Chiral Liquids

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Talk
at 3rd International Conference on New Frontiers in Physics

31 июля 2014 г.

Notion of "chiral liquids"

- Underlying field theory contains massless fermions, interacting in a chiral invariant way (classically) Quark-gluon plasma (approximately), graphene, other cond-matter systems (quasi-relativistic), as examples
- There is chiral anomaly (U(1) for simplicity)

$$\partial_\mu j_5^\mu = (\vec{E}\vec{B})\frac{e^2}{4\pi^2}$$

- Right-left asymmetric composition, or non-vanishing chiral chemical potential

$$\mu_5 \neq 0$$

- Validity of hydrodynamic approximation, or expansion in derivatives (long-wave approximation)

Motivations

Volume of Springer Lectures 871 (2013) (624 page long), with reviews on various aspects (not all possible), editors D. Kharzeev, K. Landsteiner, A. Schmitt, H-Y Yee

- theory, (we will be here)
- applications, high energy
- applications, cond. - matter

Fits very well notion of “new frontiers ”, unifying

- heavy-ion collisions, ChME (chiral magnetic effect)...
- holography,
  relating strong-coupling physics to BH physics
- new, geometric formulation of thermodynamics
- (new aspects of) condensed matter physics

(very rough) Sketch of the history

– Prehistory, before (2000)
Vilenkin; Alekseev, Cheianov & Frohlich ...,
– QCD phenomenology, (2004)...
K. Fukushima, D.E. Kharzeev, L. McLerran, M. Metlitski, D. Son, H.J. Warringa, A. Zhitnitsky,..
– More theoretic:
Erdmenger et al., Banerjee et al., (holography, (2007))
Son& Surowka, Rewriting L& L hydrodynamics (2009)
K. Jensen, R. Loganayagam, A. Yarom... (geometry and thermodynamics (2012))
A. Boyarski, J. Frohlich, M. Shaposhnikov, N. Yamamoto ...Instabilities ...(2013)..
—Condensed matter
D. Haldane, S. Hartnoll, D. Son, M. Stephanov, I. Zahed...

Outline of the talk

Central point:
since Landau's times, thought that only in case of supefluidity quantum effects manifested macroscopically
Now, strong indications that quantum effects are the same important in case of "ordinary" (though, chiral) liquids

Here: quantum effects $\equiv$ chiral anomaly (one-loop effect)

"manifestations": e.g. non-dissipative electric current:

$$j^{el}_{\mu} = \frac{e^2 \mu_5}{2\pi^2} B_{\mu}$$

where $B_{\mu} \equiv (1/2)\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}u^{\nu}$ coincides with the external magnetic field in liquid rest frame ($u^{\mu} = (1,0,0,0)$)
Reminiscent of superconductivity,
no understanding microscopically yet

Outline of the talk, cnt'd

Basic (and beautiful) effects:

- Low viscosity-to-entropy ratio for holographic liquids::

$$\frac{\eta}{s} \approx \frac{1}{4\pi}\frac{\bar{h}}{k_B}$$

which is conjectured lower limit, Kovtun et al., (2005)

- Chiral magnetic effect

$$j^{el}_{\mu} = (const) B_{\mu}$$

- Chiral vortical effect:

$$j^5_{\mu} = \rho^5 u_{\mu} + \frac{\mu^2}{2\pi^2}\epsilon_{\mu\nu\rho\sigma}u^{\nu}\partial^{\rho}u^{\sigma}$$

–Possible unification of the effects above

–Conclusions

Low-viscosity holographic liquid

It is known that for a distant observer can describe physics of BH in terms of a liquid living on the horizon with

$$\eta/s \; = \; 1/4\pi$$

(in natural units; η is viscosity, s is entropy density). Holography allows to continue this ratio to the boundary of the extra dimensions, i.e. to our world.
Kovtun Son, Starinets : in terms of quasipaticles there is lower limit on η/s due to uncertainly principle:

$$\eta \; \sim \; \epsilon\tau_{min}, s \; \sim \; k_B \cdot n, \; \eta/s \; \sim \; (\epsilon/n)\tau_{min}(k_B)^{-1} \; \gtrsim \; \bar{h}/k_B$$

where ϵ is energy density, n is density of quasiparticles, τ_{min} is mean free time

Low-viscosity plasma

KSS conjectured that $\eta/s = (1/4\pi)(\hbar/k_B)$ is the lower bound.

