Holographic Glueball Decay

and other recent results from the Witten-Sakai-Sugimoto model

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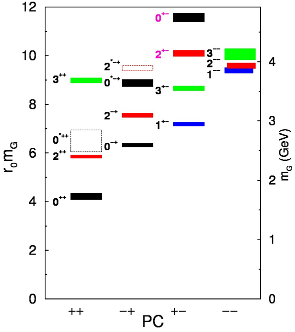
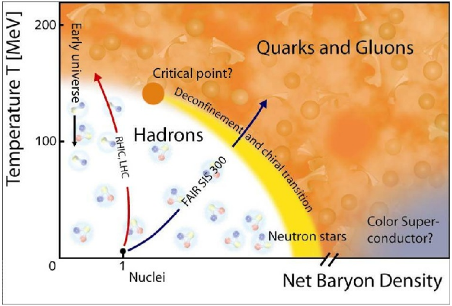
FAIR physics questions

PANDA:
Glueball spectrum *and properties*

CBM:
QCD phase diagram at *large(r)* μ_q

Morningstar & Peardon hep-lat/9901004:





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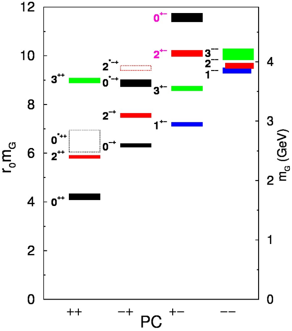

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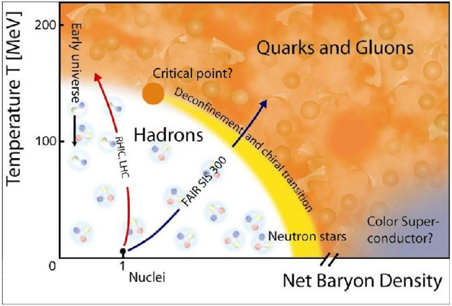


Nonperturbative first-principles information:

Lattice QCD: only spectrum — only small μ_q

new avenue: **Gauge/gravity duality**

Outline

Reviews:

- Introduction to **holographic QCD**
  top-down: **Witten**[1998]-**Sakai-Sugimoto**[2004] **model**
  = supergravity limit of conjectured full string-theoretic dual of QCD
  (2 parameters only: 1 coupling + 1 mass scale!:-)
- Glueballs from anti-de Sitter supergravity
- Chiral quarks, chiral phase transition in Sakai-Sugimoto model

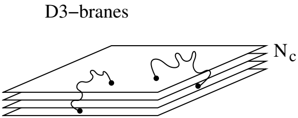
Some new results:

- Glueball decay into pions from Sakai-Sugimoto model
  (work with Frederic Brünner & Denis Parganlija)
  [Acta Phys. Polon. Supp. 7 (2014) 533]
- Inverse magnetic catalysis of chiral phase transition
  (work with Florian Preis & Andreas Schmitt)
  [JHEP 1103 (2011) 033, J. Phys. G39 (2012) 054006, Lect. Notes Phys. 871 (2013) 51]

Original AdS/CFT correspondence

J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998)

"pedestrian's guide": S. S. Gubser and A. Karch, Ann. Rev. Nucl. Part . Sci. 59, 145 (2009)



(type IIB) string theory
on 5D anti-de Sitter space $(\times S_5)$

$\Leftrightarrow$

$\mathcal{N} = 4$ SU(∞) super-YM theory
on 4D boundary of AdS$_5$

$$\frac{(\text{curvature radius})^4}{(\text{string length})^4} = \frac{R^4}{\ell_s^4} \quad = \quad g^2_{\rm YM} N_c \equiv \lambda$$ 't Hooft coupling

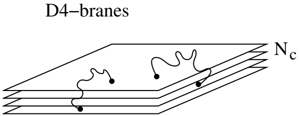
supergravity limit $\ell_s \ll R$
relatively easy

$\Leftrightarrow$

strong coupling limit $\lambda \gg 1$
impossibly difficult

Witten model: Holographic nonsupersymmetric QCD

E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998):



Type-IIA string theory with $N_c \to \infty$ $D4$ branes dual to $4+1$-dimensional super-Yang-Mills theory

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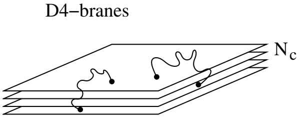


Type-IIA string theory with $N_c \to \infty$ $D4$ branes
dual to $4+1$-dimensional super-Yang-Mills theory

supersymmetry completely broken by compactification
on "thermal-like" circle $x_4 \equiv x_4 + 2\pi/M_{\rm KK}$ (Kaluza−Klein)

