Optical analogues of the event horizon

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Quantum effects in astrophysics?

Whirlpool Galaxy • M51
Hawking radiation of black holes


The purpose of this letter is to show that the evaporation of black holes cannot be described by the classical laws of thermodynamics. We observe a black hole to be a source of thermal radiation, which is emitted as it evaporates. The temperature of the radiation is given by the relation

$$T = rac{2}{m c^2},$$

where $m$ is the mass of the black hole and $c$ is the speed of light. The radiation is not in equilibrium with the black hole itself, but rather with an energy $E = 2m c^2$. The energy $E$ is the total energy of the black hole, which is the sum of its mass-energy and its rest mass.

The black hole radiation is a form of Hawking radiation, named after Stephen Hawking, who first predicted the existence of this radiation in 1974. The radiation is a consequence of the interaction between the black hole and the radiation field, and is emitted from the event horizon of the black hole. The radiation is not in thermal equilibrium with the black hole, but rather with an energy $E = 2m c^2$. The energy $E$ is the total energy of the black hole, which is the sum of its mass-energy and its rest mass.

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Black holes and event horizons

\[ k_B T = \frac{\hbar \alpha}{2 \pi}, \quad \alpha = \frac{C^3}{4GM} = \frac{C}{2R} \]

M: Mass
R: Schwarzschild Radius

Hawking radiation: connections

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The Hawking effect connects the physics of everyday, the physics of the very large, and the very small!
Hawking radiation: problems

\[ \lambda = \frac{(2\pi)^2 c}{\lambda} = 8\pi^2 R \]

\[ R = 3 \text{ km} \]

\[ T = 6 \times 10^{-8} \text{ K} \]
Contents

- Introduction to event horizons in analog systems
- Analogue horizons with water and light
- Waves encountering event horizons
- Generation of negative frequency waves in optics
- Summary and Outlook
Artificial event horizons: the black hole


Illustration: Scientific American

W. Unruh
Horizons in moving media

Illustration: Peter Hoey, Science 319, 1321 (2008)
Hydraulic jump

Photo: Piotr Pieranski
Artificial event horizons in superfluid Helium-3 and Bose Einstein Condensates
Waves and Barriers

\[ \partial_\nu \sqrt{-g} g^{\mu\nu} \partial_\mu \sigma = 0, \quad g^{\mu\nu} = \begin{pmatrix} 1 & \frac{\nu}{c} \\ -\frac{c^2 \nu}{c^2 + \nu^2} & -c^2 + \nu^2 \end{pmatrix} \]


- \( -v_0 + c > 0 \) subluminal
- \( -v_1 + c < 0 \) superluminal
- \( v_B = 0 \) white hole horizon WH
- \( -v_0 + c > 0 \) subluminal

black hole horizon BH
Water waves

Rousseaux, G; Mathis, C; Maissa, P, et al. NEW JOURNAL OF PHYSICS 10, 053015 (2008)
Waves and Barriers

Wave velocity in moving frame of water:

- $v_0 - v_1 + c < 0$
- $v_B = v_0$

Velocity profile:

- $c$
- $0$
- $v_0 - v_1 + c < 0$

white hole horizon WH  black hole horizon BH
A pulse moves at velocity $v_0$ and locally modifies the refractive index (i.e. wave velocity) by the cross-Kerr effect.
Optical event horizons

Optical fibers


Laval nozzle


Filamentation

**www.st-andrews.ac.uk/~qoi**
Conditions: Stationarity

Stationarity:

Soliton pulses in a 1-D fiber waveguide

Filaments: F. Belgiorno et al., PRL 105, 203901 (2010).

Negligible backaction: weak probe wave

Nonstationary: Pulse trapping:

A. V. Gorbach et al., Nature Photonics 1, 653 (2007)
Soliton formation

"runners«wave packet" *

ultrashort pulses in optical fibers

\[ n = n(I) \]

dispersion

Soliton formation
Soliton formation

"runners«wave packet" * dispersion

"soft mat" nonlinearity

ultrashort pulses in optical fibers

Kerr-effekt

Soliton Formation
Soliton formation

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dispersion

nonlinearity

Kerr-effekt

"soft mat"

soliton

( * Mollenauer )

n = n(I)
Horizons in moving media: trajectories

\[ \zeta \]
\[ \tau = 0 \]
\[ u = v_g \]
\[ u > v_g \]
\[ \omega_2 > \omega_m \]
\[ \omega_1 < \omega_m \]

