

Avenues of cognition of nongravitational local gauge field theories

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Abstract

This controbution is devoted to present basic fearures of a unifying local gauge field theory, prevailing up to a mass scale of approximately 10^{16} GeV , allowing the neglect of gravitational curvature effects – indicated by the attribute : 'nongravitational' in the title above .

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1 - Introduction

The two main mass generation mechanisms within a general gauge field theory in **3 + 1 uncurved space-time dimensions** – henceforth called gravitationless gauge field theory –

– minimally the neutrino mass extended standard model based on the gauge group

$SU3_c \times SU2_L \times U1_y$ and one scalar doublet with respect to $SU2_L$ –

form the basis of the present outline, supposed to be an integral part of an enveloping unifying gauge group, minimally $SO10 \equiv spin10$.

1) the Bose condensation of some components of elementary scalar fields
scalar stands here for scalar and/or pseudoscalar Yukawa couplings

2) the Bose condensation of the gauge field-strength bilinear

– gauge- and renormalization group invariant with respect to the *unbroken* gauge group $SU3_c$ –

Table 1

The key features of point 1) in table 1) are outlined in section 2.

The main topic here, point 2) in table 1, is elaborated on in section E.



2 - Bose condensation of elementary scalar fields – the Brout-Englert-Higgs effect

Recent assessments can be found in the talks of François Englert – [1-2013] and, more historically oriented, Peter Higgs [2-2013] at the Nobel Prize 2013 awards ceremony.

We base the general properties of spontaneous gauge breaking of an enveloping gravitationless gauge field theory, based on a gauge group $G_{env} \supset G_{min}$ in the sense of gauge group unification beyond the minimal case $G_{min} = SU3_c \times SU2_L \times U1_\gamma$, presented in the introduction.

A minimal such enveloping gauge group is

$$(1) \quad G_{env}^{min} = SO(10) \equiv spin(10)$$

as discussed in refs. [3-1975] and [4-2008].

Within larger **simple** enveloping groups the exceptional chain

$$(2) \quad G_{env} \rightarrow E6 \subset E8 \supset E6 \times SU(3)$$

is singled out [5-1976], [6-1980], most importantly because it offers the possibility of canceling all gauge- and gravitational anomalies in the product gauge group [7-1984], [8-2012]

$$(3) \quad G_{env} = E8 \times E8$$

Following the hypothesis of an underlying unifying gauge group the top down of gauge breaking is initiated by a primary breaking followed last by the electroweak gauge breaking . →

2-1 - Primary gauge breaking

Primary gauge breaking is linked to the unifying gauge group scale M_{env} assumed and also restricted by limits on direct observation of baryon decays and lepton flavor violation to be much larger than the electroweak scale

$$(4) \quad M_{env} \gg v = \left(\sqrt{\sqrt{2} G_F} \right)^{-1} = 246.220 \text{ GeV}$$

In eq. 4 v denotes the v.e.v. of the unique doublet scalar field using the quaternion associated basis for the local scalar fields

$$z(x) = \begin{pmatrix} \varphi^0 & -(\varphi^-)^* \\ \varphi^- & (\varphi^0)^* \end{pmatrix} (x)$$

$$\varphi^0 = 1/\sqrt{2} (Z_0 - i Z_3) ; \quad \varphi^- = 1/\sqrt{2} (Z_2 - i Z_1)$$

$$Z_\mu = (Z_\mu)^* = (Z_0, Z_1, Z_2, Z_3) = (Z_0, \vec{Z})$$

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_\mu = (\sigma_0, \sigma_1, \sigma_2, \sigma_3) = (\sigma_0, \vec{\sigma})$$

(5)



2-3

In eq. 5 the symbol $*$ denotes hermitian conjugation of individual complex and/or real field components. Thus the quantity z , defined in eq. 5, shows its quaternionic representation

$$(6) \quad z(x) = \frac{1}{\sqrt{2}} \left(Z_0 \sigma_0 + \sum_{k=1}^3 Z_k \frac{1}{i} \sigma_k \right) (x)$$

The four 2×2 matrices, displayed in eq. 5, form a 1 to 1 true representation of the base quaternions

$$(7) \quad \begin{aligned} \sigma_0 &\leftrightarrow q_0 = \mathbb{1} ; \quad \frac{1}{i} \sigma_m \leftrightarrow q_m \quad \text{for } m = 1, 2, 3 \\ q_m q_n &= -\delta_{mn} q_0 + \varepsilon_{mnr} q_r \quad \text{for } m, n, r = 1, 2, 3 \end{aligned}$$

As we will see, the final stage of the (nu-mass extended -) standard model gauge breaking involving *just* 1 doublet of scalars with respect to the electroweak part $SU(2)_w \times \mathcal{Y}_w$ represents a case for perfectly semiclassical, driven Bose condensation, eventually contrasting with intrinsic properties of primary breakdown.

