3rd International Conference on New Fronties in`Physics

Nuclear Enthalpies or Nucleon Properties inside Compressed Nuclear Matter

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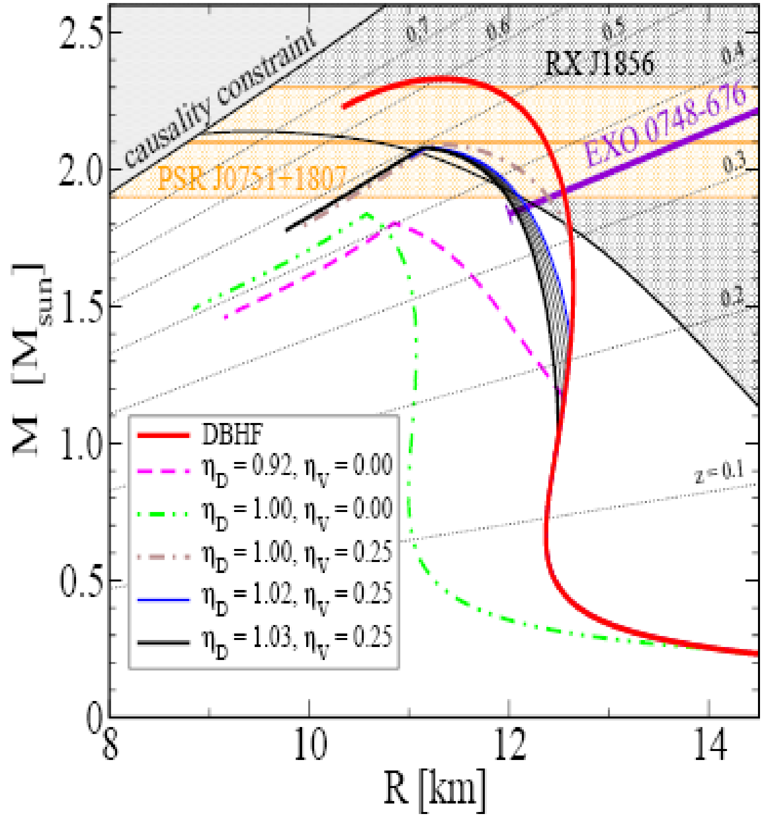
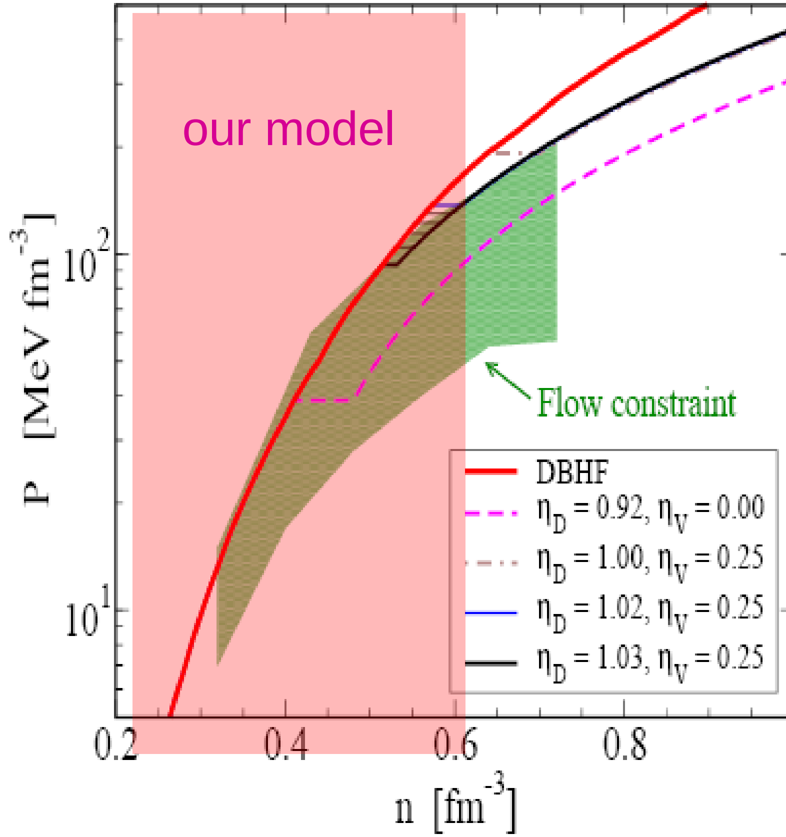
“Is it possible to maintain my volume constant when the pressure increases?”

- an nucleon when entering the compressed medium.

Nuclear Entalpies, 1311.3591; Pressure Corrections to the Equation of State in the Nuclear Mean Field, 1205.0431, Acta Phys. Pol. B Proc. Suppl. Vol. 5 No 2 (2012) 375

Mass-Radius constraint and Flow constraint (II)





- Large Mass ($\sim 2\, M_\odot$) and radius ($R \geq 12$ km) $\Rightarrow$ stiff quark matter EoS;
  Note: DU problem of DBHF removed by deconfinement! and: CFL core Hybrids unstable!
- Flow in Heavy-Ion Collisions $\Rightarrow$ not too stiff EoS !
  Note: Quark matter removes violation by DBHF at high densities

Klähn, D.B., Sandin, Fuchs, Faessler, Grigorian, Roepke, Truemper, [arxiv:nucl-th/0609067] (2006) *PRC 74*

Definitions

- **Enthalpy** is a measure of the total energy of a thermodynamic system. It includes the system's internal energy and thermodynamic potential (a state function), as well as its volume Ω and pressure p_H (the energy required to "make room for it" by displacing its environment, which is an extensive quantity).