- antisymmetric b.c. for adjoint fermions: masses $\sim M_{\rm KK}$
- adjoint scalars not protected by gauge symmetry: also masses $\sim M_{\rm KK}$

→ dual to pure-glue YM theory
3+1-dimensional at scales $\ll M_{\rm KK}$

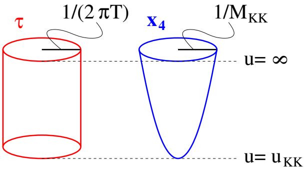
but supergravity approximation needs weak curvature,
cannot take limit $M_{\rm KK} \to \infty$

Deconfinement phase transition

Thermal circle in Euclidean time τ in addition to compactified x_4
Hawking-Page transition when $2\pi T = M_{\rm KK}$ (thus $\sim$ 1 GeV ?)

Confined phase

$$\begin{aligned} ds^2 &= \left(\frac{u}{R}\right)^{3/2}\left[d\tau^2 + d\mathbf{x}^2 + f(u)dx_4^2\right] \\ &\quad + \left(\frac{R}{u}\right)^{3/2}\left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right] \end{aligned}$$

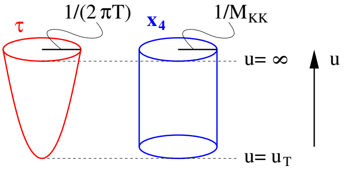


$$M_{\rm KK} = \frac{3}{2}\frac{u_{\rm KK}^{1/2}}{R^{3/2}} \qquad f(u) \equiv 1 - \frac{u_{\rm KK}^3}{u^3}$$

Cigar topology in x_4-u subspace $\longrightarrow$

Deconfined phase

$$\begin{aligned} ds^2 &= \left(\frac{u}{R}\right)^{3/2}\left[\tilde{f}(u)d\tau^2 + \delta_{ij}d\mathbf{x}^2 + dx_4^2\right] \\ &\quad + \left(\frac{R}{u}\right)^{3/2}\left[\frac{du^2}{\tilde{f}(u)} + u^2 d\Omega_4^2\right] \end{aligned}$$



$$T = \frac{3}{4\pi}\frac{u_T^{1/2}}{R^{3/2}} \qquad \tilde{f}(u) \equiv 1 - \frac{u_T^3}{u^3}$$

Cigar in τ-u = **Euclidean black hole**

Glueballs in confined phase

$\exists$ scalar and tensor glueballs corresponding to 5D dilaton Φ and graviton G_{ij}

Csaki, Ooguri, Oz & Terning 1999

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Type-IIA supergravity compactified on x_4-circle many more modes:

Constable & Myers 1999; Brower, Mathur & Tan 2000

Mode Sugra fields J^{PC}	S$_4$ G_{44} 0^{++}	T$_4$ Φ, G_{ij} $0^{++}/2^{++}$	V$_4$ C_1 0^{-+}	N$_4$ B_{ij} 1^{+-}	M$_4$ C_{ij4} 1^{--}	L$_4$ G_α^α 0^{++}
n=0	7.30835	22.0966	31.9853	53.3758	83.0449	115.002
n=1	46.9855	55.5833	72.4793	109.446	143.581	189.632
n=2	94.4816	102.452	126.144	177.231	217.397	277.283
n=3	154.963	162.699	193.133	257.959	304.531	378.099
n=4	228.709	236.328	273.482	351.895	405.011	492.171

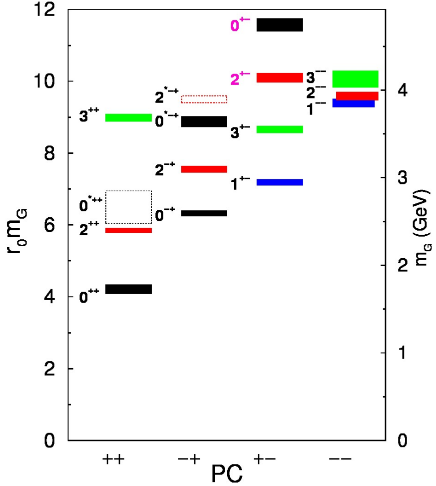
Lowest mode not from dilaton, but from "exotic polarization" – in 11D notation:

$$\delta g_{44} = -\frac{r^2}{L^2} f\, H(r) G(x), \quad \delta g_{\mu\nu} = \frac{r^2}{L^2}\left[\frac{1}{4} H(r)\eta_{\mu\nu} - \left(\frac{1}{4} + \frac{3R^6}{5r^6 - 2R^6}\right) H(r) \frac{\partial_\mu \partial_\nu}{M^2}\right] G(x)$$