This is so, because there is precisely 1 invariant

$$(8) \quad I(z, z^\dagger) = z z^\dagger = \frac{1}{2} \sum_{\mu=0}^3 (Z_\mu)^2$$

with respect to $SU(2)_w \times \mathcal{Y}_w$ of which a general invariant is a function.

This would not remain true for more than one scalar doublet.



As a consequence of eqs. 4 - 8 , the electroweak gauge breaking is *driven and semiclassical*

$$(9) \quad \langle \Omega | z(x) | \Omega \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} \quad \text{with } v = 246.220 \text{ GeV, independent of } x$$

We shall discuss 2 types of primary gauge breaking denoted a) and b) below .

The v.e.v. v in eq. 9 corresponds to the classical minimum of the quartic potential , uniquely restricted to depend on two parameters

$$(10) \quad V(z, z^\dagger) = \left[-\mu^2 z z^\dagger + \lambda (z z^\dagger)^2 \right]_{11}$$

The minimum conditions become

$$(11) \quad \begin{aligned} \partial_{Z_\nu} V &= \left(-\mu^2 + \lambda |Z|^2 \right) Z_\nu = 0 \quad \longrightarrow \quad z z^\dagger |_{11} = \frac{1}{2} \mu^2 / \lambda \\ V|_{min} &= -\frac{1}{4} (\mu^4 / \lambda) \end{aligned}$$

The second derivatives with respect to Z at the minimum of the potential become

$$(12) \quad \frac{1}{2} \partial_{Z_\rho} \partial_{Z_\sigma} V = \lambda Z_\rho Z_\sigma |_{min} = \lambda v^2 \delta_{0\rho} \delta_{0\sigma} ; \quad Z_\nu |_{min} = (v, \vec{0})$$

Expanding the deviation of the potential up to quadratic terms around the minimum thus yields \longrightarrow

2-5

$$(13) \quad (\Delta V)^{(2)} = \lambda v^2 (\Delta Z_0)^2 \quad ; \quad Z_0(x) = v + \Delta Z_0(x)$$

It is customary to denote the shifted hermitian field $\Delta Z_0(x)$

$$(14) \quad Z_0 = v + \Delta Z_0 \quad ; \quad \Delta Z_0(x) = H(x)$$

From eq. 14 we read off the mass of the field $H(x)$ as well as the vanishing of the mass of the other three fields

$$(15) \quad m_H^2 = 2\lambda v^2 = 2\mu^2 \quad ; \quad m_{\vec{Z}} \Big|_{from V} = 0 \quad ; \quad \vec{Z}(x) = (Z_1, Z_2, Z_3)(x)$$

However in the case at hand the existence of would-be long range forces represented by the $SU2_L \times U1_Y$ gauge field interactions does not permit the existence of goldstone-modes .

The three would-be Goldstone fields, defined in eq. 15, through their space-time gradient, mix with the gauge bosons to become massive, vis. W^\pm and \tilde{Z}

$$(16) \quad \partial_{x^\tau} Z_m(x) \leftrightarrow W_\tau^\pm(x), \tilde{Z}_\tau(x)$$

in such a way as to obtain masses of the 3 massive gauge bosons $m_W, m_{\tilde{Z}}$ and physically form the longitudinal spin components of the resulting massive states (resonances) . In eq. 16 the neutral massive gauge boson is denoted \tilde{Z} not to confuse it with the sclar field components Z_0, \vec{Z} . \longrightarrow

2-6

In tree approximation the mass-square of the H-field is twice the value of the Z_0 field in the unbroken case, i.e. for $\mu^2 \rightarrow -\mu^2$

$$(17) \quad m_H^2 = 2\mu^2 \rightarrow \mu = \frac{1}{\sqrt{2}} m_H = 88.388 \text{ GeV for } m_H = 125 \text{ GeV}$$

The detailed description of the mixing as stated in eq. 16 is not given here. A complete derivation can be found in the textbook [9-1995] .

The simplicity of the presumably lowest in scale gauge breaking relative to the SM gauge group $SU2_L \times U1_Y$ prompted most discussions of primary gauge breaking to be of the same type b) presented below, i.e. driven - semiclassical: for an example of primary gauge breakdown patterns see e.g. ref. [10-1013] .

This brings us to the two types a) and b) – of an enveloping gravitationless gauge field theory. For both types the scalar field variables turn out to involve a *complex* ensemble of irreducible representations of the enveloping gauge group – as e.g. $SO10$, in particular for the generation of masses for neutrino flavors, both light and heavy, as discussed e.g. in refs. 4 = [4-2008] and 10 = [10-1013] through Yukawa couplings to basic fermion bilinears .

Two recent papers deserve mention here by Guido Altarelli ref.11 i = [11-2014] and Ferruccio Feruglio, Ketan M. Patel and Denise Vicino ref. 12 = [12-2014]

We introduce notation for the ensemble of scalar fields adapted to primary gauge breaking, generalizing $SU2_L$ doublets as defined in eqs. 5 - 7 .



