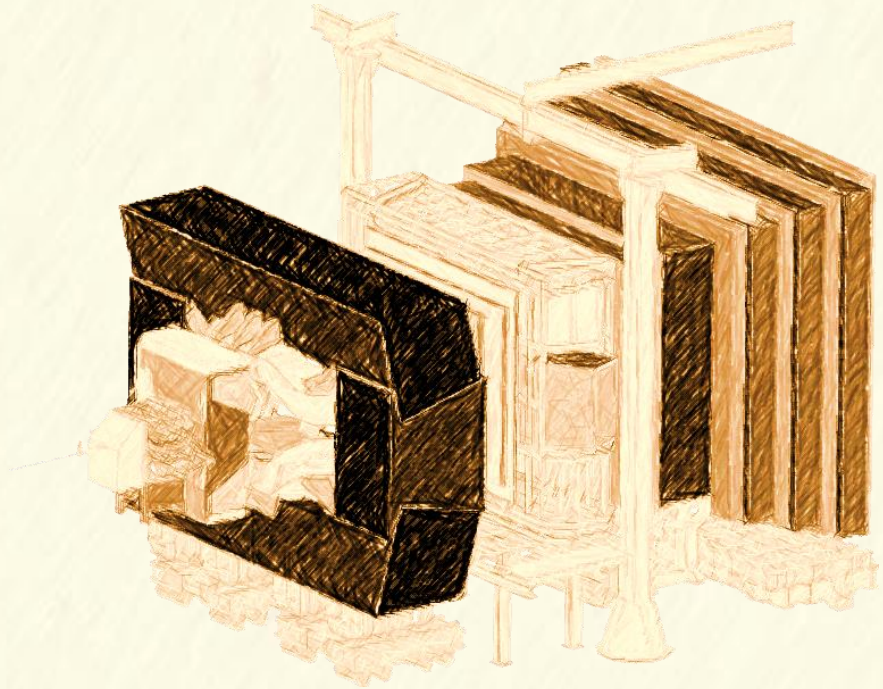
Recent LHCb results on CP Violation

Diego Martínez Santos
(on behalf of LHCb collaboration)

Introduction

- The LHCb experiment
  - Detector
  - Indirect searches for New Physics
- Measurement of ϕ_s
  - Introduction
  - Results / status
  - Prospects
- Other CPV measurements
  - The CKM angle γ
  - $\phi_s^{\phi\phi}$ from $B_s \to \phi\phi$
  - CPV in B→3h
  - CPV in charm

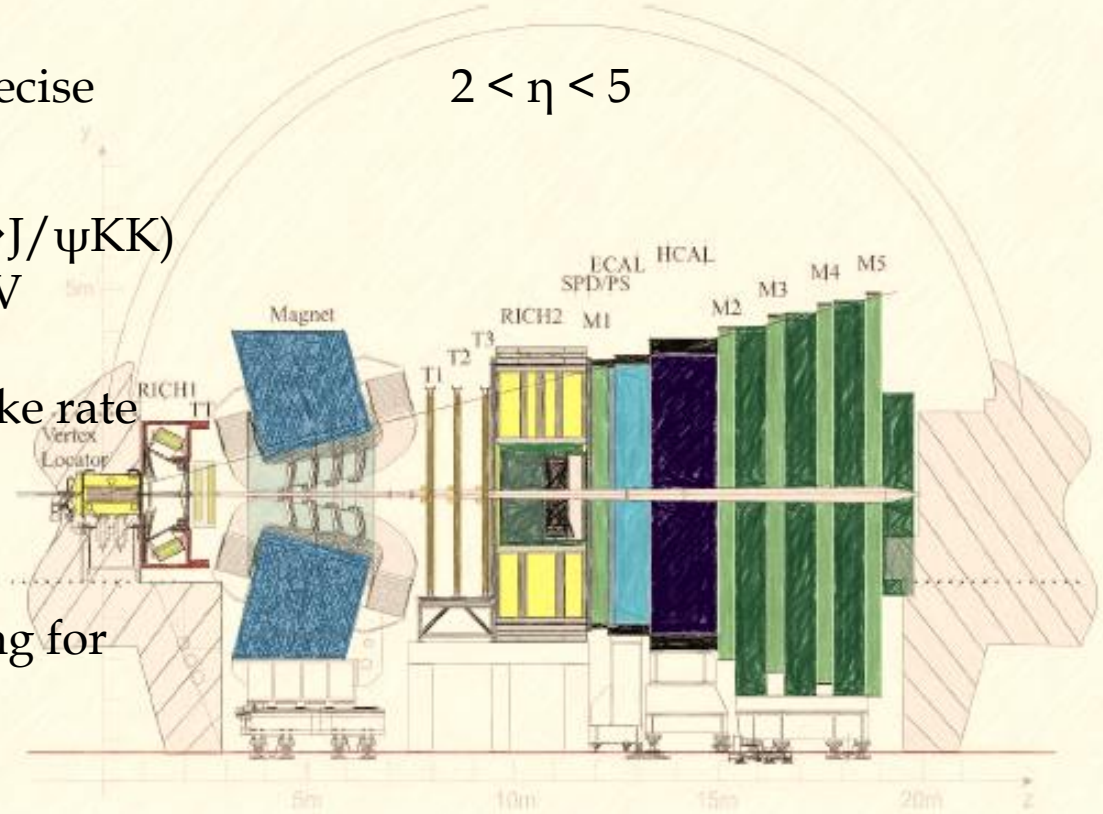


The LHCb experiment

Forward spectrometer with very precise tracking and PID

$2 < \eta < 5$

- Decay time resolution ~40 fs ($B \rightarrow J/\psi KK$)
- Invariant mass resolution ~8 MeV ($B \rightarrow J/\psi KK$)
- 95% (K-π) ID efficiency for 5% fake rate



Efficient and flexible trigger
ε ~80% $B \rightarrow J/\psi X$ decays interesting for physics studies

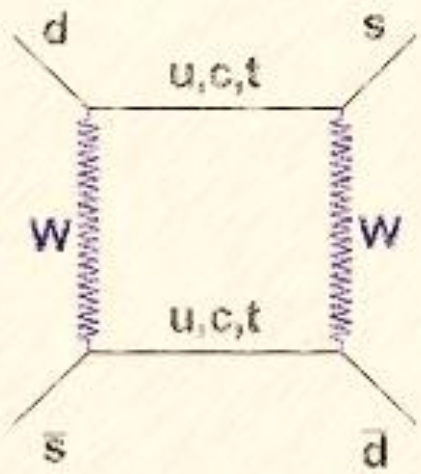
Recorded luminosity: 3 fb^{-1}

1 fb^{-1} at 7 TeV (2011)
2 fb^{-1} at 8 TeV (2012)

Also, took 13nb^{-1} of pA data

The LHCb experiment

- The **LHCb physics program** focuses mostly on CP violation and rare decays
- Both correspond to **indirect searches for New Physics** (i.e, new particles),
- Indirect approach has been very successful in the past
  - Neutral Currents
    (Z^0 inferred ten years before direct observation)
  - Kaon mixing
    (top-quark inferred 30 years before direct observation)



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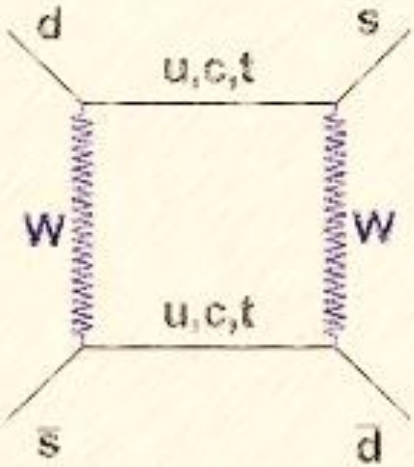
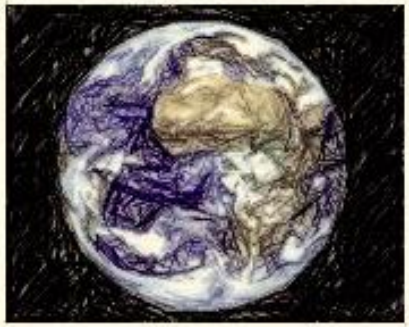
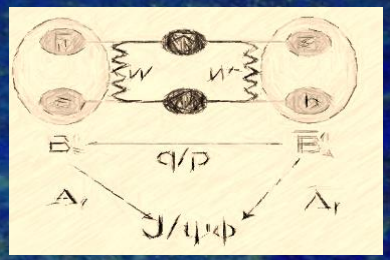


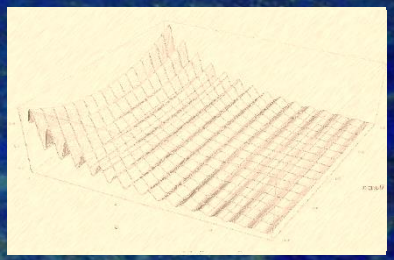
(you may also notice Earth' radius was inferred indirectly 2.3k years before direct observation…)

Ερατοσθένης

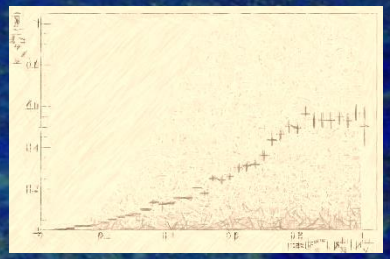
~2.3 K years till the direct observation…

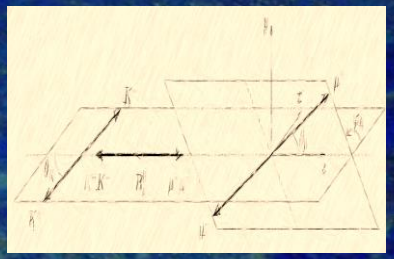




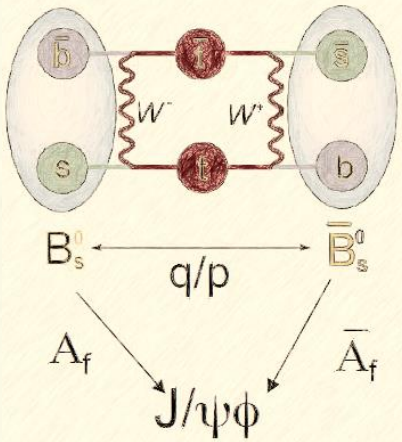


What (and why) Φ_s





Φ_s from $B_s \to J/\psi\ (\to\mu\mu)\ KK$



B_s mass eigenstates:

$$|B_L^s\rangle = p|B_s\rangle + q|\bar{B}_s\rangle$$
$$|B_H^s\rangle = p|B_s\rangle - q|\bar{B}_s\rangle$$

