Nuclear pairing
from microscopic forces
(with applications)

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L.N. Cooper considered the scattering of two particles which have an attractive interaction in the presence of a Fermi sea (restricting the possible momenta where the particles can scatter). He showed that even for arbitrarily small interactions, pairs will be formed in the system.

Instability of the Fermi sea: pair formation happens for any non-zero, attractive interaction.

L. Cooper, Phys. Rev. 104 (1956) 1189

D. Brink and R. Broglia, Nuclear Superfluidity, Cambridge Press
BCS gap equation

- Quasi-particle energy

\[ E(k)^2 = \epsilon(k)^2 + |\Delta(k)|^2 \]

- NN potential (partial waves expansion)

\[ \langle k|V|k'\rangle = 4\pi \sum_L (2L + 1) P_L(\hat{k} \cdot \hat{k}') V_L(k, k') \]

- Partial waves expansion of the gap function

\[ \Delta(k) = \sum_{L,M} \sqrt{\frac{4\pi}{2L + 1}} Y_{LM}(\hat{k}) \Delta_{LM}(k) \]

- Angle-average approximation

\[ |\Delta(k)|^2 \to D(k)^2 \equiv \frac{1}{4\pi} \int d\hat{k} |\Delta(k)|^2 = \sum_{L,M} \frac{1}{2L + 1} |\Delta_{LM}(k)|^2 \]

the different M-components become uncoupled and all equal.

- Gap equation

\[ \Delta_L(k) = -\frac{1}{\pi} \int_0^\infty k'^2 dk' \frac{V_L(k, k')}{\sqrt{\epsilon(k')^2 + \left[ \sum_{L'} \Delta_{L'}(k')^2 \right]}} \Delta_L(k') \]

No dependence on the quantum number M, but L components are coupled by tensor terms

we used the method suggested by Khodel where the original gap equation is replaced by a coupled set of equations for the dimensionless gap function \( \chi(p) \) defined by \( \Delta(p) = \Delta_F \chi(p) \) and a non-linear algebraic equation for the gap magnitude \( \Delta_F = \Delta(p_F) \) at the Fermi surface.

V.V. Khodel, V.A. Khodel, J.W. Clark, NPA 679 (2000) 827
**Microscopic forces**

The nuclear force at large distances is governed by the exchange of one or multiple pions.

The short-range part of the nuclear force is driven by physics not resolved explicitly in reactions with typical nucleon momenta of the order of $M\pi c$. It can be mimicked by zero-range contact interactions with an increasing number of derivatives.

$$V_{\text{eff}} = \sum_{\nu} \left[ V_{\text{short-range}}^{(\nu)} + V_{\text{long-range}}^{(\nu)} \right]$$

**Potential**

**Short-range**

**Medium-range**

**Long-range**
Microscopic forces

\[ V = V_{2B} + V_{3B} \simeq V_{2B} + V_{2B}^{\text{eff}}(\rho) \]

In-medium NN interaction generated

- by the two-pion exchange component \( (c_1, c_3, c_4) \)
- and by the one-pion exchange \( (c_D) \) and short-range component \( (c_E) \)

In-medium nucleon propagator

\[ -2\pi\delta(k_0)\theta(k_f - |\vec{k}|) \]

3 body \( \rightarrow \) 2 body density dependent Holt et al., [PRC 81 (2010) 024002]
How to renormalize NN forces

One can use RG transformations to evolve to lower $\Lambda$ while preserving the truncation error of the original Hamiltonian $\Rightarrow$ eliminate coupling between high- and low-momenta components ($k_F \leq 2 \text{ fm}^{-1}$)

Renormalization Group could help for
1. Strong short-range repulsion (*hard core*)
2. Iterated tensor ($S_{12}$) interaction
3. Near zero-energy bound states

**Features**
1. Decoupling
2. EFT
3. Universality
4. Perturbativeness
5. Many-body
6. Cutoff-dependence
How to renormalize NN forces

\[
\frac{d}{d\Lambda} T_\Lambda(p, q; q^2) = 0
\]

Bogner, Kuo and Schwenk, PR 386 (2003) 1

The SRG is based on a continuous sequence of unitary transformations that suppress off-diagonal matrix elements, driving the Hamiltonian towards a band-diagonal form.

Bogner, et al., PPNP 65 (2010) 94-147

The SRG is based on a continuous sequence of unitary transformations that suppress off-diagonal matrix elements, driving the Hamiltonian towards a band-diagonal form.
\[ \Delta(k) = -\frac{1}{2} \int \frac{d^3k'}{(2\pi)^3} \tilde{V}_{k,k'} \frac{\Delta(k')}{\sqrt{(\epsilon(k') - \mu)^2 + \Delta(k')^2}} \tanh \frac{\beta}{2} \sqrt{(\epsilon(k') - \mu)^2 + \Delta(k')^2} \]

- \text{FINITE TEMPERATURE}
- first calculations at finite T, still in progress
- relevant for astrophysics and neutron stars

S. Maurizio, J. W. Holt and Paolo Finelli, submitted to PRC and in preparation
Applications - BCS/BEC

A. J. Leggett

Coherence length $\xi$

Coupling constant strength $g$

$g$ large

$\kappa_F \xi << 1$

$g$ small

$\kappa_F \xi >> 1$

For nuclear systems the following parameters have been suggested:

$\Delta F / \epsilon_F$

$\xi_{RMS} / d$

$P(d)$

$\rho(r) = |\psi(r)|^2 r^2$

$P(r) = \int_0^r dr' \rho(r')$

The probability density to find two nucleons at a distance $r$ in the $^1S_0$ neutron matter channel evaluated using $V_{NN}$ only.

Paolo Finelli et al, in preparation
Applications - Finite nuclei

\[ \left( \begin{array}{c} h - \mu \\ \Delta \end{array} \right) \left( \begin{array}{c} \Delta \\ -h + \mu \end{array} \right) = E_k \left( \begin{array}{c} U \\ V \end{array} \right) \]

In the Hamiltonian \( h \) we have \( V_{ph} \): particle-hole potential

\( \mu \): chemical potential

\( \Delta \): pairing field

State-dependent single-particle states

\[ \bar{\Delta} = \frac{\sum_k \Delta_k v_k^2}{\sum_k v_k^2} \]

Occupation factors

\[ \Delta^{(5)}(N_0) = -\frac{1}{8} [E(N_0 + 2) - 4E(N_0 + 1) + 6E(N_0) - 4E(N_0 - 1) + E(N_0 - 2)] \]

Theory

Empirical Estimates

With N3LO pairing gaps in reasonable agreement with exp. data.

With the inclusion of three-body forces pairing gaps are reduced by 30/40% [See also PRC 80 (2009) 044321].

Binding energies in very good agreement.

Paolo Finelli et al, PRC 86 (2012) 034327
Neutrino emission from the formation (and breaking) of Cooper pairs

Superfluid properties

Equations of state

Minimal cooling model

Envelope composition

Stellar mass

$k_B T_c \approx 0.57 \Delta_0$

http://www.astroscu.unam.mx/neutrones/NSCool/
Cooling of a 1.4 $M_\odot$ neutron star

$^3$PF$_2$ gaps from microscopic forces can describe recent data for cooling of neutron stars (NSCool code).

Data trend is not correctly reproduced.

Preliminary!
The end