Lets fix for definiteness

$$(18) \quad G_{env}^{min} = SO10 \equiv spin10$$

in the following. A general irreducible representation of $SO10$ shall be denoted $[D]$, where D is equivalenced to its dimension. As entry point we take the $[16]$ representation for one fermion family (the lightest in mass) in the left chiral basis

$$(19) \quad [16] : \left(\begin{array}{cccc|cccc} u^1 & u^2 & u^3 & \nu_e & \mathcal{N}_e & \hat{u}^3 & \hat{u}^2 & \hat{u}^1 \\ d^1 & d^2 & d^3 & e^- & e^+ & \hat{d}^3 & \hat{d}^2 & \hat{d}^1 \end{array} \right)^{\hat{\gamma} \rightarrow L} \\ = (f)^{\hat{\gamma}}$$

It is instructive to illustrate the use of the basis defined in eq. 19 considering

the Majorana logic characterized by $\mathcal{N}_{e,\mu,\tau}$

derived from the subgroup decomposition

$$(20) \quad spin10 \rightarrow SU5 \times U1_{J_5}$$

Among the 3 generators of $spin10$ commuting with $SU3_c$, I_{3L} , I_{3R} , $B - L$ and forming part of the Cartan subalgebra of $spin10$ there is one combination, denoted J_5 in eq. 20, commuting with its largest unitary subgroup $SU5$. The charges $Q(J_5)$ form the pattern as in eq. 21 \longrightarrow

$$(21) \quad [16] : Q (J_5) = \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & -3 & 5 & 1 & 1 & 1 \\ 1 & 1 & 1 & -3 & 1 & -3 & -3 & -3 \end{array} \right)$$

$Q (J_\mu)$ with charges as given in eq. 21 represents a *hermitian* generator of the Cartan subalgebra of $spin10$, unique up to a (real) multiplicative factor, which commutes with the the $SU5$ subgroup of $spin10$. The flavors within one family sharing the same Q-charges in fact form irreducible representations of $SU5$, which shall be labeled

$$(22) \quad \mathcal{D} (SU5) \rightarrow \{ \mathcal{D} (SU5) \}_Q$$

with $\mathcal{D} (SU5)$ equivalenced with the dimension of the representation. The suffix Q is added in eq. 22, since in the present context $SU5$ multiplets necessarily occur *embedded* in $SO (10)$ representations. Thus the Q values of the [16] representation on the right han side of eq. 21 translate to

$$(23) \quad [16] = \{1\}_5 + \{10\}_1 + \{\bar{5}\}_{-3}$$

The sequence of Q values : $Q = 5, 1, -3$ as arranged in the sequence on the right hand side of eq. 23 – decreasing in steps of 4 – is related to the properties of binomial coefficients

$$(24) \quad \binom{10}{n} \quad \text{for } n = 0, 4, 8 \quad \text{and } n = 10, 6, 2$$



This is derived in ref. 4 – [4-2008] .

It is instructive to decompose the Q-value pattern in eq. 21 into an $SU2_{L+R}$ invariant part and the remainder proportional to $I_{3 L+R}$

$$(25) \quad \begin{pmatrix} 1 & 1 & 1 & -3 & | & 5 & 1 & 1 & 1 \\ 1 & 1 & 1 & -3 & | & 1 & -3 & -3 & -3 \\ 1 & 1 & 1 & -3 & | & 3 & -1 & -1 & -1 \\ 1 & 1 & 1 & -3 & | & 3 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & | & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & | & -2 & -2 & -2 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & -3 & | & 3 & -1 & -1 & -1 \\ 1 & 1 & 1 & -3 & | & 3 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & | & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & | & -2 & -2 & -2 & -2 \end{pmatrix} +$$

From eq. 25 we obtain the identification of the $spin(10)$ Cartan subalgebra hermitian components (charges)

$$(26) \quad Q = 3(B - L) - 4I_{3 R}$$

In eq. 26 B and L denote baryon and lepton number respectively .



Two remarks shall follow , concerning the recognizable key features inherent to spontaneous gauge breaking of an enveloping gravitationless gauge field theory , based on a gauge group

$$G_{env} = spin(10)$$

1) primary gauge breakdown

must be much different than on the lowest – i.e. electroweak – scale level pertaining to

$$G_{SM} = SU3_c \times SU2_L \times U1_\gamma .$$

This is so because empirically well established candidate symmetries , like baryon and lepton number conservation are broken on the primary level and imply very large scale of unification

$$M_{env} = O(10^{16} \text{ GeV}) .$$

As examples let me quote the upper limits of the $\mu^+ \rightarrow e^+ \gamma$ and $\mu^+ \rightarrow e^+ + e^+ e^-$ branching fractions

$$(27) \quad Br(\mu^+ \rightarrow e^+ + \gamma) < 2.4 \cdot 10^{-12}$$

$$Br(\mu^+ \rightarrow e^+ + e^+ e^-) < 1.0 \cdot 10^{-12} \quad \text{in ref. 11 – [13-2013]}$$