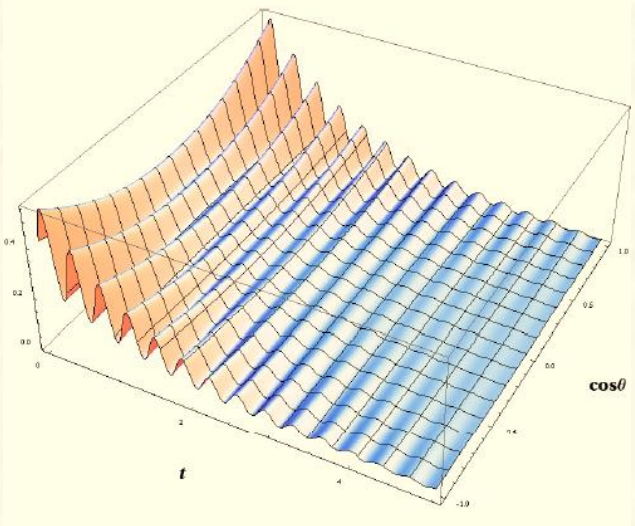
Weak eigenstates (mix via box diagram)

- q/p: complex number. $|q/p| \neq 1$ → CPV in mixing
- $A_f, \overline{A_f}$ complex amplitudes. $|A_f/\overline{A_f}| \neq 1$ → CPV in decay

Even if not CPV in mixing or decay, you can generate CPV in the interference if

$$\sin(\boldsymbol{\phi_s}) \equiv \sin\left(-arg\left(\frac{q}{p}\frac{A_f}{\overline{A_f}}\right)\right) \neq 0$$

Main (but not only) experimental signature of a non-zero $\boldsymbol{\phi_s}$: it generates **wiggles** in the time-dependent angular distribution of the $B_s \to J/\psi\ \phi \to \mu\mu KK$ final state particles. The frequency of the (potential) wiggles is known: Δm_s.



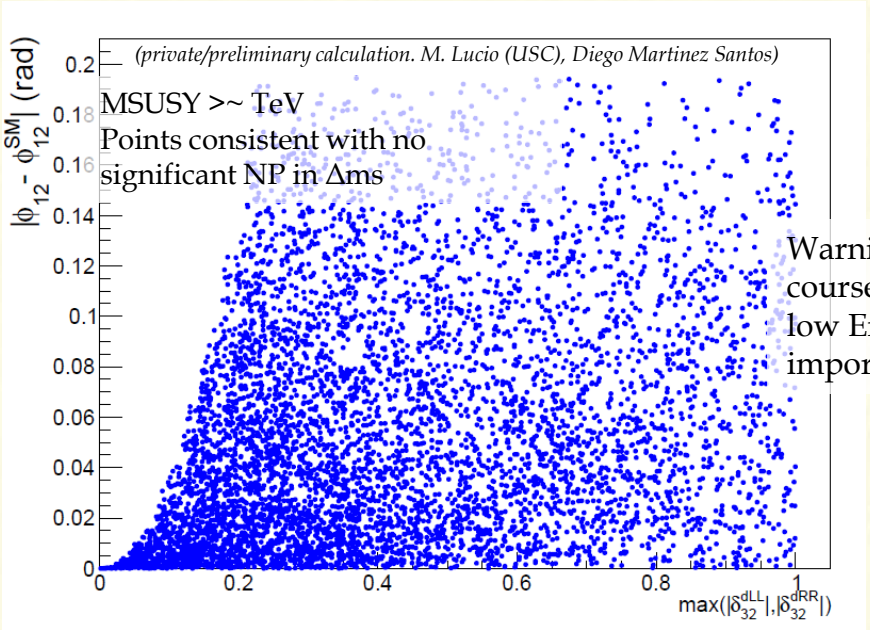
Φ_s : Standard Model and New Physics sensitivity

SM prediction: $\Phi s = -2\arg\left(-\frac{V_{cb}V_{cs}^*}{V_{tb}V_{ts}^*}\right) = -0.036 \pm 0.002^{(*)}$

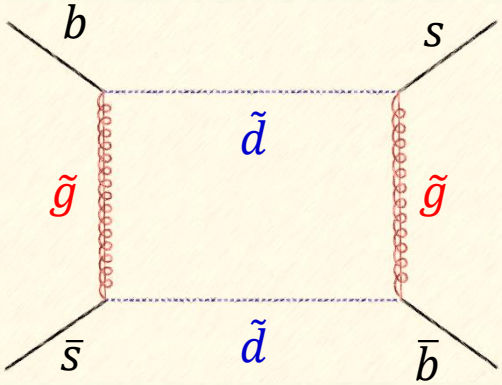
(*)Neglecting penguin contributions. Work is ongoing to confirm/reject such assumption

It is very precise, and sensitive to Physics Beyond the SM, specially to non–MFV New physics …. which is accessible even if the NP is at a high scales

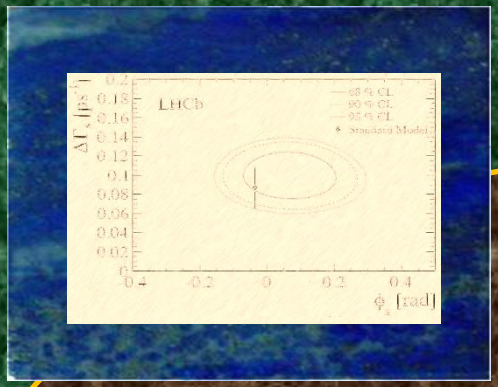
→ Illustrative (brute force) test: calculate non-MFV SUSY contributions setting all particle masses ~10 TeV

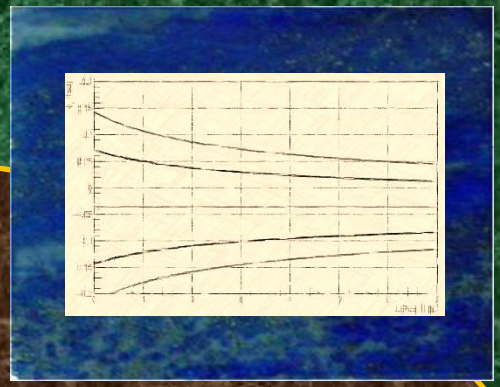


Warning: at this point of course interplay with other low Energy observables is important

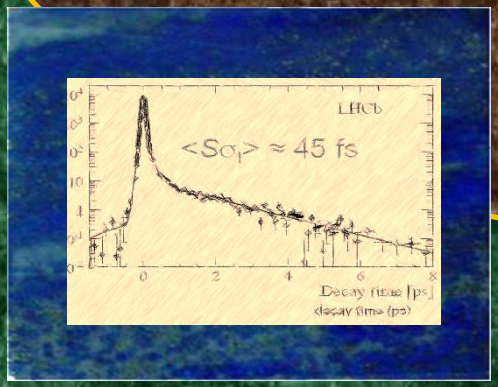


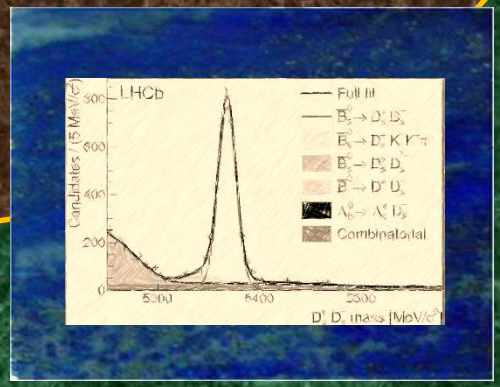
Those potential effects are within reach of current experimental precision!





Results and prospects



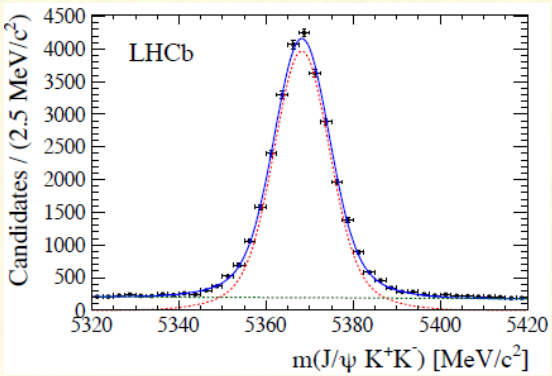


Φ_s from $B_s \rightarrow J/\psi\ (\rightarrow \mu\mu)\ KK$

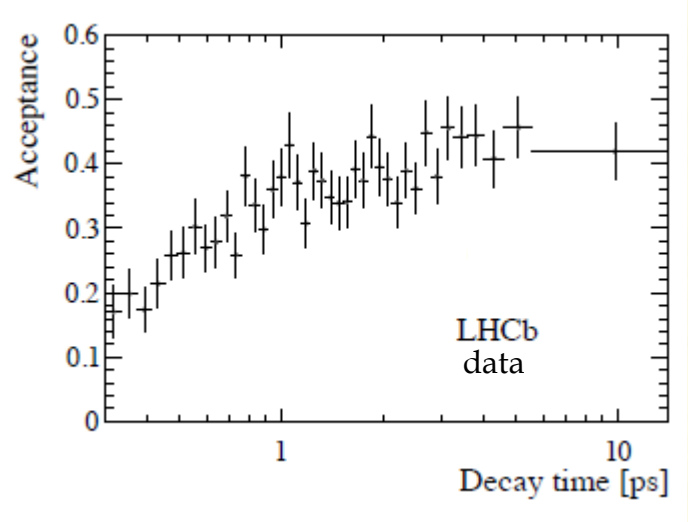
Phys. Rev. D 87, 112010 (2013)
arXiv:1304.2600

Analysis strategy: Fit the time dependent angular distribution, considering experimental effects:

- **Background**: Events are weighted according to position in J/ψKK mass spectrum



- Angular distributions are distorted on data because of **non-flat angular acceptance**. Simulation (weighted according to kinematics seen on data) is used to correct for this



- **Lifetime acceptance**. Samples from different trigger lines are used to unfold trigger biases. Simulation is used for selection/reconstruction biases

