

*QUANTUM THEORY,
NAMELY
WHERE PHYSICS AND
INFORMATION MEET*

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ESSAY

'A Practical Tool,' but Puzzling Too

By JOHN ARCHIBALD WHEELER

What is the greatest mystery in physics today? Different physicists have different answers. My candidate for greatest mystery is a question now a century old, "How come the quantum?"

What is this thing, the "quantum"? It's a bundle of energy, an indivisible unit that can be sliced no more. Max Planck showed us a hundred years ago that light is emitted not in a smooth, steady flow, but in quanta. Then physicists found quantum jumps of energy, the quantum of electric charge and more. In the small-scale world, everything is lumpy.

How come the quantum?



The New York Times

John A. Wheeler

MATHEMATICAL VS OPERATIONAL FOUNDATIONS

In every textbook Quantum Theory is presented with abstract axioms about Hilbert spaces:

“Each physical system is associated with a Hilbert space H . Unit rays in H describe the pure states of the system.”

“Physical quantities are represented by self-adjoint operators on H ”.

Why did Nature choose this particular piece of mathematics?

Can we reconstruct QT from “first principles”?

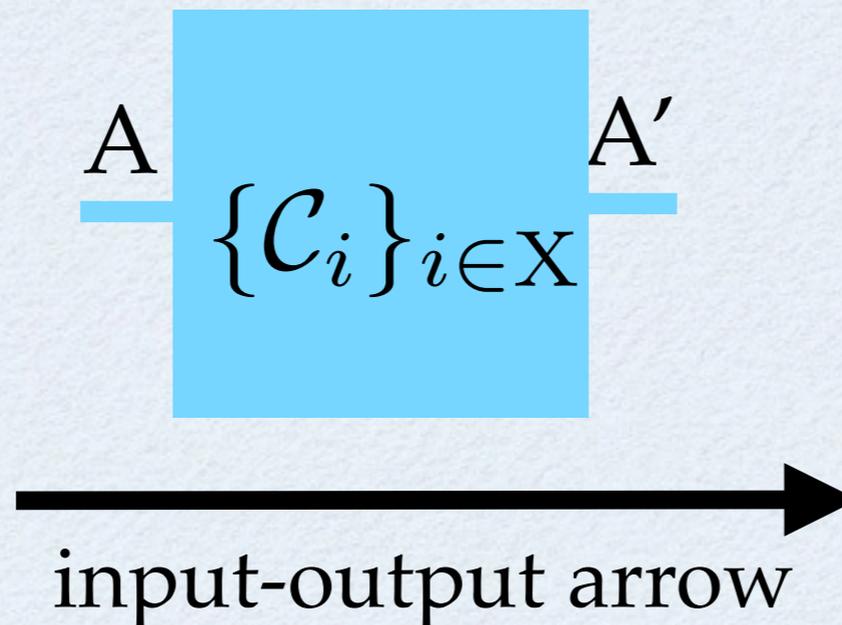
THE FRAMEWORK:
OPERATIONAL-PROBABILISTIC
THEORIES

OPERATIONAL STRUCTURE

SYSTEMS AND TESTS

-Systems: $A, B, C, \dots + I =$ trivial system (nothing)

-Tests: a test represents one use of a physical device

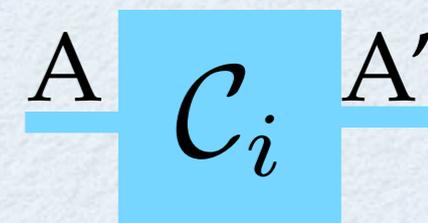


A : input system

A' : output system

i : outcome, in some outcome set X

C_i : possible process, graphically represented as

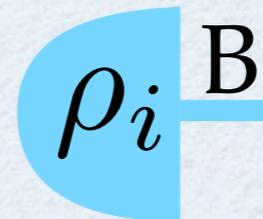


PREPARATIONS AND MEASUREMENTS

Special cases of tests:

- trivial input: preparation

ρ_i : "state"



- trivial output: measurement

a_i : "effect"



PROBABILISTIC STRUCTURE

PROBABILITY ASSIGNMENT

- Preparation + measurement = probability distribution

$$\rho_i \xrightarrow{A} a_j = p(a_j, \rho_i)$$

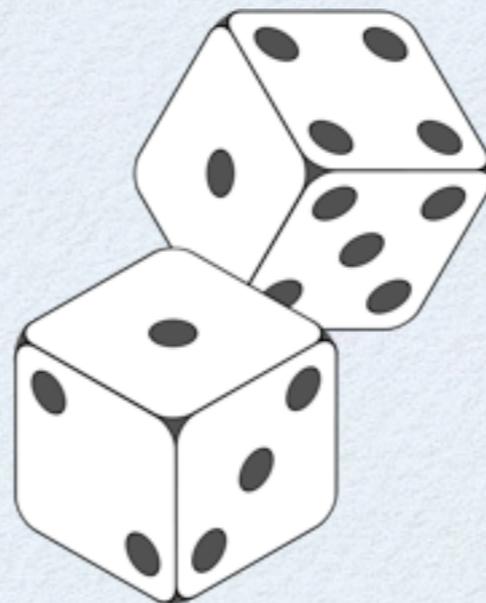
$$\left\{ \begin{array}{l} p(a_j, \rho_i) \geq 0 \\ \sum_{i \in X} \sum_{j \in Y} p(a_j, \rho_i) = 1 \end{array} \right.$$

INDEPENDENT EXPERIMENTS

- Experiments performed in parallel are statistically independent:

$$\begin{array}{c} \rho_i \text{---} \text{A} \text{---} a_j \\ \sigma_k \text{---} \text{B} \text{---} b_l \end{array} = p(a_j, \rho_i) p(b_l, \sigma_k)$$

e.g. the roll of two dice



OPERATIONAL-PROBABILISTIC THEORIES

OPERATIONAL-PROBABILISTIC THEORIES (OPTS)

Operational-probabilistic theory
=
operational structure
+
probabilistic structure

Examples:

- classical theory
- quantum theory
- quantum theory on real Hilbert spaces
- quantum theory with only LOCC operations
- ...



AXIOMS

CAUSALITY

The choice of tests performed in the future cannot influence the probabilities of outcomes of tests performed in the present.

FORMAL STATEMENT

For every pair of measurements $\{a_j\}_{j \in Y}$ and $\{b_k\}_{k \in Z}$

$$\sum_{j \in Y} \rho_i \text{---}^A a_j = \sum_{k \in Z} \rho_i \text{---}^A b_k \quad \forall \rho_i$$

Equivalent condition: there exists a **unique deterministic effect**

$$\sum_{j \in Y} \text{---}^A a_j = \sum_{k \in Z} \text{---}^A b_k =: \text{---}^A e$$

