QCD in dense matter - prospects for and beyond NJL-kind effective models

THOMAS KLÄHN

Collaborators: D. Zablocki, J. Jankowski, C.D. Roberts, R. Lastowiecki, D.B. Blaschke
QCD Phase Diagram

- dense baryonic matter

HIC in collider experiments
Won't cover the whole diagram
Hot and ‘rather’ symmetric

NS as a 2nd accessible option
Cold and ‘rather’ asymmetric

Problem is more complex than
It looks at first gaze

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Neutron Stars

- Variety of scenarios regarding inner structure: with or without QM
- Question whether/how QCD phase transition occurs is not settled
- Most honest approach: take both (and more) scenarios into account and compare to available data
Neutron Stars = Quark Cores?

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Hybrid Star
- Inner Crust
  - heavy ions
  - relativistic electron gas
  - superfluid neutrons
- Inner Core
  - (neutrons, protons)
  - electrons, muons
  - hyperons
  - bosonic condensates
  - deconfined quark matter

Neutron Star
- Outer Crust
  - ions
  - electron gas
- Core
  - neutrons, protons
  - electrons, muons
  - superconducting protons
  - strange quark matter

Strange Star
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Neutron Star Data

- Data situation in general terms is good (masses, temperatures, ages, frequencies)
- Ability to explain the data with different models in general is good, too.
  ... which sounds good, but becomes tiresome if everybody explains everything ...
- For our purpose only a few observables are of real interest
- Most promising: High Massive NS with 2 solar masses (Demorest et al., Nature 467, 1081-1083 (2010))
NS masses and the (QM) Equation of State

- NS mass is sensitive mainly to the sym. EoS (In particular true for heavy NS)

- Folcloric: QM is soft, hence no NS with QM core

- Fact: QM is softer, but able to support QM core in NS

- Problem: (transition from NM to) QM is barely understood

\[ M(n) \text{ correlated to } E_0(n) \]

stiff: higher \( M_{\text{max}} \) at smaller densities

soft: smaller \( M_{\text{max}} \) at higher densities
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(plied “universal” $\beta^2 E_S$ (error bars!))
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- traditional: two-phase construction

“masquerade” problem: quark and hadron eos almost identical!
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Dense Nuclear Matter in terms of Quark DoF is barely understood
Problem is attacked in vacuum Faddeev Equations

\[
\begin{align*}
\Psi^a & = \Gamma^a \\
p_q & = p_d
\end{align*}
\]

Baryons as composites of confined quarks and diquarks

Bethe Salpeter Equations
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Bethe Salpeter Equations
QCD in dense matter

- LQCD fails in dense (like DENSE) matter (Fermion-sign problem)
- Perturbative QCD fails in non-perturbative domain
  DCSB is explicitly not covered by perturbative approach:

\[
B(p^2) = m \left(1 - \frac{\alpha_s}{\pi} \ln \left[ \frac{p^2}{m^2} \right] \right) \lim_{m_0 \to 0} \sum_s (p^2, m_0^2) = 0
\]

- Solution: ‘some’ non-perturbative approach ‘as close as possible’ to QCD
  some = solvable; as close as possible = if possible DCSB, if possible confinement
- State of the art: Nambu-Jona-Lasinio model(s) (+bag models, +hybrids)
NJL type models

- S: DCSB
- V: renormalizes $\mu$
- D: diquarks $\rightarrow$ 2SC, CFL
- TD Potential minimized in mean-field approximation
- Effective model by its nature; can be motivated (1g-exchange) doesn’t have to though and can be extended (KMT, PNJL)
- possible to describe nucleons; not to be confused with confinement!

Effective Lagrangian

$$\mathcal{L}_{\text{int}} = G_S \eta_D \sum_{a,b=2,5,7} (\bar{q} i \gamma_5 \tau_a \lambda_b C \bar{q}^T) (q^T C i \gamma_5 \tau_a \lambda_a q)$$

$$+ G_S \sum_{a=0}^8 [(\bar{q} \tau_a q)^2 + \eta_V (\bar{q} i \gamma_0 q)^2]$$

Thermodynamical potential

$$\Omega(T, \mu) = \frac{\phi_u^2 + \phi_d^2 + \phi_s^2}{8 G_S} + \frac{\Delta_{ud}^2 + \Delta_{us}^2 + \Delta_{ds}^2}{4 G_D}$$

$$\quad - \int \frac{d^3 p}{(2\pi)^3} \sum_{n=1}^{18} \left[ E_n + 2 T \ln \left(1 + e^{-E_n/T}\right) \right] + \Omega_{\text{lep}} - \Omega_0$$
Conclusion: NS may or may not support a significant QM core. Other interaction channels won’t change this if their coupling strengths are not precisely known.
Beyond NJL

- NJL model can be understood as an approximate solution of Dyson-Schwinger equations.