2) the power of the set of scalar fields

involved in primary gauge breaking does not follow any principle of minimal selection of $spin(10)$ representations pertaining to scalar fields .



specific notation for scalar field variables

We proceed defining notation for scalar field variables suitable for primary gauge breaking

$$(28) \quad \{ \underline{z} \} = \underline{z} \left[\begin{array}{c} \left(\mathcal{D}^{(1)} \quad , \quad \otimes n_1 \right) \\ \left(\mathcal{D}^{(2)} \quad , \quad \otimes n_2 \right) \\ \left(\dots \quad , \quad \otimes \dots \right) \end{array} \right] \quad | \text{ broken down to real coordinates}$$

In eq. 28 $\mathcal{D}^{(\nu)}$; $\nu = 1, 2, \dots$ denote a complete set of unitary irreducible representations of $spin(10)$, finite dimensional; constructively defined through the method of Peter and Weyl [14-1927]. n_ν stands for the multiplicity of a given representation $\mathcal{D}^{(\nu)}$.

For $\mathcal{D}^{(\nu)}$, $\overline{\mathcal{D}}^{(\nu)}$ being a pair of inequivalent, relative complex conjugate representations the coordinates $\underline{z}(\mathcal{D}^{(\nu)})$, $\underline{\bar{z}}(\overline{\mathcal{D}}^{(\nu)})$ broken down to real and imaginary parts count as $2(\text{complex}) \dim(\mathcal{D})$ components over real numbers. This is the meaning of the attribute 'broken down to real coordinates' on the right hand side of eq. 28.

Thus choosing real values for the components of $\underline{z} \in R^M$ as defined in eq. 28 it follows

$$(29) \quad \underline{z} = (z_1, \dots, z_r, \dots, z_M) \quad ; \quad z_r : \text{ hermitian fields}$$



we find

$$(30) \quad M = \sum_{\nu} n_{\nu} \left\{ \begin{array}{l} \dim (\mathcal{D}^{(\nu)}) \text{ for } \mathcal{D}^{(\nu)} \text{ real} \\ 2 \text{ complex } \dim (\mathcal{D}^{(\nu)}) \text{ for } \mathcal{D}^{(\nu)} \text{ complex} \end{array} \right\} < \infty$$

to get an idea of the power of the set of scalars

We illustrate the order of M , in eqs. 29 , 30 by the representations of the fermion bilinears from left and right chiral bases, adapting the scalar variables to their definition in eq. 29 , allowing for complex linear combinations applied to complex representations (from section E and ref. 4 – [4-2008]) .

$\mathcal{H}_{fermion\ mass} \longleftarrow$

$$(31) \quad \left(\bar{z}^{\overline{126}} F G \right)_{\bar{\xi}} (f_{a\ 16\ F})_{\dot{\gamma}} (f_{b\ 16\ G})^{\dot{\gamma}} C \left(\begin{array}{c|cc} 126 & 16 & 16 \\ \xi & a & b \end{array} \right) + h.c.$$

$\left(\bar{z}^{\overline{126}} F G \right)_{\bar{\xi}} : \text{(pseudo-) scalar fields in the } \overline{126} \text{ representation of SO (10)}$

$F , G = I, II, III : \text{fermion family indices}$



2-13

In eq. 31 $C \left(\begin{array}{c|cc} 126 & 16 & 16 \\ \xi & a & b \end{array} \right)$ denotes the *spin* (10) Clebsch-Gordan coefficients projecting the product of two (fermionic) 16 representations on irreducible spin (10) representations . We reproduce the 4 product representations of 16 and $\overline{16}$ representations from ref. 4 (Appendix E) – [4-2008])

(32)

		[16]	[16]
[16]	$s :$	[10] + [126]	[1] + [45] + [210]
			$, a : [120]$
$\overline{[16]}$	$s :$	[1] + [45] + [210]	[10] + $\overline{[126]}$
			$, a : [120]$

The real and complex representations in the multiplication table in eq. 32 are denoted $\mathcal{D}_R, \mathcal{D}_C$ respectively

(33)

$$\mathcal{D}_R : [10]_R, [120]_R, [1]_R, [45]_R, [210]_R$$

$$\mathcal{D}_C : [126]_C, \overline{[126]}_C$$



Choosing minimally multiplicities 2 for real and 1 for complex representations in eqs. 32 and 33 yields

$$(34) \quad M = 772 + 252 = 1028$$

We conclude from the example multiplicities leading to eq. 34

$$(35) \quad M \geq O(1000)$$

2-1-a - Primary gauge breaking - type a)

The gauge breaking in type a) *gravitationless gauge field theory* is

1) driven

by pre-established quadratic, cubic and quartic scalar field self interactions

2) *not* reducible to semiclassical approximation

for vacuum expected values for scalar variables and their composite operators, necessarily include gauge variant ones in order to qualify for *gauge breaking*

