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Supersymmetry: Basic Theoretical Introduction

Outline

I. Basic Introduction to SUSY Phenomenology

1. SM Open Questions
2. Avenues Beyond the SM
3. Grand Unification
4. Supersymmetry
5. MSSM
6. Motivations

1. Open Questions about the SM

Summary of the SM (1)

• The Standard Model (SM) is a Quantum Field Theory (QFT) describing the known particles and their interactions (except for gravity)

• It obeys different types of symmetries:

- (External) Global Space-Time Symmetries:
  - described by the Poincaré group, formed by the:
    - space-time translations and
    - Lorentz group (boosts & space-time rotations)

- (Internal) Local (aka Gauge) Symmetries:

4-D

$$G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

Strong Interaction

Electroweak (EW) Interaction

- (Mixed) Discrete Global Symmetries: CPT

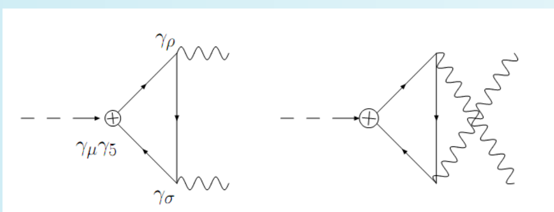
charge conjugation

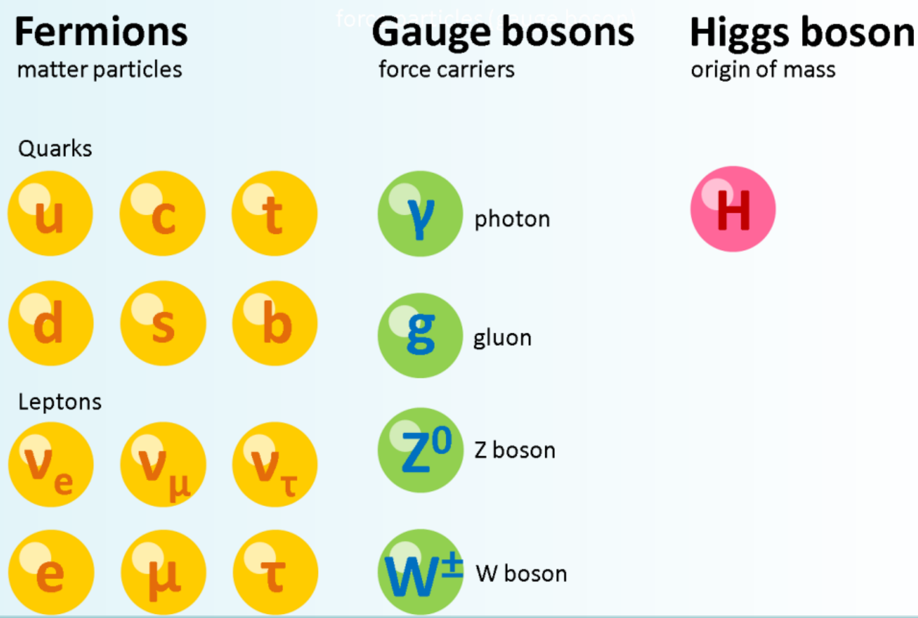
space reversal

time reversal

Summary of the SM (2)

- It's a subtle and powerful way to classify the known elementary particles:
  - Gauge Bosons:
    - mediate the fundamental interactions, which are explained as consequences of some gauge symmetries
    - they sit in the adjoint representation of the G_{SM} gauge group
    - have integer spin (S=1) and obey Bose-Einstein statistics
  - Fermions:
    - are the actual « building blocks » of matter
    - they sit in the fundamental representation of the G_{SM} gauge group
    - have half-integer spin (S=1/2) and obey Fermi-Dirac statistics (Pauli exclusion principle)
    - chiral => anomalies

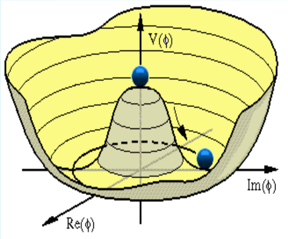




Summary of the SM (3)

- All of this looks fine, … BUT… it predicts that all these particles are massless!
- Higgs mechanism:
  - introduce an $SU(2)_L$ doublet of complex scalar (S=0) fields $\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$
  - this represents 4 d° of freedom
  - introduce an ad-hoc potential

$$V = -\mu^2 \Phi^2 + \lambda \Phi^4$$



$$< 0 | \Phi | 0 > = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

  - the higgs boson acquires a vev => spontaneously breaks the EW symmetry

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_C \otimes U(1)_{EM}$$

  - 3 of the Higgs boson d° of freedom serve to give mass to W^+, W^-, Z^0
  - there's just 1 d° of freedom left… it corresponds to the Higgs boson
  - the Higgs boson provides a mass to:
    - W, Z: through gauge interactions $m(W^\pm) = \frac{g_2 v}{\sqrt{2}}$ $m(Z^0) = \frac{1}{\cos\theta_W} \frac{g_2 v}{\sqrt{2}}$
    - H: through self-interaction $m(H^0) = \sqrt{-2\mu^2}$
    - Fermions: through Yukawa interactions $m(f_i) = \frac{y_i v}{\sqrt{2}}$

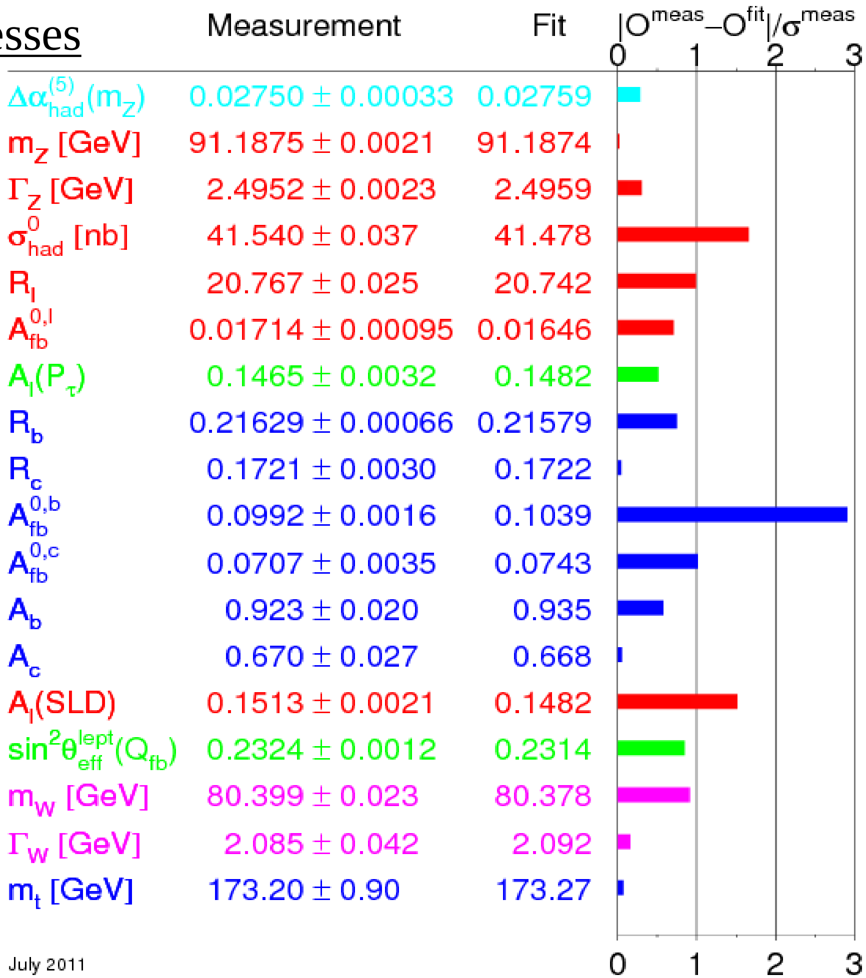
Summary of the SM (4)

Successes

• The SM is extremely successful in describing most phenomena in particle physics

• This theoretical model was proposed in the 1960's (Glashow, Salam, Weinberg) and firmly consolidated in the early 1970's (G. T'Hooft and M. Veltman). Still, more than 40 years later, it remains largely unchallenged by the numerous experiments carried out ever since!