Φ_s from $B_s \rightarrow J/\psi\ (\rightarrow\mu\mu)\ KK$

Phys. Rev. D 87, 112010 (2013)
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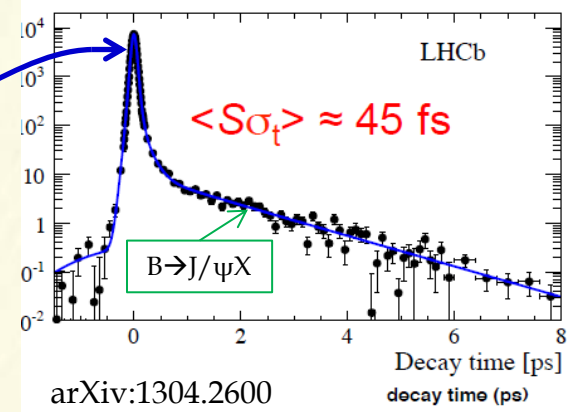
Analysis strategy: Fit the time dependent angular distribution, considering experimental effects:

- **Lifetime resolution**: Non-perfect time resolution (45 fs, still much smaller than oscillation period, 350 fs) convolved with the pdf. Main effect is a ~25% dilution of the amplitude of the wiggles. Measured on data using prompt J/ψ events



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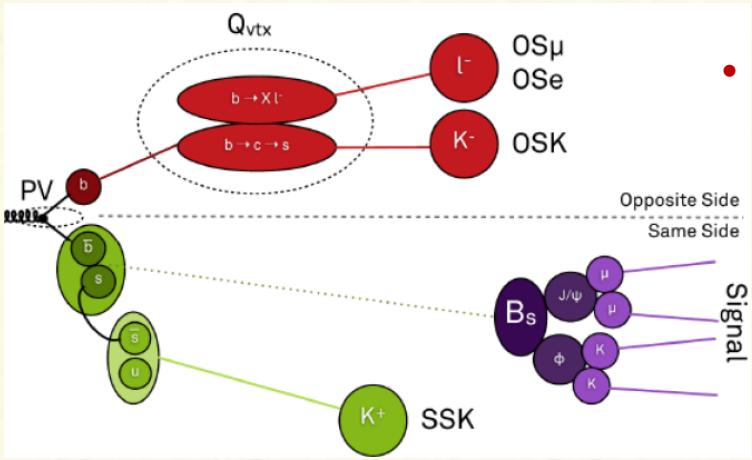

- **Flavour tagging**: The initial flavour of the B_s is determined either by a muon/kaon from the other B, and/or by a kaon from the fragmentation. The performance of these taggers is calibrated with control samples such as $B^+ \rightarrow J/\psi K^+$, $B_d \rightarrow D^{*+}\mu\upsilon$ and $B_s \rightarrow D_s^- \pi^+$

Φ_s from $B_s \rightarrow J/\psi\ (\rightarrow\mu\mu)\ KK$

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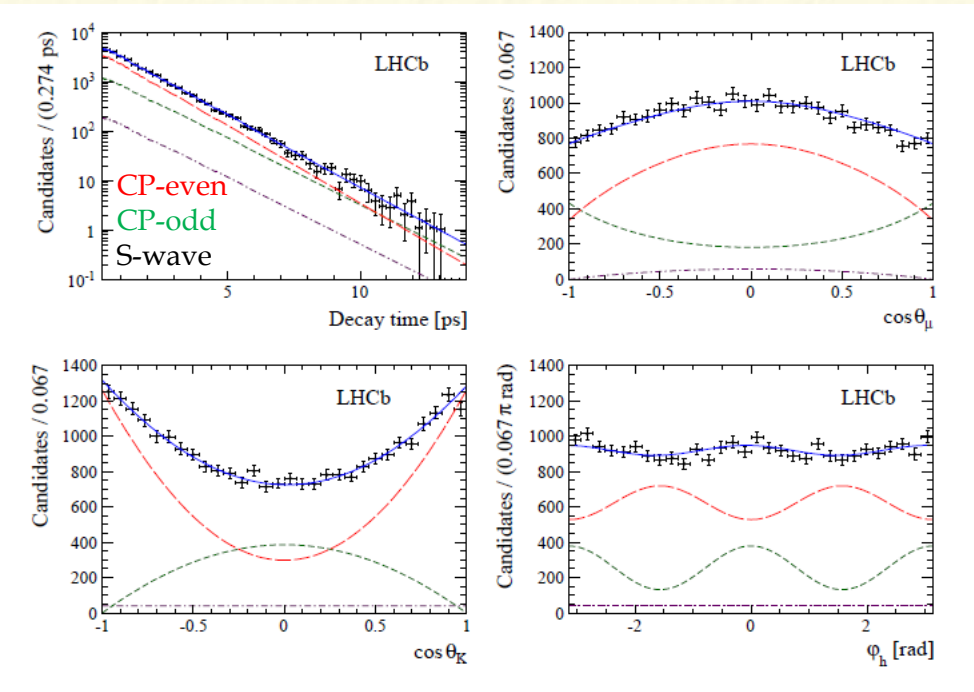
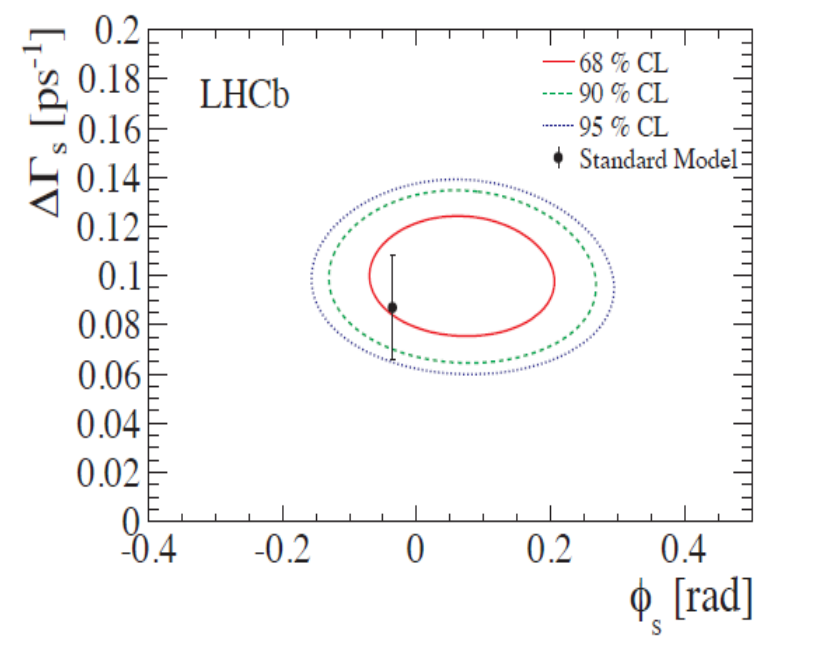
ϕ_s ($B_s \rightarrow J/\psi KK$), 1 fb^{-1} = 0.07±0.09±0.01 rad



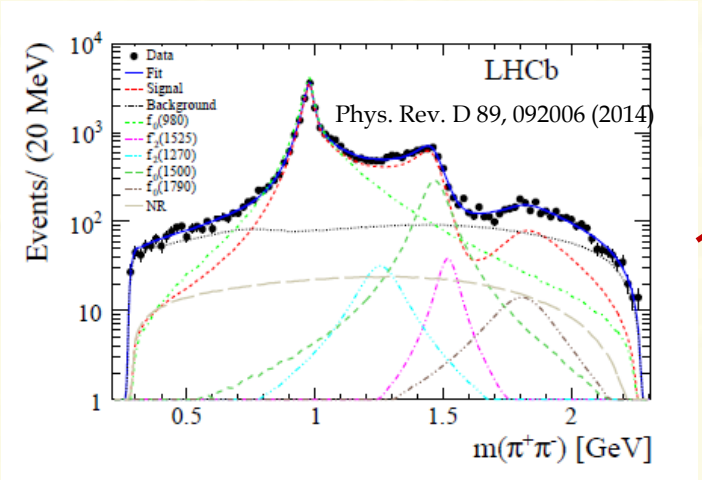


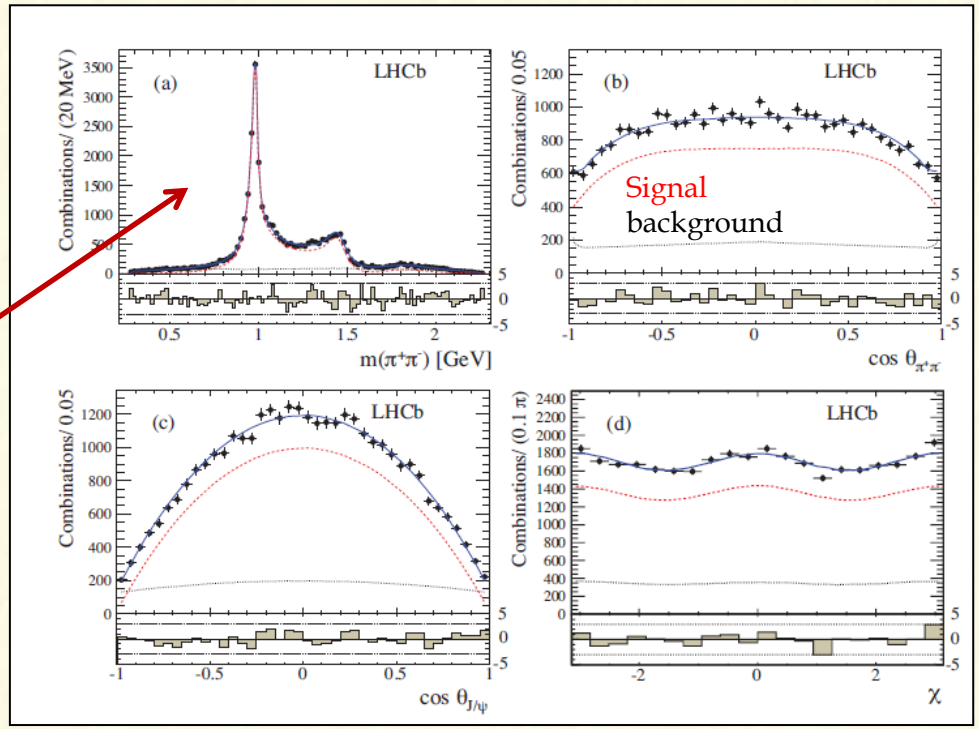
3 fb^{-1} update will come soon. Expected statistical uncertainty is 50 mrad!