For clarity let me remark that condition 2) above is necessary, since in the case of *exclusively* gauge invariant vev's for composite scalar operators they are part of an alternative case – in conjunction with other gauge invariant composite field variables – of spontaneous mass generation without gauge breaking. This will be discussed within QCD with 3 light flavors – without scalars – elsewhere . →

2-1-b - Primary gauge breaking - type b)

The gauge breaking in type b) *gravitationless gauge field theory* is

1) driven

by pre-established quadratic, cubic and quartic scalar field self interactions like in type a)

2) reducible to semiclassical approximation

for vacuum expected values for scalar variables and their composite operators. This is the usual case discussed in the literature, as e.g. in refs. 10 – [10-1013] – and 15 – [15-2013] , while in the latter reference the main topic is electroweak gauge breaking .

The semiclassical approximation – with respect to vev's of scalar fields and their composite local operators as well as composite operators involving other fields – means

$$(36) \quad \langle \Omega | f(\underline{z}) | \Omega \rangle = f \left(\langle \Omega | \underline{z} | \Omega \rangle \right)$$



E1

E – The spin10 product representations

$$(16 \oplus \overline{16}) \otimes (16 \oplus \overline{16})$$

We follow the spin10 decomposition discussed in section 2-1 (eq. 37 repeated below)

$$(37) \quad \text{spin}(10) \rightarrow \text{SU5} \times \text{U1}_{J_5}$$

Further let us denote representations of spin10 as opposed to those pertaining to SU5 and associated J_5 quantum number by

$$(38) \quad \text{spin}(10) : [dim] \quad ; \quad \text{SU5} \times \text{U1}_{J_5} : \{dim\}_{J_5}$$

Thus we obtain

$$(39) \quad \begin{aligned} [16] &= \{1\}_{+5} + \{10\}_{+1} + \{\overline{5}\}_{-3} \\ [\overline{16}] &= \{1\}_{-5} + \{\overline{10}\}_{-1} + \{5\}_{+3} \end{aligned}$$

In turn SU5 representations shall be decomposed along the standard model gauge group

$\text{SU3}_c \otimes \text{SU2}_L \otimes \text{U1}_{\mathcal{Y}}$, where \mathcal{Y} denotes the electroweak hypercharge (with a factor $\frac{1}{2}$ included)

$$(40) \quad \mathcal{Y} = Q_{e.m.} / e - I_{3L}$$



E2

$$(41) \quad \{dim\} \rightarrow \sum](dim\mathbf{SU3}_c, dim\mathbf{SU2}_L) \gamma[$$

The brackets on the right hand side of eq. 41 are reversed in order not to confuse spin10 - and standard model representations.

Then the base $16 (\overline{16})$ decompose to

$$(42) \quad [16] \rightarrow \left[\begin{array}{l} \{1\}_{+5} \rightarrow \left\{ \begin{array}{l}](1, 1)_0 [\end{array} \right\}_{+5} \\ \{10\}_{+1} \rightarrow \left\{ \begin{array}{l}](3, 2)_{+\frac{1}{6}} [+ \\](\overline{3}, 1)_{-\frac{2}{3}} [+ \\](1, 1)_{+1} [\end{array} \right\}_{+1} \\ \{\overline{5}\}_{-3} \rightarrow \left\{ \begin{array}{l}](\overline{3}, 1)_{+\frac{1}{3}} [+ \\](1, 2)_{-\frac{1}{2}} [\end{array} \right\}_{-3} \end{array} \right.$$

The product representations $(16 \oplus \overline{16}) \otimes (16 \oplus \overline{16})$ generate all SO10 antisymmetric tensor ones, of which we encountered the fivefold antisymmetric in section 2-1 →

E3

To elaborate we specify the n-fold antisymmetric tensors obtained from the 10-representation of SO (10)

$$\begin{aligned}
 [t_0] &\sim 1 \\
 [t_1]^A &\sim z^A \quad ; \quad A = 1, 2, \dots, 10 \quad \leftrightarrow \quad [t_1] = \{\bar{5}\}_2 \oplus \{5\}_{-2} \\
 [t_2]^{[A_1 A_2]} &\sim \frac{1}{2} \left(z_1^{A_1} z_2^{A_2} - z_1^{A_2} z_2^{A_1} \right) \\
 &\dots \\
 [t_n]^{[A_1 A_2 \dots A_n]} &\sim \frac{1}{n!} \sum \text{sgn} \begin{pmatrix} 1 & \dots & n \\ \pi_1 & \dots & \pi_n \end{pmatrix} z_1^{A_{\pi_1}} z_2^{A_{\pi_2}} \dots z_n^{A_{\pi_n}} \\
 n &\leq 10
 \end{aligned}
 \tag{43}$$

The quantities $[t_n]$ defined in eq. 43 form irreducible real representations of SO (10) except for $n = 5$, which is composed of the relatively complex *irreducible* representations 126 and $\overline{126}$

The tenfold antisymmetric invariant corresponds to $[t_{n=10}]$. The product of two full Clifford algebras pertaining to spin10 contains all $[t_n]$; $n = 0 \dots 10$ representations exactly once . →

E4

Treating the $n = 5$ tensor as one representation – it is reducible only over \mathbb{C} – the dimensions of the $[t_n]$ representations follow Pascal's triangle .

