Long range order in QCD. Applications to heavy ions and cosmology.
1. Motivation. Structure of the talk.

The main goal of this talk is to argue that QCD exhibits the topological long range order, which is known to occur in many condensed matter systems realized in nature (topologically ordered phases).

Furthermore, there is a novel type of energy associated with this long range order. This energy has “non-dispersive” nature, and can not be expressed in terms of conventional propagating degrees of freedom.

All these novel effects are due to the nontrivial topological sectors in the system and tunnelling transitions between them (subject of this talk).

The presence of this new type of energy is supported by the lattice numerical computations.
It can be also studied in simplified weakly coupled gauge theories where all computations are under complete theoretical control.

It can be also studied in a tabletop experiment where there is an extra contribution to the Casimir vacuum pressure. A novel extra term can not be expressed in terms of propagating physical photons with two transverse polarizations.

It is very likely that the local violation of P-symmetry as observed at RHIC and the LHC is manifestation of these effects (*subject of this talk*).

It may also have profound consequences for cosmology as well (*subject of this talk*).
2. Topological susceptibility

A convenient way to explain the nature of new type of vacuum energy is to study the topologically susceptibility (it is the key element in the resolution of the so-called U(1) problem in QCD, Witten, Veneziano, 1979)

\[ \chi_{YM} = \int d^4x \langle q(x), q(0) \rangle \neq 0 \]

\[ \frac{\partial^2 E_{\text{vac}}(\theta)}{\partial \theta^2} = \chi_{YM} \]

\[ \chi_{YM} \text{ does not vanish, though } q(x) \sim \partial_\mu K^\mu(x) \text{. It has ``wrong sign'', see below. It can not be related to any physical propagating degrees of freedom. Furthermore, it has a pole in momentum space} \]

\[ \lim_{k \to 0} \int d^4xe^{ikx} \langle K_\mu(x), K_\nu(0) \rangle \sim \frac{k_\mu k_\nu}{k^4} \]

There is a massless pole, but there are no any physical massless states in the system.
\[ \chi_{\text{dispersive}} \sim \lim_{k \to 0} \sum_n \frac{\langle 0|q|n \rangle \langle n|q|0 \rangle}{-k^2 - m_n^2} < 0, \]

Conventional physical degrees of freedom always contribute with sign (-) while one needs sign (+) to satisfy WI and resolve the U(1) problem

\[ \chi_{\text{non-dispersive}} = \int d^4x \langle q(x), q(0) \rangle = \frac{1}{N^2} |E_{\text{vac}}| > 0 \]

Conventional terms (related to propagating degrees of freedom) always produce \( \exp (-\Lambda_{QCD} L) \) behaviour at large distances.

Witten simply postulated this term, while Veneziano assumed the unphysical field, the so-called the “Veneziano ghost” to saturate “wrong” sign in \( \chi \).

In “deformed QCD” this contact non-dispersive term with “wrong” sign (+) can be explicitly computed. It is originated from the tunnelings between the degenerate topological sectors of the theory.
The topological susceptibility $\chi(r)$ as a function of $r$. Wrong sign for $\chi$ is a well-established phenomenon; it has been tested on the lattice (plot above is from C. Bernard et al, LATTICE 2007). This $\chi(r = 0)$ contribution is not related to any physical degrees of freedom, and can be interpreted as a contact term.
3. Some important features of “non-dispersive” contributions to the energy

These contributions can not be described in terms of conventional degrees of freedom (wrong sign);

They are inherently non-local in nature as they are related to the tunnelling processes which are formulated in terms of the non-local large gauge transformation operator and holonomy;

These terms may exhibit the long range features even through QCD has a gap (similar to the CM topologically ordered systems);

The \( \theta \) -dependent portion of energy \( E_{\text{vac}}(\theta) \) (which is generated due to the tunnelling transitions) has all these unusual features as \( \frac{\partial^2 E_{\text{vac}}(\theta)}{\partial \theta^2} = \chi Y M \)
4. Possible consequences for heavy ions.

I want to argue that local violation of P, CP invariance in QCD as observed at RHIC and the LHC is a consequence of the $\theta$-dependent energy $E_{\text{vac}}(\theta)$.