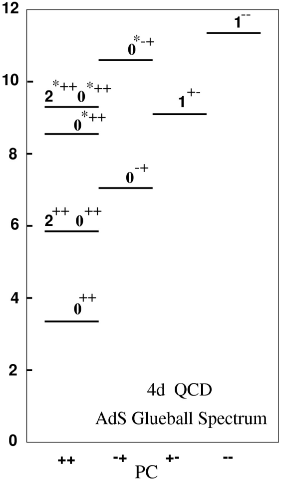
$$\delta g_{11,11} = \frac{r^2}{L^2}\frac{1}{4} H(r) G(x), \quad \delta g_{rr} = -\frac{L^2}{r^2} f^{-1} \frac{3R^6 H(r) G(r)}{5r^6 - 2R^6}, \quad \delta g_{r\mu} = \frac{90 r^7 R^6 H(r) \partial_\mu G(x)}{M^2 L^2 (5r^6 - 2R^6)^2}$$

Lattice glueballs vs. supergravity glueballs

Morningstar & Peardon hep-lat/9901004:



Brower, Mathur & Tan 2000:



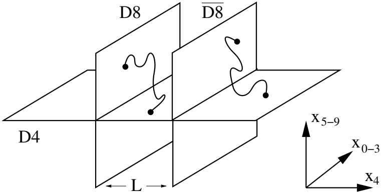
(mass scales matched on 2^{++}) → seemingly good qualitative agreement!

Sakai-Sugimoto model: Adding chiral quarks

T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)

add N_f $\mathrm{D8}$- and $\overline{\mathrm{D8}}$-branes, separated in x_4, $N_f \ll N_c$ (probe branes)

	0	1	2	3	4	5	6	7	8	9
D4	x	x	x	x	x					
D8/$\overline{\mathrm{D8}}$	x	x	x	x		x	x	x	x	x



4-8, 4-$\bar{8}$ strings
$\rightarrow$ fundamental, massless chiral fermions

flavor symmetry
$U(N_f)_L \times U(N_f)_R$

for now: maximal separation in x_4 (antipodal on x_4 circle): $L = \pi/M_{\mathrm{KK}}$

Massless pions, massive vector mesons, massive η'

D8 brane action:

$$S_{\mathrm{D8}} = -T_{\mathrm{D8}} \mathrm{Tr} \int d^9x e^{-\Phi} \sqrt{-\det(\tilde{g}_{MN} + (2\pi\alpha')F_{MN})} + S_{\mathrm{CS}}$$

$$= \frac{g_{\mathrm{YM}}^2 N_c^2}{216\pi^3} \int d^4x\, dz\, \mathrm{Tr} \left[\frac{1}{2}(1+z^2)^{-1/3} F_{\mu\nu}^2 + (1+z^2) F_{\mu z}^2 \right] + \ldots$$

- massless pions in $A_z = \phi_0(z)\pi(x^\nu)$, rho meson in $A_\mu^{(1)} = \psi_1(z)\rho_\mu(x^\nu)$,
- more massive vector mesons and axial vector mesons in tower of $A_\mu^{(n)}$ modes

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eigenvalue of ψ_1 implies $m_\rho=\sqrt{0.669314}M_{\rm KK}$

$\Rightarrow$ matching $m_\rho\approx 776$ MeV fixes $\boxed{M_{\rm KK}=949\text{ MeV}}$ ($\Rightarrow T_{deconf}=151$ MeV)

matching $f_\pi=\frac{\lambda N_c}{54\pi^4}M_{\rm KK}^2$ gives $\boxed{\lambda=g_{\rm YM}^2N_c\approx 16.6}$

yields e.g.

- $m_{a_1}^2/m_\rho^2\approx 2.4$ (versus 2.5 from experiment!)
- nonzero η' mass from anomaly inflow: Witten-Veneziano formula with $m_{\eta'}=\frac{\sqrt{N_f/N_c}}{3\sqrt{3}\pi}\lambda M_{\rm KK}\approx 967$MeV for $N_f=3$ (exp.: 958 MeV !)