$[t_0]$	$[t_2]$	$[t_4]$	$[t_6]$	$[t_8]$	$[t_{10}]$
	$[t_1]$	$[t_3]$	$[t_5]$	$[t_7]$	$[t_9]$
1	45	210	210	45	1
	10	120	252	120	10

(44)

This corresponds to the following products of $16 + \overline{16}$

(45)

		$[16]$		$[\overline{16}]$
$[16]$	$s :$	$[10] +$ $[126]$	$, a : [120]$	$[1] + [45] +$ $[210]$
$[\overline{16}]$	$s :$	$[1] + [45] +$ $[210]$	$, a : [120]$	$[10] +$ $[\overline{126}]$



E5

The correspondence of product representations of the $16 + \overline{16} = 32$ associative Clifford algebra with the sum of antisymmetric tensor ones follows from the completeness of all products of γ matrices forming the spin (10) algebra i.e. are of dimension

$$(46) \quad (32)^2 = (2^5)^2 = 2^{10}$$

We proceed to reduce the $[16] \otimes [16]$ product with respect to J_5 , SU5 and $SU3_c \times SU2_L \times U1_\gamma$.

The individual products are $\left(\begin{smallmatrix} s \\ a \end{smallmatrix} \right) : (\mathbf{a})\text{symmetric}$

	$\{1\}_5$	$\{10\}_1$	$\{\overline{5}\}_{-3}$
$\{1\}_5$	$\{1\}_{10_s}$	$\{10\}_6$	$\{\overline{5}\}_2$
$\{10\}_1$	$\{10\}_6$	$\left(\begin{smallmatrix} \{\overline{5}\}_2^+ \\ \{\overline{50}\}_2 \end{smallmatrix} \right)_s$	$\left(\begin{smallmatrix} \{\overline{45}\}_2 \\ \{45\}_{-2} \end{smallmatrix} \right)_a$
$\{\overline{5}\}_{-3}$	$\{\overline{5}\}_2$	$\left(\begin{smallmatrix} \{5\}_{-2}^+ \\ \{45\}_{-2} \end{smallmatrix} \right)_s$	$\left(\begin{smallmatrix} \{\overline{15}\}_{-6} \\ \{\overline{10}\}_{-6} \end{smallmatrix} \right)_a$

(47)



E6

We proceed to decompose the diagonal $\{\text{SU5}\}_{J_5}$ representations (eq. 42)

$$(\{10\}_1 \otimes \{10\}_1)_s = \{\bar{5}\}_2 + \{\bar{50}\}_2 \quad \downarrow$$

s	$\left] (3, 2) + \frac{1}{6} \left[\begin{matrix} +1 \end{matrix} \right. \right]$	$\left] (\bar{3}, 1) - \frac{2}{3} \left[\begin{matrix} +1 \end{matrix} \right. \right]$	$\left] (1, 1) +1 \left[\begin{matrix} +1 \end{matrix} \right. \right]$
(48)	$\left] (3, 2) + \frac{1}{6} \left[\begin{matrix} +1 \end{matrix} \right. \right]$	$\left(\begin{matrix} \left] (6, 3) + \frac{1}{3} \left[\begin{matrix} 2 \\ + \end{matrix} \right. \right] \\ \left] (\bar{3}, 1) + \frac{1}{3} \left[\begin{matrix} 2 \end{matrix} \right. \right] \end{matrix} \right)$	$\left(\begin{matrix} \left] (8, 2) - \frac{1}{2} \left[\begin{matrix} 2 \\ + \end{matrix} \right. \right] \\ \left] (1, 2) - \frac{1}{2} \left[\begin{matrix} 2 \end{matrix} \right. \right] \end{matrix} \right)$
	$\left] (\bar{3}, 1) - \frac{2}{3} \left[\begin{matrix} +1 \end{matrix} \right. \right]$	$\left] (\bar{6}, 1) - \frac{4}{3} \left[\begin{matrix} 2 \end{matrix} \right. \right]$	$\left] (\bar{3}, 1) + \frac{1}{3} \left[\begin{matrix} 2 \end{matrix} \right. \right]$
	$\left] (1, 1) +1 \left[\begin{matrix} +1 \end{matrix} \right. \right]$		$\left] (1, 1) +2 \left[\begin{matrix} 2 \end{matrix} \right. \right]$

→

E7

$$\left(\{\bar{5}\}_{-3} \otimes \{\bar{5}\}_{-3} \right)_s = \{\bar{15}\}_{-6} \quad \downarrow$$