Φ_s from $B_s \rightarrow J/\psi\ (\rightarrow \mu\mu)\ \pi\pi$

arXiv:1405.4140

- Similar analysis methodology than $B_s \rightarrow J/\psi KK$. Some differences:
  - Deal with several $\pi^+\pi^-$ resonances (implies a time dependent Dalitz analysis)
  - Almost no sensitivity to $\Delta\Gamma_s$ → less sensitive to decaytime acceptance





Φ_s from $B_s \rightarrow J/\psi\ (\rightarrow\mu\mu)\ \pi\pi$

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ϕ_s ($B_s \rightarrow J/\psi\pi\pi$), $3fb^{-1}$ = 0.075±0.067±0.008 rad

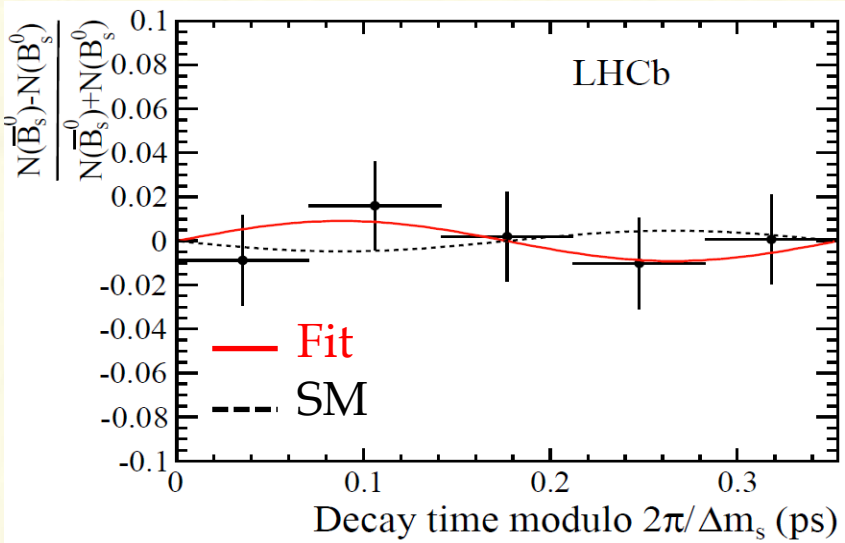
0.070±0.068±0.008 rad allowing $|\lambda|$ to vary

This is currently world best measurement, until the 3 fb^{-1} Bs → J/ψKK comes out

Combined with $B_s \rightarrow J/\psi KK$ (1 fb^{-1})

Φ_s = 0.070±0.055 radians

Compatible within two sigma with SM
(i.e, excluded at 95% CL) : -0.036±0.002



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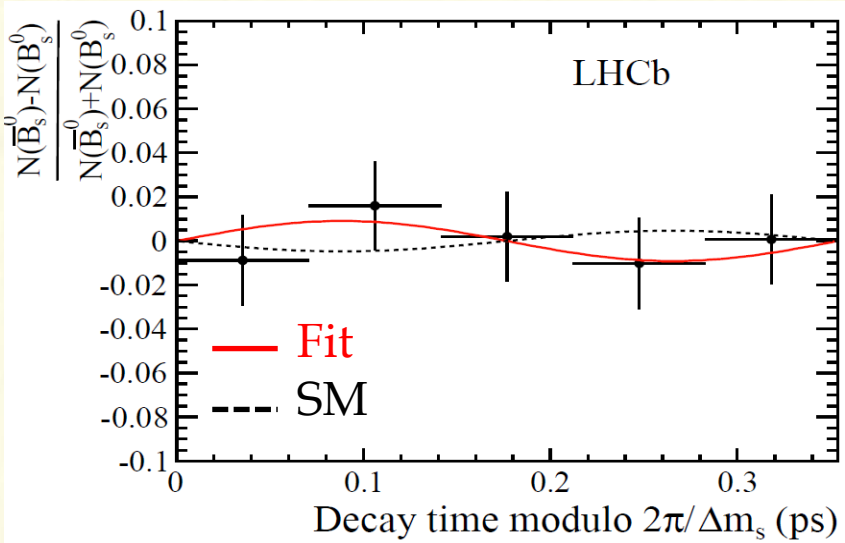
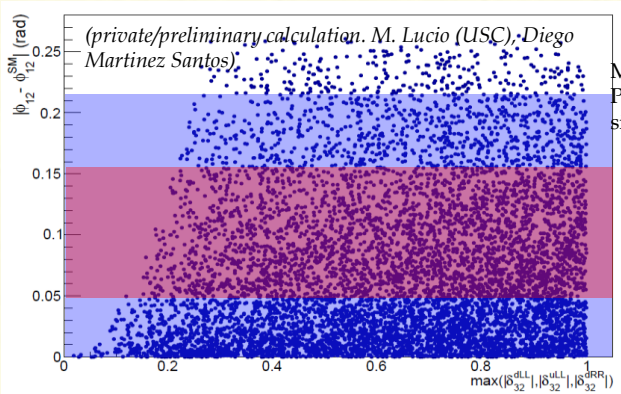


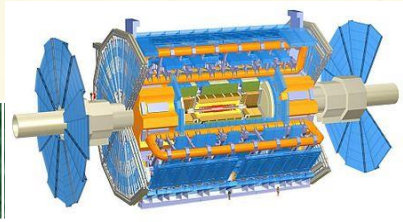
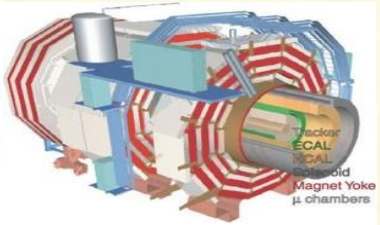


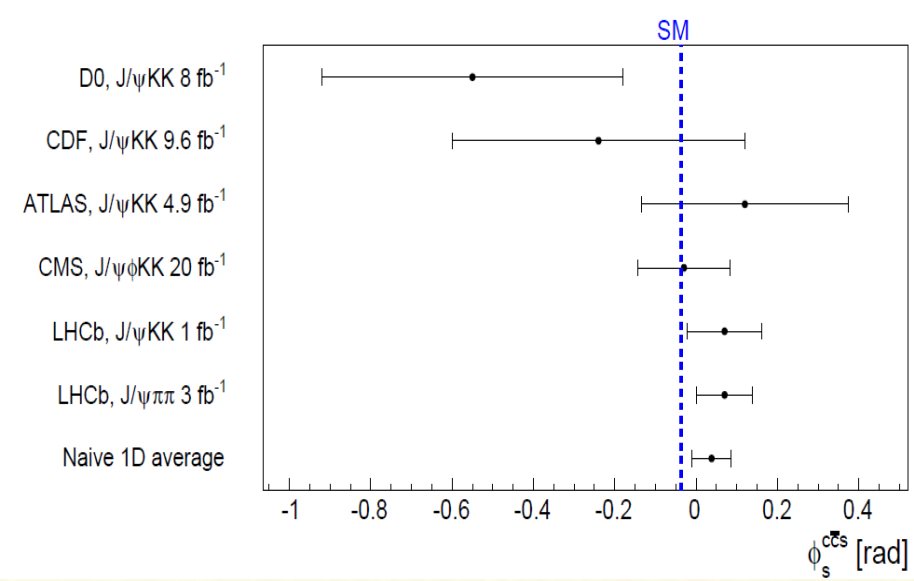
MSUSY >~ TeV
Points consistent with no significant NP in Δms

Precision will go further to ~0.04 once Bs → J/ψKK is out

Φ_s (ATLAS/CMS)

ATLAS and CMS also study
$B_s \to J/\psi\, \phi \to \mu\mu KK$

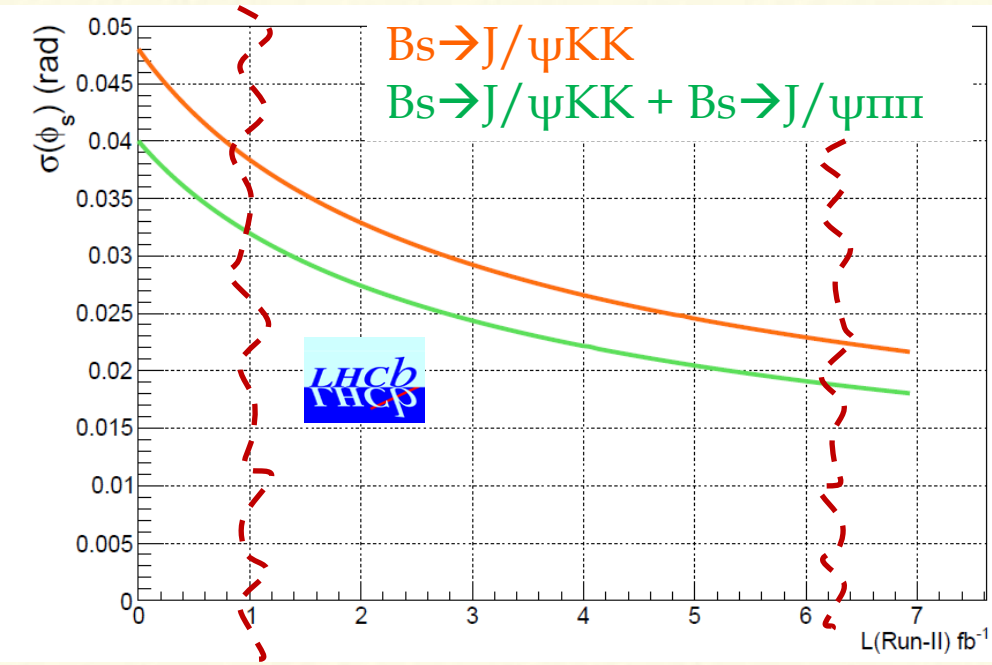
Experiment		
Lumi. (fb^{-1})	4.9	20.0
$\Delta\Gamma_s$ (ps^{-1})	0.053±0.021±0.010	0.096±0.014±0.007
Φ_s	0.12±0.25±0.05 arxiv.org/abs/1407.1796	-0.03±0.11±0.03 CMS-PAS-BPH-13-012