ρ meson decay rate

D8 mode corresponding to ρ-meson stable,
but can calculate effective action for mesons, in particular:

$$\mathcal{L}_{\rho\pi\pi} = -g_{\rho\pi\pi}\epsilon_{abc}(\partial_\mu \pi^a)\rho^{b\mu}\pi^c$$

$$g_{\rho\pi\pi} = \sqrt{2}\int dz \frac{1}{\pi(1+z^2)}\psi_1(z) = 33.98\,\lambda^{-\frac{1}{2}}\,N_c^{-\frac{1}{2}}$$

gives

$$\Gamma_\rho/m_\rho = \frac{g_{\rho\pi\pi}^2}{48\pi} \approx 0.1535 \qquad \text{(exp.: 0.191(1))}$$

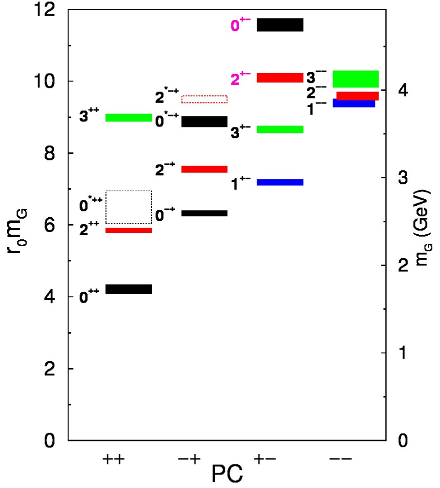
encourages **calculation of gluon decay rates**
which could not be easily obtained from (Euclidean!) lattice QCD

Lattice vs. supergravity glueballs

seemingly good qualitative agreement by matchup up 2^{++}

(but AdS spectrum somewhat stretched. . .)

Morningstar & Peardon hep-lat/9901004:



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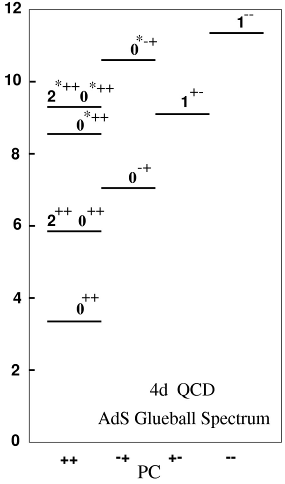

Lattice vs. supergravity glueballs in Sakai-Sugimoto model

Sakai-Sugimoto model: glueball masses $\propto M_{\mathrm{KK}} = 949$ MeV fixed by m_ρ

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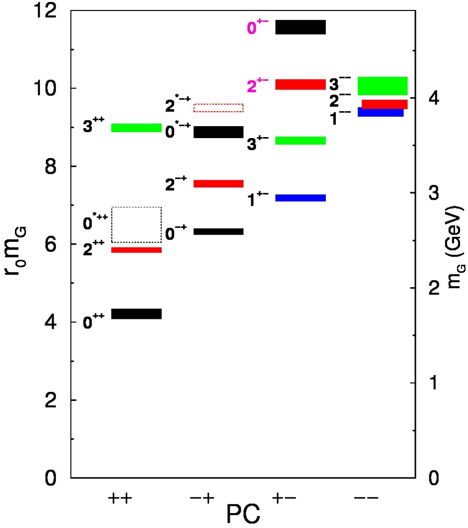

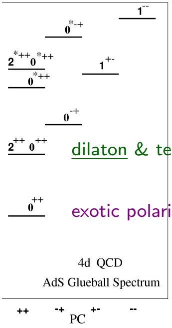

dilaton & tensor 1487 MeV

exotic polarization 855 MeV

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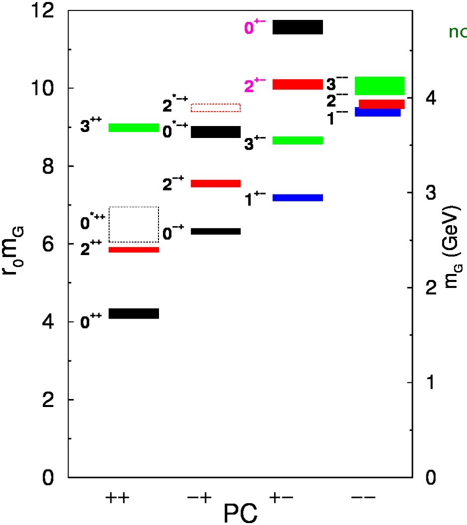
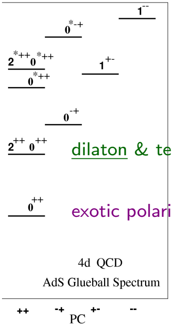


now good match lowest $0^{++} \leftrightarrow$ dilaton



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Lattice vs. supergravity glueballs in Sakai-Sugimoto model

Should exotic polarization be excluded as lowest glueball mode?