• The most spectacular success is the discovery a Higgs boson in 2012 at CERN



Open Questions about the SM (1)

- It has 19 unpredicted parameters
  - 3 gauge coupling constants: g_1, g_2, g_3 (or g_2, g_3 and $\sin^2\theta_W$)
  - 9 Yukawa couplings (e,μ,τ,u,d,s,c,b,t)
  - μ and λ from Higgs potential (or λ and m_H)
  - 3 elements of the CKM matrix + 1 phase
  - 1 parameter θ_{PC} could (or should?) cause CP violation in QCD

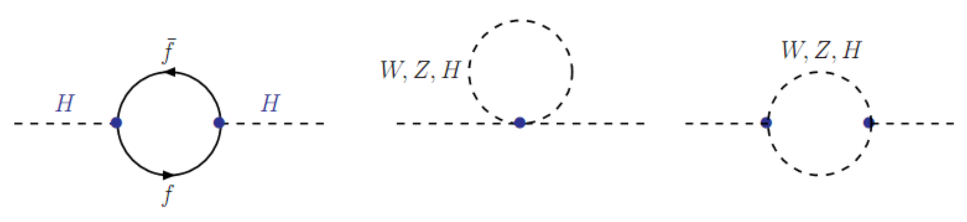
- One can regroup the open problems into 2 main categories:
  - Flavor problems:
    - Why are there 3 generations?
    - What's the origin of the CKM angles?
    - What's the origin of CP violation?
    - Are quarks and leptons elementary?,…
  - Hierarchy & Unification problems:
    - How to stabilize the Higgs boson mass?
    - Is Grand Unification: does strong and EW interaction unify?
    - Is there a theory for quantum gravity?
    - Is there a unified theory of the 4 fundamental interactions (TOE)?
    - Do this theory require extra spatial dimensions?
    - Do elementary particles have spatial extensions?
    - Why is electric charge quantized?,…

Open Questions about the SM (2)

- Naturalness:
  - Definition (t'Hooft): « a physical parameter can naturally be small if it is null due to a symmetry » (it may acquire a small value through radiative corrections)
  - In the SM most masses are protected from non-naturally large radiative corrections by some symmetries, remember:
    - gauge bosons masses are protected by the gauge symmetry
    - charged leptons are protected by the chiral symmetry
      - $\Rightarrow$ corrections:
        1. depend on the fermion bare mass
        2. are logarithmic
    - only the Higgs boson mass is not protected

$$\delta m_f \approx m_f^0 \mathrm{Log}\frac{\Lambda}{m_f^0}$$

Λ: high energy (UV) cut-off



$$m_H^2 = (m_H^0)^2 + \frac{3\Lambda^2}{8\pi^2 v^2}\left(m_H^2 + 2m_W^2 + m_Z^2 - 4m_t^2\right)$$ => m_H diverges quadratically wrt Λ !

- Hierarchy problem:
  - In presence of any large Λ, m_H is destabilized by this quadratic divergence
  - There's no guarantee to keep it within the EW range O(0.1-1 TeV),
  - Unless one uses unatural fine-tuning:
    - GU: $\Lambda=10^{16}$ GeV => 16 orders of magnitude fine-tuning, at each P.O. !

Open Questions about the SM (3)

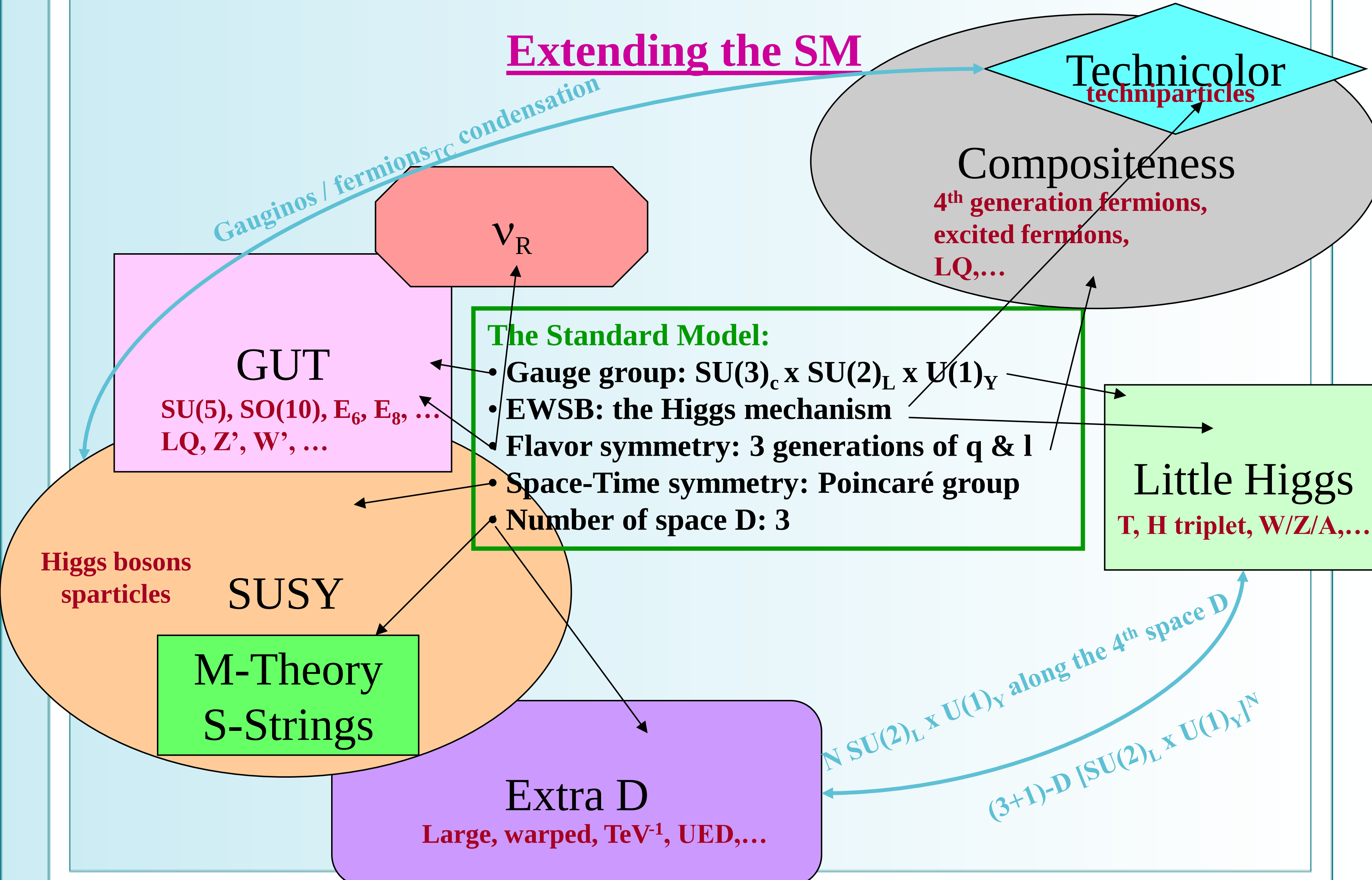
- Hints of New Physics Beyond the SM:

  - In order to account for non-zero ν masses
    - one has to introduce ν_R or
    - to introduce L non-conserving interactions

  - Non-baryonic dark matter:
    - required by galaxy rotation curves
    - confirmed by the WMAP experiment
    - no SM particle fits these observations
    - must consist of some BSM neutral weakly interacting massive particle (WIMP)

2.   Avenues Beyond the SM

Avenues Beyond the SM



3. Grand Unification

Grand Unification (1)

- Grand Unification is the unfication of the EW and strong interactions at high energy
- Georgi and Glashow SU(5) Model in 1974
  - They looked for a larger gauge group to embed G_{SM}

    The rank is the nber of generators that can be diagonalized simultaneously

    For G_{SM}: there are 2 in $SU(3)_C$, and 2 (T_3 & Y) in $SU(2)_L xU(1)_Y$ => rank =4

    => G_{GUT} group of rank >= 4

    They picked the simpler rank=5-1 group w/ complex representations for chirality

  - Gauge group: SU(5)
    - has 5^2-1=24 generators

  - SM Quarks and Leptons sit in irreducible representations (IR-REPS) of SU(5):

$$5 \qquad \bar{5} = (1,\bar{2}) \oplus (\bar{3},1) \qquad 10 = (1,1) \oplus (\bar{3},1) \oplus (3,2)$$

$$\psi_R = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ e^+ \\ -\bar{\nu}_e \end{pmatrix}_R \qquad \psi_L = \begin{pmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \\ e^- \\ -\nu_e \end{pmatrix}_L \qquad \psi_L = \begin{pmatrix} 0 & \bar{u}_3 & \bar{u}_2 & u_1 & d_1 \\ \bar{u}_3 & 0 & \bar{u}_1 & u_2 & d_2 \\ \bar{u}_2 & \bar{u}_1 & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & -e^+ & 0 \end{pmatrix}_L$$

