse varies along the machine, but its area (**the emittance ε**) remains constant at **a given energy**.

Why introduce β(s) and ε ?

The β function and the emittance are fundamental parameters, because they are directly related to the beam size (**measurable quantity !**):

Beam size [m]

$$\sigma_{x,y}(s) = (\varepsilon.\beta_{x,y}(s))^{1/2}$$

σ (IP) = 17 μm

at 7 TeV (β=0.55 m)

The emittance ε characterises the quality of the injected beam (kind of measure how the particules depart from ideal ones). It is an **invariant** at a given energy.

ε = beam property

β = machine property (quads)