MARGINAL STATES

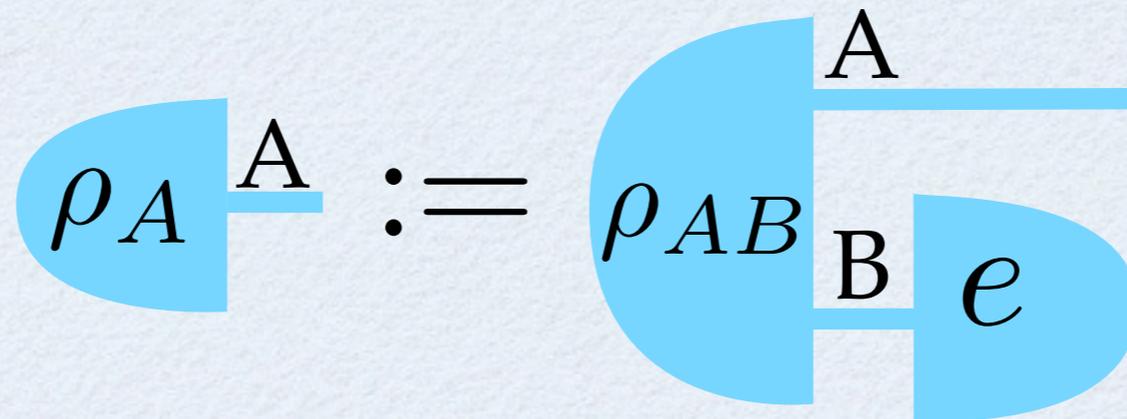
Uniqueness of the
deterministic effect



only one way
to discard a system



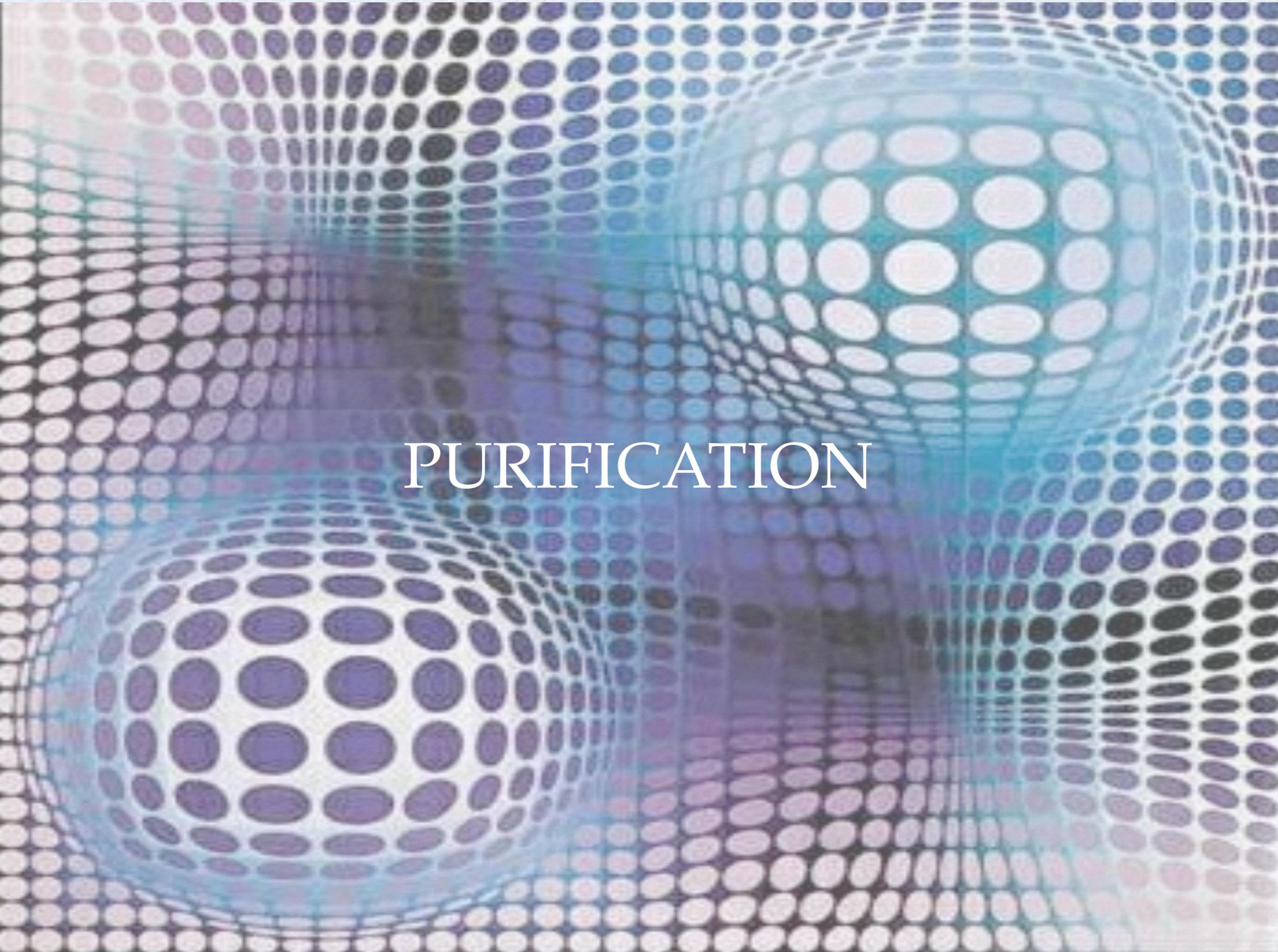
marginal states are
uniquely defined



e.g. in QT:

The diagram shows a light blue oval labeled ρ with a horizontal line extending from its right side. This line connects to another