$$H_A = E_A + p_H \Omega_A \qquad \text{Nuclear Enthalpy} \qquad (1)$$

$$H_N = M_{pr} + p_H \Omega_N \qquad \text{Nucleon Enthalpy} \qquad (2)$$

Specific Enthalpies

$$h_A(\rho) = H_A/E_A = 1 + p_H/(\rho\, \varepsilon(\rho))$$

$$h_N(\rho) = H_N/M_{pr} = 1 + p_H/(\rho_{cp}\, M_{pr}(\rho)) \qquad (3)$$

Enthalpy vs Hugenholz - van Hove relation with chemical potential

$$\mu \doteq (\partial M_A/\partial A)_{\Omega_A} \equiv (\partial H_A/\partial A)_{p_H} = \varepsilon_A + \frac{p_H}{\varrho} = H_A/A$$

$$E_F \doteq P_N^0(P_F) = (\partial M_A/\partial A)_{\Omega_A} = \varepsilon_A + p_H/\varrho = \mu$$

In the NM in equilibrium $p_H = 0$ therefore $H_A = E_A$. Dividing H_A by A we obtain the following relation between single particle enthalpy h_A and $\varepsilon_A = E_A/A$,

$$h_A = \varepsilon_A + p_H/\varrho. \qquad (1a)$$

Please note that the same equation fulfills a Fermi energy $E_F \equiv P_N^0(p_F) = \varepsilon_A + p_H/\varrho$ of nucleon with a Fermi momentum p_F; well-known as the HvH relation, also proven in the self-consistent RMF approach [ ]. It turns out that definitions of the Fermi energy or a single particle enthalpy have the same energy balance.

We will argue that the enthalpy rather than the rest energy should be used in the momentum distribution sum rules (MSR) in NM and in a nucleon.

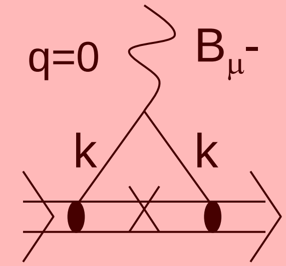
RMF and Momentum Sum Rule

The relativistic nuclear dynamics of nucleons in the nucleus is described by the Light Cone (LC), momentum distribution function $f_N(y)$ [Birse], where $y = AP_N^+/P_A^+$, a fraction of longitudinal momentum of A nucleons in the nucleus is Lorentz invariant. Let us now focus our attention on the sum rule for longitudinal momenta $P_N^+ = P_N^0 + P_N^Z$. Do they sum in the rest frame to the nuclear energy E_A, or rather to nuclear enthalpy H_A? To answer this question we can examine the distribution

$$f_N(y) = \int \frac{d^4 P_N}{(2\pi)^4} \delta\left(y - \frac{AP_N^+}{P_A^+}\right) Tr\left[\gamma^+ S(P_N, P_A)\right]. \quad (4)$$

Frankfurt, Strikman Phys. Reports 160 (1988)

Finally with a good normalization of S_N we have:



Flux Factor

$$f_N(y) = \frac{4}{\varrho}\int_0^{P_F} \frac{S_N(P_N)\,d^3\boldsymbol{P}_N}{(2\pi)^3}\left(1+\frac{P_N^3}{E_N^*}\right)\delta\left(y - AP_N^+/P_A^0\right) =$$

$$= (3/4)[P_A^0/(AP_F)]^3[(AP_F/P_A^0)^2 - (y - AE_F/P_A^0)^2]. \quad (5)$$

$P^0_A = E_A = A\varepsilon_A$

and Momentum Sum Rule

$$\frac{1}{A}\int dy\, y f_N(y) = \frac{E_F}{P_A^0} \equiv \frac{\frac{\partial}{\partial A}(E_A)_{\Omega_A}}{P_A^0} = \frac{\varepsilon_A + p_H/\varrho}{P_A^0}. \quad (6)$$

$$\int dy\, y f_N(y) = \frac{E_F}{h_A} = 1. \; .$$

Fermi Energy

Enthalpy/A

Bag Model in Compress Medium

$p_H=0$

$$E^0_{Bag}(R) = \frac{3\omega_0 - Z_0}{R} + \frac{4\pi}{3}B(\varrho_0)R^3 \sim 1/R,$$

$$p_H = p_B = \frac{3\omega_0 - Z_0}{4\pi R^4} - B(\varrho) \quad \rightarrow \quad (B(\varrho)+p_H)R^4 = const$$

$$R = \left[\frac{3\omega_0 - Z_0}{4\pi(B(\varrho) + p_H)}\right]^{1/4}, \qquad (12)$$

$$M_{pr} = E_{Bag} = 4\pi R^3\left[\frac{4}{3}(B + p_H) - \frac{p_H}{3}\right] = E^0_{Bag}\frac{R_0}{R} - p_H\Omega_N.$$