(49)

s	$\left[(\bar{3}, 1) + \frac{1}{3} \left[\begin{smallmatrix} - \\ -3 \end{smallmatrix} \right] \right] (1, 2) - \frac{1}{2} \left[\begin{smallmatrix} - \\ -3 \end{smallmatrix} \right]$
$\left[(\bar{3}, 1) + \frac{1}{3} \left[\begin{smallmatrix} - \\ -3 \end{smallmatrix} \right] \right]$	$\left[(\bar{6}, 1) + \frac{2}{3} \left[\begin{smallmatrix} - \\ -6 \end{smallmatrix} \right] \right] (\bar{3}, 2) - \frac{1}{6} \left[\begin{smallmatrix} - \\ -6 \end{smallmatrix} \right]$
$\left[(1, 2) - \frac{1}{2} \left[\begin{smallmatrix} - \\ -3 \end{smallmatrix} \right] \right]$	$\left[(1, 3) - 1 \left[\begin{smallmatrix} - \\ -6 \end{smallmatrix} \right] \right]$

↓

complex e.w. triplet coupling to

$$\frac{1}{2} \left(\nu_{\dot{F}}^* \right)^\alpha \left(\nu_{\dot{G}}^* \right)_\alpha$$

Next we assemble the (anti)symmetric products $\left([16] \otimes [16] \right)_s = [10] \oplus [126]$ **and**
 $\left([16] \otimes [16] \right)_a = [120]$ **with respect to** $SU5 \otimes U1_{J_5}$ →

E8

using eq. 47

$$\begin{aligned}
 & ([16] \otimes [16])_s = [10] \oplus [126] \quad \downarrow \\
 (50) \quad & = \left\{ \begin{array}{c} \left[\begin{array}{c} \{5\}_{-2} + \\ \{\bar{5}_I\}_2 \end{array} \right] \\ \oplus \left[\begin{array}{c} \{1\}_{10} + \{\bar{5}_{II}\}_2 + \{10\}_6 + \{\bar{15}\}_{-6} \\ + \{45\}_{-2} + \{\bar{50}\}_2 \end{array} \right] \end{array} \right\}
 \end{aligned}$$

$$([16] \otimes [16])_a = [120] \quad \downarrow$$

$$= \left[\begin{array}{c} \{5\}_{-2} + \{\bar{5}\}_2 \\ + \{10\}_6 + \{\bar{10}\}_{-6} \\ + \{45\}_{-2} + \{\bar{45}\}_2 \end{array} \right]$$

The roman indices I, II in eq. 50 indicate that appropriate linear combinations of the two $\{\bar{5}\}_2$ representations form parts of $[10]$ and $[126]$ respectively . →

E9

It remains to decompose the $SU_5 \otimes U_1$ representations in eq. 50 with respect to $SU_3_c \times SU_2_L \times U_1_Y$. We do this associating according to the product representations as they appear in eq. 50

$$\begin{array}{l}
 [10] [120] \quad \{5\}_{-2} \quad \left] (3, 1) - \frac{1}{3} \left[\begin{array}{c} + \\ +3 \end{array} \right] (1, \bar{2}) + \frac{1}{2} \left[\begin{array}{c} + \\ +3 \end{array} \right] \\
 [10] [126] [120] \quad \{\bar{5}\}_{+2} \quad \left] (\bar{3}, 1) + \frac{1}{3} \left[\begin{array}{c} + \\ -3 \end{array} \right] (1, 2) - \frac{1}{2} \left[\begin{array}{c} + \\ +3 \end{array} \right] \\
 \hline
 [126] \quad \{1\}_{+10} \quad \left] (1, 1)_0 \left[\begin{array}{c} + \\ +10 \end{array} \right] \\
 [126] [120] \quad \{10\}_{+6} \quad \left] (3, 2) - \frac{1}{6} \left[\begin{array}{c} + \\ +6 \end{array} \right] (\bar{3}, 1) - \frac{2}{3} \left[\begin{array}{c} + \\ +6 \end{array} \right] (1, 1)_{+1} \left[\begin{array}{c} + \\ +6 \end{array} \right] \\
 [120] \quad \{\bar{10}\}_{-6} \quad \left] (\bar{3}, \bar{2}) + \frac{1}{6} \left[\begin{array}{c} + \\ -6 \end{array} \right] (3, 1) + \frac{2}{3} \left[\begin{array}{c} + \\ -6 \end{array} \right] (1, 1)_{-1} \left[\begin{array}{c} + \\ -6 \end{array} \right]
 \end{array}$$

(51)