Grand Unification (2)

- Gauge bosons:
  - 12 LeptoQuarks bosons X, Y
    - charged under $SU(2)_L$ and $SU(3)_C$
    - have fractional electric/colour charges Q(X)=+/-4/3, Q(Y)=+/-1/3
    - new interactions do not conserve B nor L
    - however all interactions conserve (B-L)

$$(u,d)_L \rightarrow e^+ + (\overline{X},\overline{Y})$$

$$24 = \underset{\text{gluons}}{(8,1)} \oplus \underset{W^{\pm},\,Z}{(1,3)} \oplus \underset{\gamma}{(1,1)} \oplus \underset{X,Y\text{ bosons}}{(3,2) \oplus (\overline{3},2)}$$

$$G = \begin{pmatrix} G_3 + \frac{G_8}{\sqrt{3}} - \sqrt{\frac{2}{5}}B & G_1 - iG_2 & G_4 - iG_5 & X_1 & Y_1 \\ G_1 + iG_2 & -G_3 + \frac{G_8}{\sqrt{3}} - \sqrt{\frac{2}{5}}B & G_6 - iG_7 & X_2 & Y_2 \\ G_4 + iG_5 & G_6 + iG_7 & -\frac{2}{\sqrt{3}}G_8 - \sqrt{\frac{2}{5}}B & X_3 & X_3 \\ X_1 & X_2 & X_3 & \frac{W_3}{\sqrt{2}} + \sqrt{\frac{3}{10}}B & W^+ \\ Y_1 & Y_2 & Y_3 & W^- & -\frac{W_3}{\sqrt{2}} + \sqrt{\frac{3}{10}}B \end{pmatrix}$$

Grand Unification (3)

- SU(5) Spontaneous Symmetry Breaking:
  - a 24-D Higgs multiplet 24_H spontaneously breaks SU(5) gives mass to X and Y of O(Λ_{GUT}):

$$M_X^2 = M_Y^2 = \frac{5}{6} \cdot g_5^2 \cdot \langle 0 | 24_H | 0 \rangle^2$$

  - a 5-D Higgs multiplet 5_H or $\bar{5}_H$, containing a colour triplet and the usual SM Higgs doublet is responsible for the EWSB

$$5_H = \begin{pmatrix} T \\ \Phi \end{pmatrix}$$

$$M_W^2 \approx \frac{g_2^2}{4} \cdot \langle 0 | 5_H | 0 \rangle^2$$

$$M_Z^2 \approx \frac{g_2^2}{4 \cdot \cos^2\theta_W} \cdot \langle 0 | 5_H | 0 \rangle^2$$

- SU(5) → $SU(3)_C$ x $SU(2)_L$ x $U(1)_Y$ → $SU(3)_C$ x $U(1)_{EM}$

$\langle 0 | 24_H | 0 \rangle$ $\langle 0 | 5_H | 0 \rangle$

Grand Unification (4)

- SM:
  - θ_W is not predicted by the theory
  - it has to be measured from experiment (Z to W mass ratio)
  - there is not a full unification of weak & EM interactions

- GUT:
  - the unified interactions (EW+strong) all have the same single coupling constant

$$g_3 = g_2 = g_1 = g_{GUT} \qquad \mu_R = \Lambda_{GUT}$$

  - weak and EM gauge couplings are related by a Clebsch-Gordan coefficient C predicted by the GUT group

$$\tan\theta_W(\mu) = \frac{1}{C}\frac{g_1(\mu)}{g_2(\mu)}$$

  - to estimate of Λ_{GUT}, use renormalization group equations (RGE):

$$\frac{1}{g_3^2(\mu_0)} = \frac{1}{g_3^2(\mu_R)} + \frac{\beta_3}{2\pi}\cdot \mathrm{Log}\frac{\mu_R}{\mu_0}$$

  - Charge Quantization: in the 5-D IR-REP, using:

$$\mathbb{Q} = \mathbb{T}_3 + \frac{\mathbb{Y}}{2} \qquad 3Q(d) + Q(e^+) = 0$$

  - Quatum numbers of the SU(5) fermions enable to avoid anomalies

Grand Unification (5)

- Gauge coupling evolutions at 1-loop:

gauge bosons loops

- SU(N): $$\beta_N(g_N) = \frac{g_3}{(4\pi)^2}\left[\frac{-11}{3}N + \frac{4}{3}N_{gen}\right]$$

fermions loops

- SU(3)$_C$: $$\beta_3(g_3) = \frac{g_3}{(4\pi)^2}\left[-11 + \frac{4}{3}N_{gen}\right]$$

- SU(2)$_L$: $$\beta_2(g_2) = \frac{g_2}{(4\pi)^2}\left[\frac{-22}{3} + \frac{4}{3}N_{gen} + \frac{1}{6}N_H\right]$$

Higgs loops

- U(1)$_Y$: $$\beta_1(g_1) = \frac{g_1}{(4\pi)^2}\left[\frac{4}{3}N_{gen} + \frac{1}{10}N_H\right]$$

$$g_i^2(\Lambda_{GUT}) = g_{GUT}^2(\Lambda_{GUT}) \qquad \frac{1}{g_i^2(\mu_0)} = \frac{1}{g_{GUT}^2} + \frac{\beta_i(g_i)}{2\pi}\cdot Log\frac{\Lambda_{GUT}}{\mu_0} \Rightarrow \Lambda_{GUT} \approx 2\times 10^{16}\ GeV$$

- GUT: $$\sin^2\theta_W = \frac{g_1^2(Q)}{g_1^2(Q) + C^2 g_2^2(Q)} \qquad C^2 = \frac{5}{3} \quad \Rightarrow \sin^2\theta_W(\Lambda_{GUT}) = \frac{3}{8}$$

SU(5) EW CGC

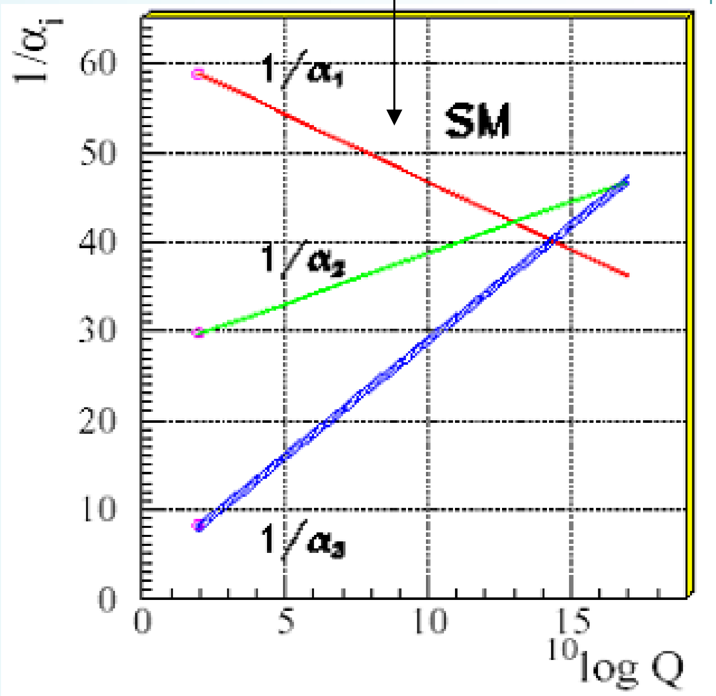
Grand Unification (6)

- Prediction of unification in standard SU(5) GUT:
  - Inputs @ EW Scale:

$$\alpha_3(M_Z) = 0.1184 \pm 0.0031$$
$$\sin^2\theta_W = 0.23146 \pm 0.00017$$
$$\alpha_1^{-1}(M_Z) = 128.978 \pm 0.027$$

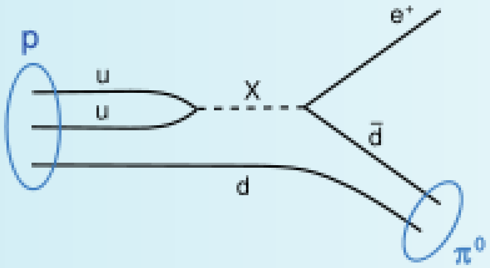
  - Output:
    - Unification fails!