$$H_N = E^0_{Bag}\frac{R_0}{R} \sim 1/R. \qquad (7)$$

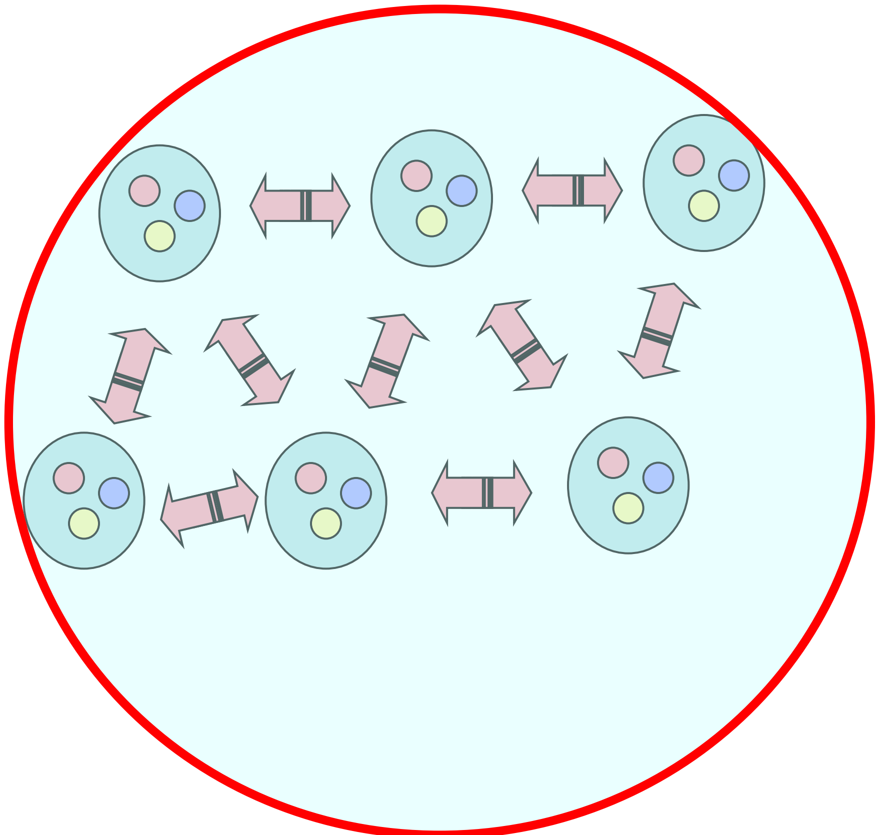
Two Scenarios

for NN repulsion with qq attraction

- **Constant Mass**

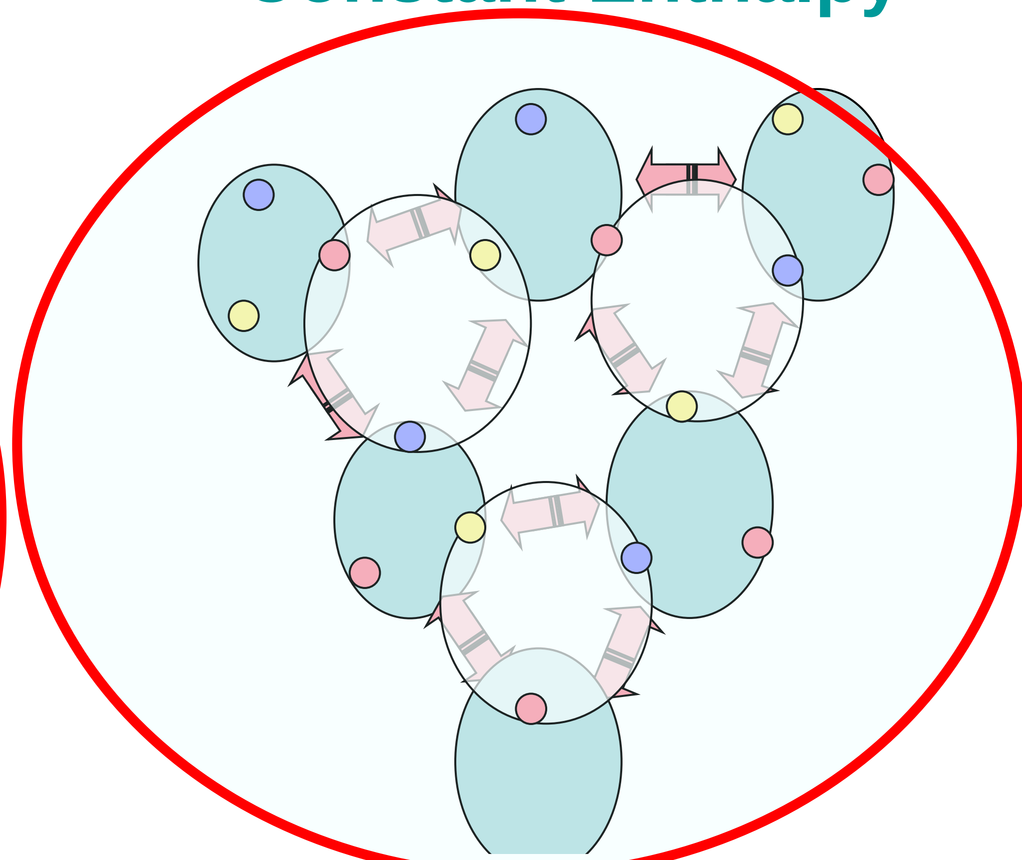
= Increasing Enthalpy

1/R



- **Constant Volume**

= Constant Enthalpy

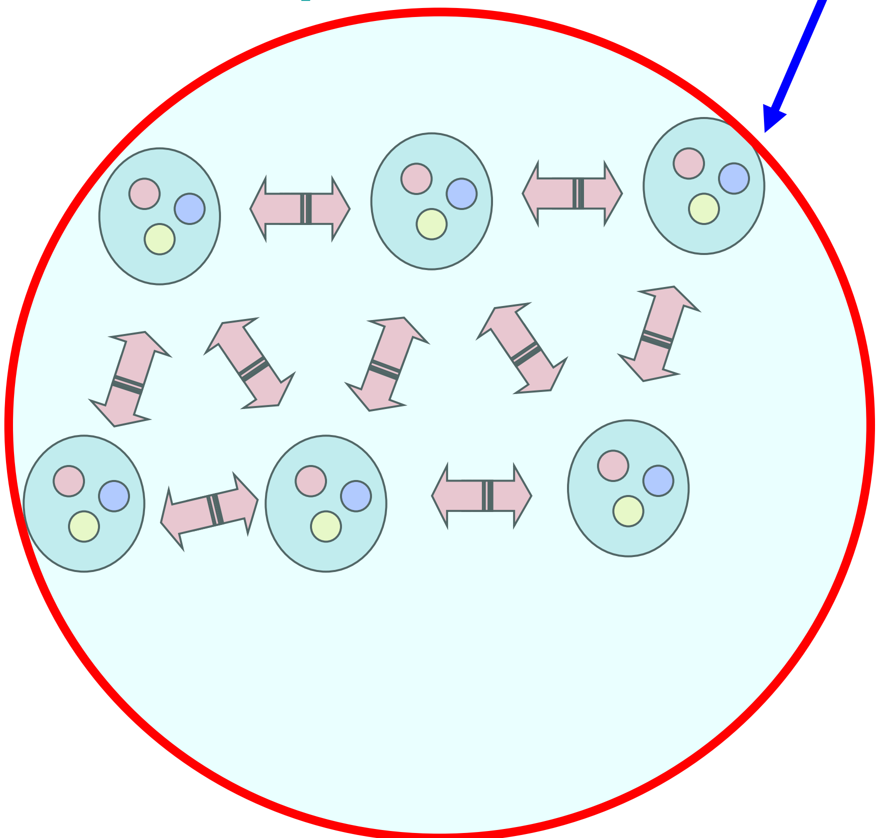


Two Scenarios

affecting nuclear compressibility K^{-1}

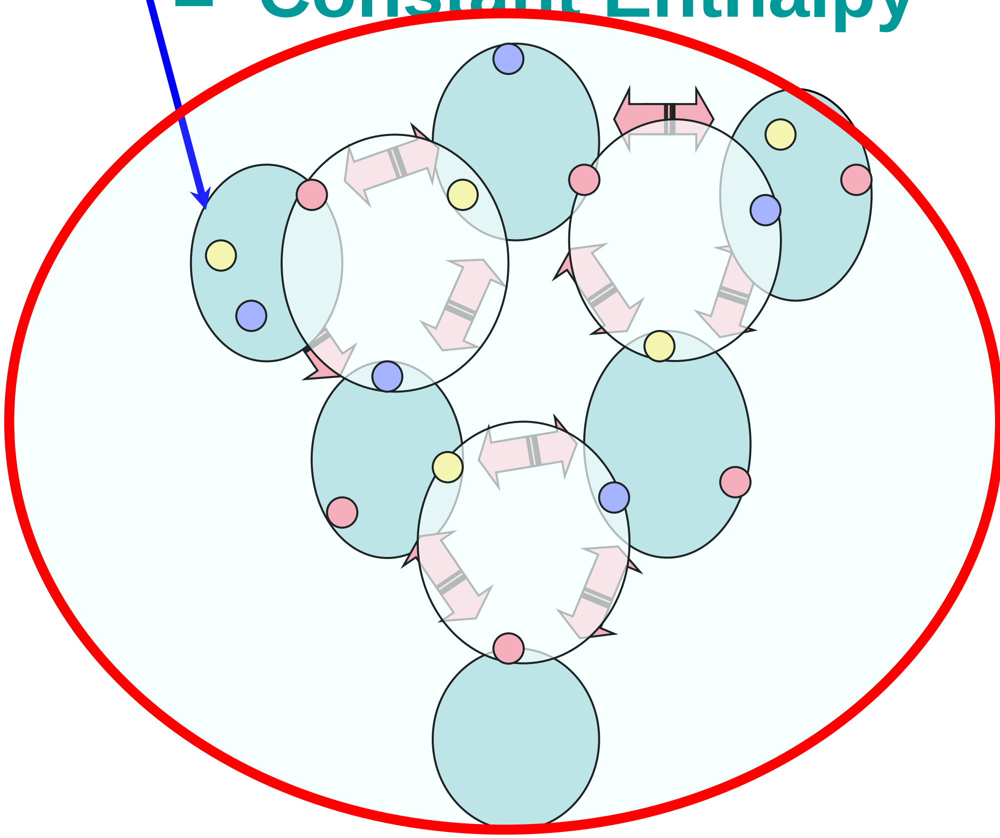
- **Constant Mass**

= Increasing Enthalpy 1/R