- possibly not part of spectrum of holographic QCD in limit $M_{\mathrm{KK}} \to \infty, \lambda \to 0$ (already asked by Constable & Myers)
- simpler bottom-up AdS/QCD have dilaton mode as dual for lowest glueball

Dual operators:

Dilaton: Tr $F_{\mu\nu}F^{\mu\nu}$ and $\mathcal{L}_{\mathrm{massive\ adj.matter}}$

Exotic polarization: Tr $F_{\mu\nu}F^{\mu\nu}$, T^{00} and adj.matter composite operators

Glueball decay rates in Sakai-Sugimoto model

Gravitational modes stable in confined background, but
can calculate effective action for glueball-meson interactions

done for lowest (exotic) mode by
Hashimoto, Tan & Terashima, Phys.Rev. D77 (2008) 086001, arXiv:0709.2208

revisited, corrected, and extended to other modes by
Brünner, Parganlija & AR, Acta Phys. Polon. Supp. 7 (2014) 533 and in prep.

Decay into two pions:

Exotic mode: $\Gamma_{G\to\pi\pi}/M \approx \frac{13.79}{\lambda N_c^2} \approx 0.092 \quad (M_G \approx 855\mathrm{MeV})$

Dilaton mode: $\Gamma_{D\to\pi\pi}/M_D \approx \frac{4.076}{\lambda N_c^2} \approx 0.027 \quad (M_D \approx 1487\mathrm{MeV})$

Excited dilaton: $\Gamma_{D^*\to\pi\pi}/M_{D^*} \approx \frac{4.898}{\lambda N_c^2} \approx 0.033 \quad (M_{D^*} \approx 2358\mathrm{MeV})$

NB: relative width of lowest (exotic) scalar mode much larger than next ones!?

Experimental candidate: $\Gamma^{(\mathrm{ex})}(f_0(1500) \to \pi\pi)/(1505\mathrm{MeV}) = 0.025(3)$
would fit perfectly to dilaton mode

Glueball decay rates in Sakai-Sugimoto model (cont'd)

Tensor mode: $\Gamma_{T\to\pi\pi}/M_T \approx \dfrac{2.174}{\lambda N_c^2} \approx 0.0145 \quad (M_T \approx 1487\mathrm{MeV})$

Experimental candidate: $\Gamma^{(\mathrm{ex})}(f_J(2200))/(2231\mathrm{MeV}) = 0.010(4)$

Mass of holographic tensor mode off by 33%
but dimensionless Γ/M remarkably close again!

Glueball decay rates in Sakai-Sugimoto model (cont'd)

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Branching ratios:
General pattern

- Narrow widths $\Gamma_{Glueball\to\pi\pi} \propto \lambda^{-1}N^{-2}$
- Strong suppression for $\Gamma_{Glueball\to 4\pi} \propto \lambda^{-3}N^{-4}$
  not really suppressed in $f_0(1500)$, $f_J(2200)$
  would have to be due to mixing in of $q\bar{q}$
- Even stronger suppression for $Glueball \to 4\pi^0$
  (direct $\Gamma_{GB\to 4\pi^0} \propto \lambda^{-7}N^{-4}$ from F^4 terms in DBI action
  also $\Gamma_{GB\to GB+2\pi^0\to 4\pi^0} \propto \lambda^{-6}N^{-3}$, but kinematically suppressed)

Summary – Glueballs in Witten-Sakai-Sugimoto model

After fitting just m_ρ to fix $M_{\mathrm{KK}} = 949$ MeV

- good prediction of higher vector and axial vector mesons masses,
- good prediction of deconfinement/chiral transition temperature,
- good prediction of glueball masses if “exotic mode” discarded;

after fitting f_π to also fix 't Hooft coupling at $\lambda = 16.6$

- good prediction of rho decay rates
- good prediction of anomalous $m'_\rho \propto N_c^{-\frac{1}{2}} \lambda M_{\mathrm{KK}}$
- surprisingly good decay rates $glueball \to \pi\pi$
- strong suppression of $glueball \to 4\pi$, in particular $\to 4\pi^0$

Warrants further studies!