SM RGEs



Grand Unification (7)

- Proton lifetime:
  - LeptoQuark interactions may cause the p decay:



$$\tau_p \approx \frac{1}{\alpha^2}\frac{\Lambda^4_{GUT}}{m_p^5} \approx 10^{28-30} \text{ years}$$

- main theoretical uncertainty coming from QCD corrections

  - However the experimental bound is: $\tau_p > 8.2 \times 10^{33}$ years

- this could be accomodated by using an extended Higgs sector
- but at the expense of spoiling the fermions electric dipole moment

  - This discrepancy kills minimal SU(5) GUT!

4. Supersymmetry

Supersymmetry (1)

- To make a SUSY transformation upon a particle of spin S:
  - Apply a fermionic operator Q on this particle

$$Q|S\rangle = |S \pm 1/2\rangle$$

$$\begin{cases} Q|F\rangle = |B\rangle \\ Q|B\rangle = |F\rangle \end{cases}$$

- Extended SUSY Models:
  - up to $\mathcal{N}$=4 supersymmetries, if SUSY is global
  - up to $\mathcal{N}$=8 supersymmetries, if SUSY is local

- In the following we'll consider models w/ only $\mathcal{N}$=1 supersymmetry
  - Important note: only $\mathcal{N}$=1 SUSY can accomodate chiral representations!

- SUSY algebra contains several commutation AND anti-commutations relations:
  - Reminder: $[A,B] = A \cdot B - B \cdot A$ $\quad \{A,B\} = A \cdot B + B \cdot A$
  - $\{Q,Q\} = \gamma^{\mu} \cdot P_{\mu}$ → 2 consecutives SUSY transformations are equivalent to a translation in space-time!

Supersymmetry (2)

- How to extend the SM while including SUSY?
  1. Consider only $\mathcal{N}$=1 SUSY generators
  2. Associate 1 bosonic SUSY d° of freedom to 1 fermionic SM d° of freedom

  Example:
  - start from an electron, S=1/2 => (2S+1)=2 d° of freedom
  - It will be associated to 2 scalar electrons (aka selectrons)

$$e^{\pm} \leftrightarrow \begin{cases} \tilde{e}_L^{\pm} \\ \tilde{e}_R^{\pm} \end{cases}$$

Fermion	Boson
Fermi-Dirac Statistics Pauli Exclusion Principle	Bose-Einstein Statistics

- If SUSY was a good low energy (up to O(1 TeV)) symmetry then:
  - Mass: $M(e^{\pm}) = M(\tilde{e}^{\pm})$
  - There would strange atoms with selectrons gathering in the same quantum states!!!

- **If SUSY exists, it MUST be a broken symmetry**

Supersymmetry (3)

- In the early days of SUSY model building, in the 1970's, people (P. Fayet, J. Illiopoulos, L. O'Raifertaigh) tried to break a global SUSY. This led to many phenomenological problems unableing to reproduce the SM

- These problems culminated with the supertrace theorem by S. Ferrara et al.:

- Supertrace theorem (1979):
  - ***If spontaneous SUSY breaking is mediated through 1-loop renormalisable interactions then the following sum rule applies in each super-multiplet:***

$$\sum_{S}(-1)^{2S}\cdot(2S+1)\cdot M_S^2=0$$

  - In our example: $M^2_{\tilde{e}_R}+M^2_{\tilde{e}_L}-2M^2_e=0$

  - This is clearly not confirmed by experiment! Therefore it was a show-stopper for attempts to model global SUSY spontaneous breaking

- SUSY as a local symmetry:
  - $\{Q,Q\}=\gamma^{\mu}\cdot P_{\mu}$ invariance wrt to local SUSY is equivalent to invariance wrt change of local coordinates: ie General Relativity!
  - That's why local SUSY is called SuperGravity

5. The MSSM

Gravity Mediated SUSY Breaking

TeV Scale | HE Scale

Visible Sector
M(SM) << M(SUSY)

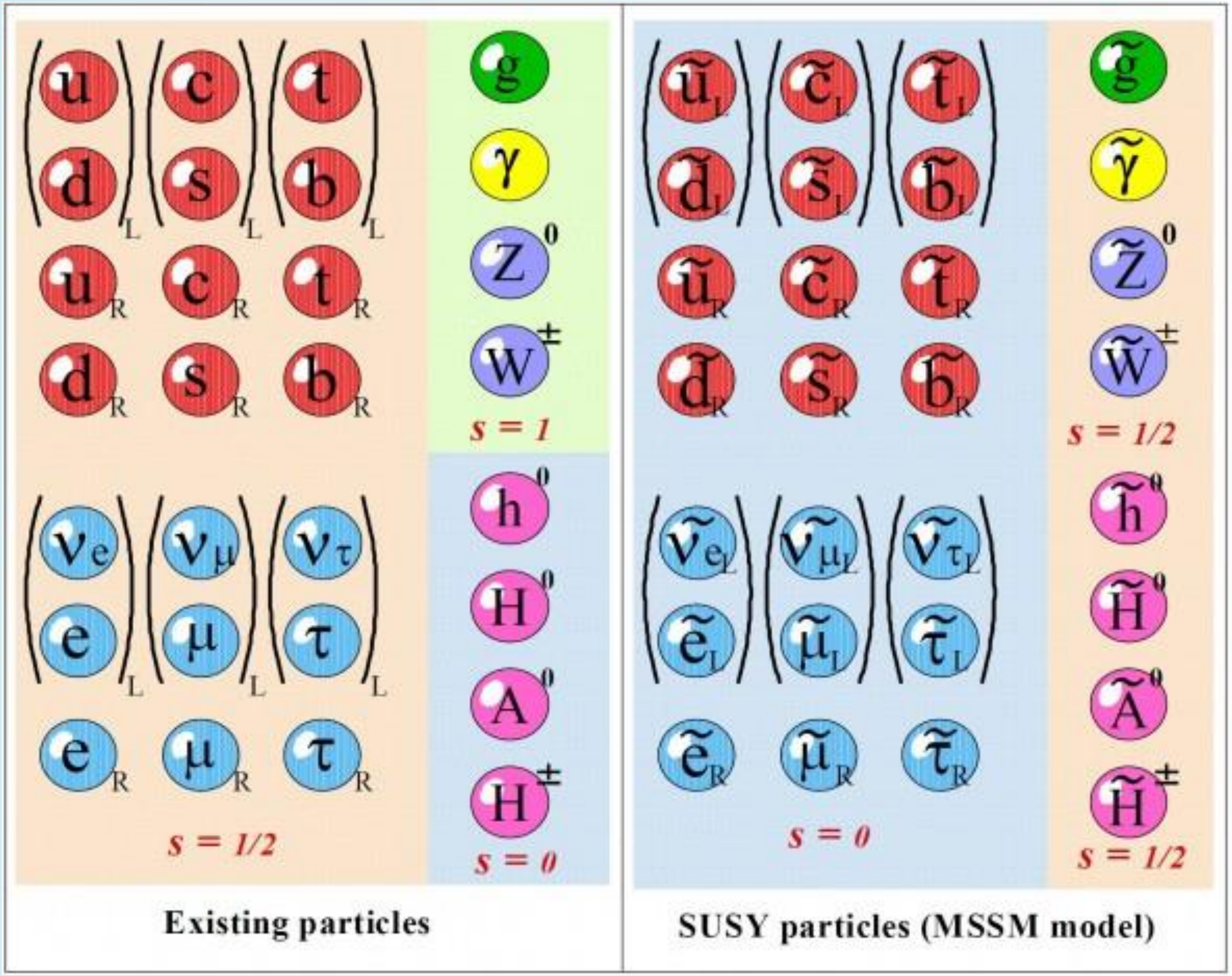
Hidden Sector
~~SUSY~~

Flavor & Charge
Blind Interactions:
Gravity

- MSSM (H. Georgi, S. Dimopoulos, 1981):
  - *Minimal Supersymmetric Standard Model*

  - What do we keep of the SM?
    - Its gauge group: $G_{MSSM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$
      - No new fundamental interactions
      - Fermions chirality in EW interactions
      - Higgs mechanism to break EW symmetry: $SU(3)_C \otimes U(1)_{EM}$
  - Its SUSY extension is miniman:
    - $\mathcal{N}$=1
    - Minimal extension of the particle content
      - This requires not 1, but 2 Higgs doublets in order to cancel anomalies

MSSM Particle Content (1)



MSSM Particle Content (2)

S=1	S=1/2
Gluons g (8,1,0)	Gluinos $\tilde{g}$
Weak Isospin bosons $W^{\pm}$ W^{0} (1,3,0)	Winos $\tilde{W}^{\pm}$ $\tilde{W}^{0}$
Weak Hyperch. boson B^{0} (1,1,0)	Bino $\tilde{B}^{0}$