E10

$$[126] \quad \{\overline{15}\}_{-6} \quad \left[(\overline{6}, 1) + \frac{2}{3} \left[\begin{smallmatrix} - \\ -6 \end{smallmatrix} + \right] (\overline{3}, 2) - \frac{1}{6} \left[\begin{smallmatrix} - \\ -6 \end{smallmatrix} + \right] (1, 3) -_1 \left[\begin{smallmatrix} - \\ -6 \end{smallmatrix} \right. \right.$$

$$[126] [120] \quad \{45\}_{-2}$$

c.c. \updownarrow

$$[120] \quad \{\overline{45}\}_{+2}$$

$$\left[\begin{array}{l} \left(\begin{array}{l} \left[\begin{smallmatrix} - \\ -6 \end{smallmatrix} \right] (6, 1) + \frac{1}{3} \left[\begin{smallmatrix} - \\ 2 \end{smallmatrix} + \right] \\ \left[\begin{smallmatrix} - \\ -6 \end{smallmatrix} \right] (\overline{3}, 3) + \frac{1}{3} \left[\begin{smallmatrix} - \\ 2 \end{smallmatrix} \end{array} \right) + \left(\begin{array}{l} \left[\begin{smallmatrix} - \\ -6 \end{smallmatrix} \right] (8, 2) - \frac{1}{2} \left[\begin{smallmatrix} - \\ 2 \end{smallmatrix} + \right] \\ \left[\begin{smallmatrix} - \\ -6 \end{smallmatrix} \right] (1, 2) - \frac{1}{2} \left[\begin{smallmatrix} - \\ 2 \end{smallmatrix} \end{array} \right) + \\ \left[\begin{smallmatrix} - \\ -6 \end{smallmatrix} \right] (3, 2) + \frac{7}{6} \left[\begin{smallmatrix} - \\ 2 \end{smallmatrix} + \right] (3, 1) - \frac{4}{3} \left[\begin{smallmatrix} - \\ 2 \end{smallmatrix} + \right] (\overline{3}, 1) + \frac{1}{3} \left[\begin{smallmatrix} - \\ 2 \end{smallmatrix} \end{array} \right. \end{array} \right]$$

$$[126] \quad \{\overline{50}\}_{+2}$$

$$\left[\begin{array}{l} \left(\begin{array}{l} \left[\begin{smallmatrix} - \\ -6 \end{smallmatrix} \right] (6, 3) + \frac{1}{3} \left[\begin{smallmatrix} - \\ 2 \end{smallmatrix} + \right] \\ \left[\begin{smallmatrix} - \\ -6 \end{smallmatrix} \right] (\overline{3}, 1) + \frac{1}{3} \left[\begin{smallmatrix} - \\ 2 \end{smallmatrix} \end{array} \right) + \left[\begin{smallmatrix} - \\ -6 \end{smallmatrix} \right] (8, 2) - \frac{1}{2} \left[\begin{smallmatrix} - \\ 2 \end{smallmatrix} + \right] (3, 2) + \frac{7}{6} \left[\begin{smallmatrix} - \\ 2 \end{smallmatrix} \right. \\ \left. + \left[\begin{smallmatrix} - \\ -6 \end{smallmatrix} \right] (\overline{6}, 1) - \frac{4}{3} \left[\begin{smallmatrix} - \\ 2 \end{smallmatrix} + \right] (1, 1) +_2 \left[\begin{smallmatrix} - \\ 2 \end{smallmatrix} \right. \end{array} \right]$$

(52)

\rightarrow

E11

$$(\{10\}_1 \otimes \{10\}_1)_s = \{\overline{45}\}_2 \quad \downarrow$$

a	$](3, 2) + \frac{1}{6} [_{+1}$	$](\overline{3}, 1) - \frac{2}{3} [_{+1}$	$](1, 1) +1 [_{+1}$
(53)	$\left(\begin{array}{l}](6, 1) + \frac{1}{3} [_{2^+} \\](\overline{3}, 3) + \frac{1}{3} [_{2^-} \end{array} \right)$	$\left(\begin{array}{l}](8, 2) - \frac{1}{2} [_{2^+} \\](1, 2) - \frac{1}{2} [_{2^-} \end{array} \right)$	$](3, 2) + \frac{7}{6} [_{2^-}$
	$](\overline{3}, 1) - \frac{2}{3} [_{+1}$	$](3, 1) - \frac{4}{3} [_{2^-}$	$](\overline{3}, 1) + \frac{1}{3} [_{2^-}$
	$](1, 1) +1 [_{+1}$		--

I end the collection of representation decompositions with the adjoint $[45]$ representation of SO10 →

E12

$$([10] \otimes [10])_a = [45] \quad \downarrow$$

(54)

a	$\{5\}_{-2}$	$\{\bar{5}\}_{+2}$
$\{5\}_{-2}$	$\{10\}_{-4}$	$\left\{ \begin{array}{l} \{1\}_0 \leftrightarrow J_5 \\ \{24\}_0 \leftrightarrow \text{adjoint SU5} \end{array} \right\}$
$\{\bar{5}\}_{+2}$		$\{\bar{10}\}_{+4}$

It should be noted that despite coinciding dimensions the following entities are most distinct

(55)

$$[10] \neq \{10\}_{-4}, \{10\}_6$$

$$[45] \neq \{45\}_{-2}; \dots$$

— Thank you for your attention —

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