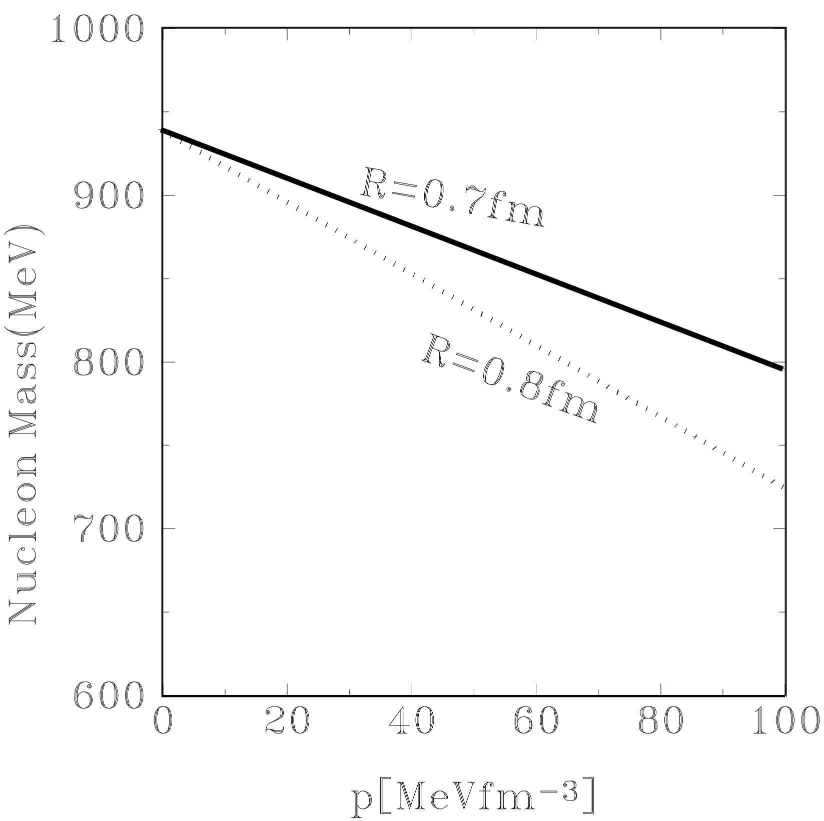
- **Constant Volume**

= Constant Enthalpy



Nucleon Mass for different nucleon radii in compressed NM

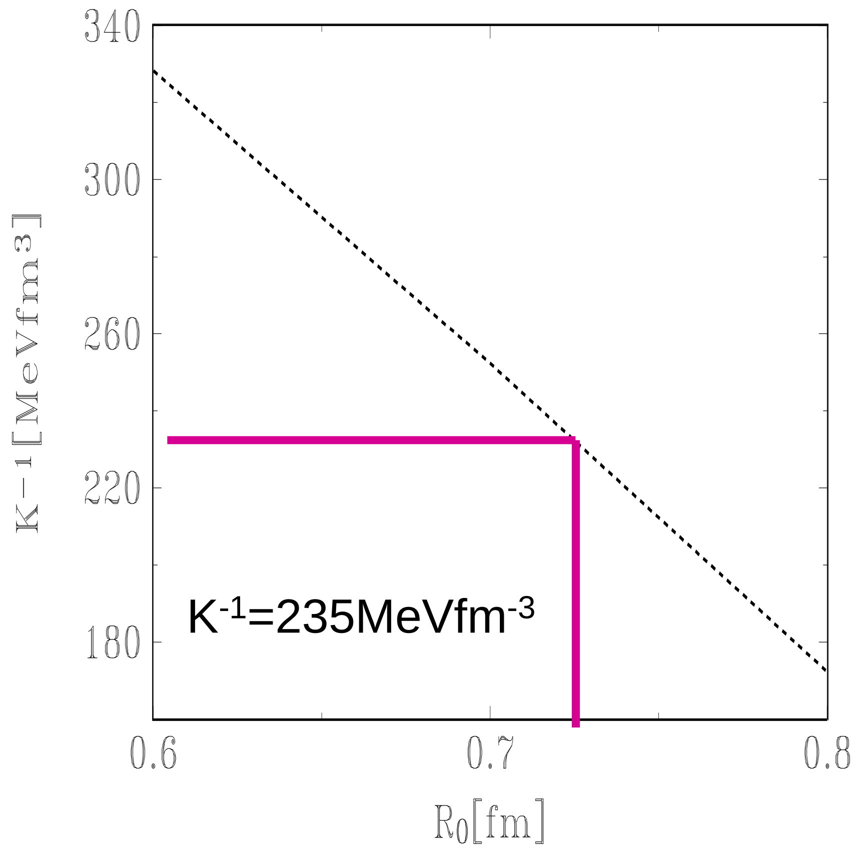
$$M_{pr}(\varrho) = M_N - p_H(\varrho)\Omega_N,$$
$$p_H(\varrho) = \varrho^2 \varepsilon'_A(\varrho)/(1-\varrho\Omega_N).$$



Nuclear compressibility for different constant nucleon radii in compressed NM

$$H_A^T/A = \varepsilon_A - (\partial M_A/\partial\Omega_{A-})_A/\varrho = \varepsilon_A + p_H/\varrho = E_F$$

Our version of Hugenholz-Van Hove relation for finite nucleons in NM



RMF Equation of State for const Enthalpy

$$\varepsilon_A = C_1^2 \varrho + \frac{C_2^2}{\varrho}(M_{pr} - M_{pr}^*)^2 + \frac{\gamma}{\varrho}\int_0^{P_F} \frac{d^3p}{(2\pi)^3}\sqrt{\boldsymbol{P}_N^2 + M_{pr}^{*2}}$$

$$M_{pr}^* = M_{pr} - \frac{\gamma}{2C_2^2}\int_0^{P_F} \frac{d^3p}{(2\pi)^3}\frac{M_{pr}^*}{\sqrt{\boldsymbol{P}_N^2 + M_{pr}^{*2}}}, \qquad (16)$$