light blue oval labeled e , representing the discarding effect. To the right of this diagram is the equation $= \text{Tr}[\rho]$.

$$\rho_A := \text{Tr}_A[\rho_{AB}]$$

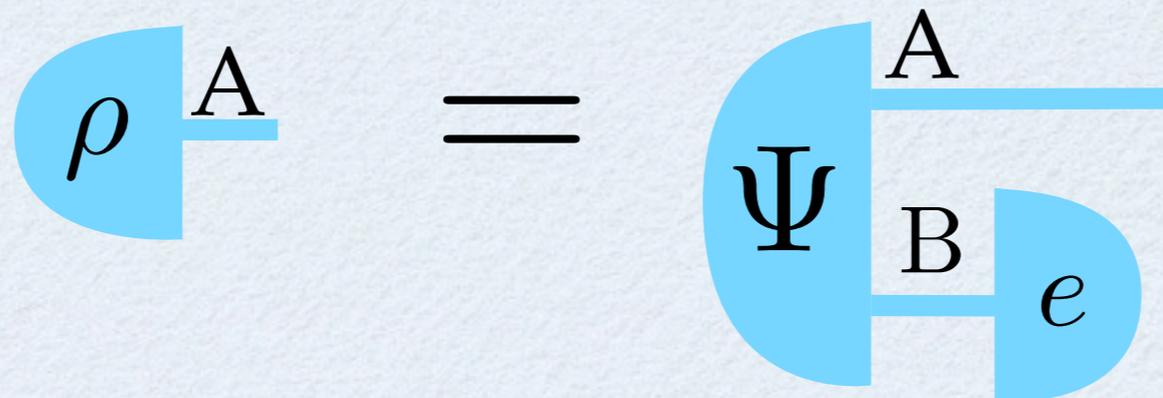


PURIFICATION

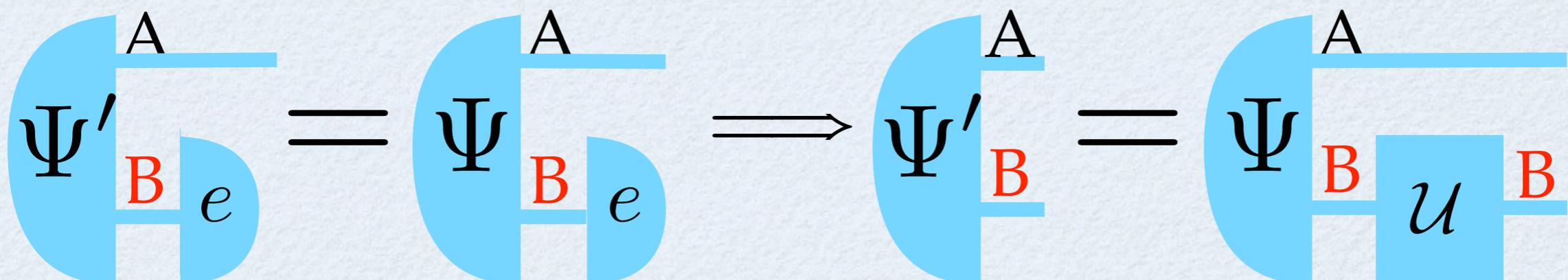
Every mixed state can be modeled as the marginal of a pure state in an essentially unique way.