Soliton
Horizons in moving media: trajectories

\[ A = A e^{i \varphi} \quad \varphi = \int (k \Delta z - \omega dt) \quad \text{where} \quad k = \frac{n \omega}{c}, \quad n = n_0(\omega) + \delta n \]

\[ \tau = t - \frac{z}{c} \quad \{ \text{moving frame} \} \]

\[ \xi = \frac{z}{\omega} \]

\[ \rightarrow \varphi = - \int \omega dt - \omega' \xi \]

\[ \omega' = \omega - uk \]

\[ \omega' = (1 - \frac{um}{c}) \omega \quad \text{Doppler} \]

Wavelength (air) [nm]

1490 1500 1510

Nonlinear index \( \delta n \times 10^6 \)

1.5

1.2

0.9

0.6

0.3

0

1245 1255 \( \omega_1 \) 1265 \( \omega_2 \) Frequency \( \omega \) [THz]
Blue shifting at the white hole


Optical horizon physics

- Characteristic and efficient mode conversion can be observed

- Fast waves can tunnel through the horizon

- Light can be trapped and released behind horizons

- Generation of waves with *negative frequency*
Horizons and modes in the dispersion relation

Transformation to the frame moving with the pulse:

\[ \omega' = \gamma(\omega - vk) \]
\[ k' = \gamma(k - v/c^2(\omega)) \]

\[ \beta(\omega) - k - \beta_0 - (\omega - \omega_0)/v = 0 \iff \omega'_0 - \omega'_0 = 0 \]

Relative wave vector:
\[ \beta(\omega) = k + \beta_0 + (\omega - \omega_0)/v \]

Moving frame frequency:
\[ \omega' = \omega - v \beta(\omega) \text{ (Doppler relation)} \]

Conservation of momentum in the lab frame

Conservation of energy in the moving frame
Excitation of the RR mode in fibers

Resonant radiation from solitons:
- Wai et. al. Optics Lett. 11, 464 (1986)
- Karpman, PRE, 47, 2073, (1993)
- Skryabin et. al., Science, 301, 1705 (2003)
- Chang et al., Opt. Express 19, 6635 (2011)
Negative frequency?

Light always oscillates with positive and negative frequencies:

\[ A(t) = \int_{-\infty}^{\infty} \tilde{A}(\omega) e^{-i\omega t} d\omega \]

For real fields \( A \) therefore: \( \tilde{A}(-\omega) = \tilde{A}^*(\omega) \)

**Negative frequencies are redundant !?**

Quantum fields:

\[ \hat{A} = \sum_k \left( A_k \hat{a}_k + A_k^* \hat{a}_k^\dagger \right) \]

Annihilation / creation amplitudes

Hermitian

Image: APS/Alan Stonebraker
Negative frequency waves

S. Weinfurtner et al., PRL 106 021302 (2011)
Experiments in fibers

NRR spectra

NRR spectra

Conclusion

- Optical pulses in fibers form analogue black and white hole horizons

- Characteristic and efficient mode conversion can be observed

- Negative frequencies are physical because we can couple positive and negative frequency modes to generate new waves

- Conversion between negative and positive frequencies leads to photon creation/amplification

- First steps towards astrophysical particle creation in optical analogues
Applications of trapping and optical horizons

Frequency conversion – classical and quantum

‘Frequency conjugation’

Dispersion management

Ultrafast optical delay lines
Outlook: Mixing of $A_k$ and $A_k^*$

Mixing of amplitudes creates photon pairs, even from the quantum vacuum state. In astrophysics, the particle creation by mixing of positive and negative frequencies occurs:

- **At the event horizon of black holes: Hawking effect**
  
  \[ \text{(Hawking, Nature 248, 30 (1974))} \]

- **In accelerated systems: Unruh effect**
  
  \[ \text{(Unruh, Phys. Rev. D 14, 870 (1976))} \]

- **Novel parametric amplifier**

- **“Black hole Laser”**
  
  \[ \text{(S. Corley and T. Jacobson, PRD, 59:124011, 1999)} \]
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Thank you!

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