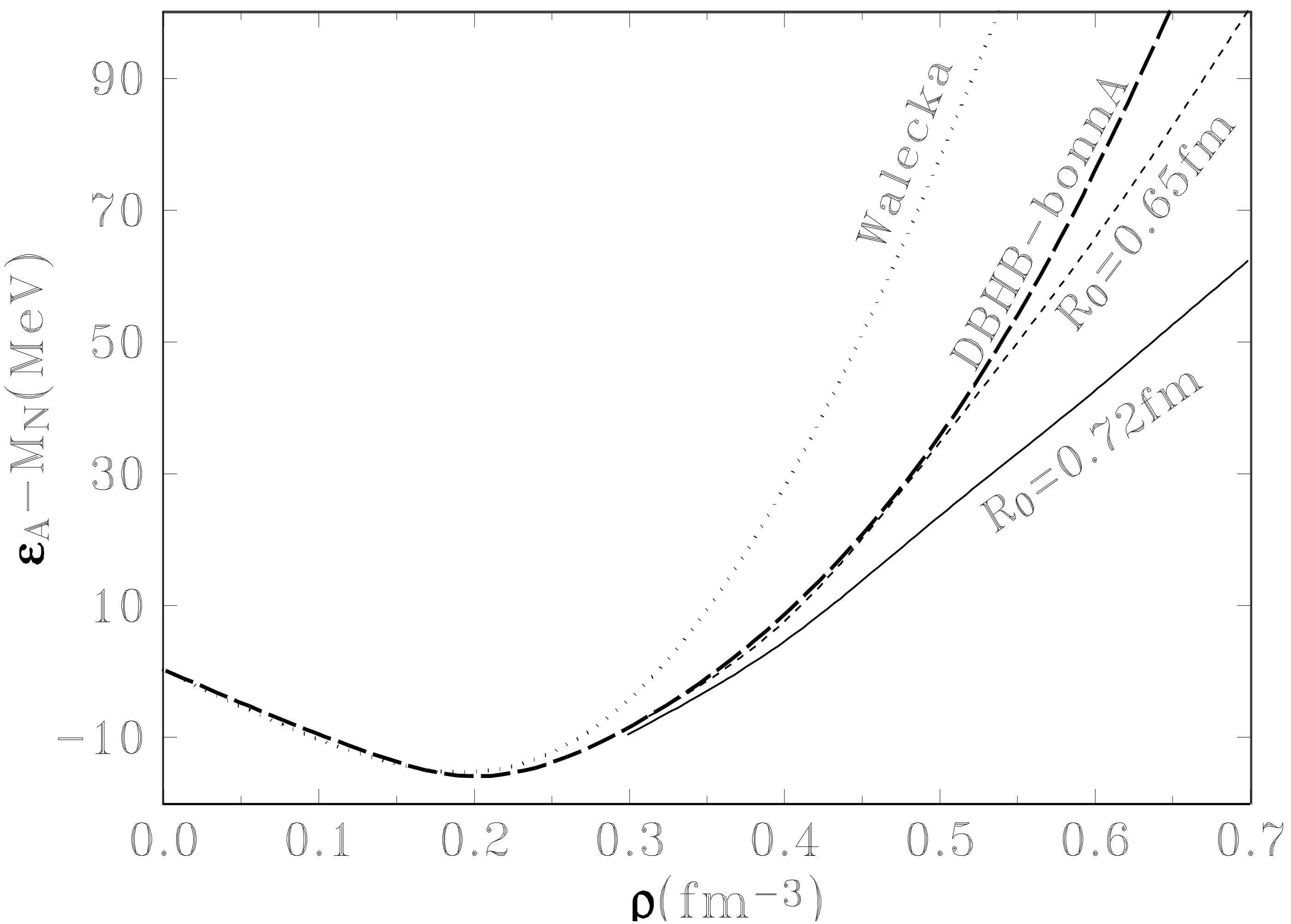
(8)

$$M_{pr}(\varrho) = M_N - p_H(\varrho)\Omega_N$$
$$p_H(\varrho) = \varrho^2 \varepsilon'_A(\varrho)/(1 - \varrho\Omega_N).$$

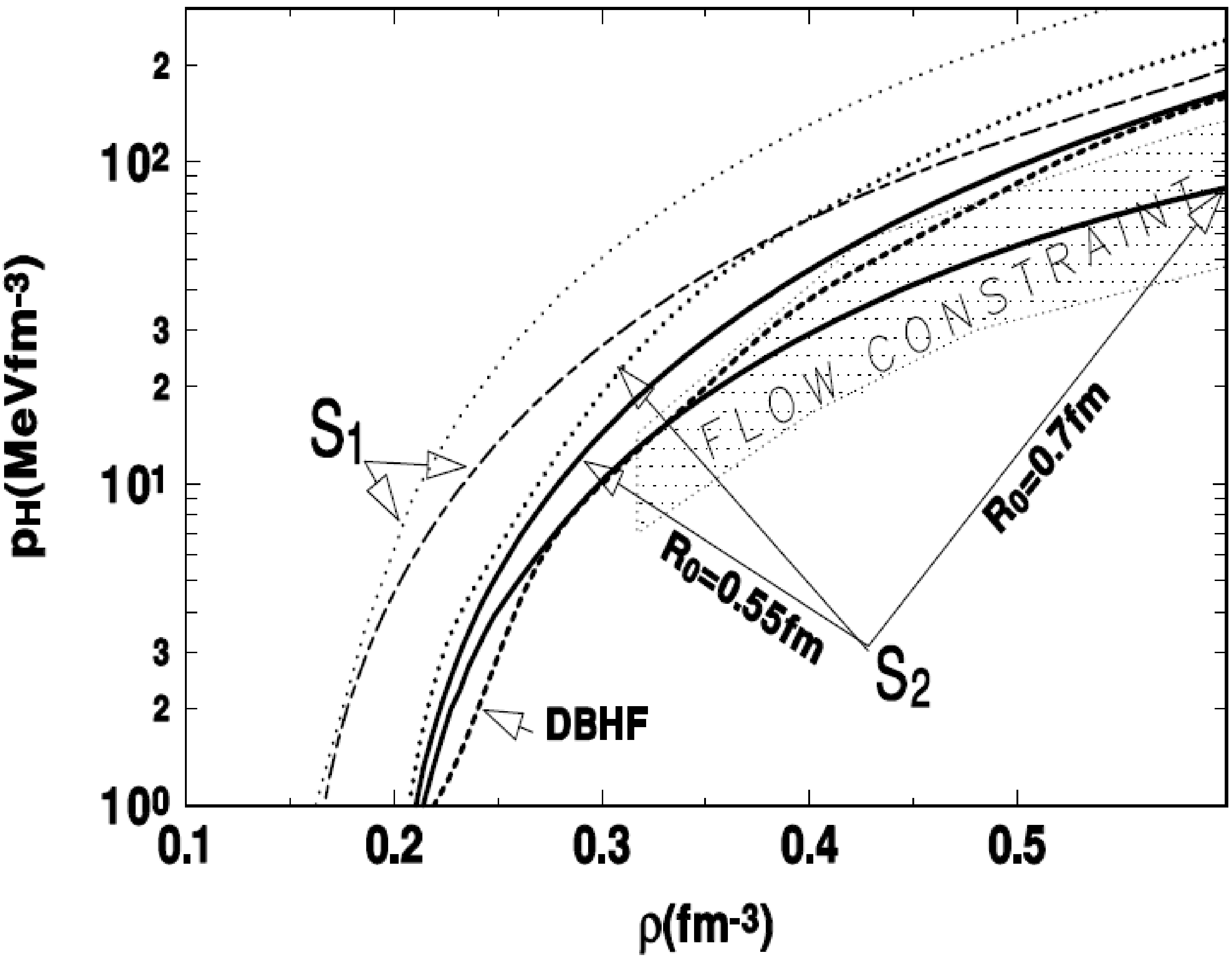
(9)