S=1/2	S=0
Quarks $\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ (3,2,1/3) u_R (3,1,4/3) d_R (3,1,-2/3)	Squarks $\begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$ $\tilde{u}_R$ $\tilde{d}_R$
Leptons $\begin{pmatrix} \ell^{\pm}_L \\ \nu_L \end{pmatrix}$ (1,2,-1) $\ell^{\pm}_R$ (1,1,-2)	Sleptons $\begin{pmatrix} \tilde{\ell}^{\pm}_L \\ \tilde{\nu}_L \end{pmatrix}$ $\tilde{\ell}^{\pm}_R$
Higgsinos $\tilde{\Phi}_1 = \begin{pmatrix} \tilde{\varphi}_1^{+} \\ \tilde{\varphi}_1^{0} \end{pmatrix}$ (1,2,1) $\tilde{\Phi}_2 = \begin{pmatrix} \tilde{\varphi}_2^{0} \\ \tilde{\varphi}_2^{-} \end{pmatrix}$ (1,2,-1)	Higgs $\Phi_1 = \begin{pmatrix} \varphi_1^{+} \\ \varphi_1^{0} \end{pmatrix}$ $\Phi_2 = \begin{pmatrix} \varphi_2^{0} \\ \varphi_2^{-} \end{pmatrix}$

MSSM Lagrangian

- In constructing the minimal supersymmetric extension of the SM:

$$\mathcal{L}_{MSSM} = \mathcal{L}_{SUSY} + \mathcal{L}_{Soft-X}$$

- Includes $\mathcal{L}_{SM}$ (invariant under G_{SM})
- Global SUSY
- SM particles + superpartners
- Generates mass for SM particles
- Renormalizable
- Interactions (between SM & SUSY):
    - gauge interactions
    - Yukawa interactions

Remnants of SUSY X:
- explicitely violation of SUSY
- no quadratic divergences
- masses of SUSY particles

(Lots of new parameters)

MSSM R-Parity

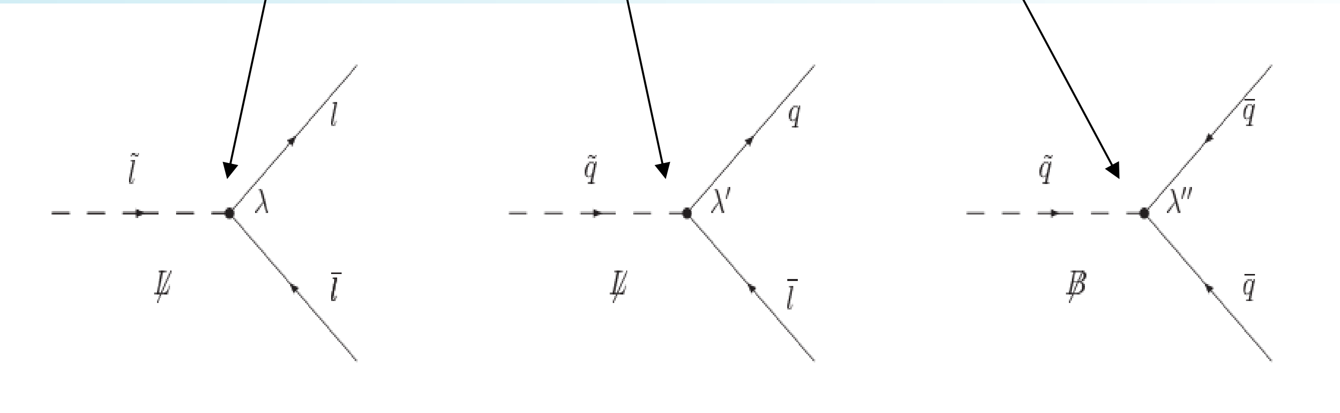
$$\mathcal{L}_{SUSY}$$

- Superpotential: $W = y_u \overline{U} Q \Phi_2 + y_d \overline{D} Q \Phi_1 + y_e \overline{E} L \Phi_1 + \mu \Phi_1 \Phi_2$

higgs-higgsino mass term

  - Most general form, renormalizable, gauge & SUSY invariant
  - However there exist additional terms w/ the same properties:

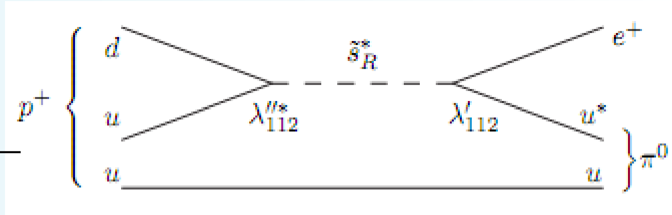
$$W_{\not{R}P} = \frac{1}{2}\lambda_{ijk} L_L^i L_L^j E_R^k + \lambda'_{ijk} L_L^i Q_L^j D_R^k + \frac{1}{2}\lambda''_{ijk} D_L^i D_L^j U_R^k$$



Conservation of R-Parity

- But $W_{\not R_P}$ enables a too fast proton decay

$$|\lambda'\lambda''| < O(10^{-9})$$ ←



  - 2 possible solutions:
    - RPC: either cancel those terms by imposing a new discrete parity:

$$R_P = (-1)^{L+2S+3B}$$

→ +1 for SM particles

→ -1 for SUSY particles

    - RPV: or accept small values of a few of these couplings

- RPC:
  - Conservation of R_P (RPC) has 3 crucial pheno. consequences:
    - Sparticles are pair-produced
    - A given Sparticle decays into an odd number of lighter Sparticles
    - The lightest SUSY particle (LSP):
      - is stable
      - if it does not carry either charge or colour

      => it's a good candidate for Cold Dark Matter (CDM)

MSSM Higgs Sector (1)

Extended Higgs Sector

- 2 $SU(2)_L$ doublets of complex scalar fields
- 2 x 2 x 2 x 1 = 8 d° of freedom, 3 are « eaten » to give mass to **W^+, W^-, Z^0**
- left with 5 physical d° of freedom: **h^0, A^0, H^0, H^+, H^-**

- Actual EWSB in the MSSM, when:

$$\langle 0|\Phi_1|0\rangle = v_1/\sqrt{2}$$
$$\langle 0|\Phi_2|0\rangle = v_2/\sqrt{2}$$

$$\tan\beta = \frac{v_2}{v_1} \qquad \left(0 < \beta < \frac{\pi}{2}\right)$$

- Related to EWSB in the SM:

$$v^2 = v_1^2 + v_2^2 = \frac{4M_Z^2}{g_1^2 + g_2^2} = (246\,\text{GeV})^2$$

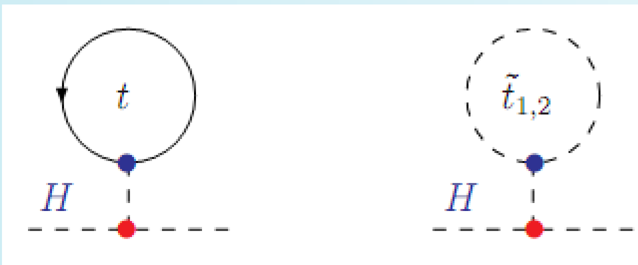
- Tree level mass relations:

$$M_{W^\pm} \le M_{H^\pm} \qquad M_{h^0} \le M_{A^0} \le M_{H^0}$$

$$M_{Z^0} \le M_{H^0} \qquad 0 < M_{h^0} \le M_{Z^0}\left|\cos(2\beta)\right|$$

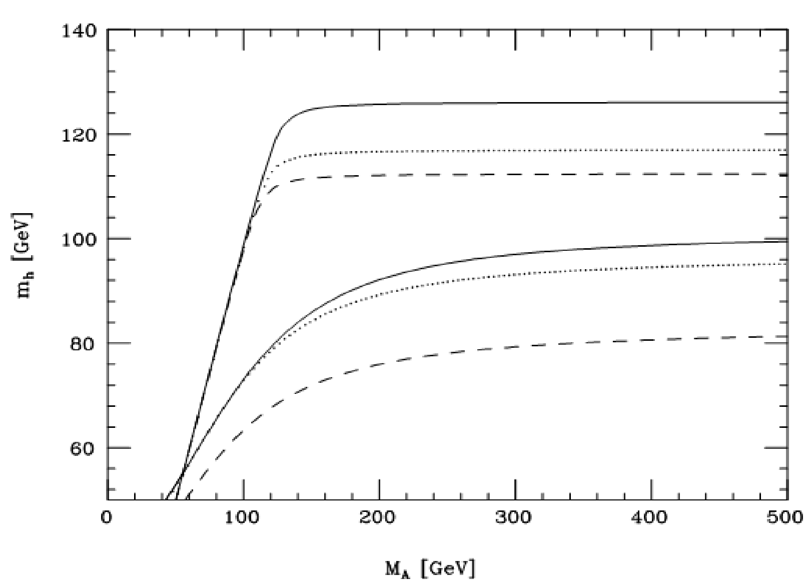
MSSM Higgs Sector (2)