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- inclusion of nonzero mass for strange quark
- mixing with quarkonia (suppressed by $N_c^{-\frac{1}{2}}$)
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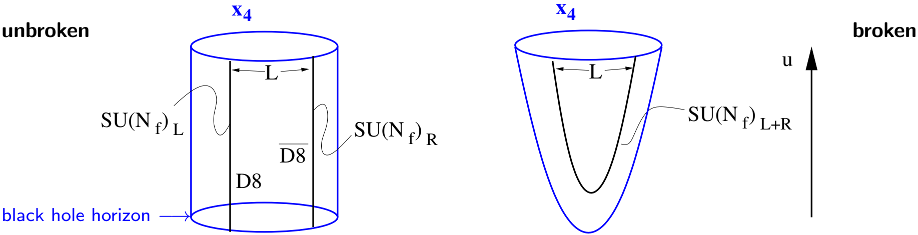
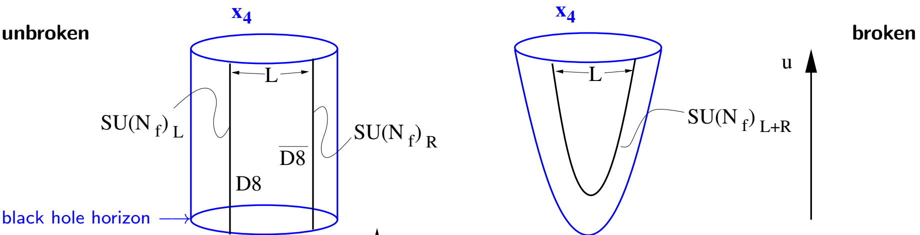
NEXT TOPIC: chiral symmetry restoration at finite T and μ

Chiral symmetry breaking/restoration



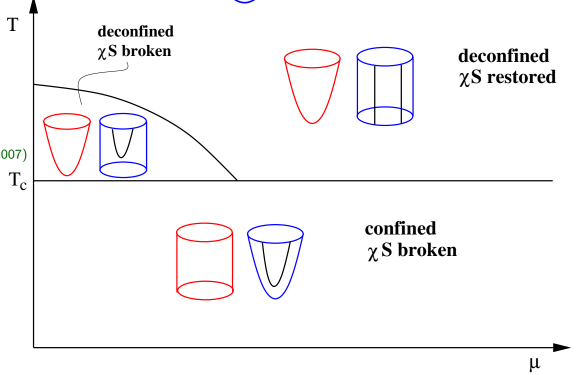
Chiral symmetry breaking/restoration



cigar topology in x_4-u subspace:
χS necessarily broken

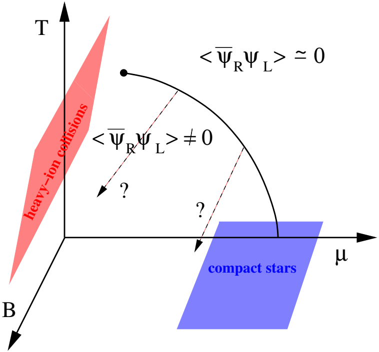
Aharony, Sonnenschein & Yankielowicz, Ann.Phys. 322 (2007)
Horigome & Tanii JHEP0701:
D8-$\overline{\mathrm{D8}}$ branes *not* maximally separated:
difference between
deconfinement and χS restoration

see de Boer, Chowdhury, Heller
& Jankowski, PRD87 (2013)
for possibility of quarkyonic phase
(confined, χS restored) below T_c



QCD at finite T and μ and B

superstrong magnetic fields $B \sim \Lambda^2_{QCD} \simeq (150\,\mathrm{MeV})^2 \sim 10^{18}\,\mathrm{G}$
will have effects on QCD phase diagram



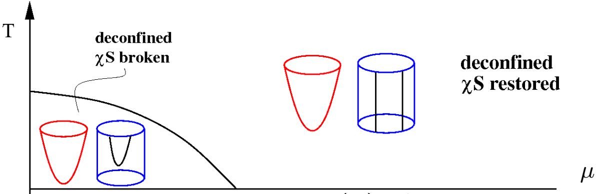
Of actual interest in

1. **Heavy ion collisions:**
   up to perhaps 10^{19} G
   for earliest stage of collisions
2. **Astrophysics:** magnetars have
   up to $B \sim 10^{15}$ G at surface
   interior perhaps up to 10^{18-20} G

Dense holographic matter with magnetic field

F. Preis, AR & A. Schmitt 2010

Small brane separation: chiral phase transition separate from deconfinement transition sensitive to magnetic field



— comparable to NJL models; gluon (de)confinement ignored

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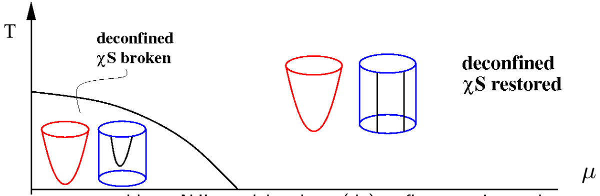
F. Preis, AR & A. Schmitt 2010