Please note the positive absence of the **nonlinear terms** introduced by Boguta & Stocker (1983) fitted to get good value of K^{-1} and symmetry energy (2 extra parameters).

Equation of state - different models



Results



pH(MeVfm-3)
ρ(fm-3)
0.1
0.2
0.3
0.4
0.5
10^0
10^1
10^2
2
3
S1
S2
DBHF
FLOW CONSTRAINT
R0=0.55fm
R0=0.7fm

The toy model for phase transition

It is easy to show that the equality of these specific enthalpies at a certain density ϱ_{cr}

$$h_A^T(\varrho_{cr}) = h_N(\varrho_{cr})$$

is equivalent to a following condition for the critical density ϱ_{cr}

$$\varrho_{cr}\, \varepsilon_A(\varrho_{cr}) = \varrho_{cp}(\varrho_{cr})\, M_{pr}(\varrho_{cr}). \quad (10)$$

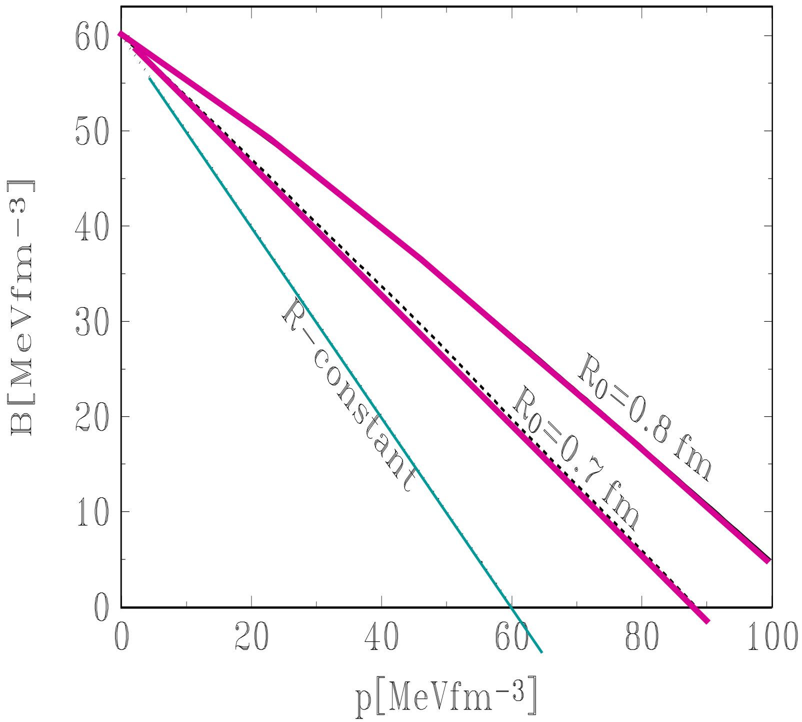
where the alignment of energy densities, outside and inside nucleon, takes place. Another word, energy density $(\varrho\,\varepsilon_A)$, which includes a space Ω_{A-} between nucleons, reaches the energy density of a quark plasma $(\varrho_{cp} M_{pr})$ inside nucleon therefore an ultimate de-confinement transition to the Quark-Gluon-Plasma (QGP) will take place when condition (10) is satisfied. This self-consistent condition for the will discussed in two selected regimes: a constant nucleon radius (subsection "A") and a constant nucleon mass (subsection "B").

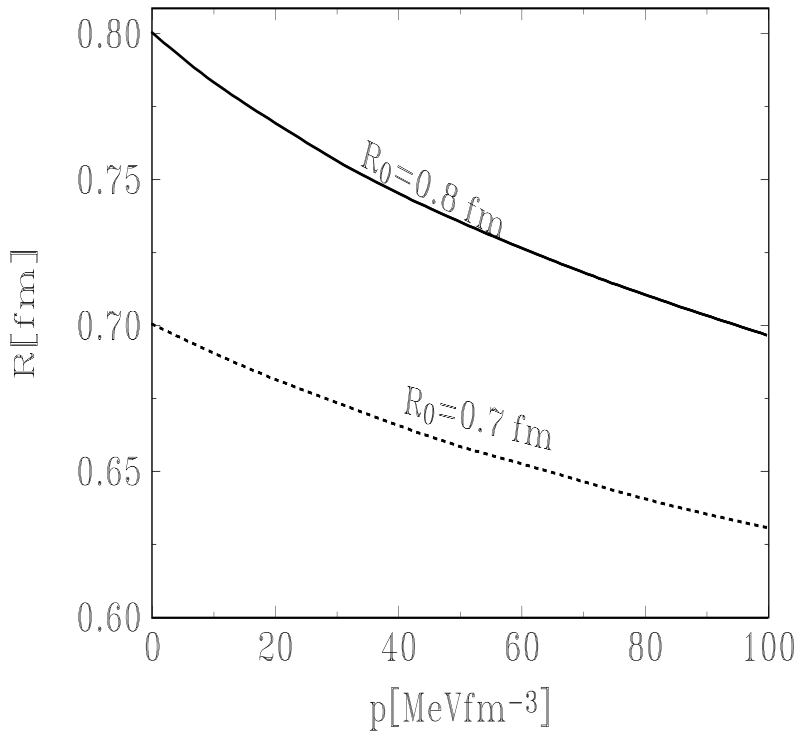
Bag constant in function of nuclear pressure

Nucleon radius in compressed NM for a constant nucleon mass

$$B = B(\varrho_0)(R_0/R)^4 - p.$$

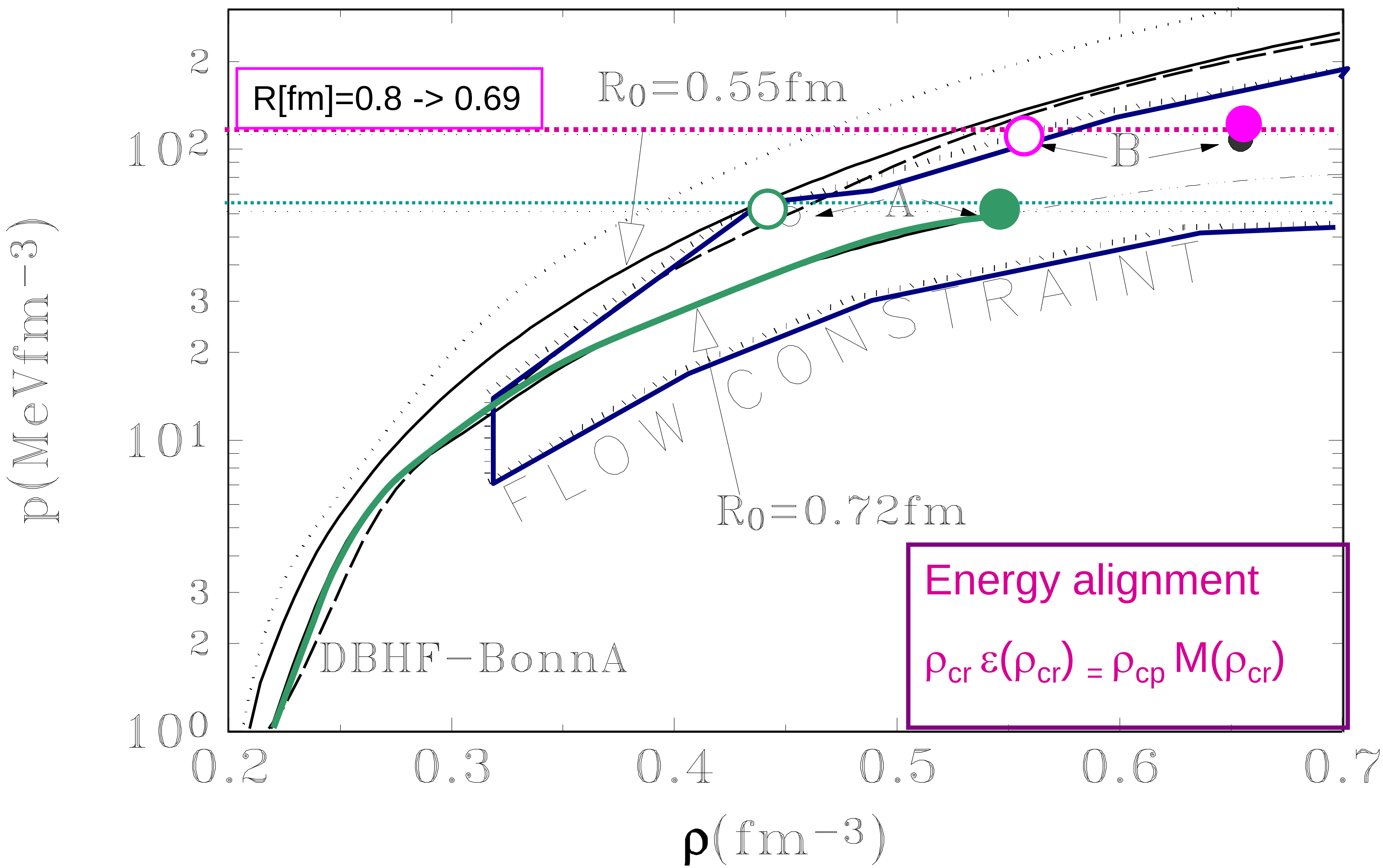
$$M_N R_0/R = M_N + 4/3\pi R^3 p$$





Two possible scenario of phase transition

A - constant nucleon radius, B - constant nucleon mass



Conclusions

A. Constant nucleon mass
requires increasing enthalpy

STIFFER EOS

Critical pressure 120MeVfm^{-3}

B. Constant nucleon volume give
the constant enthalpy with
decreasing nucleon mass

lower compressibility

SOFTER EOS

Critical pressure 60MeVfm^{-3}