Note that in the weak coupling limit η/s is much larger

Quark-gluon plasma turns to have lowest η/s ratio known,

probably not far from the conjectured lower bound

and, hence, one talks about
"strongly interacting quark-gluon plasma"

Chiral particles in magnetic field,oversimplified

Begin with massive, spinning particles.
Because of the interaction

$$H_{Pauli} = -\vec{\mu} \cdot \vec{B}$$

Spin of positively charged particles looks along $\vec{B}$, and of negatively charged — in the opposite direction.

Now let particles move (go to massless limit) and consider all particles, say, left-handed, i.e. move along their spins

Then charges get separated and electric current arises

Moreover, magnetic field produces no work,
hence no dissipation
Conclusions correct, derivation can be improved

Chiral magnetic effect and hydrodynamics

Hydrodynamics is a universal framework. It uses only conservation laws and expansion in derivatives, i.e. long-wave approximatiion.
It turns out that ChME can be derived using only these general conservation laws in presence of external electric and magnetic fields:

$$\partial_\mu T^{\mu\nu} = F^{\nu\rho} j^{el}_\rho$$

$$\partial_\mu j^\mu_{el} = 0, \quad \partial_\mu j^\mu_5 = \frac{\alpha_{el}}{4\pi} \epsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta}$$

$$\partial^\mu s_\mu \geq 0$$

where s_μ is the entropy current

Non-renormalizability of the ChME

It turns out that all the currents, including the modified entropy current, are uniquely determined in terms of the anomaly, which is not renormalized by virtue of the Adler-Bardeen theorem (Son&Surowka)

$$j_\mu^{el} = \frac{e^2 \mu_5}{2\pi^2} B_\mu , \quad j_\mu^5 = \ldots , \quad s_\mu = \ldots$$

Reservations:
fermion mass assumed small on the hydrodynamic scale;
external fields can be screened in media

No dissipation

Eqns

$$\vec{j}_{el} = \sigma_E \vec{E} \quad and \quad \vec{j}_{el} = \sigma_B \vec{B}$$

change with opposite signs under time reflection

$$t \rightarrow -t$$

Implication: dissipation in ChME is forbidden by time invariance (Kharzeev and Yee)
Analogy: superconducting current in the London limit,

$$\vec{j}_{el} = (const) m_\gamma^2 \vec{A}$$

is invaraiant under $(t \rightarrow -t)$ and dissipation is forbidden
No such selection rule in case of, say, superfluidity (see also discussion above)

Chiral vortical effect

The same mashinary of hydrodynamics allowed
to establish a new term in axial current
in terms of the anomaly which
survives in the limit of vanishing external fields,
(although one starts with the chiral anomaly $\partial_\mu j_5^\mu \sim F\tilde{F}$)

$$\delta j_5^\mu = \frac{\mu^2}{2\pi^2}\omega^\mu \quad ,$$

$\omega^\mu = (1/2)\epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$, and u^μ is local 4-velocity

New term is non-vanishing for macroscopic helical motion

Hydrodynamics as an effective theory

Hydrodynamics is an "effective theory" but not (only) in the sense of Wilson (integrating out short distances) Rather, one changes Hamiltonian

$$H_0 \rightarrow H_0 - \mu Q$$

$Q = \int d^3x\, j_0$ is charge associated with chemical potential μ
Moreover, $\delta L = -\delta H$
To observe relativistic invariance, $\mu j_0 \rightarrow u^\mu j_\mu$ As a result,

$$eA_\mu \rightarrow eA_\mu + \mu u_\mu$$

Recent studies within geometric picture of thermodynamics confirmed that this substitution is exact as far as anomaly is concerned (plus temperature-related terms due to modification of the Christoffel's symbols)