FORMAL STATEMENT

- **Existence:** For every state ρ of A there is a system B and a pure state Ψ of $A \otimes B$ such that



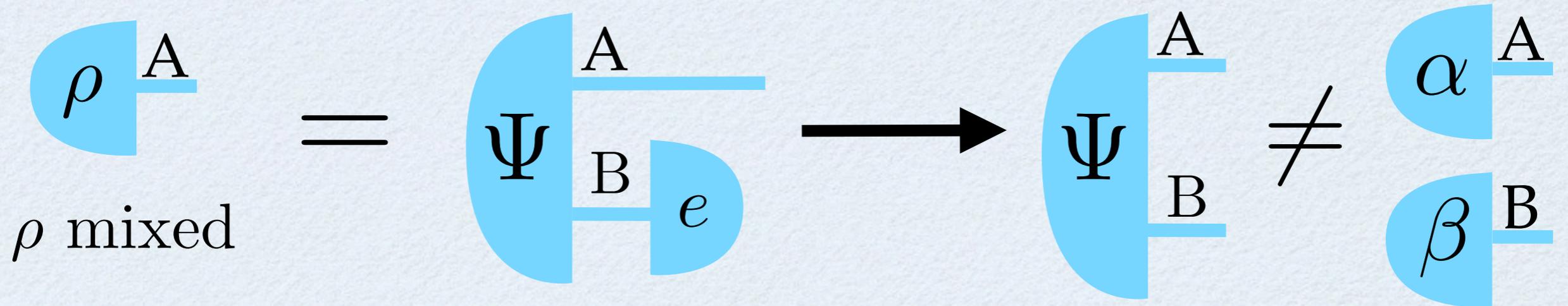
- **Uniqueness:** two purifications of the same state are equivalent up to a reversible transformation