The effect is normally formulated in terms of the correlations $\langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle$ between two particles that are not related to the reaction plane (RP) orientation (S. Voloshin, 2004).
Similar analysis is done for the LHC energies (Pb+Pb collisions at 2.76 TeV), ALICE.
Basic (sufficiently old) idea is: a large domain with effective $\theta_{\text{ind}} \neq 0$ may be induced in heavy ion collisions (D. Kharzeev et al, 98, A. Zhitnitsky et al, 98). It may result in local $P$ and $CP$-odd fluctuations (apparently observed at RHIC and the LHC).

$$\theta \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu a} = \theta \partial_{\mu} K^\mu$$

is total derivative, does not change the equation of motion. Still, it leads to the physically observable effects.

Why does the size of $P$-odd domain with $\theta_{\text{ind}} \neq 0$ could be large in heavy ion collisions? The understanding of this long range order in QCD is a key element of this talk. Conventional $\mu_5$ (used in many papers on the subject) is related to $\mu_5 = \theta_{\text{ind}}$
5. Charge separation effect. CME.

For the uniform magnetic field the electric field will be induced along B in the presence of large domain $\theta$ (assuming a large coherent effect)

$$L^2 E_z^{\text{ind}} = -\left(\frac{e \theta}{2\pi}\right) l, \quad \text{where} \quad l = \frac{e}{2\pi} \int d^2 x_\perp B_z^{\text{ext}}$$

$$[Q(z = +L) - Q(z = -L)] \sim \left(\frac{e \theta}{2\pi}\right) l$$

The induced electric field will lead to the induced currents and to the separation of charges along B (CME) on macroscopical scale:

$$\vec{J} = (\mu_L - \mu_R) \frac{e \vec{B}}{2\pi^2}, \quad \text{where} \quad (\mu_L - \mu_R) = \dot{\theta}$$

A similar phenomenon happens when the system is rotating, chiral vortical effect (CVE), chiral separation effect (CSE), etc. Therefore: an upper hemisphere can thus have either excess of quarks over anti-quarks or vice-versa on scale “L”.
**Crucial Question:** why the size $L$ of $P$ odd domains with $\theta_{\text{ind}} \neq 0$ is large, much larger than conventional $L \gg \Lambda_{\text{QCD}}^{-1}$ scale? This is a required feature for effective Lagrangian to be justified.

**Important:** any conventional finite size $\sim \Lambda_{\text{QCD}}^{-1}$ fluctuations would always produce non-coherent exponentially small effect $\exp(-\Lambda_{\text{QCD}}L)$ while the observed large intensities for asymmetries apparently require a coherent effect on scale $\sim L$.

**How come? Hidden long range order in QCD:**

\[ \lim_{k \to 0} \int d^4 x e^{ikx} \langle K_\mu(x), K_\nu(0) \rangle \sim \frac{k_\mu k_\nu}{k^4} \quad \Rightarrow \quad L^{-p} \]
If a typical emission time $c \Delta t \leq \lambda$ is smaller than the size of the domain with $\theta \neq 0$ than the P-violating correlations can be observed as emission effectively happens in P-odd environment. It can be interpreted as a coherent accumulation of asymmetry from a large region $\sim L$.

$\lambda \sim L$ is typical size of a domain with $\theta \neq 0$

$\Delta t \sim \Lambda_{QCD}^{-1}$ is a typical emission time (for pions) from region
6. Possible applications for cosmology

We speculate that the same long range order may have profound consequences for cosmology as well.

In this case a relatively small parameter $L^{-1} \sim (10 \text{ fm})^{-1}$ (from heavy ion collisions) is replaced by a drastically smaller parameter in expending universe, $L^{-1} \sim H \sim 10^{-33} \text{ eV}$

In both cases the algebraic sensitivity to large distances $L^{-p}$ is due to the long range fluctuations (formulated in terms of the massless auxiliary topological fields) with an algebraic Casimir-like decay at large $L$ in the gapped QCD.
What is the evidence for an algebraic (linear) dependence on cosmological scale (which is the Hubble constant \( L^{-1} \sim H \sim 10^{-33} \text{eV} \))?

1. A number of analytical computations in simplified models (shall not be discussed here).

2a. Lattice numerical simulations. In this case the computations of a real part of the energy-momentum tensor \( \text{Re}\langle T_{\mu\nu}\rangle \) is a hard problem.

2b. However, the **imaginary** (absorptive) portion of the energy-momentum tensor \( \text{Im}\langle T_{\mu\nu}\rangle \) due to particle production, can be computed, see plot below.

2c. Absorptive part corresponds to a production of real particles in a time-dependent background.
The plots from A. Yamamoto, arxiv 1405.6665.

1. The expansion in Euclidean space-time was parametrized by the “imaginary” Hubble constant when the lattice action is positively defined;

2. Red curve describes the particle production rate per unit volume per unit time in the background $H_I$;

3. The linear dependence on $H_I$ has been computed, $\text{Im}[\langle T_{\mu\nu} \rangle] \sim H_I$. It strongly supports our arguments.
We assume that the non-dispersive $\theta$ dependent portion of the vacuum energy $E_{\text{vac}}(\theta)$ shows the same linear correction (as absorptive $\sim Im[\langle T_{\mu\nu} \rangle]$) with respect to Hubble “H”, i.e.

$$E_{\text{FLRW}} = c_0 \Lambda_{\text{QCD}}^4 + c_1 H \Lambda_{\text{QCD}}^3 + O(H^2 \Lambda_{\text{QCD}}^2) + ...$$

We also assume that the relevant (gravitating) energy which enters the Friedman’s equation is the difference $\Delta E = (E_{\text{FLRW}}(H) - E_{\text{Mink}})$ similar to computations of the Casimir energy, when the difference $\Delta E$ is observed. This assumption was, in fact, originally formulated by Zeldovich in 1967.

With these assumptions the Friedman’s equation exhibits a solution with the de-Sitter behaviour

$$H^2 = \frac{8\pi G}{3} \left( c_1 H \Lambda_{\text{QCD}}^3 + \rho_{\text{DM}} \right) \longrightarrow H_0 \sim \frac{\Lambda_{\text{QCD}}^3}{M_{\text{PL}}^2}, \quad a(t) \sim \exp(H_0 t)$$
This energy has the same “non-dispersive” nature, which cannot be expressed in terms of any propagating degrees of freedom.

This energy cannot be formulated in terms of any local effective matter fields nor curvature as it has inherently non-local nature. Crucial role of nontrivial holonomy in dynamics.

The de Sitter behaviour is pure quantum effect describing the dynamics of the topological sectors of strongly coupled QCD in expanding background.

With these assumptions the non-dispersive contribution to energy (at large \( a(t) \sim \exp(\Lambda t) \to \infty \)) is

\[
H \sim \frac{\Lambda_{\text{QCD}}^3}{M_{\text{PL}}^2} \sim 10^{-33} \text{eV}, \quad \rho_{\text{DE}} \sim H\Lambda_{\text{QCD}}^3 \sim (10^{-3} \text{eV})^4
\]

It is amazingly close to the observed values
Conclusion

We interpret a relatively large observed intensities of the asymmetries in heavy ion collisions as a coherent vacuum phenomenon when a measured asymmetry is accumulated on large scales $L \sim 10\text{ fm}$, rather than a result of the conventional fluctuation $\sim 1\text{ fm}$ scale.

We speculate that a similar phenomena could occur on cosmological scales, $L^{-1} \sim H \sim 10^{-33}\text{ eV}$.

This fundamentally new type of the vacuum energy can not be expressed in terms of the physical local propagating degrees of freedom. It emerges as a result of (nonlocal) tunnelling processes between degenerate topological sectors, and formulated in terms of the (non-dispersive) contact terms and non-local operators (non-trivial holonomy).
Proposal: Instead of theoretical speculations I suggest to conduct a real tabletop experiment to study this new type of energy:

When the Maxwell system is formulated on four-torus there will be an extra contribution to the Casimir pressure, not related to the physical propagating photons with two transverse polarizations (4-torus has nontrivial holonomy).

This setting based on 4-torus topology should be contrasted with conventional setting when the Casimir energy is generated between two conducting plates (trivial holonomy).

The Maxwell system on the 4-torus shows all signs (degeneracy, etc) which are normally attributed to the topologically ordered systems (AZ, 2013).