HFAG private/unofficial combination yields

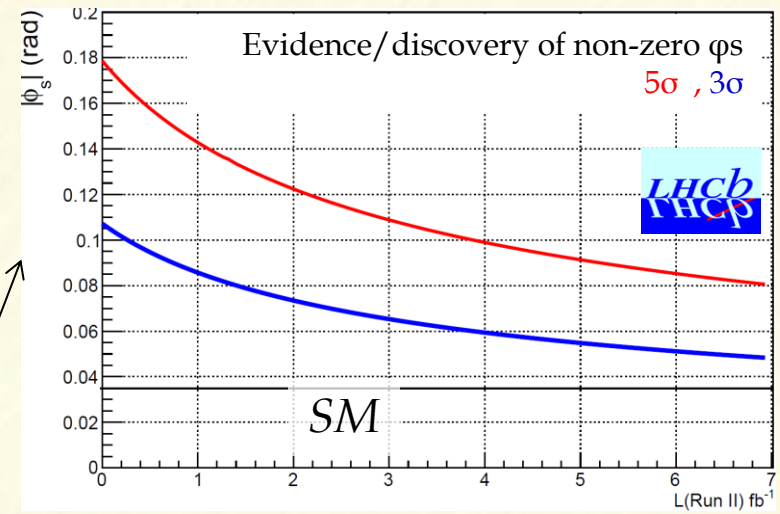
LHCb: 0.070 ± 0.055 rad
LHC: 0.054 ± 0.049 rad
World: 0.038 ± 0.048 rad

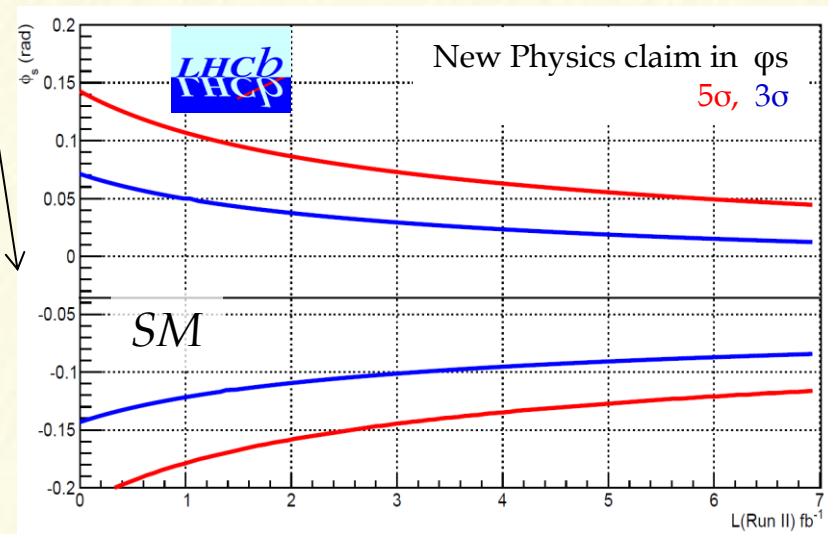
Prospects



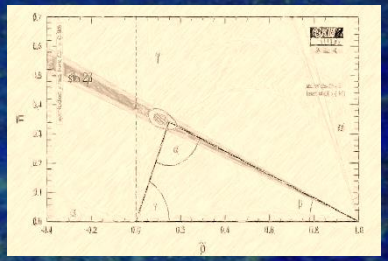
~2016

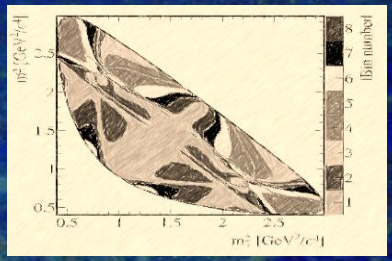
~ end of Run-II



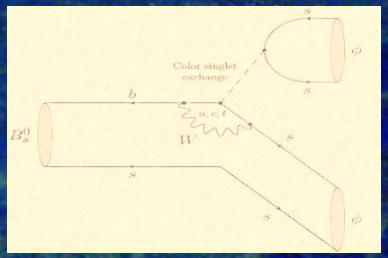


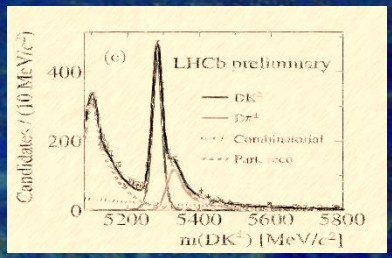
… and with LHCb upgrade the sensitivity can go below 0.01 rad





Other CPV measurements

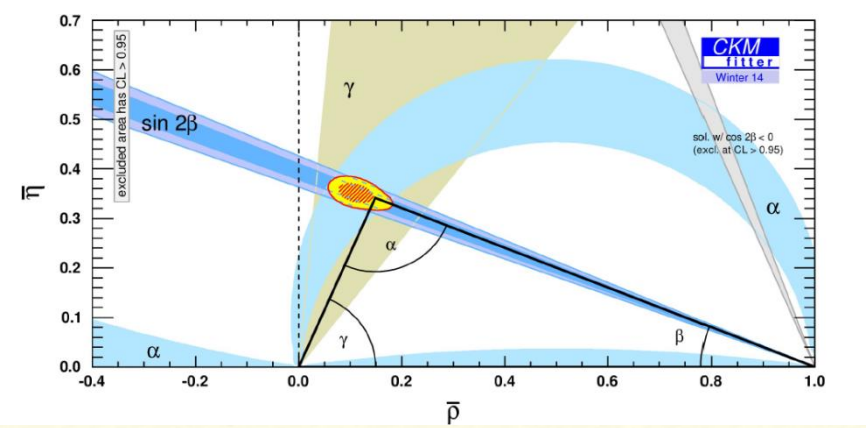




The CKM angle γ

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix}$$

$$\begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



- The precision of the SM prediction is very high, $\delta\gamma/\gamma \sim 10^{-7}$ (JHEP 1401(2014)051)
- Comparison between different measurements (specially those from tree-level decays with loop-level decays) **can be used to test SM /NP**

Experiment	ref	γ (o)
BaBar	PRD87(2013)05015	69^{+17}_{-16}
Belle	arXiv:1301.2033	68^{+15}_{-14}
LHCb	LHCb-CONF-2013-006	67 ± 12

CKM fitter (from global fit): $66.5^{+1.3}_{-2.5}$

The CKM angle γ

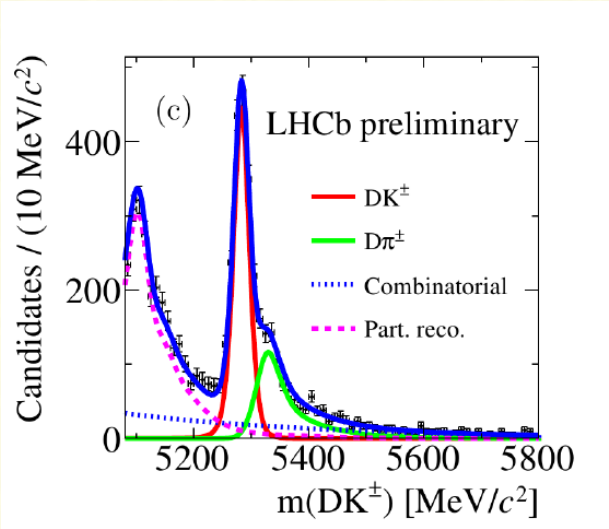
LHCb-PAPER-2014-041

- $B^{\pm} \to DK^{\pm}$, full Run-I dataset

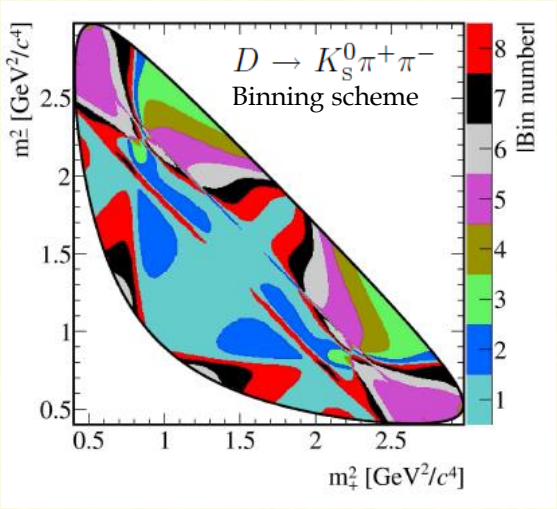
Measured by comparing the Dalitz plot of the D decays between D's from B^+ and D's from B^-.

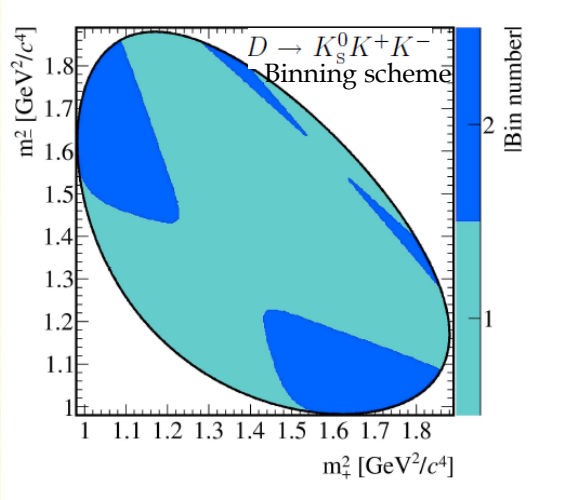
$$A_+(m_+^2, m_-^2) \equiv \bar{A}(m_+^2, m_-^2) + r_B e^{i(\delta+\gamma)} A(m_+^2, m_-^2)$$

$$A_-(m_+^2, m_-^2) \equiv A(m_+^2, m_-^2) + r_B e^{i(\delta-\gamma)} \bar{A}(m_+^2, m_-^2)$$



The (m_+^2, m_-^2) Dalitz planes are binned in a non-trivial way in order to maximize sensitivity to γ