$0 < M_{h^0} \leq M_{Z^0} \left|\cos(2\beta)\right|$ is strongly modified by radiative corrections



$$\delta M_{h^0}^2 = \frac{3}{2^{3/4} v \pi^2} \cdot M_t^4 \cdot Log\left(\frac{M_S^2}{M_t^2}\right)$$

$$M_S^2 \approx M_{\tilde{t}_1} M_{\tilde{t}_2}$$



$$M_{h^0} \leq 130\,\text{GeV}$$

The existence of a light Higgs boson is a generic prediction of SUSY models

MSSM Strong Gaugino Sector

$\tilde{g}$: $SU(3)_C$ octet of fermion partners of the gluons

At tree level the gluino sector has 1 parameter: M_3

MSSM EW Gaugino Sector (1)

- Mass matrix:

$$M_{\chi^\pm} = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{pmatrix}$$

- Mass eigenstates:
  - Charginos are linear combinations of charged higgsinos and charged winos

$$M^2_{\tilde{\chi}_1^\pm / \tilde{\chi}_2^\pm} = \frac{1}{2}\left[M_2^2 + \mu^2 + 2M^2_{W^\pm} \mp \sqrt{(M_2^2 - \mu^2)^2 + 4M^2_{W^\pm}\cos^2(2\beta) + 4M^2_{W^\pm}[M_2^2 + \mu^2 + 2M_2\mu\sin(2\beta)]} \right]$$

$$M_{\tilde{\chi}_1^\pm} < M_{\tilde{\chi}_2^\pm}$$

At tree level the chargino sector
has 3 indept parameters: M_2, μ, $\tan\beta$

MSSM EW Gaugino Sector (2)

$$M_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -M_Z \cos\beta \sin\theta_W & M_Z \sin\beta \sin\theta_W \\ 0 & M_2 & M_Z \cos\beta \cos\theta_W & -M_Z \sin\beta \cos\theta_W \\ -M_Z \cos\beta \sin\theta_W & M_Z \cos\beta \cos\theta_W & 0 & -\mu \\ M_Z \sin\beta \sin\theta_W & -M_Z \sin\beta \cos\theta_W & -\mu & 0 \end{pmatrix}$$

- Mass eigenstates: $M_{\tilde{\chi}_1^0} < M_{\tilde{\chi}_2^0} < M_{\tilde{\chi}_3^0} < M_{\tilde{\chi}_4^0}$
  - Neutralinos are linear combinations of neutral higgsinos, winos and bino

$$\left|\tilde{\chi}_1^0\right\rangle = c_{0,1} \cdot \left|\tilde{\varphi}_1^0\right\rangle + c_{0,2} \cdot \left|\tilde{\varphi}_2^0\right\rangle + c_{0,3} \cdot \left|\tilde{W}^0\right\rangle + c_{0,4} \cdot \left|\tilde{B}^0\right\rangle$$

At tree level the chargino sector
has 4 indept parameters: M_1, M_2, μ, $\tan\beta$

MSSM Scalar Fermions Sector

- Mass matrix:

$$M^2_{\tilde{f}_{LL}} = M^2_{\tilde{f}_L} + M_Z^2 (I_3 - Q\sin^2\theta_W)\cos(2\beta) + M_f^2$$

$$M^2_{\tilde{f}} = \begin{pmatrix} M_f^2 + M^2_{\tilde{f}_{LL}} & X_f M_f \\ X_f M_f & M_f^2 + M^2_{\tilde{f}_{RR}} \end{pmatrix}$$

$$M^2_{\tilde{f}_{RR}} = M^2_{\tilde{f}_R} + M_Z^2 (I_3 - Q\sin^2\theta_W)\cos(2\beta) + M_f^2$$

$$X_f = M_f [A_f - \mu(\tan\beta)^{-2I_3}]$$

- Mass eigenstates: $$M^2_{\tilde{f}_1/\tilde{f}_2} = M_f^2 + \frac{1}{2}\left[M^2_{LL} + M^2_{RR} \mp \sqrt{(M^2_{LL} - M^2_{RR})^2 + 4X_f^2 M_f^2}\right]$$

- Mixing: significant only for Spartners of heavy fermions (t, b, τ)

At tree level the stop sector has 4 indept parameters:

$$M_{\tilde{t}_L}, M_{\tilde{t}_R}, \mu, \tan\beta, A_t$$

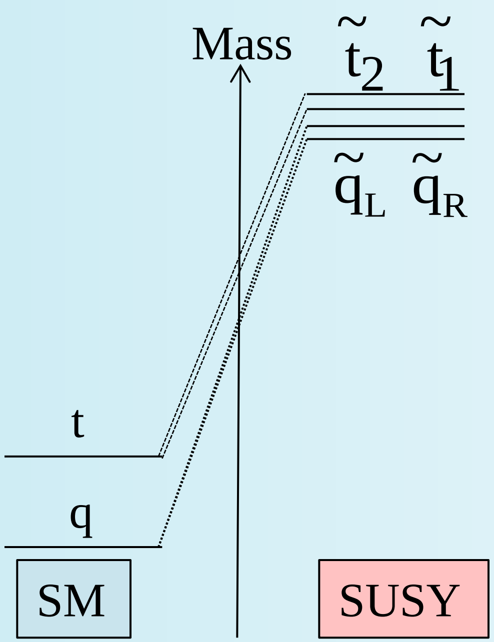
Relation between Higgs and Stop

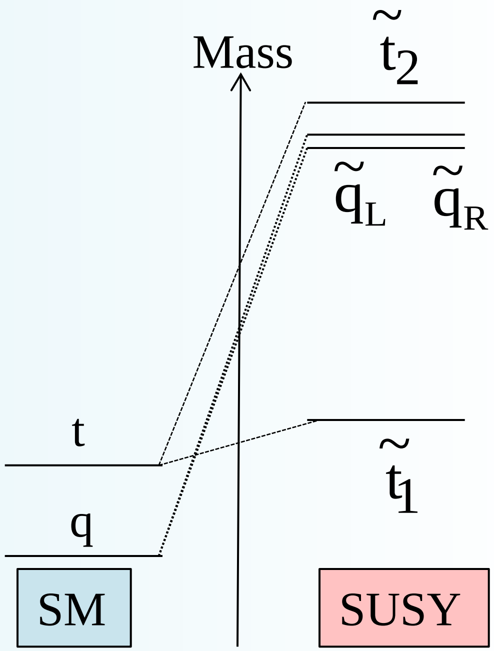
$$M^2_{\tilde{t}} = \begin{pmatrix} M_t^2 + M^2_{\tilde{t}_L} + D^t_L & M_t\left(A_t - \mu \cdot \mathrm{cotan}\beta\right) \\ M_t\left(A_t - \mu \cdot \mathrm{cotan}\beta\right) & M_t^2 + M^2_{\tilde{t}_R} + D^t_R \end{pmatrix}$$

$$\begin{cases} D^t_L = \left(\frac{1}{2} - \frac{2}{3}\sin\theta^2_W\right) M^2_Z \cos(2\beta) \\ D^t_R = \frac{2}{3}\sin\theta^2_W M^2_Z \cos(2\beta) \end{cases}$$

Diag. >> Off-Diag.

Diag. ≈ Off-Diag.