Small brane separation: chiral phase transition separate from deconfinement transition sensitive to magnetic field



— comparable to NJL models; gluon (de)confinement ignored

consequences of chiral anomaly:

- χS restored: $j_5 \propto \mu B$ (in QCD: Metlitski & Zhitnitsky, Son & Newman '05)
- χS broken: $\nabla\pi^0 \propto \mu B$

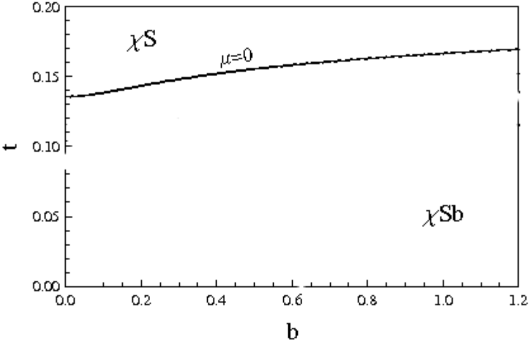
  real QCD: $\nabla\pi^0 \neq 0$ ground state for $B > B_1 \sim 10^{19}$ G $\quad (\propto m_\pi)$

  $n_b \propto B\nabla j$ nonzero baryon number even for $\mu < m_{baryon}$ (Son & Stephanov '07)

in Sakai-Sugimoto ($L = L_{max}$): Son & Thompson; Bergman, Lifshytz & Lippert; AR, Schmitt & Stricker, '08

Magnetic catalysis at $\mu = 0$

$\mu = 0$: Critical temperature for chiral symmetry restoration increases with B



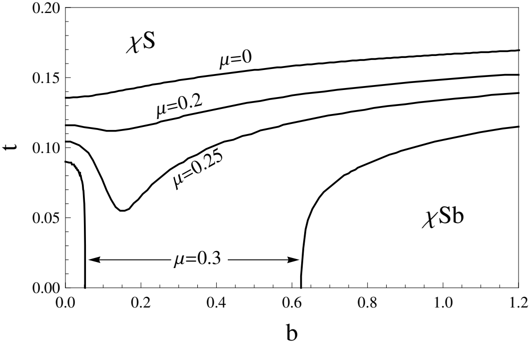
I.e.: Magnetic field enhances (catalyses) chiral symmetry breaking

Gross-Neveu and NJL models: Klimenko '92; Gusynin, Miransky & Shovkovy '94

AdS_5/CFT_4: Filev et al.; Erdmenger et al. '07; *Sakai-Sugimoto:* Johnson & Kundu '08

Inverse magnetic catalysis at nonzero μ

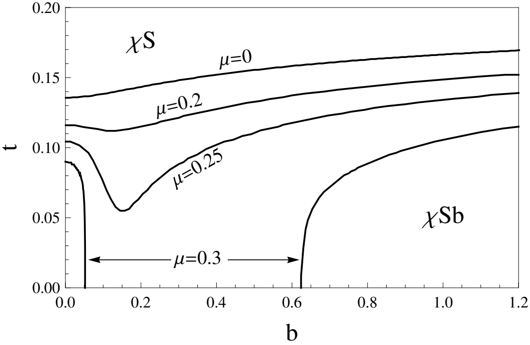
$\mu > 0$: [F. Preis, AR & A. Schmitt, JHEP 1103]

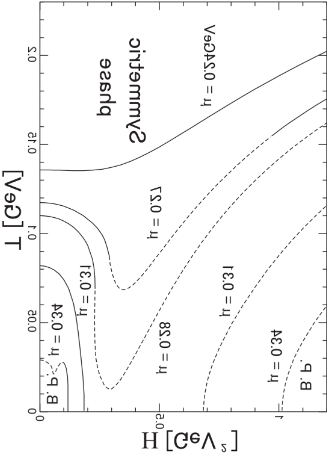


inverse phenomenon at large μ,
not too large B

Inverse magnetic catalysis at nonzero μ

$\mu > 0$: [F. Preis, AR & A. Schmitt, JHEP 1103]