CONSEQUENCES
OF
PURIFICATION

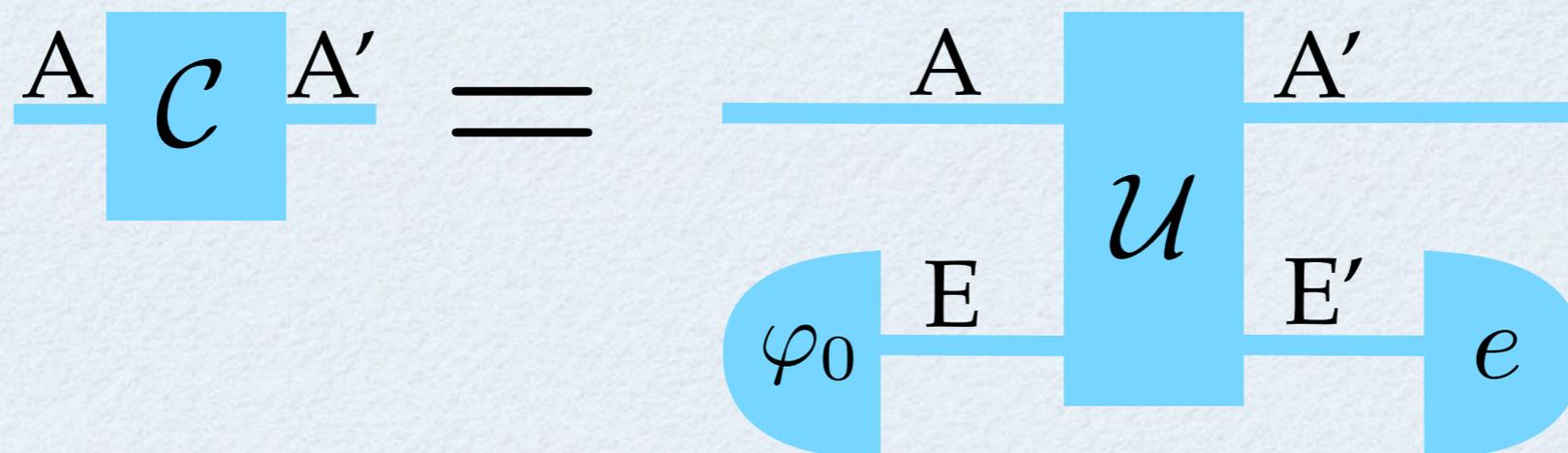
ENTANGLEMENT

The purification of a mixed state is **entangled**



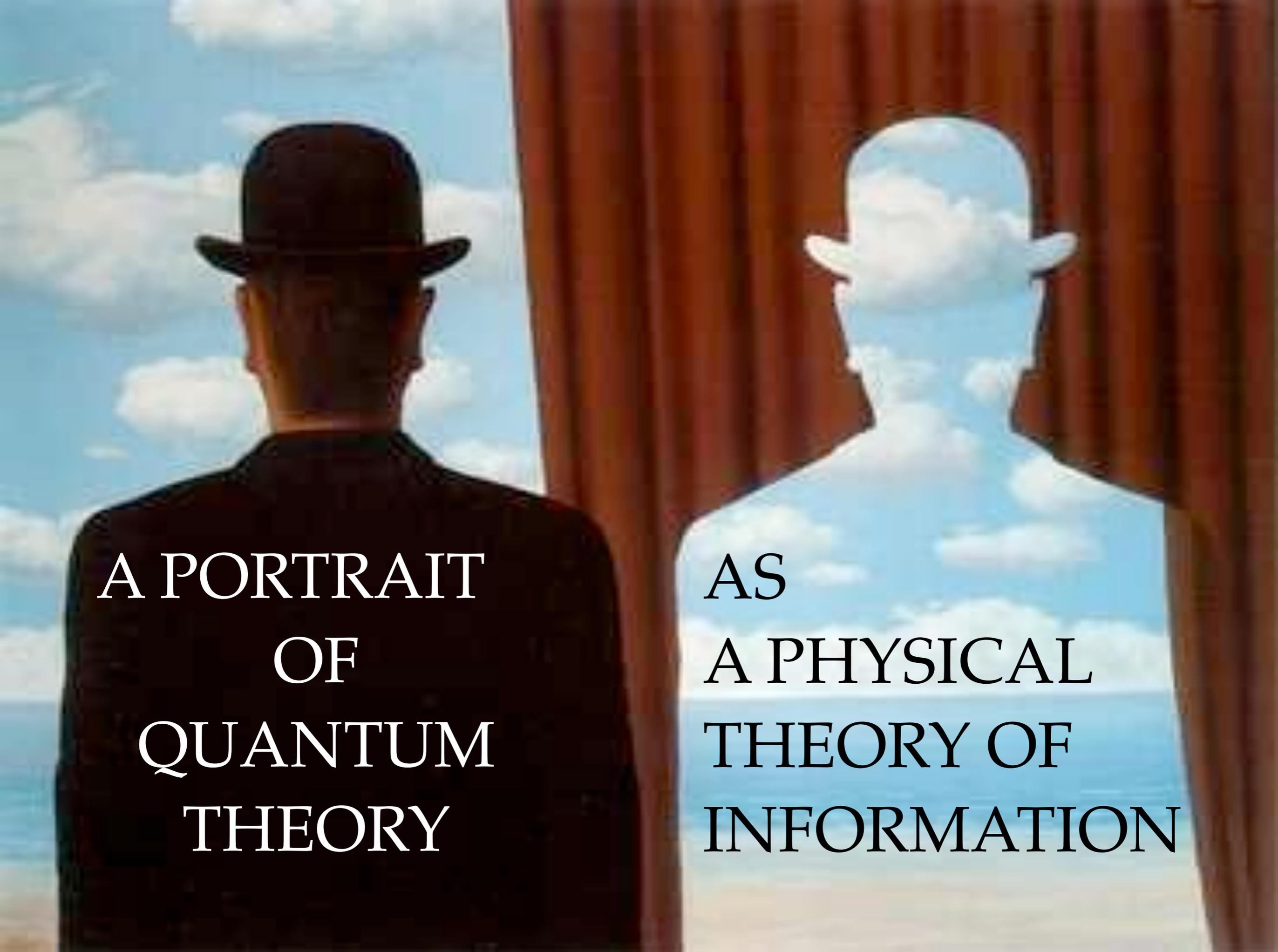
PURE AND REVERSIBLE SIMULATION OF IRREVERSIBLE PROCESSES

Every irreversible process can be modeled as the result of an **reversible** interaction of the system with an environment:

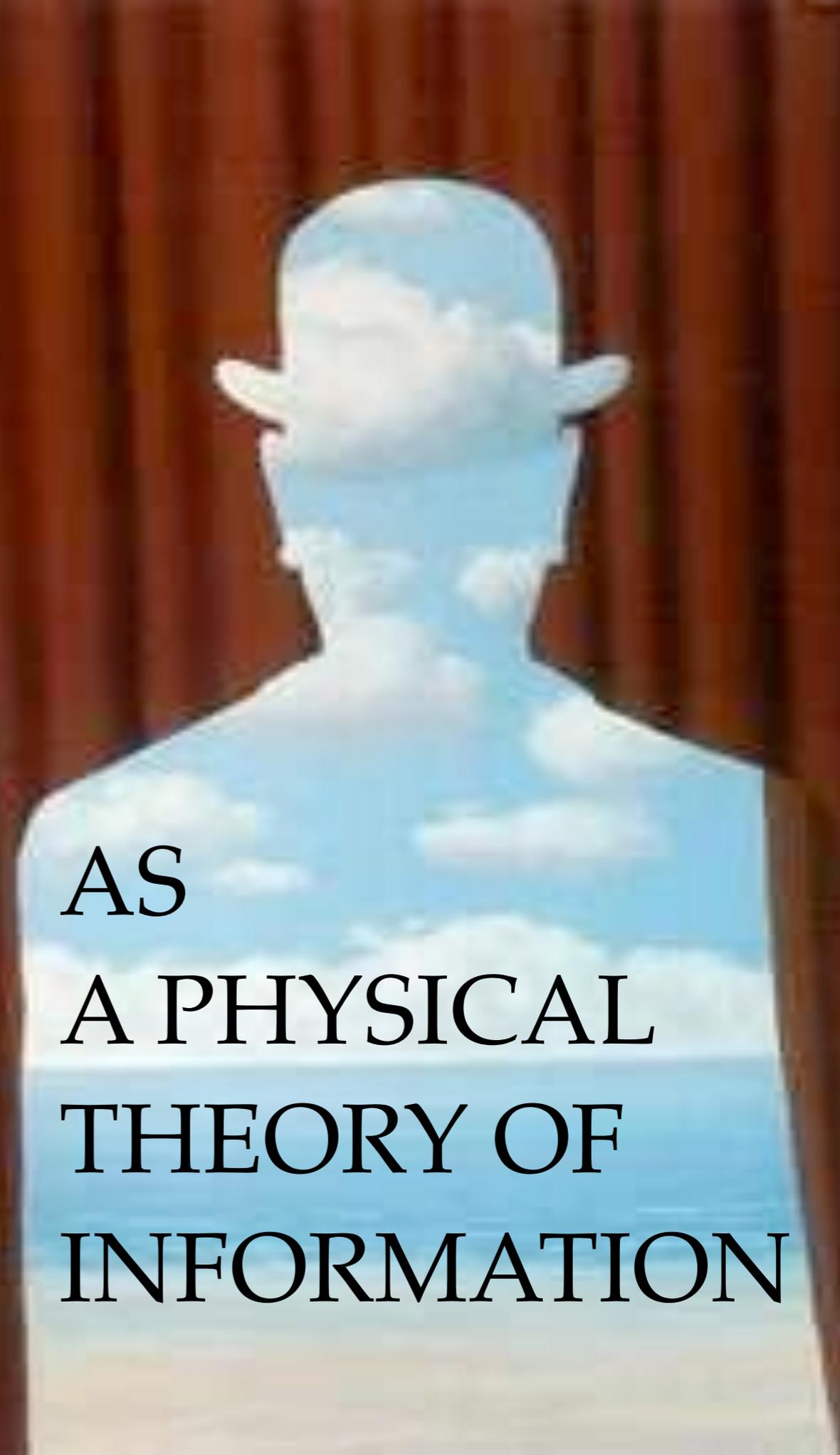


Conservation of Information:

Information cannot be erased, it can only be discarded.

A man in a black suit and bowler hat is seen from behind, looking out a window. The window shows a blue sky with white clouds and a glimpse of a beach. The man's silhouette is dark against the bright light from the window.

A PORTRAIT
OF
QUANTUM
THEORY

A white silhouette of a man in a bowler hat is seen from behind, looking out a window. The window shows a blue sky with white clouds and a glimpse of a beach. The silhouette is filled with the light from the window, making it appear as if the man is made of light.

AS
A PHYSICAL
THEORY OF
INFORMATION

THE RESULT (CDP 2010)

- Causality
- Fine-Grained Composition