Conserved axial current

Chiral (U(1)) anomaly can be reformulated as an expression for a conserved axial current:

$$Q^A_{conserved} = Q^A_{naive} + \frac{e^2}{4\pi^2}\mathcal{H}, \qquad \frac{d}{dt}Q^A_{conserved} = 0$$

where Q^A_{naive} counts chiral constituents, $Q^A_{naive} = n_L - n_R$, and $\mathcal{H}$ is the so called magnetic helicity:

$$\mathcal{H} = \int d^3x \vec{A} \cdot \vec{B}$$

However, there is a change brought in by hydrodynamics (as is discussed, $eA_\mu \rightarrow eA_\mu + \mu \cdot u_\mu$)

Conserved axial current, hydrodynamics

Using this substitution:

$$Q^A_{hydro} = Q^A_{naive} + Q^A_{fluid\ \ helicity} + Q^A_{mixed} + Q^A_{magnetic\ \ helicity}$$

where $Q^A_{fluid\ \ helicity} = (1/4\pi^2) \int d^3 x j^0_{fluid\ \ helicity}$

$$j^\mu_{fluid\ \ helicity} = (1/2)\epsilon^{\mu\nu\rho\sigma}\omega_{\nu\rho}(\mu u)_\sigma$$

$$\omega_{\nu\rho} = (\mu u_\nu)_{,\rho} - (\mu u_\rho)_{,\nu}$$

In $Q^A_{fluid\ \ helicity}$ we recognize axial charge associated with the vortical effect, or helical macroscopic motion.
Note:Only the last term, $Q^A_{magnetic\ \ helicity}$ is quadratic in charge and related to the anomaly on fundamental level

Analogy to the Einstein-de Haas effect

In hydrodynamics:

$$Q^A_{naive} \rightarrow Q^A_{naive} + Q^A_{fluid\ \ helicity} + Q^A_{mixed\ \ helicity}$$

In other words, chirality of microscopical constituents can be transformed into macroscopic helical motion. Somewhat similar to the Einstein- de Haas effect, when spin of electrons is transformed into angular momentum of a rotating body. For the analogy to be true, helical motion is to be conserved. Which is not the case, generally speaking. We see that this is to be true for chiral liquids (classically). Solution for the conservation of macroscopic helicities is known since long (in magneto-hydrodynamic).
Namely, dissipation is to vanish in chiral liquids:

$$\eta_{chiral\ \ lquids} = 0\ ,\ \sigma_{chiral\ \ liquids} \rightarrow \infty$$

Reservations

Formal counting powers of e^2 can be invalidated by instabilities (if e.m. is the ony interaction). One may have

$$< Q^A_{naive} >_{liquid} \sim \frac{e^2}{4\pi^2} < \mathcal{H} >_{liquid}$$

In other words, $< \mathcal{H} >$ can be generated in absence of external fields. The corresponding negative mode is known explicitly (Akamatsu&Yamamoto (2013)).
Instability develops at large distances, $d \sim e^{-1}$
Two consistent regimes possible:

- $\sigma_E \rightarrow \infty$ and then $< \vec{E} \cdot \vec{B} >_{liquid} \rightarrow 0$ (screening of anomaly, no instabilities)
- or: σ_E finite, growing $< \vec{E} \cdot \vec{B} >_{liquid}$

Conclusions

Chiral liquids are predicted to exhibit a number of striking effects. Our path through effects:

- Chiral magnetic effect

$$\vec{j}^{el} = (const)\vec{B}$$

  is non-dissipative (at any temperature !) and exactly calculable (non-renormalizable)

- Chiral vortical effect: transfer of chirality from elementary constituents to helical macroscopic motion:

$$\frac{d}{dt}\left((n_L - n_R) + \frac{\mu^2}{4\pi^2}\int d^3x \vec{v}\,\vec{rot}v\right) = 0$$

- (Classical) conservation of j_5^μ might be related to low viscosity

Conclusions (cnt'd)

- Chiral liquids appear to be an example of low-dissipation media (dissipation on quantum level)
- There are further examples in cond-matter phyisics
- Holographycally, low -dissipation media might be related to BH quantum mechanics