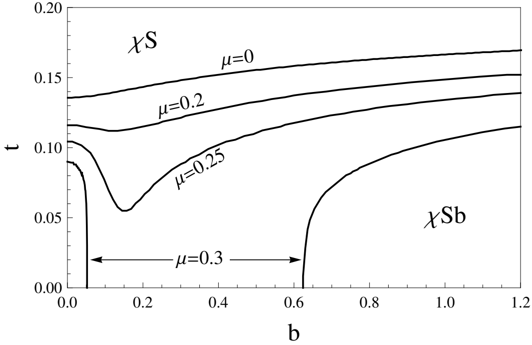


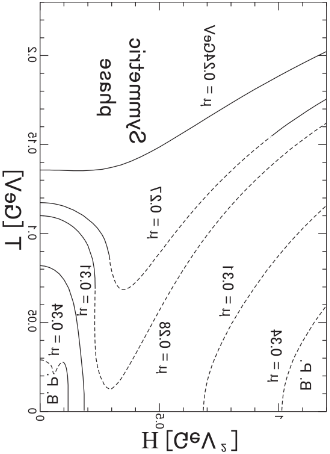
inverse phenomenon at large μ,
not too large B

seen before in NJL models: Inagaki, Kimura & Murata, Prog.Theor.Phys. 111 (2004)

Inverse magnetic catalysis at nonzero μ

$\mu > 0$: [F. Preis, AR & A. Schmitt, JHEP 1103]





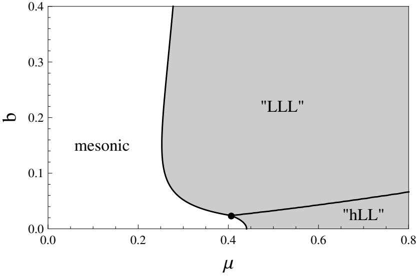
inverse phenomenon at large μ,
not too large B

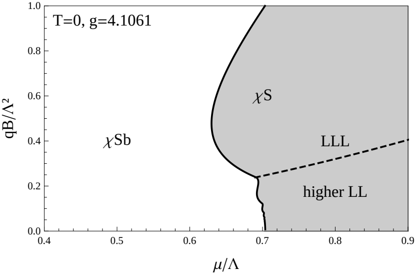
seen before in NJL models: Inagaki, Kimura & Murata, Prog.Theor.Phys. 111 (2004)

Recently also found in lattice simulations but at $\mu = 0$ (and not yet understood) [Bali et al, JHEP 1202]

Comparison with NJL model at zero temperature

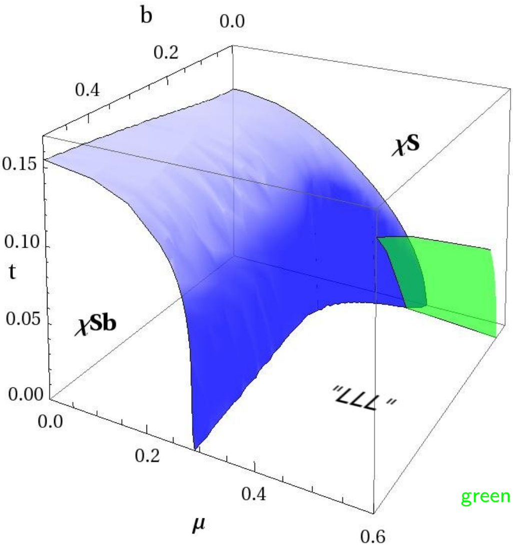
F. Preis, AR, A. Schmitt: Lect. Notes Phys. 871 (2013) 51 [arXiv:1208.0536]





inverse magnetic catalysis in NJL and related models:
D. Ebert, K. G. Klimenko, M. A. Vdovichenko and A. S. Vshivtsev, PRD 61, 025005 (2000)
T. Inagaki, D. Kimura and T. Murata, Prog. Theor. Phys. 111, 371 (2004)
B. Chatterjee, H. Mishra and A. Mishra, PRD 84, 014016 (2011)
S. S. Avancini, D. P. Menezes, M. B. Pinto and C. Providencia, PRD 85, 091901 (2012)
J. O. Andersen and A. Tranberg, JHEP 1208, 002 (2012)

Phase diagram of Sakai-Sugimoto model at finite T, μ, B



blue: chiral phase transition
green: transition into lowest Landau level

Conclusions

- Witten-Sakai-Sugimoto model is an interesting holographic model of low-energy nonsupersymmetric QCD with chiral quarks
  - only 2 parameters: mass scale and coupling
  - top-down (from clear-cut superstring construction)
  - supergravity approximation expected to be continuous deformation of real dual of QCD
- good semi-quantitative predictions for mesons and glueballs, including decay rates
- interesting phase structure at finite T and μ
- interesting effects of strong magnetic fields for non-maximal D8 brane separation