- Perfect Distinguishability
- Ideal Compression
- Local tomography

+

PURIFICATION

=

Quantum Theory
(in finite dimensions)

TAKE HOME MESSAGE

Quantum Theory =
the only pure and reversible theory of information

Physics

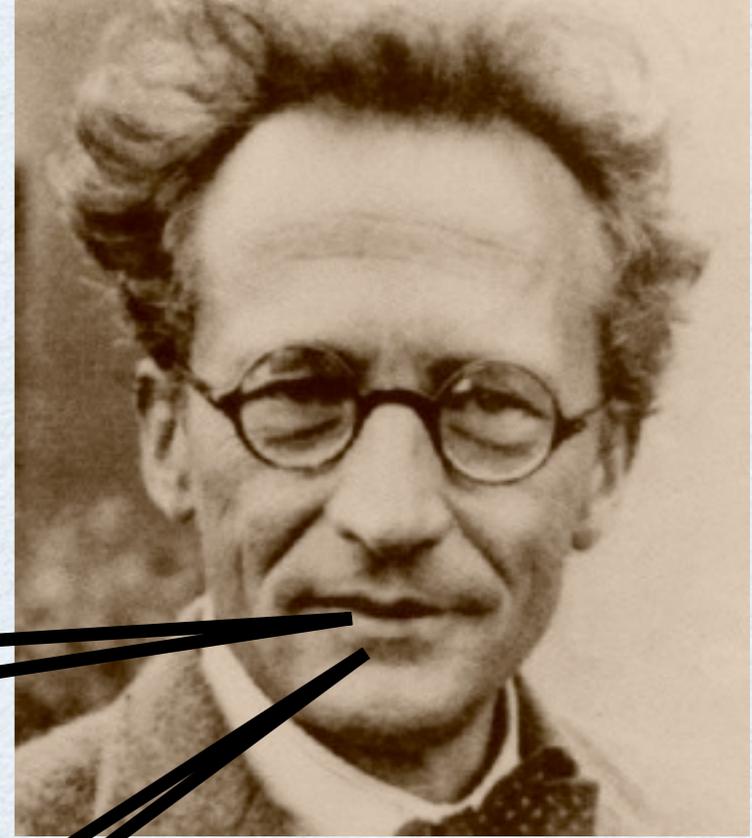
- pure states
(maximal knowledge
about the system)
- reversible dynamics

Quantum Theory

Information Theory

- mixed states
(information sources)
- noisy channels

SCHROEDINGER AND PURIFICATION



The best possible knowledge of a whole does not necessarily imply the best possible knowledge of its parts.

I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought.

...and now we know that he was right!

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