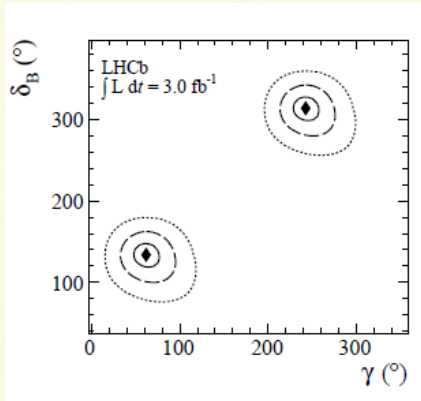
The CKM angle γ

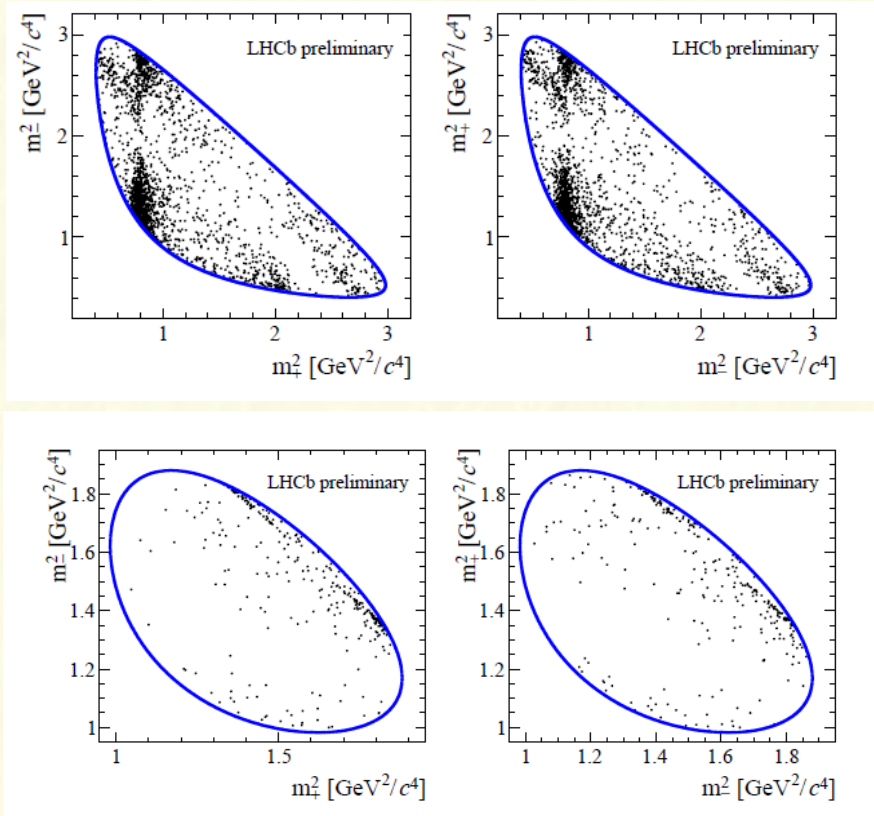
LHCb-PAPER-2014-041

$$A_+(m_+^2, m_-^2) \equiv \bar{A}(m_+^2, m_-^2) + r_B e^{i(\delta+\gamma)} A(m_+^2, m_-^2)$$

$$A_-(m_+^2, m_-^2) \equiv A(m_+^2, m_-^2) + r_B e^{i(\delta-\gamma)} \bar{A}(m_+^2, m_-^2)$$

Detector efficiency modelled with data from B→D*μυ





$$\gamma = (62^{+15}_{-14})^\circ$$ (modulo 180°)

Other LHCb measurements of γ from tree level decays:

B → DK, different methodology ,1fb^{-1} (arXiv:1407.6211 [hep-ex]): $\gamma = (84^{+49}_{-42})^\circ$

Bs →DsK 1fb^{-1} (arXiv:1407.6127 [hep-ex]): $\gamma = (115^{+28}_{-43})^\circ$

$\phi_s^{\phi\phi}$ from $B_s \to \phi\phi$

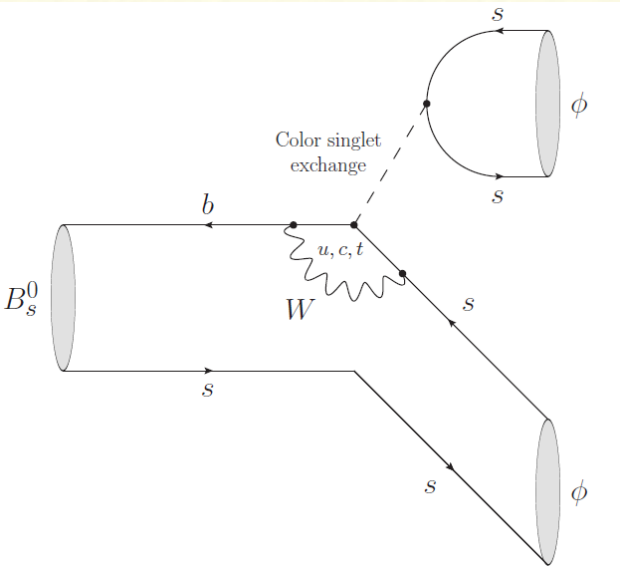
arxiv:1407.2222

$$\phi_s^{\phi\phi} \equiv arg\left(\frac{q}{p}\frac{A\,(\overline{B_s} \to \phi\phi)}{A\,(B_s \to \phi\phi)}\right)$$

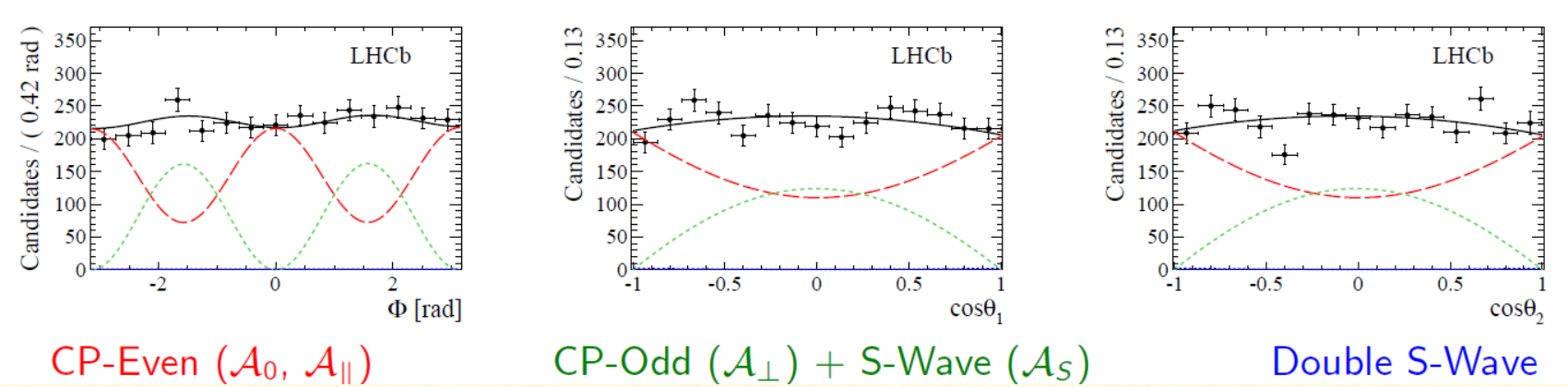
different quantity than the Φs I presented at the beginning of my talk

SM expectation is $\phi_s^{\phi\phi} < 0.02$

arXiv:0810.0249
arXiv:hep-ph/0612290
arXiv:0910.5237



Also measured through time dependent angular analysis. We have analysed the full 3 fb^{-1} dataset:



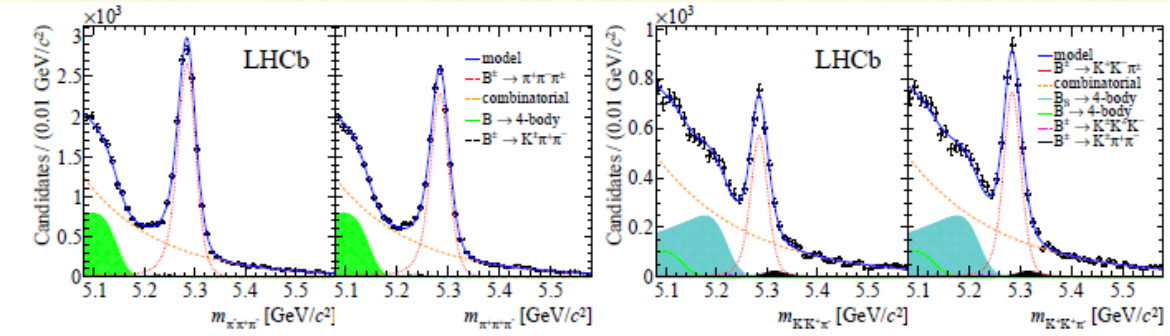
$$\boxed{\phi_s^{\phi\phi} = -0.17 \pm 0.15 \pm 0.03}$$

In very good agreement with SM

CPV in B→3h

LHCb-PAPER-2014-044

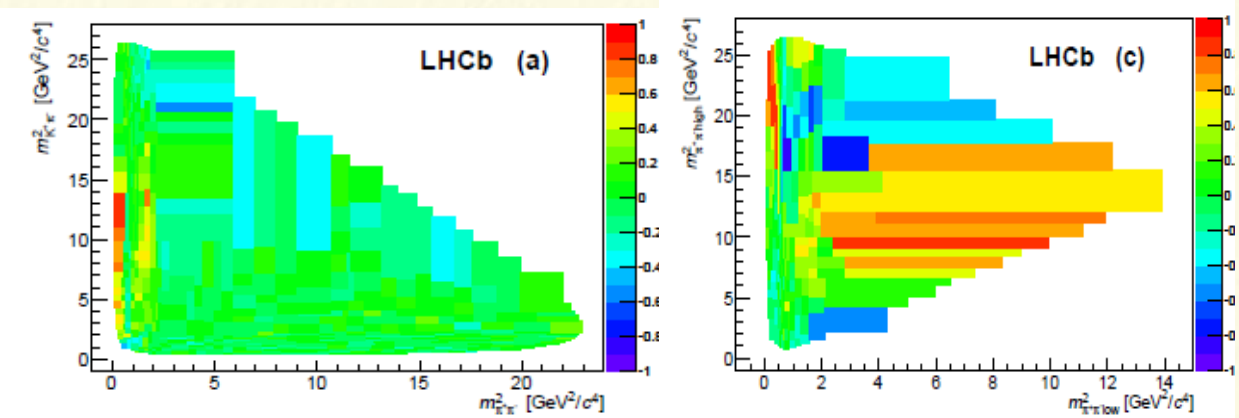
Study of CP asymmetries across the B→3h Dalitz plane
Overall CP asymmetries are found to be significant



$$
\begin{aligned}
A_{CP}(B^\pm \to K^\pm\pi^+\pi^-) &= +0.025 \pm 0.004\,(\text{stat}) \pm 0.004\,(\text{syst}) \pm 0.007(J/\psi\, K^\pm)\,, \\
A_{CP}(B^\pm \to K^\pm K^+K^-) &= -0.036 \pm 0.004\,(\text{stat}) \pm 0.002\,(\text{syst}) \pm 0.007(J/\psi\, K^\pm)\,, \\
A_{CP}(B^\pm \to \pi^\pm\pi^+\pi^-) &= +0.058 \pm 0.008\,(\text{stat}) \pm 0.009\,(\text{syst}) \pm 0.007(J/\psi\, K^\pm)\,, \\
A_{CP}(B^\pm \to \pi^\pm K^+K^-) &= -0.123 \pm 0.017\,(\text{stat}) \pm 0.012\,(\text{syst}) \pm 0.007(J/\psi\, K^\pm)\,,
\end{aligned}
$$