**Quark**

**Gluon**

**q-g-Vertex**
Beyond NJL

- NJL model can be understood as an approximate solution of Dyson-Schwinger equations.

\[ g^2 D_{\mu\nu}(k) = \left( \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{G(k^2)}{k^2} \]

\[ \Gamma^a_\mu(k,p)_{\text{bare}} = \gamma_\mu \frac{\lambda^a}{2} \]
**single particle: quark self energy**

Inverse Quark Propagator:
\[
S(p; \mu)^{-1} = Z_2(i \vec{\gamma} \vec{p} + i \gamma_4 (p_4 + i \mu) + m_{\text{bm}}) + \Sigma(p; \mu)
\]
\[
= i \gamma p \quad \text{revokes Poincaré covariance}
\]

Renormalised Self Energy:
\[
\Sigma(p; \mu) = Z_1 \int \frac{d^4q}{q^2} \delta^4(\mu) D_{\rho\sigma}(p-q, \mu) \frac{\lambda^a}{2} \gamma_\rho S(q, \mu) \Gamma_\sigma^a(q, p; \mu)
\]

Loss of Poincaré covariance increases complexity
\[
\rightarrow \text{technically and numerically more challenging} \rightarrow \text{no surprise, though}
\]

**General Solution:**

**Vacuum:** \( \mu = 0 \)
\[
S(p^2)^{-1} = i \gamma \ p \ A(p^2) + B(p^2)
\]

**Medium:** \( \mu \neq 0 \)
\[
S(p^2, p_4; \mu)^{-1} = i \vec{\gamma} \vec{p} \ A(p^2, p_4, \mu) + i \gamma_4 (p_4 + i \mu) \ C(p^2, p_4, \mu) + B(p^2, p_4, \mu)
\]

Similar structured equations in vacuum and medium, but in medium:

1. one more gap
2. gaps are complex valued
3. gaps depend on (4-)momentum, energy and chemical potential
Effective gluon propagator

\[ S(p;\mu)^{-1} = Z_2 (i \gamma \vec{p} + i \gamma_4 (p_4 + i \mu) + m_{\text{bm}}) + \Sigma(p;\mu) \]

\[ \Sigma(p;\mu) = Z_1 \int_q g^2 (\mu) D_{\rho\sigma} (p-q,\mu) \frac{\Lambda^2}{2} \gamma_\rho S(q,\mu) \Gamma_\sigma^a (q,p;\mu) \]

Ansatz for self energy (rainbow approximation, effective gluon propagator(s))

\[ Z_1 \int_q g^2 D_{\mu\nu} (p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma_\nu^a (q,p) \rightarrow \int_q G ((p-q)^2) D_{\mu\nu}^\text{free} (p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \frac{\lambda^a}{2} \gamma_\nu \]

Specify behaviour \( G(k^2) \)

\[ \frac{G(k^2)}{k^2} = 8 \pi^4 D^4(k) + \frac{4 \pi^2}{\omega^6} D k^2 e^{-k^2/\omega^2} + 4 \pi \frac{\gamma m \pi}{\frac{1}{2} \ln \left[ \tau + \left( 1 + k^2/\Lambda^2_{\text{QCD}} \right)^2 \right]} \]

Infrared strength running coupling for large k (zero width + finite width contribution)

Results at finite densities obtained for
1st term (Munczek/Nemirowsky (1983)) → Klähn et al. (2010)
2nd term → Chen et al. (2008, 2011)
NJL model: \[ g^2 D_{\rho\sigma} (p-q) = \frac{1}{m_G^2} \delta_{\rho\sigma} \]
delta function in configuration(!) space
NJL model within DS framework

\[ B(p) = m + \frac{16}{3m_G^2} \int \frac{d^4 q}{(2\pi)^4} \frac{B(q)}{q^2 A^2(q) + \tilde{q}_4^2 C^2(q) + B^2(q)}, \]

\[ \tilde{p}_4 A(p) = \tilde{p}_4^2 + \frac{8}{3m_G^2} \int \frac{d^4 q}{(2\pi)^4} \frac{\tilde{p}_4 q A(q)}{q^2 A^2(q) + \tilde{q}_4^2 C^2(q) + B^2(q)}, \]

\[ \tilde{p}_4^2 C(p) = \tilde{p}_4^2 + \frac{8}{3m_G^2} \int \frac{d^4 q}{(2\pi)^4} \frac{\tilde{p}_4 \tilde{q}_4 C(q)}{q^2 A^2(q) + \tilde{q}_4^2 C^2(q) + B^2(q)}. \]

\[ \frac{8}{3m_G^2} \int \frac{d^4 q}{(2\pi)^4} \frac{\tilde{q}_4 C(q)}{q^2 + \tilde{q}_4^2 C^2(q) + B^2(q)} = iK \]

\[ \tilde{p}_4^2 C = \tilde{p}_4^2 + i\tilde{p}_4 K \]

\[ \Rightarrow \tilde{p}_4 C = p_4 + i(\mu + K) \]

\[ B = m + \frac{16}{3m_G^2} \int \frac{d^4 q}{(2\pi)^4} \frac{B}{q^2 + \tilde{q}_4^2 + B^2} \]

To satisfy these equations, all gap solutions have to be momentum independent. Simplest solution: A=1

Renormalization of chem. pot. due to vector interaction

mass gap equation

This is a 1 to 1 reproduction of the (basic) NJL model
NJL model within DS framework

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\[ P[S] = \text{Tr} \ln[S^{-1}] - \frac{1}{2} \text{Tr} [\Sigma S] \]

Steepest descent approximation

\[ P(\mu) = \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \ln S^{-1}(\vec{p}^2, \vec{p}_4) + \frac{3}{4} m_G^2 K^2 - \frac{3}{8} m_G^2 B^2 \]

1 to 1 NJL (regularization issue ignored)

\[ \frac{8}{3 m_G^2} \int \frac{d^4 q}{(2\pi)^4} \frac{\tilde{q}_4 C(q)}{\vec{q}^2 + \tilde{q}_4^2 C^2(q) + B^2(q)} = iK \]

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**Model 1 (Munczek/Nemirowsky)**

\[
f_1(|\vec{p}|; \mu) = \frac{1}{4\pi} \int_{-\infty}^{\infty} d\rho_4 \, tr_D(-\gamma_4) S(p; \mu)
\]

Wigner Phase

\[
\vec{p}^2 = \mu^2 - 2\eta^2
\]

\[
\mu^2 \geq 2\eta^2 \quad \text{to obtain} \quad f_1(\vec{p}^2 = 0) = 1
\]

\( (\eta = 1.09 \text{ GeV}) \)

small' chem. Potential: \[ f_1(\vec{p}^2 = 0, \mu < \eta) \propto \mu \]

\[
P(\mu < \eta) = P_0 + \int_0^\mu \, d\mu' \, n(\mu') \propto P_0 + \text{const} \times \mu^5
\]

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model is scale invariant regarding \( \mu/\eta \)

\[
P(\mu) \propto \mu^5 \quad \text{well satisfied up to} \quad \mu/\eta \approx 1
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Model 2

\[ f_1(|\vec{p}|; \mu) = \frac{1}{4\pi} \int_{-\infty}^{\infty} dp_4 \text{tr}_D (-\gamma_4) S(p; \mu) \]

Wigner Phase Less extreme, but again, 1 particle number density distribution different from free Fermi gas distribution

Chen et al. (TK) PRD 78 (2008)
Conclusions

NJL model is a powerful tool to explore possible features of dense QCD

It possibly might be a too powerful tool for unambiguous predictions

NJL mf approximation is a gluon mf approximation in DSE

NB: Momentum independent gap solutions in their very nature result in a quasi particle picture → no confinement

Accounting for momentum dependent gap solutions enriches the model space significantly

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Thank you!