MSSM Renormalization Group Equations (1)

- Note: 1-loop $t = \mathrm{Log}\left(\frac{Q}{Q_0}\right)$
- Gauge couplings:

$$\frac{dg_i}{dt} = \frac{1}{32\pi^2} b_i g_i^3 \quad \text{with} \quad b_1 = -1 - \frac{10}{3} n_g \ , \ b_2 = 5 - 2n_g \ , \ b_3 = 9 - 2n_g$$

- Gaugino masses:

$$\frac{dM_i}{dt} = \frac{1}{16\pi^2} M_i b_i g_i^2$$

- Yukawa couplings:

$$\frac{dY_u^i}{dt} = -\frac{Y_u^i}{32\pi^2}\left[\sum_k 3(Y_u^k)^2 + (Y_d^i)^2 + 3(Y_u^i)^2 - \left(\frac{13}{9}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)\right]$$

$$\frac{dY_d^i}{dt} = -\frac{Y_d^i}{32\pi^2}\left[\sum_k \{3(Y_d^k)^2 + (Y_l^k)^2\} + (Y_u^i)^2 + 3(Y_d^i)^2 - \left(\frac{7}{9}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)\right]$$

$$\frac{dY_l^i}{dt} = -\frac{Y_l^i}{32\pi^2}\left[\sum_k \{(Y_l^k)^2 + 3(Y_d^k)^2\} + 3(Y_l^i)^2 - 3(g_1^2 + g_2^2)\right]$$

MSSM Renormalization Group Equations (2)

• Higgs parameters:

$$\frac{d\mu}{dt} = -\frac{\mu}{32\pi^2}\left[\sum_k 3\{(Y_u^k)^2 + 3(Y_d^k)^2 + (Y_l^k)^2\} - (g_1^2 + 3g_2^2)\right]$$

$$\frac{dv_1}{dt} = \frac{v_1}{32\pi^2}\left[\sum_k \{3(Y_d^k)^2 + (Y_l^k)^2\} - \frac{3}{4}\left(\frac{1}{3}g_1^2 + g_2^2\right)\right]$$

$$\frac{dv_2}{dt} = \frac{v_2}{32\pi^2}\left[\sum_k 3(Y_u^k)^2 - \frac{3}{4}\left(\frac{1}{3}g_1^2 + g_2^2\right)\right]$$

$$\frac{dm_{H_1}^2}{dt} = -\frac{1}{16\pi^2}\left[\sum_k \{3(Y_d^k)^2 P_{\tilde{d}}^k + (Y_l^k)^2 P_{\tilde{l}}^k\} - \frac{1}{2}g_1^2 \mathrm{Tr}(Ym^2) - (g_1^2 M_1^2 + 3g_2^2 M_2^2)\right] \quad (2$$

$$\frac{dm_{H_2}^2}{dt} = -\frac{1}{16\pi^2}\left[\sum_k 3(Y_u^k)^2 P_{\tilde{u}}^k + \frac{1}{2}g_1^2 \mathrm{Tr}(Ym^2) - (g_1^2 M_1^2 + 3g_2^2 M_2^2)\right] \quad (2$$

$$\frac{dB}{dt} = -\frac{1}{16\pi^2}\left[\sum_k \{3A_u^k (Y_u^k)^2 + 3A_d^k (Y_d^k)^2 + A_d^k (Y_l^k)^2\} - (g_1^2 M_1 + 3g_2^2 M_2)\right] \quad (2$$

$$[\text{with } P_{\tilde{u},\tilde{d},\tilde{l}}^k \equiv m_{H_2,H_1,H_2}^2 + m_{\tilde{Q}_k,\tilde{Q}_k,\tilde{L}_k}^2 + m_{\tilde{u}_k,\tilde{d}_k,\tilde{l}_k}^2 + (A_{u,d,l}^k)^2]$$

MSSM Renormalization Group Equations (3)

• Trilinear couplings:

$$\frac{dA_u^i}{dt} = -\frac{1}{32\pi^2}\Big[6A_u^i(Y_u^i)^2 + 2A_d^i(Y_d^i)^2 + 6\sum_k A_u^k(Y_u^k)^2 - \Big(\frac{26}{9}g_1^2M_1 + 6g_2^2M_2 + \frac{32}{3}g_3^2M_3\Big)\Big]$$

$$\frac{dA_d^i}{dt} = -\frac{1}{32\pi^2}\Big[6A_d^i(Y_d^i)^2 + 2A_u^i(Y_u^i)^2 + 2\sum_k\{A_l^k(Y_l^k)^2 + 3A_d^k(Y_d^k)^2\} - \Big(\frac{14}{9}g_1^2M_1 + 6g_2^2M_2 + \frac{32}{3}g_3^2M_3\Big)\Big]$$

$$\frac{dA_l^i}{dt} = -\frac{1}{32\pi^2}\Big[6A_l^i(Y_l^i)^2 + 2\sum_k\{A_l^k(Y_l^k)^2 + 3A_d^k(Y_d^k)^2\} - 6(g_1^2M_1 + g_2^2M_2)\Big]$$

MSSM Renormalization Group Equations (4)

• Sfermions masses:

$$\frac{dm^2_{\tilde{l}_{R_i}}}{dt} = -\frac{1}{16\pi^2}\left[2(Y^i_l)^2 P^i_{\tilde{l}} + g_1^2 \mathrm{Tr}(Ym^2) - 4g_1^2 M_1^2\right]$$

$$\frac{dm^2_{\tilde{L}_i}}{dt} = -\frac{1}{16\pi^2}\left[(Y^i_l)^2 P^i_{\tilde{l}} - \frac{1}{2}g_1^2 \mathrm{Tr}(Ym^2) - (g_1^2 M_1^2 + 3g_2^2 M_2^2)\right]$$

$$\frac{dm^2_{\tilde{d}_{R_i}}}{dt} = -\frac{1}{16\pi^2}\left[2(Y^i_d)^2 P^i_{\tilde{d}} + \frac{1}{3}g_1^2 \mathrm{Tr}(Ym^2) - \left(\frac{4}{9}g_1^2 M_1^2 + \frac{16}{3}g_3^2 M_3^2\right)\right]$$

$$\frac{dm^2_{\tilde{u}_{R_i}}}{dt} = -\frac{1}{16\pi^2}\left[2(Y^i_u)^2 P^i_{\tilde{u}} - \frac{2}{3}g_1^2 \mathrm{Tr}(Ym^2) - \left(\frac{16}{9}g_1^2 M_1^2 + \frac{16}{3}g_3^2 M_3^2\right)\right]$$

$$\frac{dm^2_{\tilde{Q}_i}}{dt} = -\frac{1}{16\pi^2}\left[(Y^i_u)^2 P^i_{\tilde{u}} + (Y^i_d)^2 P^i_{\tilde{d}} + \frac{1}{6}g_1^2 \mathrm{Tr}(Ym^2) - \left(\frac{1}{9}g_1^2 M_1^2 + 3g_2^2 M_2^2 + \frac{16}{3}g_3^2 M_3^2\right)\right]$$

6. Motivations for SUSY

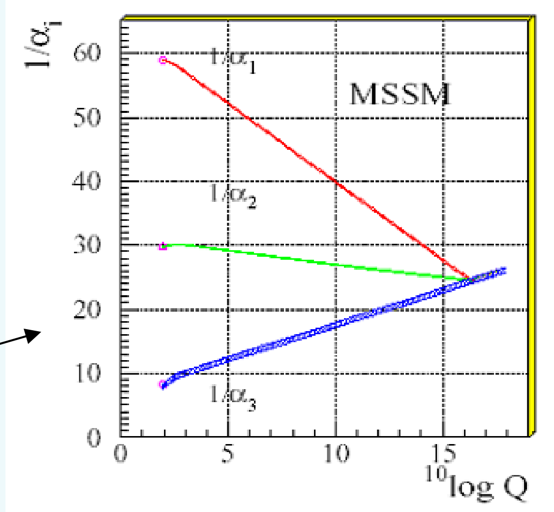
Motivations for SUSY (1)

- Largest Symmetry of Space-Time
  - Haag-Lopuszanski-Sohnius theorem (1975)

- Framework for a possible theory of quantum gravity

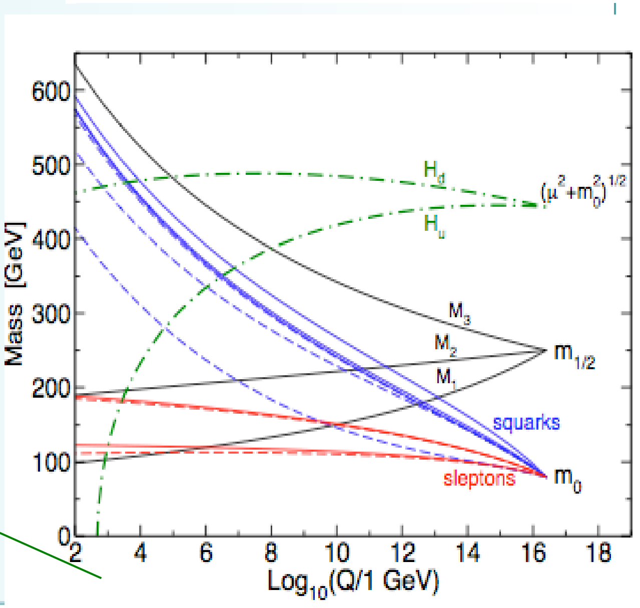
$$\{Q,Q\} = \gamma^{\mu} \cdot P_{\mu}$$

- LSP: good candidate for Cold Dark Matter
  - Provided that R_P is conserved



- Enables Grand Unification:
  - Both $\sin^2\theta_W$ and τ_p compatible with data

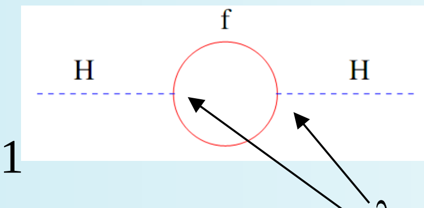
- Radiative ElectroWeak Symmetry Breaking:
  - $M^2(\Phi_2)$ is driven negative dynamically by the RGEs



Motivations for SUSY (2)

- Taming the Higgs Radiative Corrections:

SM:

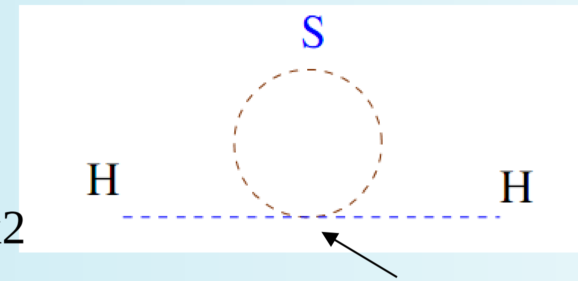


x1

λ_f

$$\delta M_H^2 = \frac{|\lambda_f|^2}{16\pi^2}\left[-2\Lambda_{UV}^2 + 6m_f^2 \mathrm{Log}\left(\frac{\Lambda_{UV}}{m_f}\right) + \mathrm{H.O.}\right]$$

SUSY:



x2

λ_S

$$\delta M_H^2 = \frac{\lambda_S}{16\pi^2}\left[\Lambda_{UV}^2 - 2m_S^2 \mathrm{Log}\left(\frac{\Lambda_{UV}}{m_S}\right) + \mathrm{H.O.}\right]$$

Exact cancellation, if and only if:

- $M_f = M_S$

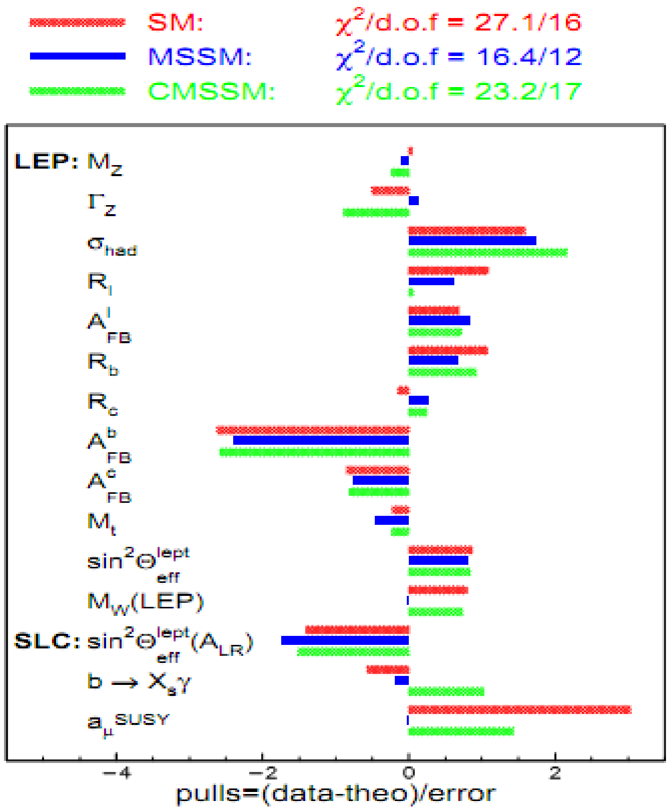
and

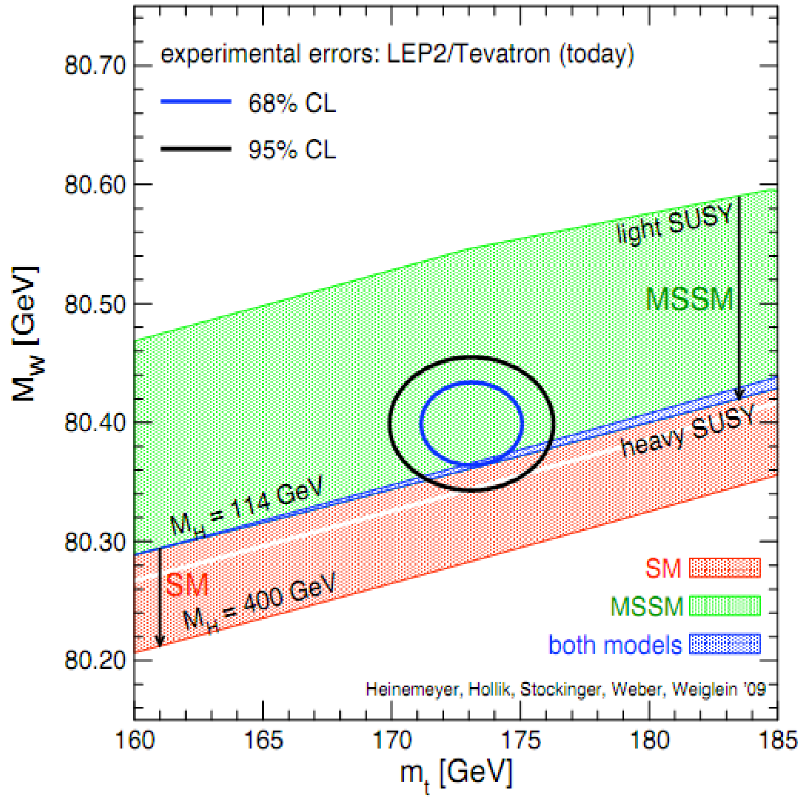
- $|\lambda_f|^2 = \lambda_S$

$$M_H = O(M_W) \Rightarrow |M_f - M_S| < O(1\,\mathrm{TeV})$$

Motivations for SUSY (3)

• The MSSM is fully compatible w/ all current data and even fits the data better than the SM





BACK-UP

• Superpotential: $W = y_u \overline{U} Q \Phi_2 + y_d \overline{D} Q \Phi_1 + y_e \overline{E} L \Phi_1 + \mu \Phi_1 \Phi_2$

$$\mathcal{L}_{Soft-X} = \frac{-1}{2}\left(\sum_{a=1,2,3} M_{\lambda}\lambda_a \lambda^a + c.c.\right) - M_{ij}^2 \varphi_j^* \varphi_i + \left(\frac{1}{2} B_{ij}\varphi_i \varphi_j + \frac{1}{6} A_{ijk}\varphi_i \varphi_j \varphi_k + c.c.\right)$$

$$= \frac{-1}{2}\left(M_1 \tilde{B}\tilde{B} + M_2 \tilde{W}\tilde{W} + M_3 \tilde{g}\tilde{g} + c.c.\right)$$

→ gaugino masses

$$-\left(\tilde{Q}^{\perp} M_{\tilde{Q}}^2 \tilde{Q} + \tilde{\overline{U}}^{\perp} M_{\tilde{\overline{U}}}^2 \tilde{\overline{U}} + \tilde{\overline{D}}^{\perp} M_{\tilde{\overline{D}}}^2 \tilde{\overline{D}}\right) - \left(\tilde{\overline{L}}^{\perp} M_{\tilde{\overline{L}}}^2 \tilde{\overline{L}} + \tilde{\overline{E}}^{\perp} M_{\tilde{\overline{E}}}^2 \tilde{\overline{E}}\right)$$

→ squarks/slepton masses

$$-\left(-A_u \tilde{\overline{U}}^{\perp} \tilde{Q}^{\perp} \Phi_2 + A_d \tilde{\overline{D}}^{\perp} \tilde{Q}^{\perp} \Phi_1 + A_e \tilde{\overline{E}}^{\perp} \tilde{L} \Phi_1 + c.c.\right)$$

→ trilinear couplings

$$-\left(M_{\Phi_2}^2 \Phi_2^* \Phi_2 + M_{\Phi_1}^2 \Phi_1^* \Phi_1\right) - B\mu\left(\Phi_1 \Phi_2 + c.c.\right)$$

→ higgs-higgsino mass

→ bilinear coupling

=> 105 new free parameters, on top of the 19 ones from the SM

=> Total of 124 free parameters!?!