(3 fb^{-1})

On top of that, the asymmetries in some regions of the Dalitz plane are huge



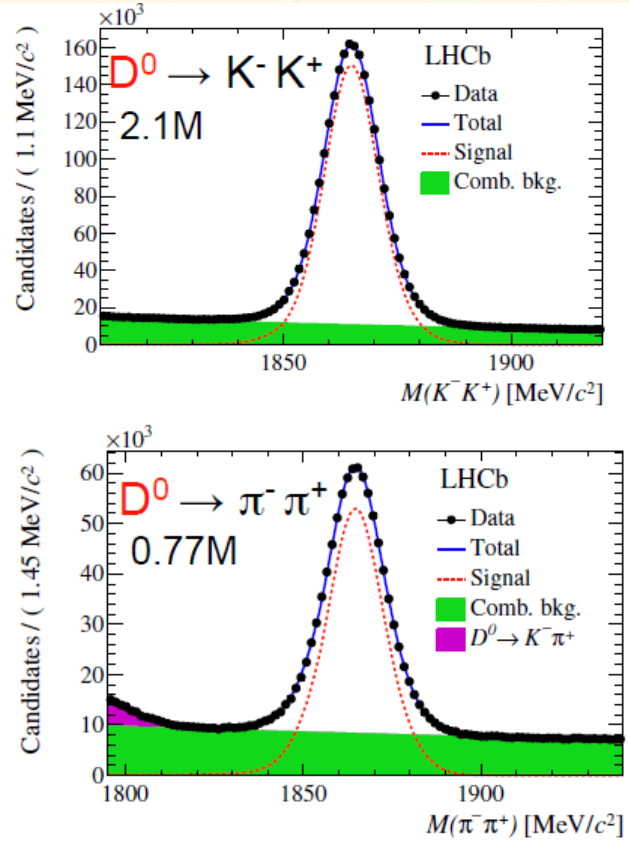
CPV in charm

$$A_{CP} \equiv \frac{N(D^0 \to h^- h^+) - N(\bar{D}^0 \to h^- h^+)}{N(D^0 \to h^- h^+) + N(\bar{D}^0 \to h^- h^+)}$$

$$\Delta A_{CP} = A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-)$$

$$\Delta A_{CP} = (+0.14 \pm 0.16 \pm 0.08)\%$$

arxiv:1405.2797



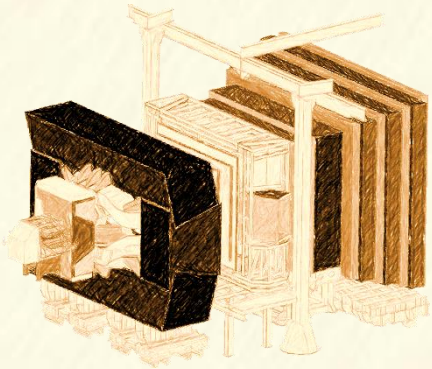
Other recent studies of CPV in charm:

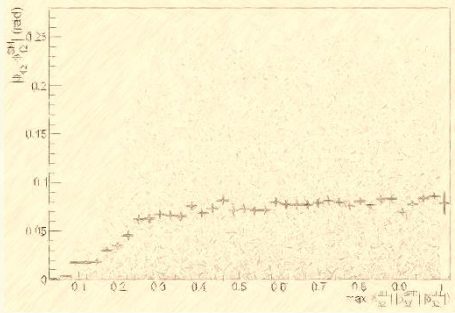
$D_{(s)}^+ \to K_S h^+$ (arXiv: 1406.2624)
$D^+ \to \pi^+\pi^-\pi^+$ PLB 728 (2014) 585
$D^0 \to K^+K^-\pi^+\pi^-$ (LHCb-PAPER-2014-046)

… all consistent for the moment with CP conservation.

Conclusions

- Flavour physics measurements are sensitive to New Physics even if the scale is higher than several TeV
- Current LHCb combined value of ϕ_s is 0.070±0.055 rad, 2σ within SM prediction
- Stay tuned for update, where the overall uncertainty will reduce to ~0.04
- Latest measurements of γ from tree decays , $\phi_s^{\phi\phi}$ and CPV in B→3h have been presented
- No evidence of CP violation in charm decays up to now





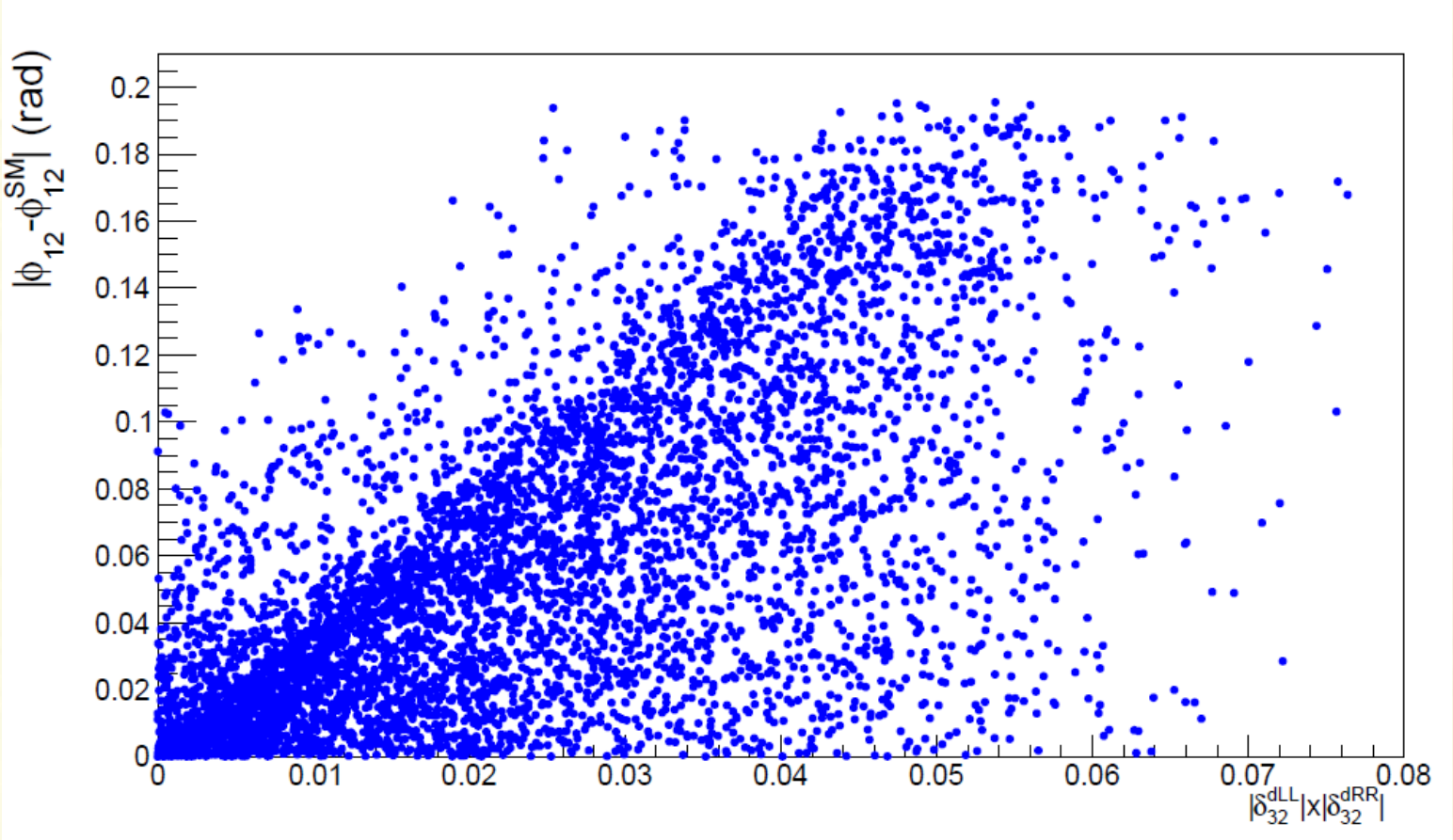
ευχαριστώ

Φ_s *from* $B_s \rightarrow J/\psi\ (\rightarrow \mu\mu)\ \pi\pi$

Sources	ϕ_s(mrad)
Decay time acceptance	± 0.6
Mass acceptance	± 0.3
Background time PDF	± 0.2
Background mass distribution PDF	± 0.6
Resonance model	± 6.0
Resonance parameters	± 0.7
Other fixed parameters	± 0.4
Production asymmetry	± 5.8
Total	± 8.4

Φ_s *from* $B_s \rightarrow J/\psi\ (\rightarrow \mu\mu)\ KK$

Source	Γ_s [ps^{-1}]	$\Delta\Gamma_s$ [ps^{-1}]	$\lvert A_\perp\rvert^2$	$\lvert A_0\rvert^2$	$\delta_\parallel$ [rad]	$\delta_\perp$ [rad]	ϕ_s [rad]	$\lvert\lambda\rvert$
Stat. uncertainty	0.0048	0.016	0.0086	0.0061	$^{+0.13}_{-0.21}$	0.22	0.091	0.031
Background subtraction	0.0041	0.002	–	0.0031	0.03	0.02	0.003	0.003
$B^0 \rightarrow J/\psi K^{*0}$ background	–	0.001	0.0030	0.0001	0.01	0.02	0.004	0.005
Ang. acc. reweighting	0.0007	–	0.0052	0.0091	0.07	0.05	0.003	0.020
Ang. acc. statistical	0.0002	–	0.0020	0.0010	0.03	0.04	0.007	0.006
Lower decay time acc. model	0.0023	0.002	–	–	–	–	–	–
Upper decay time acc. model	0.0040	–	–	–	–	–	–	–
Length and mom. scales	0.0002	–	–	–	–	–	–	–
Fit bias	–	–	0.0010	–	–	–	–	–
Decay time resolution offset	–	–	–	–	–	0.04	0.006	–
Quadratic sum of syst.	0.0063	0.003	0.0064	0.0097	0.08	0.08	0.011	0.022
Total uncertainties	0.0079	0.016	0.0107	0.0114	$^{+0.15}_{-0.23}$	0.23	0.092	0.038


$|\phi_{12}-\phi_{12}^{SM}|$ (rad)
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
$|\delta_{32}^{dLL}|\times|\delta_{32}^{dRR}|$

Penguin pollution

arXiv:0810.4248

Mainly two observables:

CKM angle

F(|V_{us}|)

$$H_f = \frac{1}{\epsilon}\left|\frac{\mathcal{A}_f}{\mathcal{A}'_f}\right|^2 \frac{\Gamma[f,t=0]'}{\Gamma[f,t=0]} = \frac{1-2a'_f\cos\theta'_f\cos\gamma+a'^2_f}{1+2\epsilon a_f\cos\theta_f\cos\gamma+\epsilon^2 a_f^2}$$

penguin stuff

f = polarization state

Experimental input. Basically $\frac{(BR\cdot f_f)_{J/\psi K^*}}{(BR\cdot f_f)_{J/\psi\varphi}}$
(modulo lifetimes)

$$\hat{A}_D^{f'} = \frac{2a'_f\sin\theta'_f\sin\gamma}{1-2a'_f\cos\theta'_f\cos\gamma+a'^2_f},$$

Direct CP asymmetry (difference of yields)

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SU(3) → a′ = a, θ′ = θ

…and plug here

Direct CP asymmetry (difference of yields)

$$\tan\Delta\phi_s^f = \frac{2\epsilon a_f\cos\theta_f\sin\gamma+\epsilon^2 a_f^2\sin 2\gamma}{1+2\epsilon a_f\cos\theta_f\cos\gamma+\epsilon^2 a_f^2\cos 2\gamma}$$

Penguin pollution

arXiv:0810.4248

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penguin stuff

This other stuff are SU(3) breaking effects which are currently poorly known

$$\left|\frac{\mathcal{A}'_0}{\mathcal{A}_0}\right|^2 = 0.42 \pm 0.27\ ,$$

$$\left|\frac{\mathcal{A}'_\parallel}{\mathcal{A}_\parallel}\right|^2 = 0.70 \pm 0.29\ ,$$

$$\left|\frac{\mathcal{A}'_\perp}{\mathcal{A}_\perp}\right|^2 = 0.38 \pm 0.16\ .$$

Penguin pollution

arXiv:0810.4248

Mainly two observables:

$$H_f \equiv \frac{1}{\epsilon}\left|\frac{\mathcal{A}_f}{\mathcal{A}'_f}\right|^2 \frac{\Gamma[f,t=0]'}{\Gamma[f,t=0]} = \frac{1-2a'_f\cos\theta'_f\cos\gamma+a'^2_f}{1+2\epsilon a_f\cos\theta_f\cos\gamma+\epsilon^2 a_f^2}$$

penguin stuff

This other stuff are SU(3) breaking effects which are currently poorly known

Or maybe not so poorly?

arXiv:1309.0313 [hep-ph]

arXiv:0810.4248

$$\left|\frac{\mathcal{A}'_0}{\mathcal{A}_0}\right|^2 = 0.42 \pm 0.27\ , \qquad = 0.858^{+0.206}_{-0.196}$$

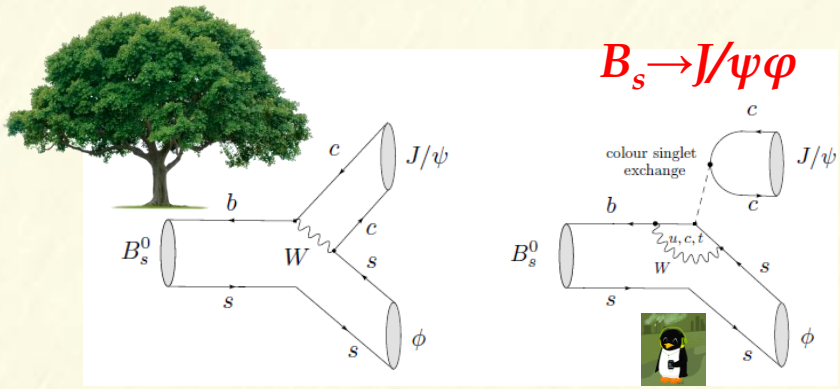
$$\left|\frac{\mathcal{A}'_\parallel}{\mathcal{A}_\parallel}\right|^2 = 0.70 \pm 0.29\ , \qquad = 0.845^{+0.210}_{-0.188}$$

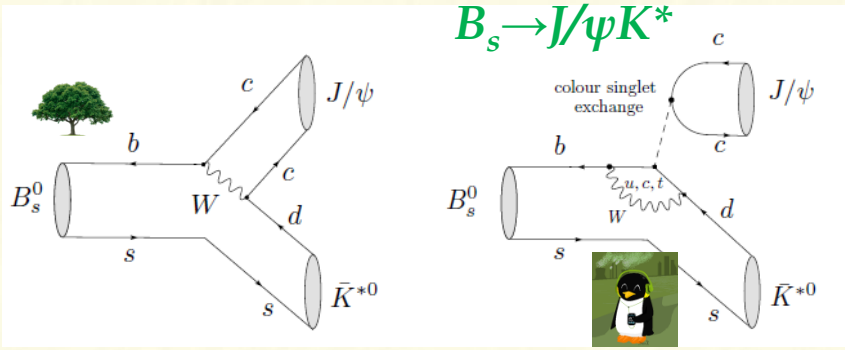
$$\left|\frac{\mathcal{A}'_\perp}{\mathcal{A}_\perp}\right|^2 = 0.38 \pm 0.16\ . \qquad = 0.869^{+0.234}_{-0.204}$$

Penguin pollution

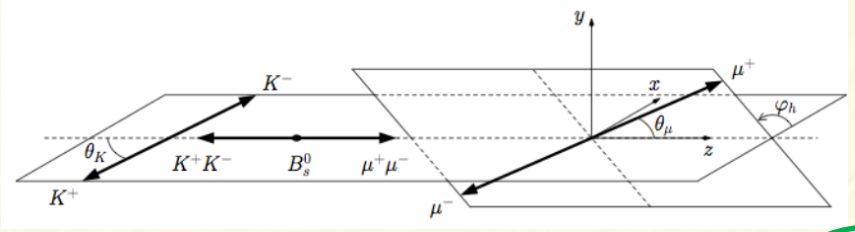
- Penguin contributions to Φ_s , are usually neglected because they are doubly Cabibbo supressed.
- However, these contributions cannot be calculated reliably from QCD
- S. Faller, R. Fleischer, T. Mannel arXiv:0810.4248 [hep-ph] propose a method to calculate the penguin pollution to Φ_s by analyzing Bs → J/ψK* and Bd → J/ψρ data

Source: google penguin pollution





Φ_s from $B_s \to J/\psi\ (\to\mu\mu)\ KK$



wiggles

$$
\begin{aligned}
|A_0|^2(t) &= |A_0|^2 e^{-\Gamma_s t}[\cosh\left(\frac{\Delta\Gamma}{2}t\right) - \cos\phi_s \sinh\left(\frac{\Delta\Gamma}{2}t\right) + \sin\phi_s \sin(\Delta m t)]\,,\\
|A_\parallel(t)|^2 &= |A_\parallel|^2 e^{-\Gamma_s t}[\cosh\left(\frac{\Delta\Gamma}{2}t\right) - \cos\phi_s \sinh\left(\frac{\Delta\Gamma}{2}t\right) + \sin\phi_s \sin(\Delta m t)]\,,\\
|A_\perp(t)|^2 &= |A_\perp|^2 e^{-\Gamma_s t}[\cosh\left(\frac{\Delta\Gamma}{2}t\right) + \cos\phi_s \sinh\left(\frac{\Delta\Gamma}{2}t\right) - \sin\phi_s \sin(\Delta m t)]\,,\\
\Im(A_\parallel(t)\,A_\perp(t)) &= |A_\parallel||A_\perp|e^{-\Gamma_s t}[-\cos(\delta_\perp - \delta_\parallel)\sin\phi_s \sinh\left(\frac{\Delta\Gamma}{2}t\right)\\
&\quad - \cos(\delta_\perp - \delta_\parallel)\cos\phi_s \sin(\Delta m t) + \sin(\delta_\perp - \delta_\parallel)\cos(\Delta m t)]\,,\\
\Re(A_0(t)\,A_\parallel(t)) &= |A_0||A_\parallel|e^{-\Gamma_s t}\cos(\delta_\parallel - \delta_0)[\cosh\left(\frac{\Delta\Gamma}{2}t\right) - \cos\phi_s \sinh\left(\frac{\Delta\Gamma}{2}t\right)\\
&\quad + \sin\phi_s \sin(\Delta m t)]\,,\\
\Im(A_0(t)\,A_\perp(t)) &= |A_0||A_\perp|e^{-\Gamma_s t}[-\cos(\delta_\perp - \delta_0)\sin\phi_s \sinh\left(\frac{\Delta\Gamma}{2}t\right)\\
&\quad - \cos(\delta_\perp - \delta_0)\cos\phi_s \sin(\Delta m t) + \sin(\delta_\perp - \delta_0)\cos(\Delta m t)]\,,\\
|A_s(t)|^2 &= |A_s|^2 e^{-\Gamma_s t}[\cosh\left(\frac{\Delta\Gamma}{2}t\right) + \cos\phi_s \sinh\left(\frac{\Delta\Gamma}{2}t\right) - \sin\phi_s \sin(\Delta m t]\,,\\
\Re(A_s^*(t)A_\parallel(t)) &= |A_s||A_\parallel|e^{-\Gamma_s t}[-\sin(\delta_\parallel - \delta_s)\sin\phi_s \sinh\left(\frac{\Delta\Gamma}{2}t\right) - \sin(\delta_\parallel - \delta_s)\cos\phi_s \sin(\Delta m t)\\
&\quad + \cos(\delta_\parallel - \delta_s)\cos(\Delta m t)]\,,\\
\Im(A_s^*(t)A_\perp(t)) &= |A_s||A_\perp|e^{-\Gamma_s t}\sin(\delta_\perp - \delta_s)[\cosh\left(\frac{\Delta\Gamma}{2}t\right) + \cos\phi_s \sinh\left(\frac{\Delta\Gamma}{2}t\right)\\
&\quad - \sin\phi_s \sin(\Delta m t)]\,,\\
\Re(A_s^*(t)A_0(t)) &= |A_s||A_0|e^{-\Gamma_s t}[-\sin(\delta_0 - \delta_s)\sin\phi_s \sinh\left(\frac{\Delta\Gamma}{2}t\right)\\
&\quad - \sin(\delta_0 - \delta_s)\cos\phi_s \sin(\Delta m t) + \cos(\delta_0 - \delta_s)\cos(\Delta m t)]\,.
\end{aligned}
$$

Apart from the wiggles, there are other terms in the pdf that have some sensitivity to Φs: