

*Dimitri Nanopoulos*



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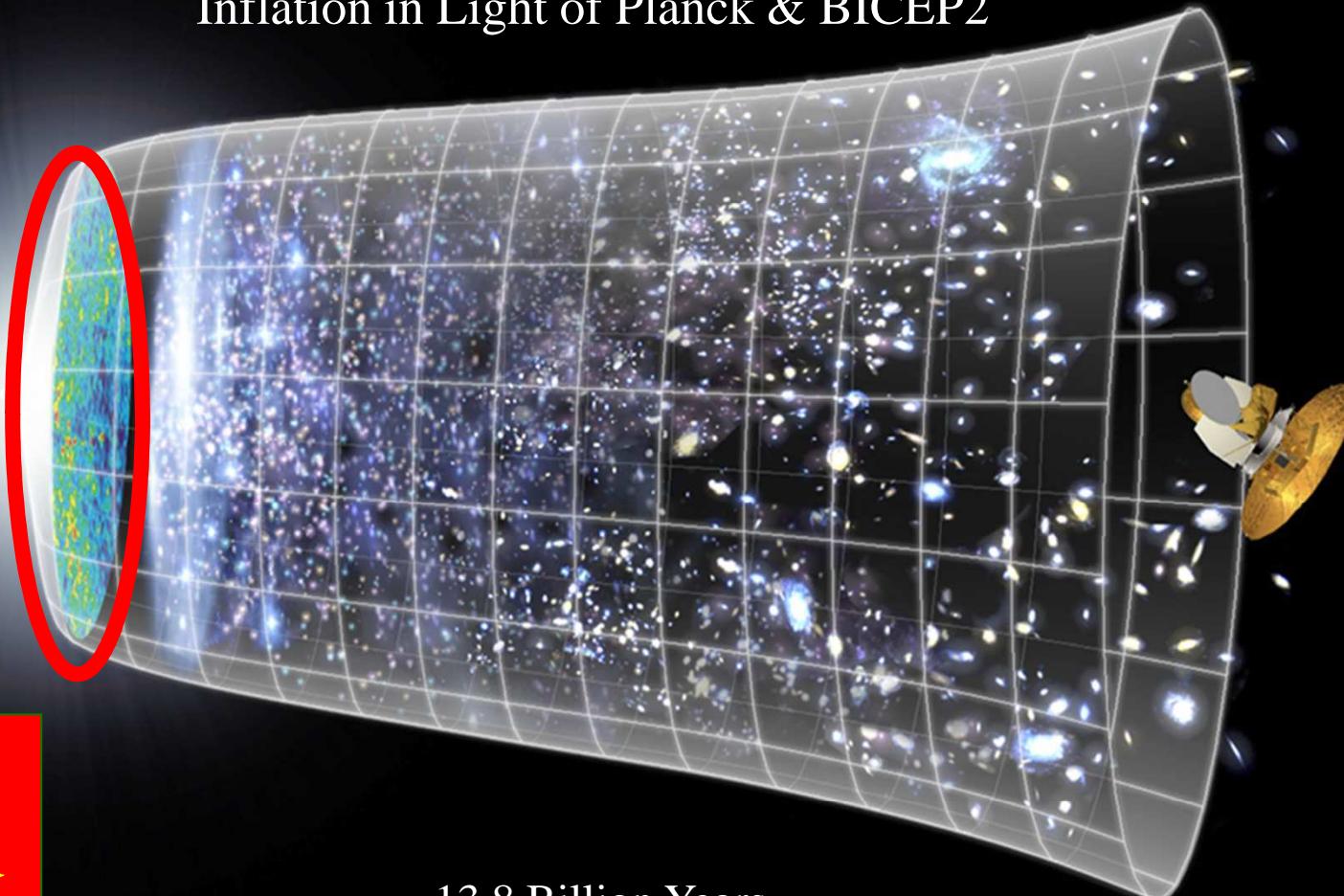


# Echo from the Big Bang

Inflation in Light of Planck & BICEP2

Big Bang

What  
happened  
here?



# Some Big Questions

- Why is the Universe so big and old?
- Why is it (almost) homogeneous on large scales?
- Why is its geometry (almost) Euclidean?
- What is the origin of structures in the Universe?

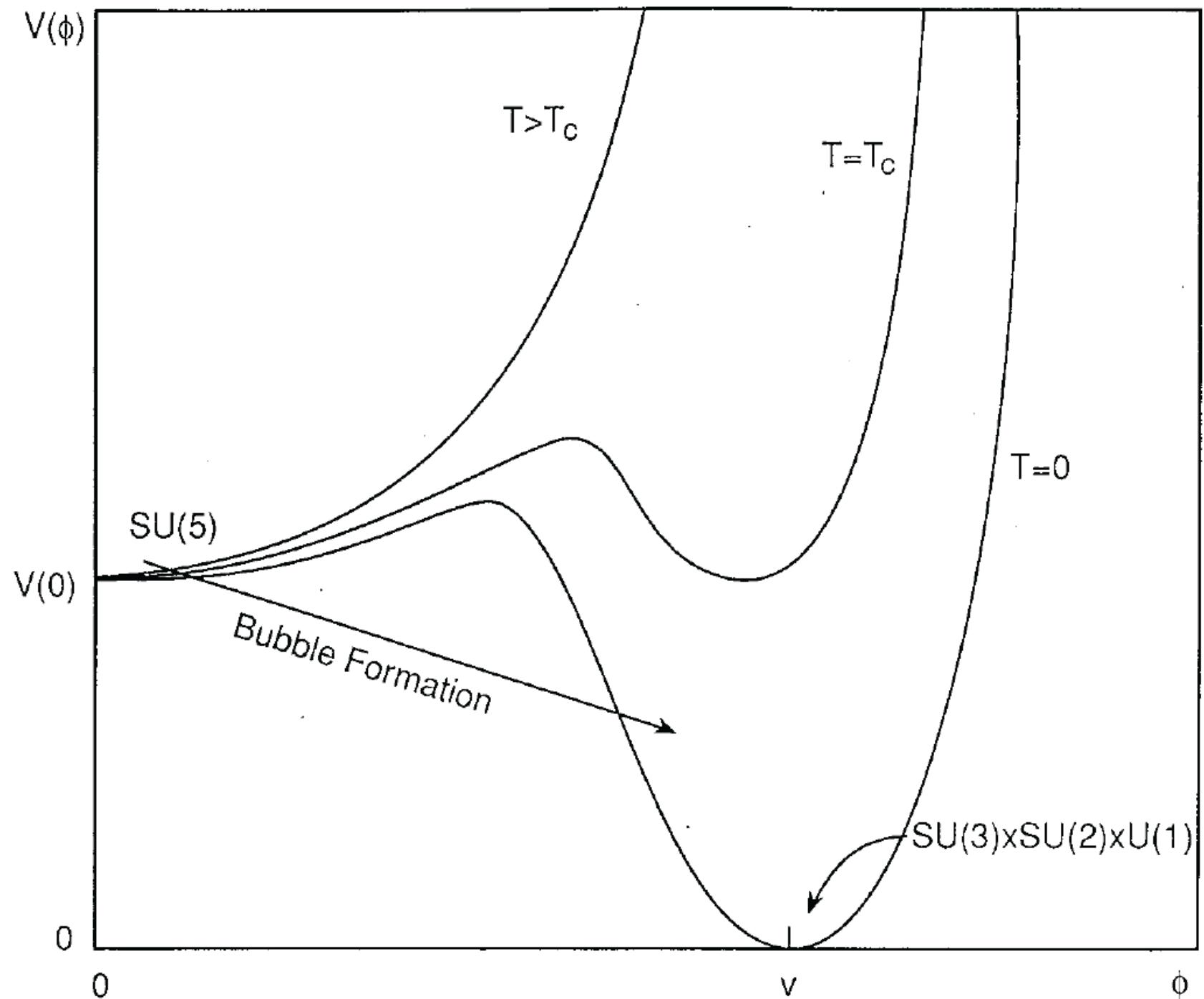
A possible answer:

## Cosmological inflation

- What do the Planck and BICEP2 data tell us?
- Models of inflation

# Inflation

- Standard cosmology assumes an adiabatically expanding Universe,  $R \sim 1/T$
- Phase transitions can violate this condition
- Expect several phase transitions in the Early Universe
  - GUTS:  $SU5 \rightarrow SU(3) \times SU(2) \times U(1)$
  - SM:  $SU(2) \times U(1) \rightarrow U(1)$
  - possibly other non-gauged symmetry breakings
- Entropy production common result
- Type of inflation will depend on the order of the phase transition



# Inflation

$$\Lambda = 8 \pi G_N V_0$$

For  $\rho \ll V_0$ ,

$$H^2 = \frac{\dot{R}^2}{R^2} \approx \frac{8\pi G_N V_0}{3} = \frac{\Lambda}{3}$$

or

$$\frac{\dot{R}}{R} \approx \sqrt{\frac{\Lambda}{3}} \quad R \sim e^{Ht}$$

For  $H\tau > 65$ , curvature problem solved

When the transition is over, the  
Universe reheats to  $T < V_0^{1/4} \sim T_i$ ,  
but  $R \gg R_i$

# Old New Inflation

Great idea based on 1-loop corrected SU(5) potential for the adjoint:

$$V(\sigma) = A\sigma^4 \left( \ln \frac{\sigma^2}{v^2} - \frac{1}{2} \right) \quad A = \frac{5625}{1024\pi^2} g_5^4$$

Linde; Albrecht  
Steinhardt

Problems:

- Vacuum structure
- destabilization through quantum fluctuations
- fine tuning (require curvature to be  $\ll M_X$ )
- density fluctuations -  $\delta\rho/\rho \sim 100 g_5^2$

# How SUSY can help

Exact Susy -  $V_{\text{1-loop}} = 0$

$$A = \frac{1}{64\pi^2 v^4} \left( \sum_B g_B m_B^4 - \sum_F g_F m_F^4 \right) = 0$$

Broken Susy -  $A = \frac{75}{32\pi^2 v^2} g_5^2 m_s^2$

fixes fine-tuning,  $\delta\rho/\rho$ , etc. -  
but isn't really a model

Ellis,  
Nanopoulos,  
Olive,  
Tamvakis

# Supergravity

Start with a Kähler Potential

$$G = K + \ln |W|^2 \quad \text{Minimal N=1 defined by } K = \phi^i \phi_i^*$$

and scalar potential

$$V = e^G [G_i (G^{-1})_j^i G^j - 3] + \text{D-terms}$$

or

$$V = e^{\phi^i \phi_i^*} \left[ \left| \frac{\partial W}{\partial \phi^i} + \phi_i^* W \right|^2 - 3|W|^2 \right] + \text{D-terms}$$

for minimal N=1

Typically,  $m^2 \sim H^2$

$\eta$ -problem!

# Supergravity

## Constructing Models

$$W = \mu^2 \sum_n \lambda_n \phi^n$$

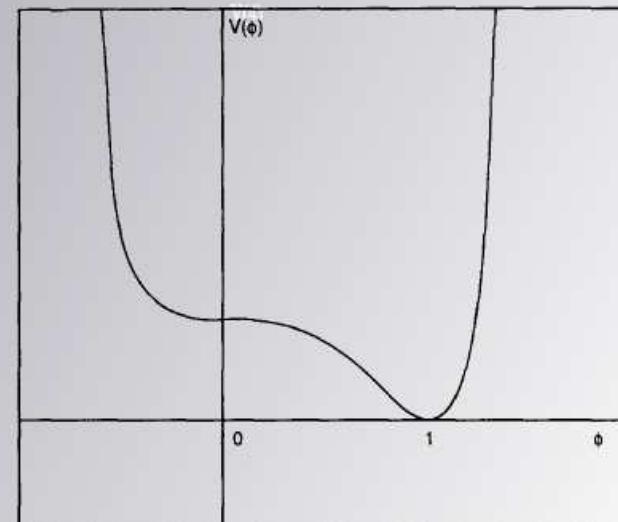
Nanopoulos,  
Olive,  
Srednicki,  
Tamvakis

$\mu^2$  fixed by amplitude of density fluctuations,  $\lambda_n \sim \mathcal{O}(1)$

Simplest example,  $W = \mu^2(1 - \phi)^2$

Holman,  
Ramond, Ross

$$\begin{aligned} V &= \mu^4 e^{|\phi|^2} \left[ 1 + |\phi|^2 - (\phi^2 + \phi^{*2}) - 2|\phi|^2(\phi + \phi^*) \right. \\ &\quad \left. + 5|\phi^2|^2 + |\phi|^2(\phi^2 + \phi^{*2}) - 2|\phi^2|^2(\phi + \phi^*) + |\phi^3|^2 \right] \\ &\simeq \mu^4 \left( 1 - 4\phi^3 + \frac{13}{2}\phi^4 + \dots \right) \end{aligned}$$



# No-Scale Supergravity

Natural vanishing of cosmological constant (tree level)  
with the supersymmetry scale not fixed at lowest order.  
(Also arises in generic 4d reductions of string theory.)

$$K = -3 \ln(T + T^* - \phi^i \phi_i^*/3)$$

$$V = e^{\frac{2}{3}K} \left| \frac{\partial W}{\partial \phi^i} \right|^2$$

Globally supersymmetric potential once  
K (canonical) picks up a vev

## No Scale SUGRA: A Case Study in Reductionism

There is a function called the Kähler potential which must be specified by the model builder in order to fix the metric of superspace, and determine the scalar potential. It is not fixed by the symmetries of the theory. There is however a particularly natural choice.

$$K = -3 \ln (T + T^* - \sum \phi_i^* \phi_i)$$



$$V_{SUGRA} = 0$$

The scalar potential is flat and vanishing. Supersymmetry is BROKEN, and there is no cosmological constant. This is all desirable at the *Tree Level*.

CONSTRAINT:  $m_0 = 0, A = 0, B = 0$        $m_{1/2} \neq 0$  for SUSY breaking

The gaugino mass  $m_{1/2}$  remains undetermined at the classical level.

All soft-terms though, are dynamically evolved in terms of only the single parameter ( $m_{1/2}$ ), which may itself be determined by radiative corrections to the potential !

- E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, Phys. Lett. B 133, 61 (1983)  
J. R. Ellis, A. B. Lahanas, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B 134, 429 (1984)  
J. R. Ellis, C. Kounnas and D. V. Nanopoulos, Nucl. Phys. B 241, 406 (1984)  
A. B. Lahanas and D. V. Nanopoulos, Phys. Rept. 145, 1 (1987)

## No-scale Supergravity (nSUGRA)

Choose a specific form for the Kähler potential:

$$K = -3\ln(T + T^* - \sum \varphi_i^* \varphi_i)$$



$$V_{SUGRA} = 0 \quad \text{At the tree-level}$$

Furthermore the gaugino mass  $m_{1/2}$  remains undetermined. Thus, the soft terms are not fixed (at the classical level) close to the Planck scale.

$$\text{So, } m_{1/2} = m_{1/2}(T_i)$$

with  $\langle T_i \rangle$  determined by radiative corrections.

$$m_{1/2}, \quad m_0 = 0, \quad A_0 = 0, \quad B = 0$$

Thus, in principle all soft-terms may be determined in terms of only **one-parameter**,  $m_{1/2}$  The One-Parameter Model

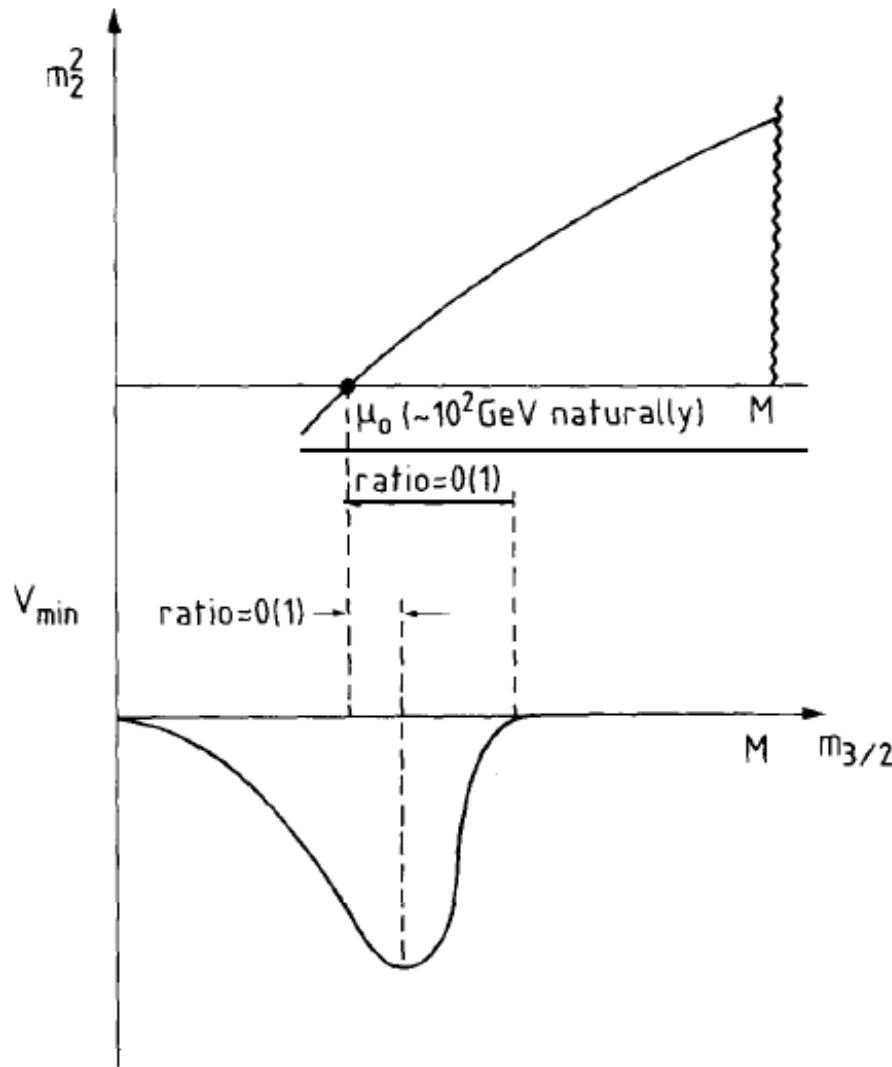


Fig. 14.  $V_{\min}$  as a function of the floating gravitino mass  $m_{3/2}$ .  $V_{\min}$  becomes deepest at  $m_{3/2} = O(1) \mu_0$  with  $\mu_0$  the critical scale at which  $m_2^2$  vanishes ( $m_3^2$  has been taken as zero) (from ref. [90]).

## Relation to String Theory

The no-scale structure emerges naturally as the infrared limit of string theory.

In particular,

- Heterotic M-theory compactifications
- Type IIB flux compactifications – **Flipped SU(5)**
- F-theory compactifications (non-perturbative limit of Type IIB)

## The nSUGRA ‘One-Parameter Model’

Strict No-scale Moduli Scenario:  $m_0 = A = B = 0$

Special Dilaton Scenario:

$$m_0 = \frac{m_{1/2}}{\sqrt{3}} \quad A = -m_{1/2} \quad B = \frac{2 m_{1/2}}{\sqrt{3}}$$

These ansatz combined with the no-scale condition define the so-called **one-parameter model** since the soft-terms are now all defined in terms  $m_{1/2}$

Subset of the mSUGRA parameter space



Highly constrained, but predictive!

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Ellis, Kounnas, and DVN, **Nucl.Phys.B247:373-395,1984**

Lopez, DVN, and Zichichi, **Phys.Lett.B319:451-456,1993**

Lopez, DVN, and Zichichi, **Int.J.Mod.Phys.A10:4241-4264,1995**

Lopez, DVN, and Zichichi, **Phys.Rev.D52:4178-4182,1995**

# No-Scale Supergravity

Constructing Models

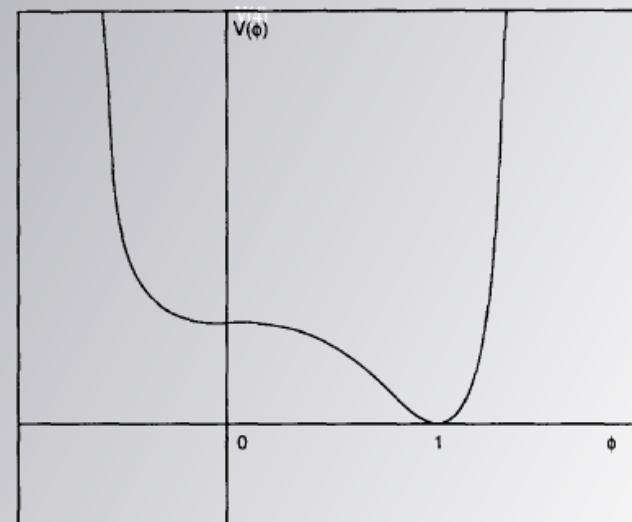
$$W = \mu^2 \sum_n \lambda_n \phi^n$$

Ellis, Enqvist,  
Nanopoulos,  
Olive,  
Srednicki

$\mu^2$  fixed by amplitude of density fluctuations,  $\lambda_n \sim \mathcal{O}(1)$

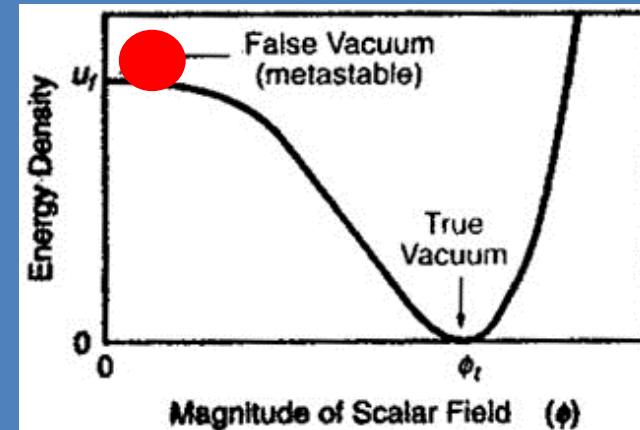
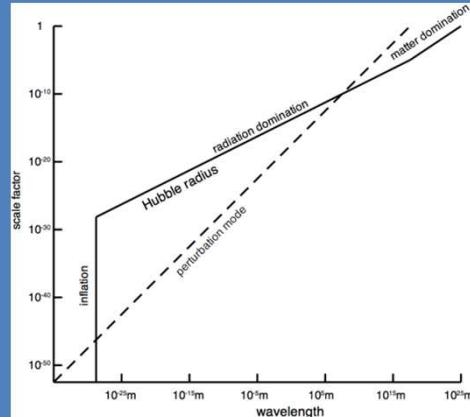
Simplest example,  $W = \mu^2(\phi - \phi^4/4)$

$$V = \mu^4 |1 - \phi^3|^2$$



# Primordial Perturbations

- “Cosmological constant” due to vacuum energy in “inflaton” field  $\phi$ :  $\Lambda \sim V(\phi) \neq 0$
- Quantum fluctuations in  $\phi$  cause perturbations in energy density (scalar) and metric (tensor)
- (Almost) independent of scale size



- Visible in cosmic microwave background (CMB)

# Inflationary Perturbations in a Nutshell

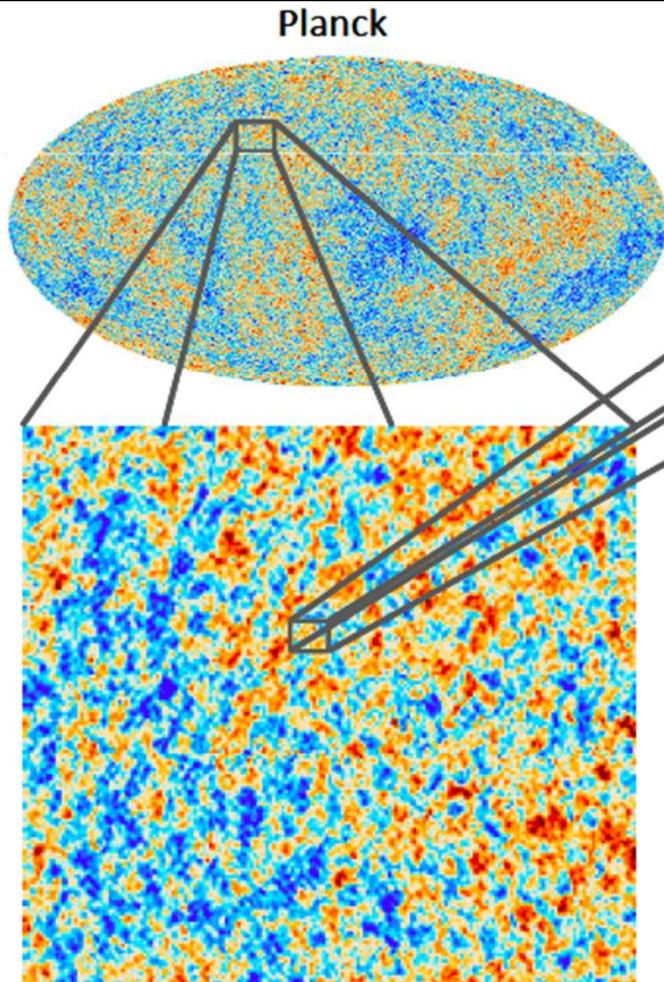
- Universe expanding like de Sitter
- Horizon  $\rightarrow$  information loss  $\rightarrow$  mixed state
- Effective ‘Hawking temperature’  $H/2\pi$
- Expect quantum fluctuations:

$$\langle \phi \phi \rangle, \quad \langle hh \rangle \sim H^2$$

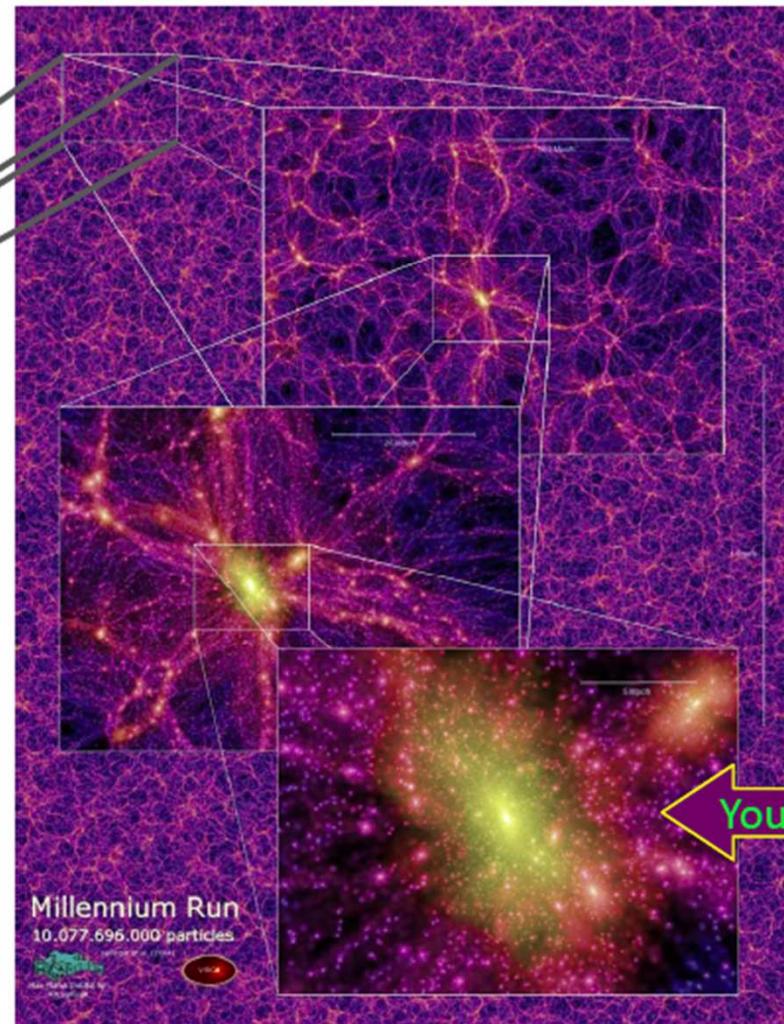
(inflaton, gravitational field:  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ )  
Scalar (density), tensor perturbations

- Should have thermal spectrum
- Scalars enhanced by evolution of  $\phi \rightarrow r \sim \varepsilon$

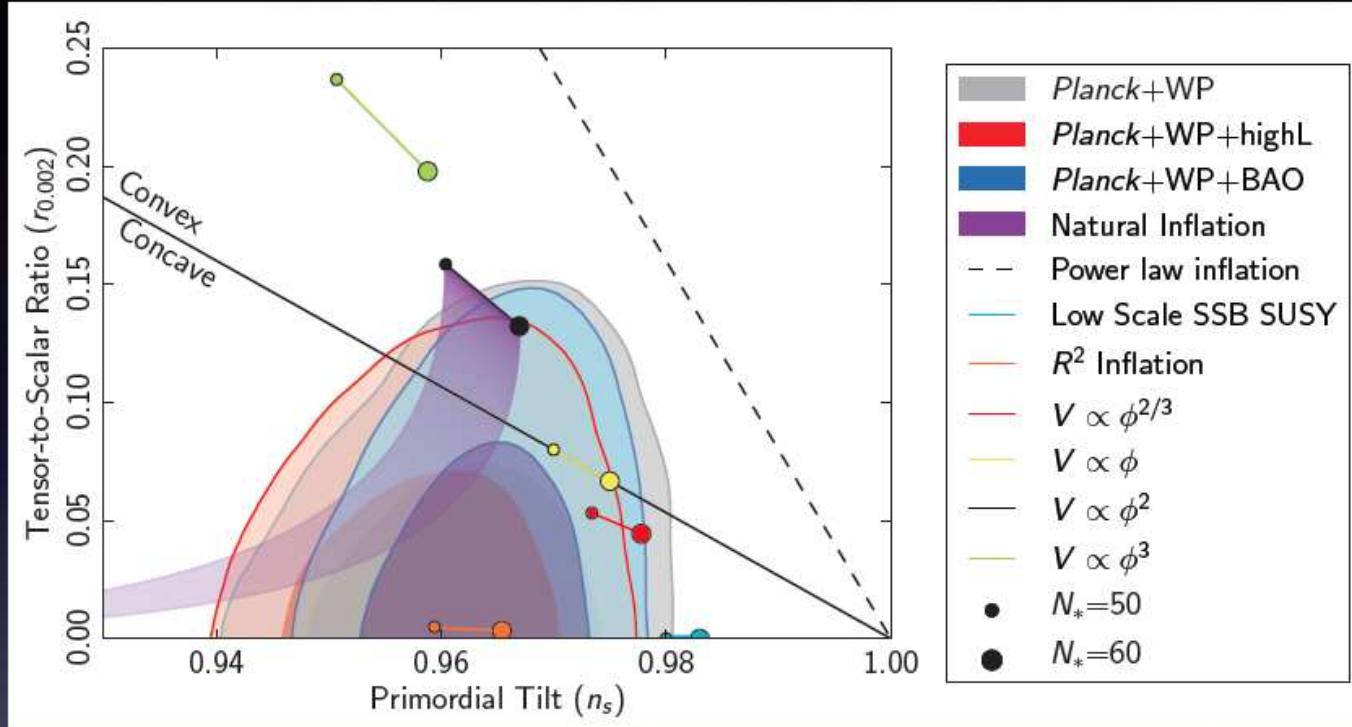
# Perturbations Generate Structures



Dark matter distribution today (simulated)



# Planck Results



$$\epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2 \quad \eta = \frac{V''}{V} \quad n_s = 1 - 6\epsilon + 2\eta \quad r = 16\epsilon$$

Trouble for simple supergravity  
and no-scale models:

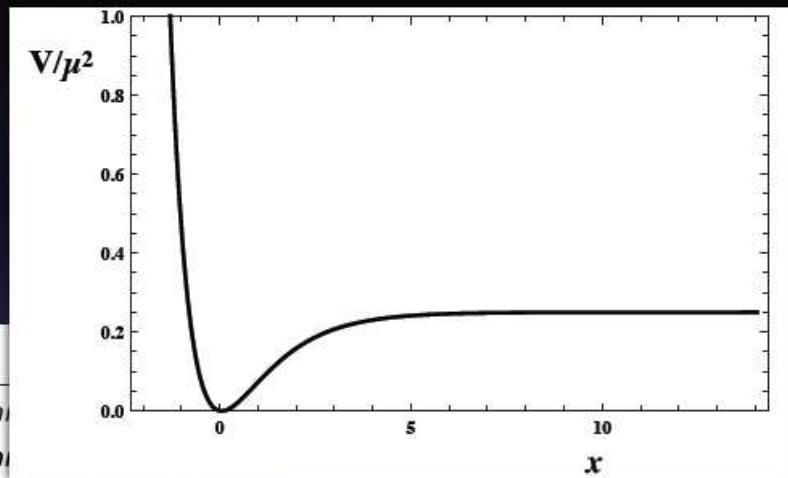
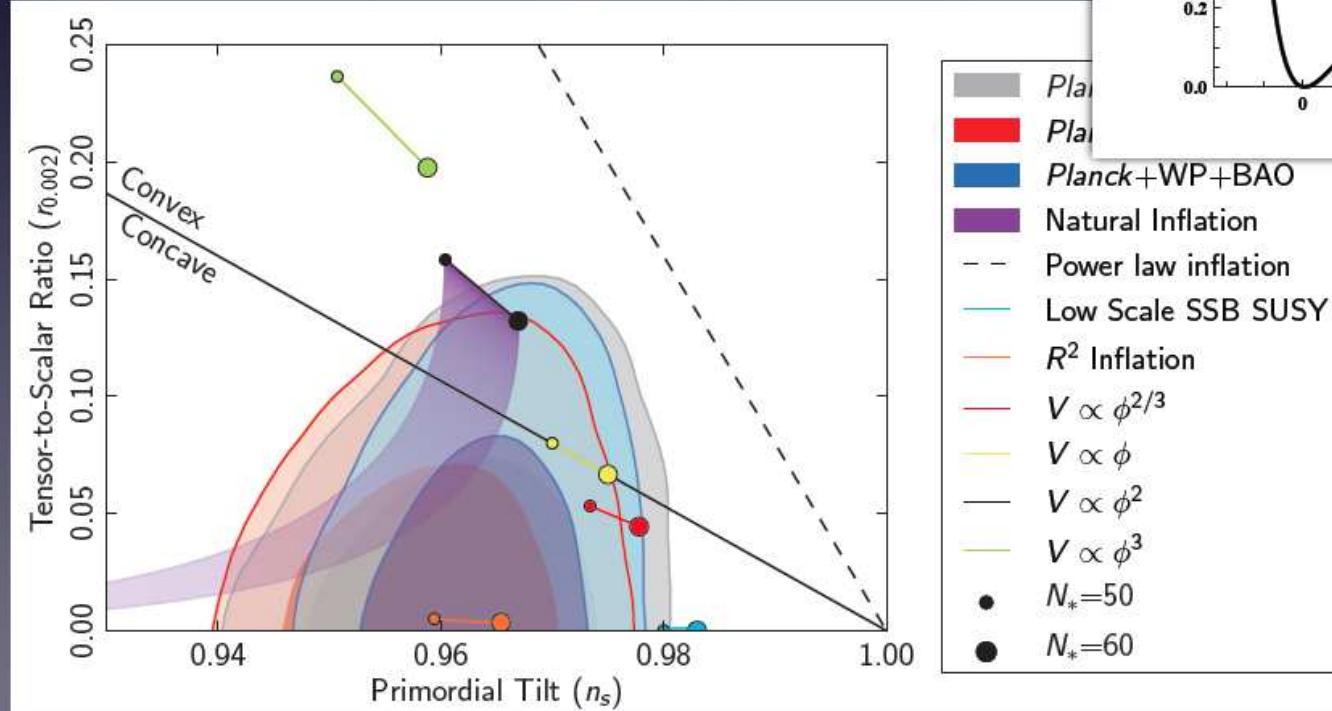
$$\epsilon \simeq \frac{1}{72N^4} \ll 1 \quad \text{OK} \quad n_s \sim .933$$

$$\eta \simeq -\frac{1}{2N} \quad \text{not OK}$$

# Planck-friendly Models

## R+R<sup>2</sup> Inflation

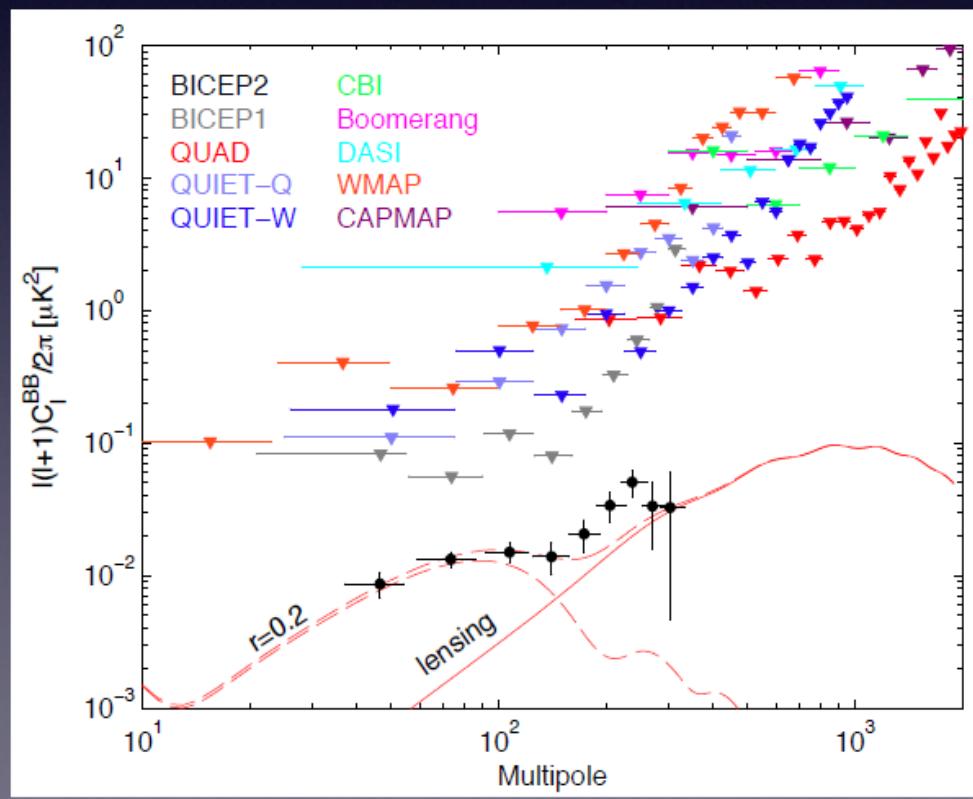
For N=55,  $n_s = 0.965$ ;  $r = .0035$



# Bicep-friendly Models

## Quadratic Chaotic Inflation

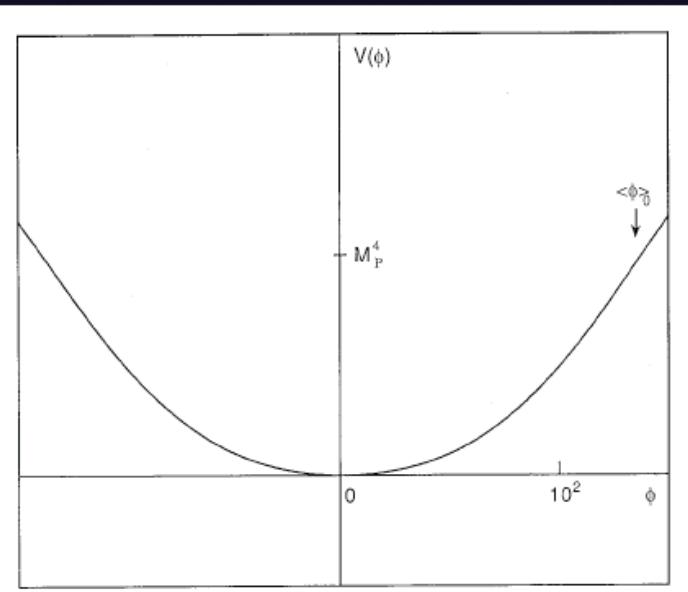
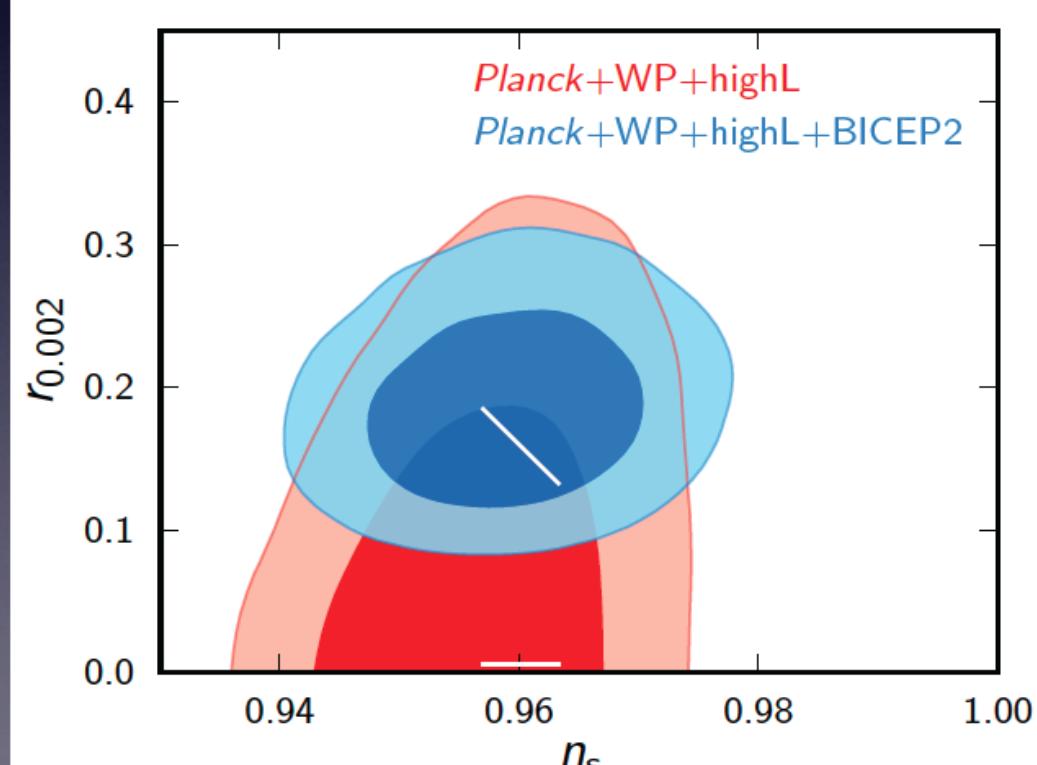
For N=55,  $n_s = 0.964$ ;  $r = .145$



# Bicep-friendly Models

## Quadratic Chaotic Inflation

For  $N=55$ ,  $n_s = 0.964$ ;  $r = .145$



← Includes running  
of the spectral index

# No-Scale models revisited

Can we find a model consistent with Planck?

Ellis, Nanopoulos, Olive

Start with WZ model:  $W = \frac{\hat{\mu}}{2}\Phi^2 - \frac{\lambda}{3}\Phi^3$

Assume now that T picks up a vev:  $2\langle\text{Re } T\rangle = c$

$$\mathcal{L}_{eff} = \frac{c}{(c - |\phi|^2/3)^2} |\partial_\mu \phi|^2 - \frac{\hat{V}}{(c - |\phi|^2/3)^2}$$

Redefine inflaton to a canonical field  $\chi$   $\hat{V} = |W_\Phi|^2$

$$\phi = \sqrt{3c} \tanh\left(\frac{\chi}{\sqrt{3}}\right)$$

# No-Scale models revisited

The potential becomes:

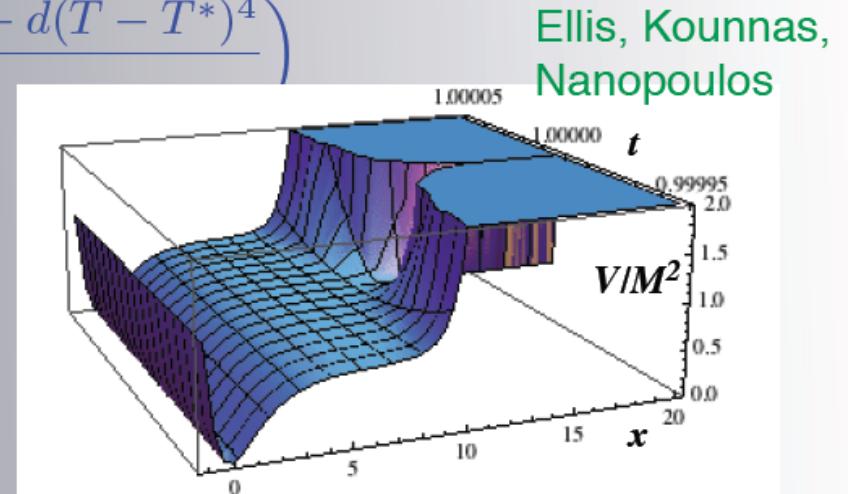
$$V = \mu^2 \left| \sinh(\chi/\sqrt{3}) \left( \cosh(\chi/\sqrt{3}) - \frac{3\lambda}{\mu} \sinh(\chi/\sqrt{3}) \right) \right|^2$$

For  $\lambda=\mu/3$ , this is exactly the  $R + R^2$  potential  
and Starobinsky model of inflation

$$V = \mu^2 e^{-\sqrt{2/3}x} \sinh^2(x/\sqrt{6})$$

Have so far assumed a vev for one of the two fields

$$K = -3 \ln \left( T + T^* - \frac{|\phi|^2}{3} + \frac{(T + T^* - 1)^4 + d(T - T^*)^4}{\Lambda^2} \right)$$



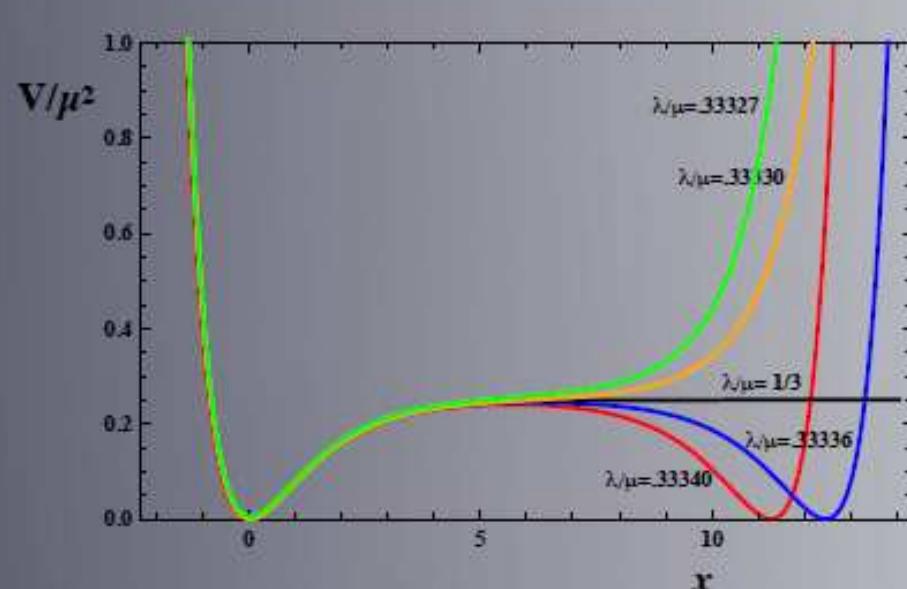
# No-Scale models revisited

The potential becomes:

$$V = \mu^2 \left| \sinh(\chi/\sqrt{3}) \left( \cosh(\chi/\sqrt{3}) - \frac{3\lambda}{\mu} \sinh(\chi/\sqrt{3}) \right) \right|^2$$
$$\hat{\mu} = \mu \sqrt{(c/3)}$$

For  $\lambda=\mu/3$ , this is exactly the  $R + R^2$  potential

$$V = \mu^2 e^{-\sqrt{2/3}x} \sinh^2(x/\sqrt{6})$$



$$\chi = (x + iy)/\sqrt{2}$$

# Classes of $R+R^2$ in No-Scale Supergravity

So is the inflaton  $T$  or  $\phi$ ?

1) T-fixed ( $\phi$ -inflaton)

Starobinsky potential found when

$$\hat{V} = M^2 |\phi|^2 |1 - \phi/\sqrt{3}|^2 \quad \text{Ellis, Nanopoulos, Olive}$$
$$W = M \left[ \frac{\phi^2}{2} - \frac{\phi^3}{3\sqrt{3}} \right]$$

2)  $\phi$ -fixed (T-inflaton)

Starobinsky potential found when

$$\hat{V} = 3M^2 |T - 1/2|^2 \quad \text{Cecotti; Kallosh, Linde}$$
$$W = \sqrt{3}M\phi(T - 1/2)$$

## Phenomenology with No-Scale (WZ) Inflation

$$K = -3 \ln \left( 1 - \frac{|y_1|^2 + |y_2|^2}{3} + \frac{|y_2|^4}{\Lambda^2} \right),$$

↑ Solves Polonyi issues

$$W = M \left[ \frac{y_1^2}{2} \left( 1 + \frac{y_2}{\sqrt{3}} \right) - \frac{y_1^3}{3\sqrt{3}} \right]$$

$y_2$  - Polonyi like field  
 $y_1$  - Inflaton

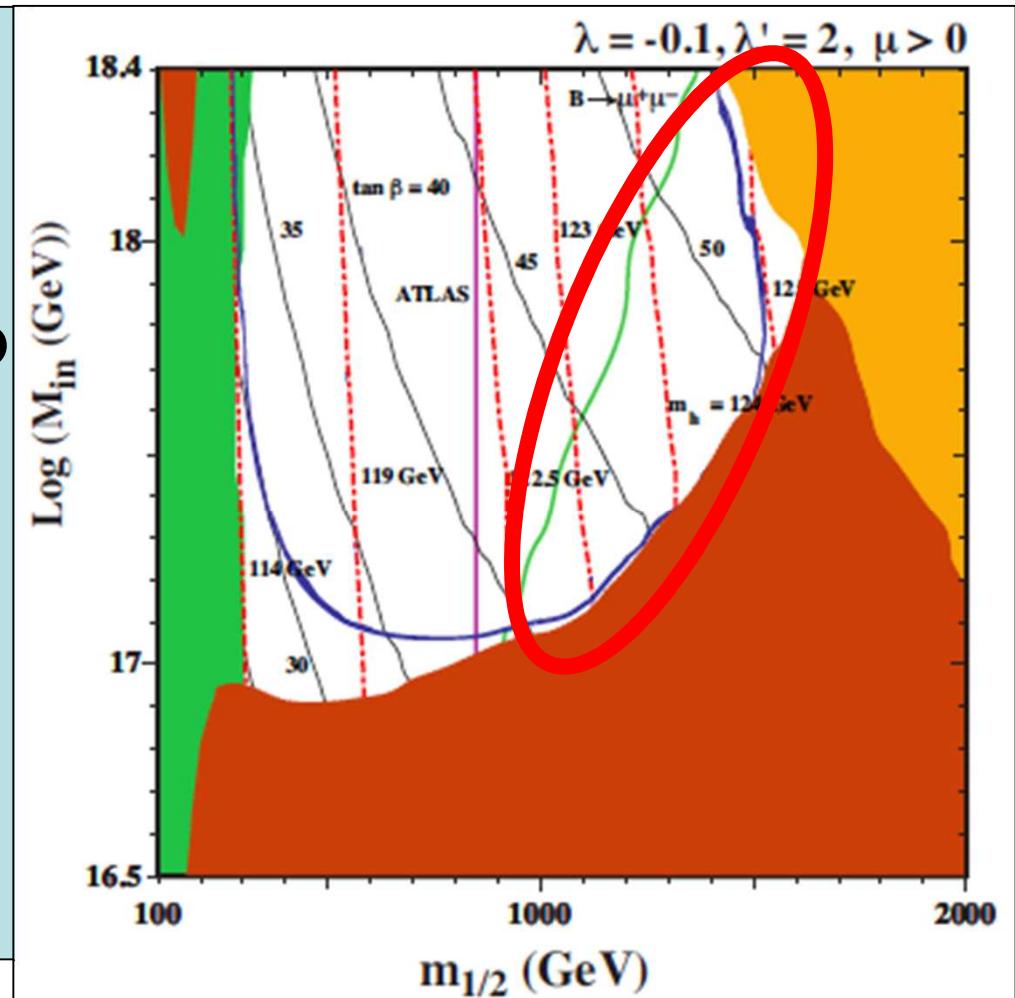
Add:  $W_Y = \lambda H_2^i L_i y_1$

$y_1$  now interpreted as a right handed sneutrino  
with mass  $M = 10^{-5} M_P$

Neutrino see-saw  $m_\nu = \frac{\lambda^2 v^2}{M}$

# No-Scale Framework for Particle Physics & Dark Matter

- Incorporating Starobinsky-like inflation, leptogenesis, neutrino masses, LHC constraints, supersymmetric dark matter,
- Stringy origin...?



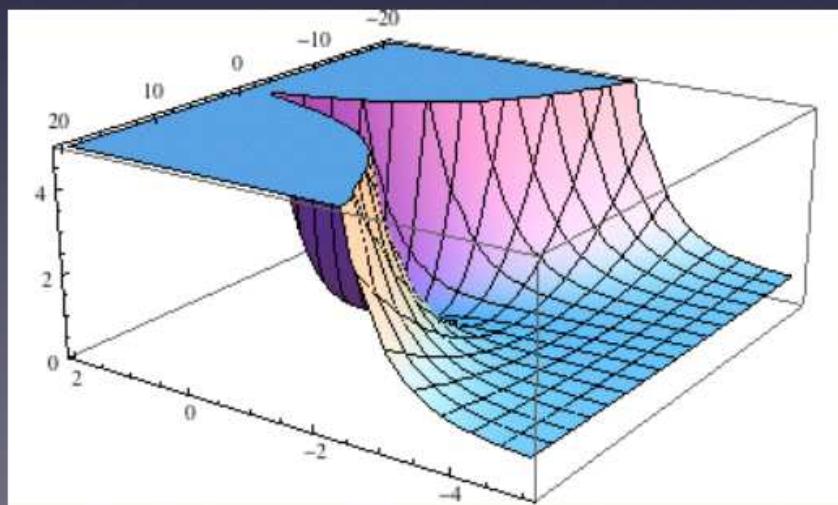
# Qadratic Chaotic Inflation:

with  $W = \sqrt{3}M\phi(T - 1/2)$

Cecotti

write  $T = \frac{1}{2} \left( e^{-\sqrt{\frac{2}{3}}\rho} + i\sqrt{\frac{2}{3}}\sigma \right)$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\rho\partial^\mu\rho + \frac{1}{2}e^{2\sqrt{\frac{2}{3}}\rho}\partial_\mu\sigma\partial^\mu\sigma - \frac{3}{4}m^2 \left(1 - e^{-\sqrt{\frac{2}{3}}\rho}\right)^2 - e^{2\sqrt{2/3}\rho} \frac{1}{2}m^2\sigma^2,$$



Ferrara, Kehagias,  
Riotto

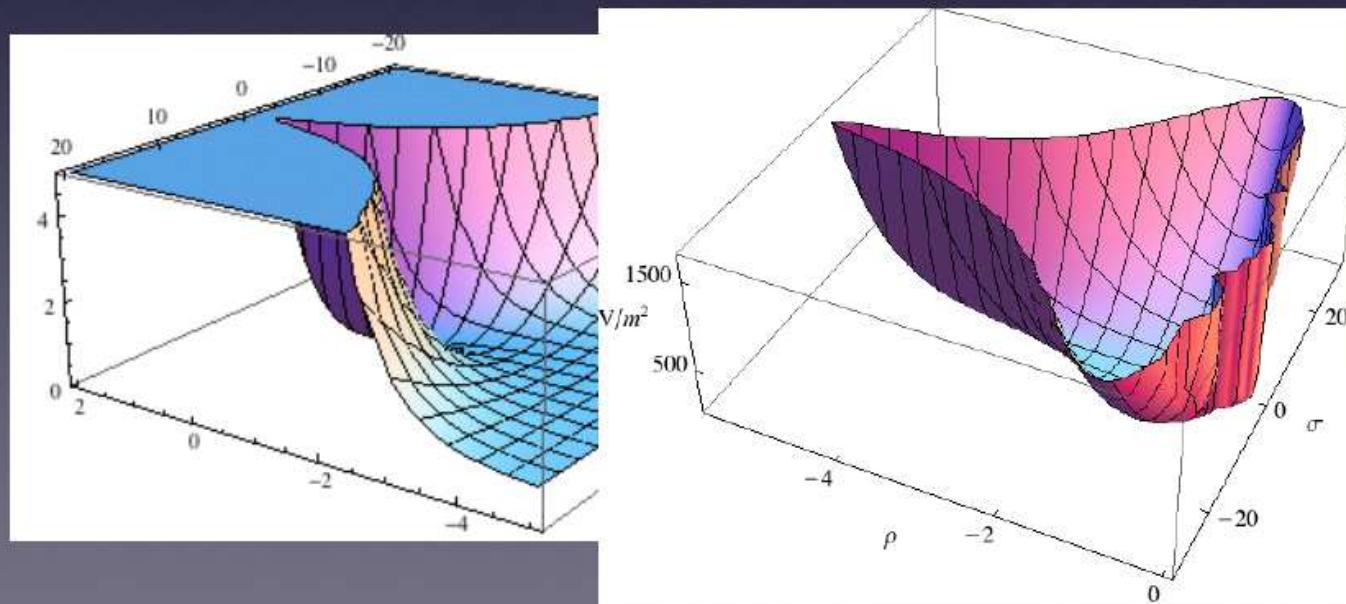
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Ferrara, Kehagias,  
Riotto

Ellis, Garcia,  
Nanopoulos, Olive

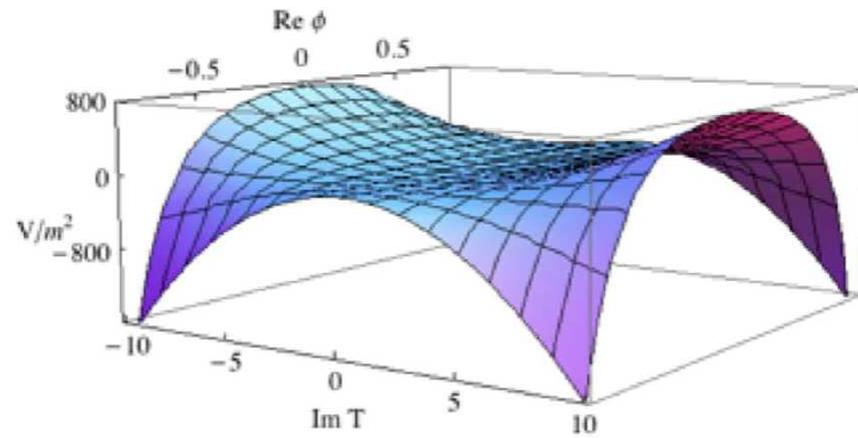


Figure 2: The scalar potential of the model (30, 33) projected onto the  $(\text{Im } T, \text{Re } \phi)$  plane with fixed values  $\text{Re } T = 1/2$  and  $\text{Im } \phi = 0$ .

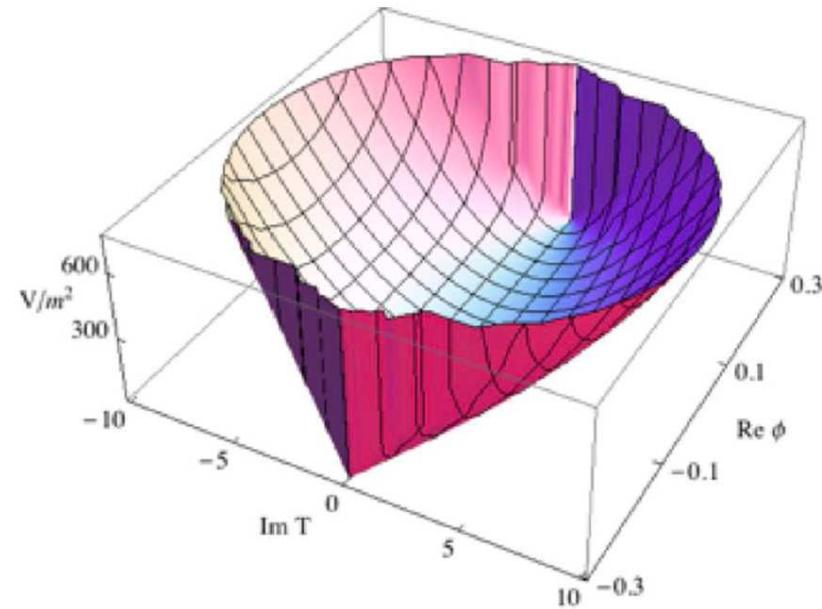


Figure 4: As in Fig. 2, but for the scalar potential of the model (30, 34).

# $SU(1,1)/U(1)$ models

Ellis, Garcia,  
Nanopoulos, Olive

$$K = -3 \ln(T + \bar{T}) + \frac{|\phi|^2}{(T + \bar{T})^w}$$

with  $W = \sqrt{3}M\phi(T - 1/2)$

$$V \propto e^{|\phi|^2/(T+\bar{T})^w} \simeq e^{|\phi|^2}$$

$$V = 3m^2|T - 1/2|^2(T + T^*)^{(w-3)}$$

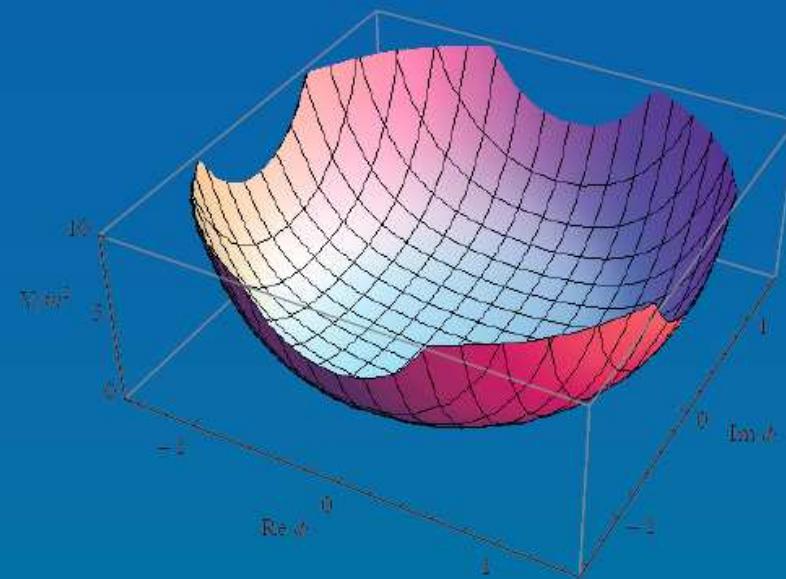
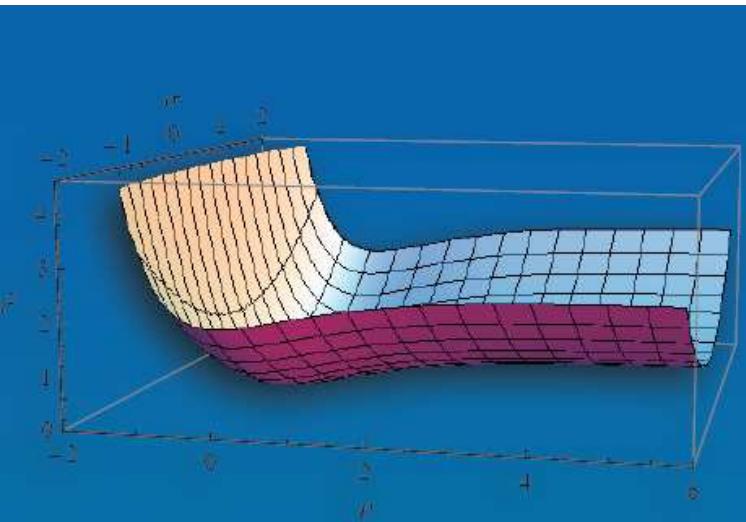
choose  $w = 3$

# One No-Scale Model to Fit Them All

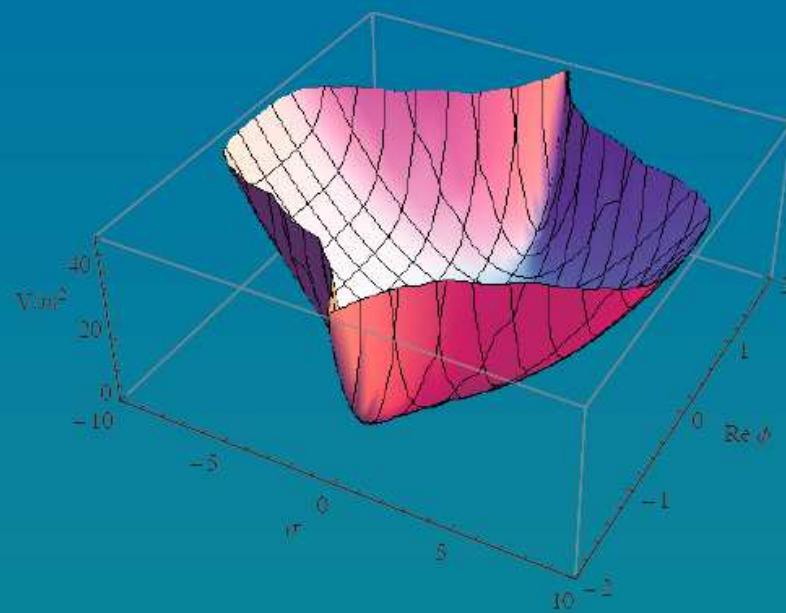
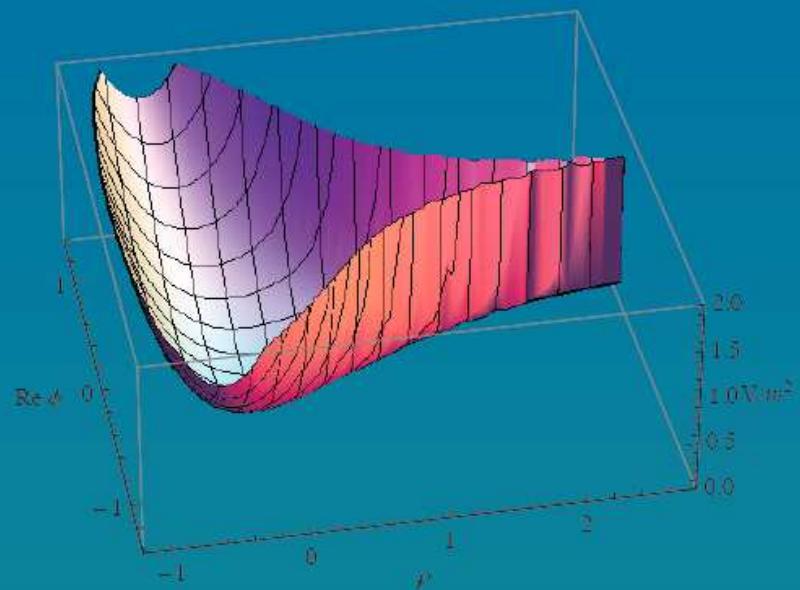
Ellis, Garcia,  
Nanopoulos, Olive

again write  $T = \frac{1}{2} \left( e^{-\sqrt{\frac{2}{3}}\rho} + i\sqrt{\frac{2}{3}}\sigma \right)$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\rho\partial^\mu\rho + \frac{1}{2}e^2\sqrt{\frac{2}{3}}\rho\partial_\mu\sigma\partial^\mu\sigma - \frac{3}{4}m^2 \left( 1 - e^{-\sqrt{\frac{2}{3}}\rho} \right)^2 - \frac{1}{2}m^2\sigma^2$$



## The Potential



$$K = -3 \log(T + T^* - c(\cos \theta(T + T^* - 1) - i \sin \theta(T - T^*))^4) + \frac{|\phi|^2}{(T + T^*)^3}.$$

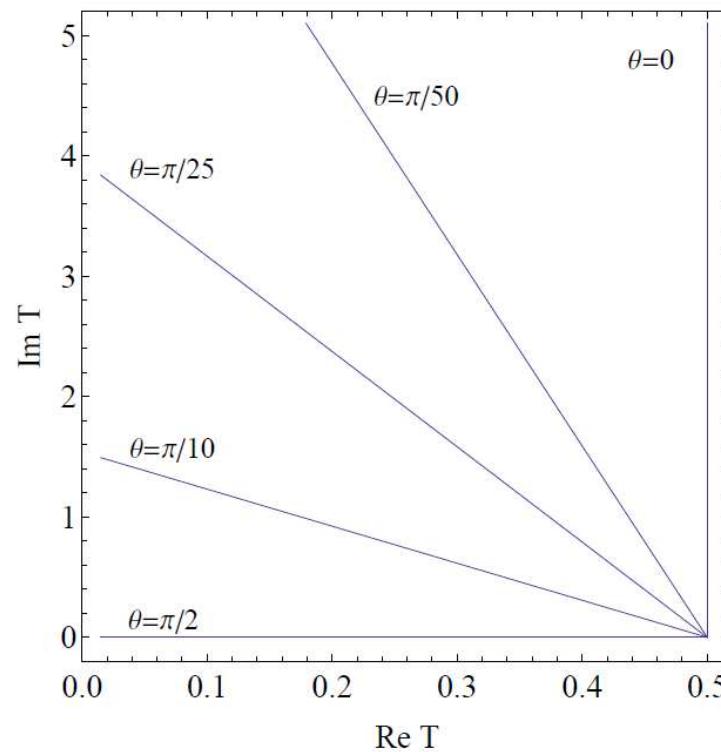


Figure 2: Inflationary directions in the  $(\text{Re } T, \text{Im } T)$  plane, labeled by the stabilization angle  $\theta$ .

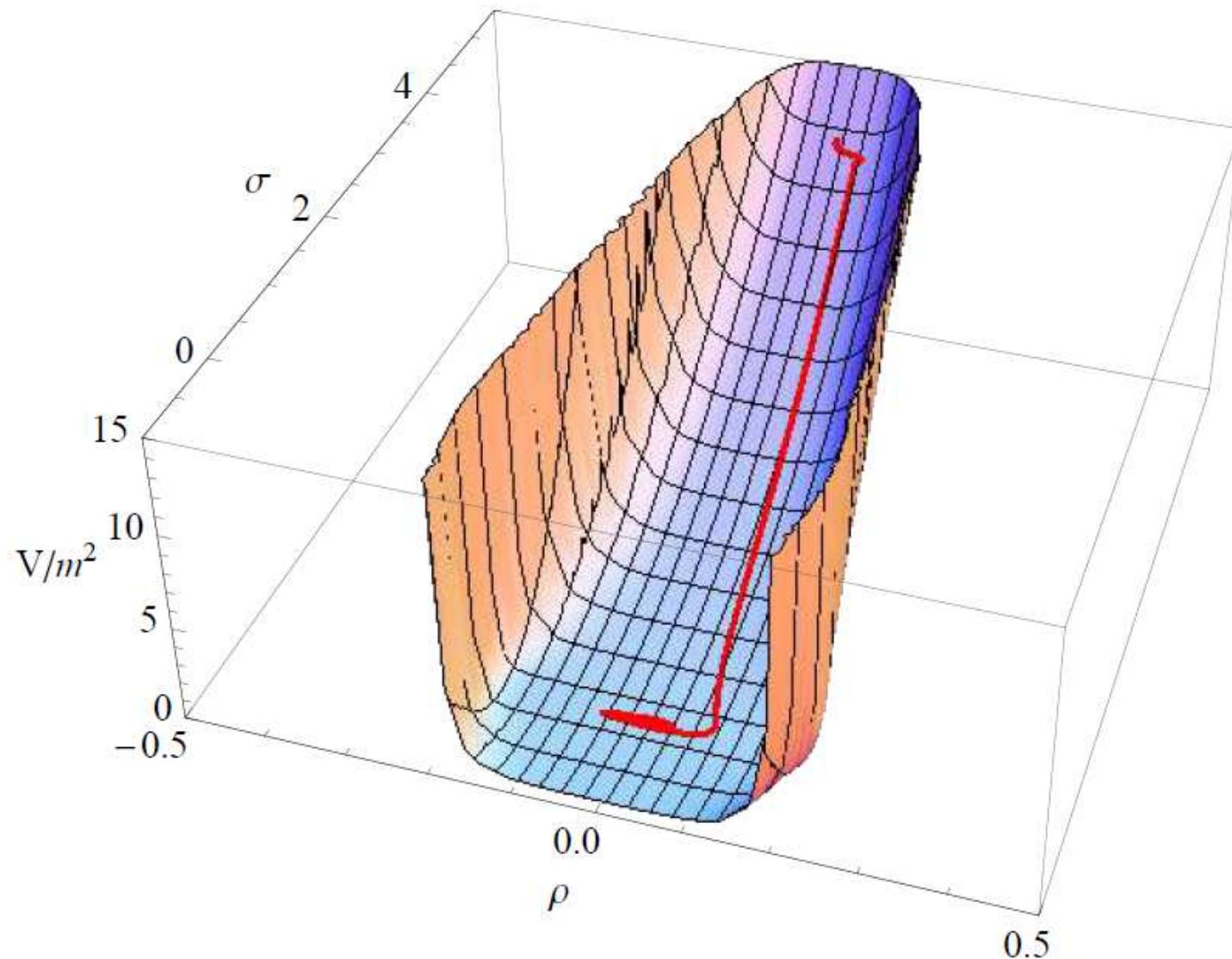


Figure 4: Numerical solution for  $\theta = 0$ ,  $c = 1000$ , with initial conditions  $\rho_0 = 0$ ,  $\sigma_0 = 5$  and  $\phi_0 = 0$ : ‘circling the drain’.

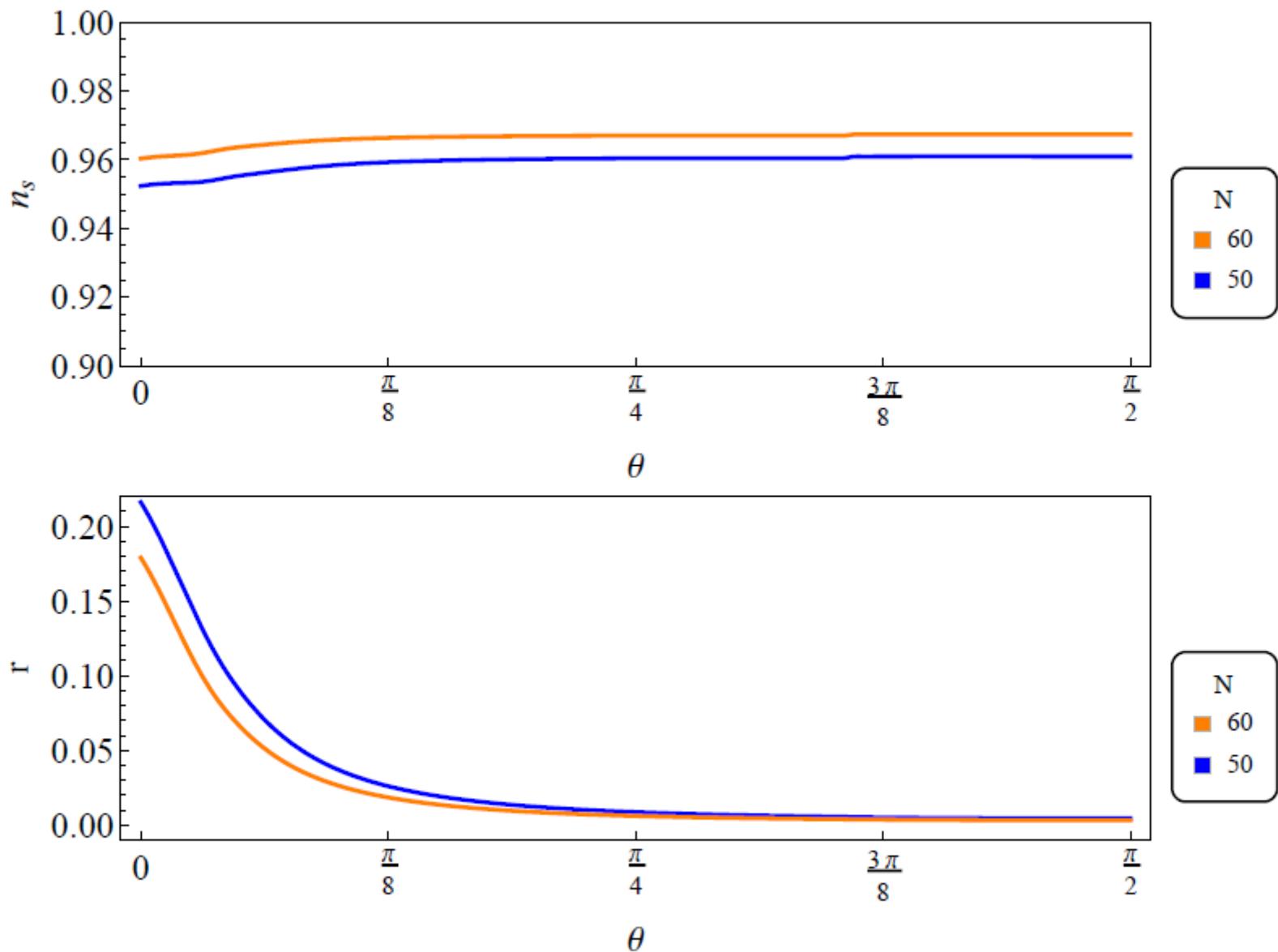


Figure 9: *Upper panel:* The scalar tilt as function of  $\theta$ . *Lower panel:* The tensor-to-scalar ratio as function of  $\theta$ .

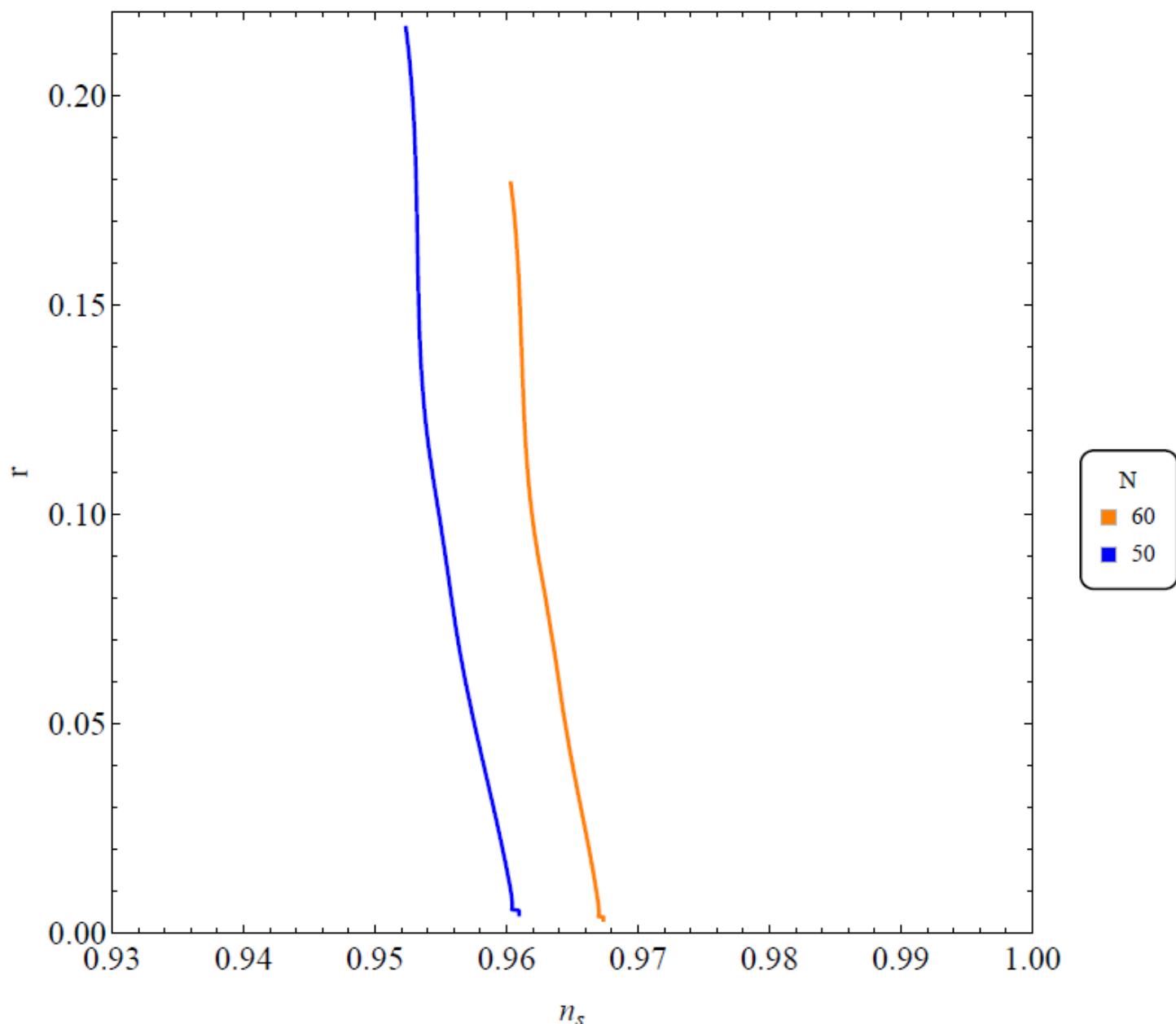


Figure 10: *The parametric curve  $(n_s(\theta), r(\theta))$ .*

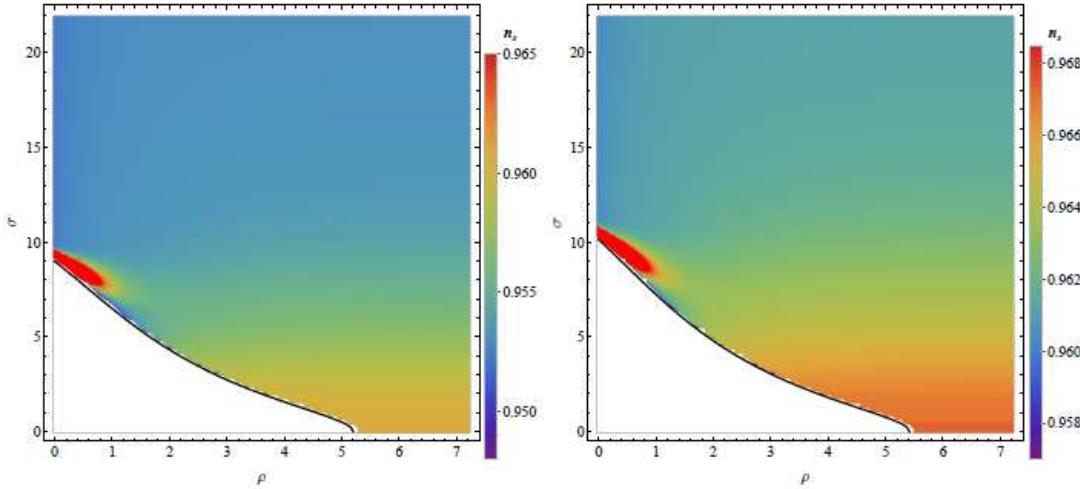


Figure 13: The scalar tilt  $n_s$  with stationary initial conditions at arbitrary points in the  $(\rho, \sigma)$  plane for  $\phi_0 = 0$ ,  $c = 10$ . Left panel: The case of  $N = 50$  e-foldings. Right panel: The case of  $N = 60$  e-foldings. In each case, the solid curve is the boundary for the specified value of  $N$ .

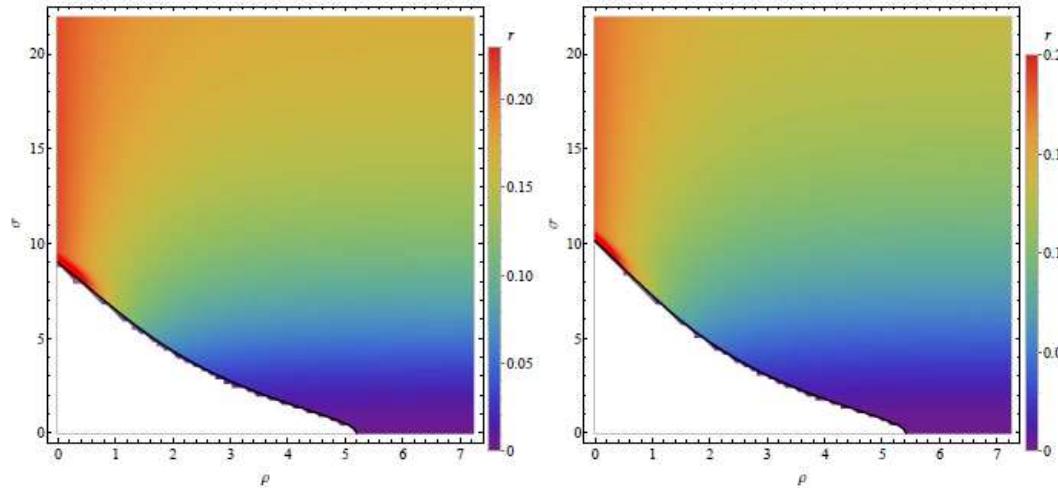
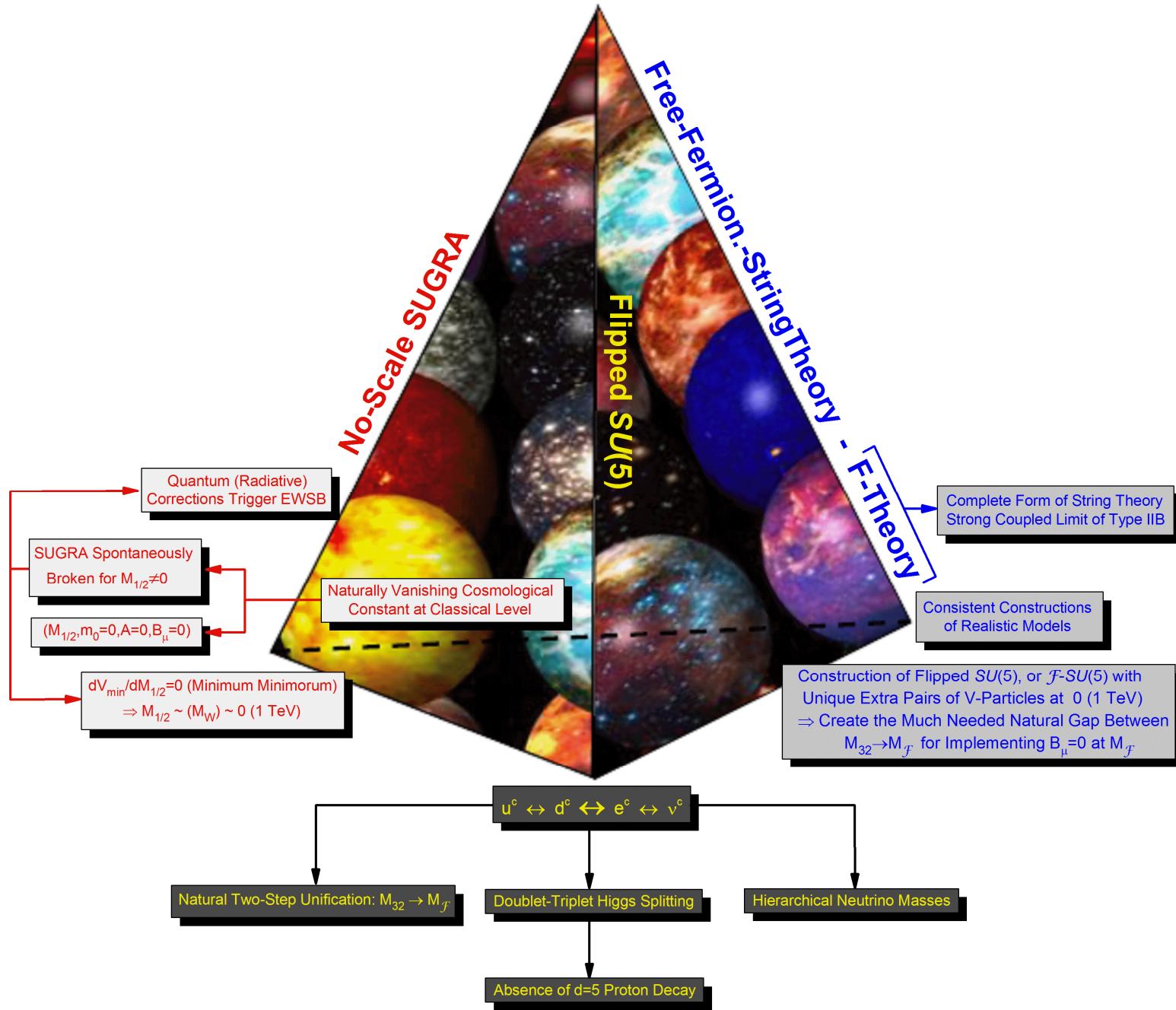


Figure 14: The tensor-to-scalar ratio  $r$  with stationary initial conditions at arbitrary points in the  $(\rho, \sigma)$  plane for  $\phi_0 = 0$ ,  $c = 10$ . Left panel: The case of  $N = 50$  e-foldings. Right panel: The case of  $N = 60$  e-foldings. In each case, the solid curve is the boundary for the specified value of  $N$ .

# $\mathcal{F}$ - $SU(5)$





Part II of The Golden Point Saga, as Featured in arXiv:1007.5100

Also Starring: Tianjun Li, James A. Maxin & Joel W. Walker

## THE GOLDEN STRIP

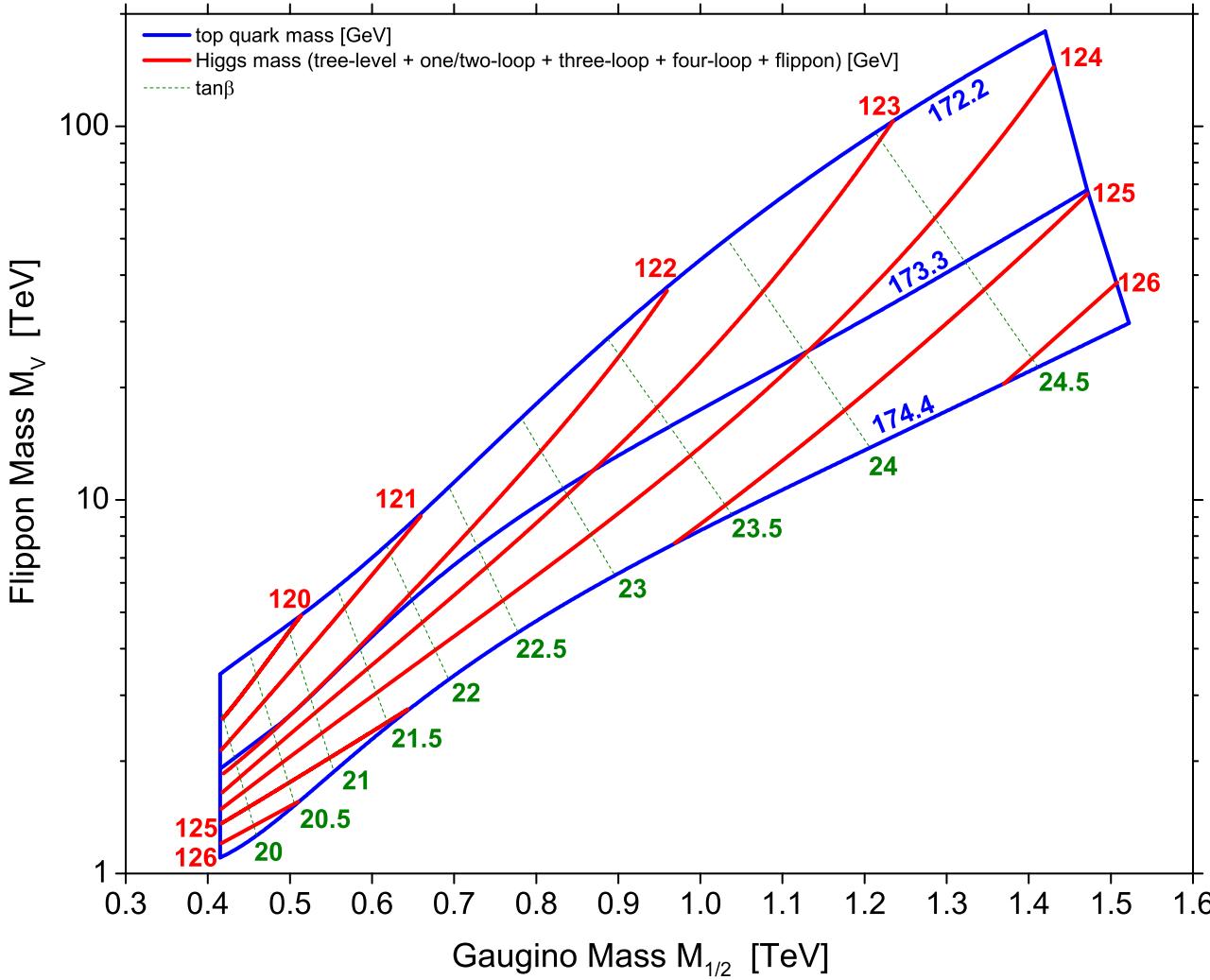
of Correlated Top Quark, Gaugino,  
and Vectorlike Mass In No-Scale, No Parameter

$F\text{-SU}(5)$

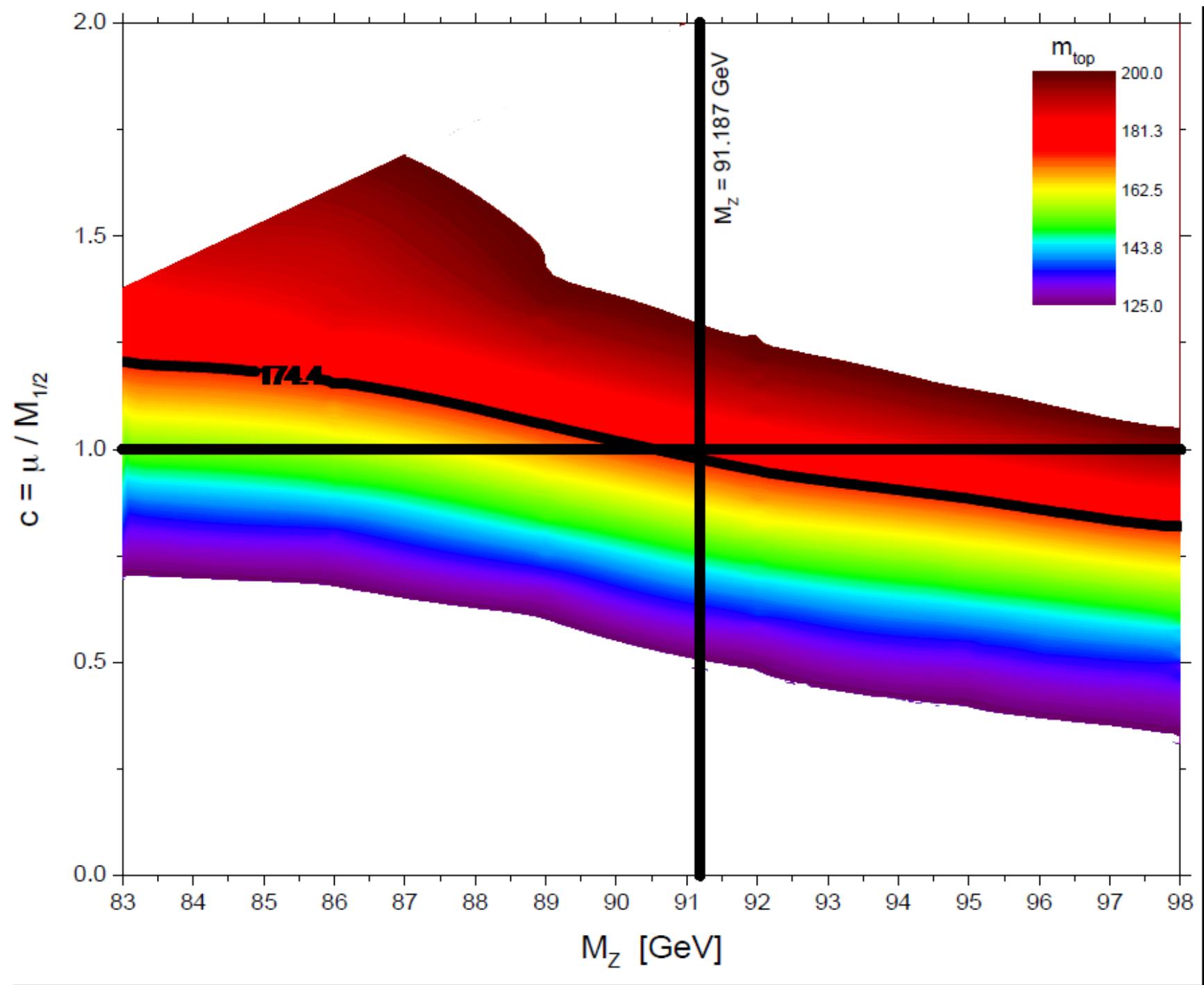
George P. and Cynthia W. Mitchell Institute for  
Fundamental Physics and Astronomy  
at Texas A&M University

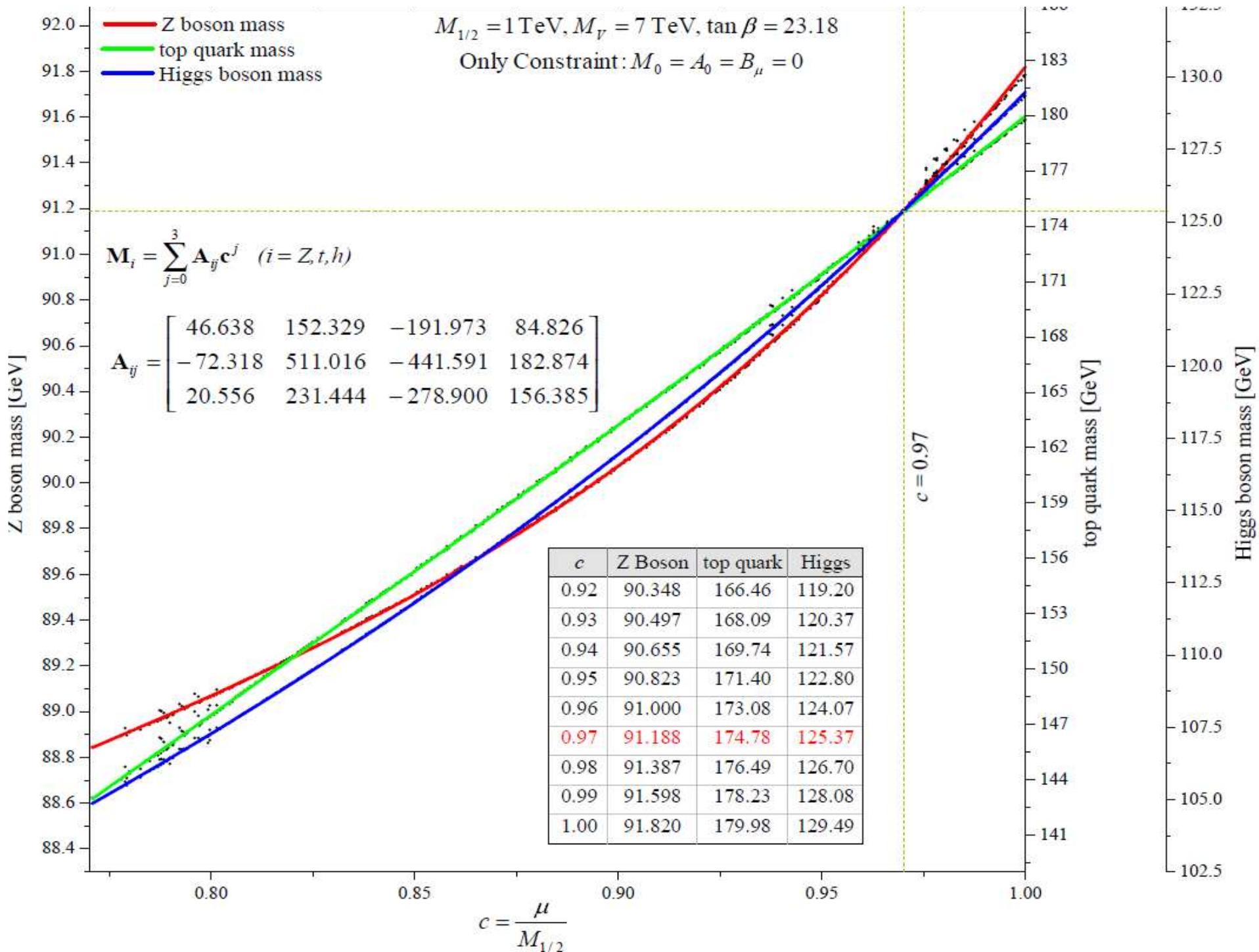
Houston Advanced Research Center (HARC)

Academy of Athens



Constrained model space of No-Scale F-SU(5) as a function of the gaugino mass  $M_{1/2}$  and flippon mass  $M_V$ . The thick lines demarcate the total Higgs boson mass gradients, including the tree-level plus one/two-loop (as computed by the SuSpect 2.34 codebase), the three-loop plus four-loop contributions, and the flippon contribution. The thin dashed lines represent gradients of  $\tan\beta$ , while the upper and lower exterior boundaries are defined by a top quark mass of  $m_t = 173.3 \pm 1.1$  GeV. The left edge is marked by the LEP constraints, while the right edge depicts where the Planck relic density can no longer be maintained due to an LSP and light stau mass difference less than the on-shell tau mass. All model space within these boundaries satisfy the Planck relic density constraint  $\Omega h^2 = 0.1199 \pm 0.0027$  and the No-Scale requirement  $B\mu = 0$ .





# Summary

- Inflation ingrained in Standard Cosmology.
- The Starobinsky model of inflation can be realized in no-scale supergravity with either modulus  $T$  or ‘matter’ field  $\phi$  with a simple WZ superpotential.
- The latter lends itself nicely to equating the inflaton with a right-handed sneutrino
- Can construct a simple potential which interpolates between the Planck-friendly (small  $r$ ) solution, or the BICEP-friendly (large  $r$ ) solution
- Can easily integrate low energy susy phenomenology