Optical analogues of the event horizon

Friedrich König

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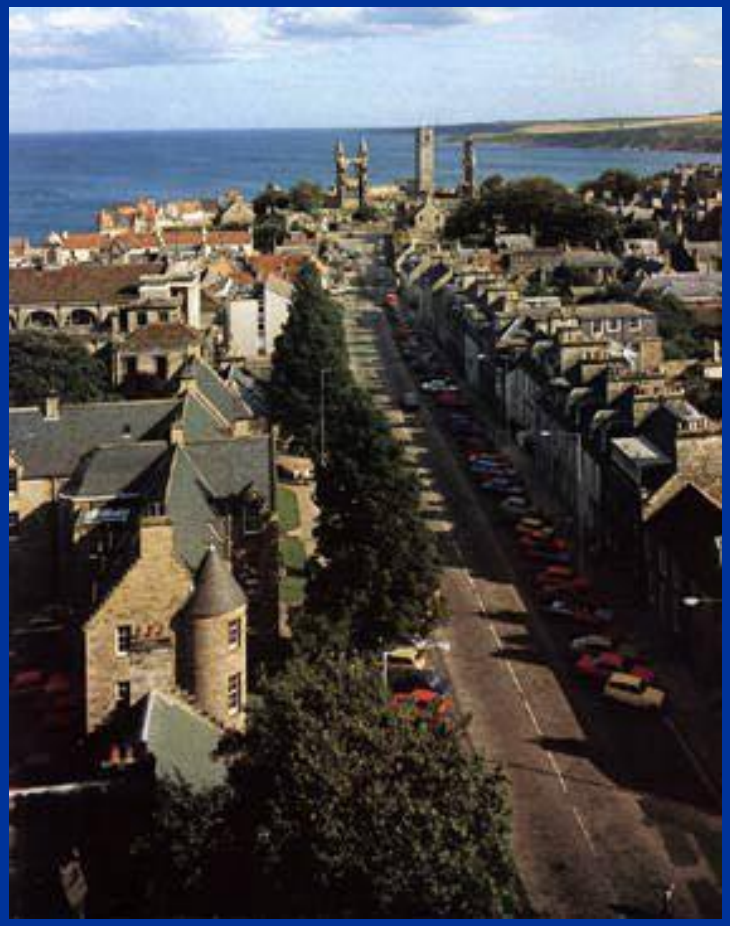

Quantum effects in astrophysics?

NASA, ESA, S. Beckwith (STScI), and The Hubble Heritage Team (STScI/AURA) • Hubble Space Telescope ACS • STScI-PRC05-12a

Hawking radiation of black holes

they must produce a response in the scintillator comparable with that produced by a conventional charged particle. As mentioned earlier, the EAS studied by Ramana Murthy were almost two orders of magnitude less energetic than those used by us and also a much smaller time range was examined. Since the peak in our distribution occurs after the 18 μs time interval investigated by Ramana Murthy, the results cannot be directly compared. But a statistical examination of his published data using David's technique indicates that there is less than 5% probability that the data is from a uniform distribution. We note that, subjectively, the observed distribution seems to rise in the period prior to 13 μs before the shower arrival. This is not inconsistent with our observations.

It is possible than an explanation may be found for these results without invoking the existence of tachyons. A. G. Gregory has pointed out to us that fission or spallation in the interstellar medium or production of associated particles at the source might account for the correlated arrival of cosmic rays. We note, however, that unless particles are produced with closely similar rigidity and velocity vectors they are unlikely to remain associated for long in the interstellar or source magnetic fields. A closely related problem has been discussed by Weekes[8] in connection with pulsar periodicities in cosmic-ray arrival times.

We conclude that we have observed non-random events preceding the arrival of an extensive air shower. Being unable to explain this result in a more conventional manner, we suggest that this is the result of a particle travelling with an apparent velocity greater than that of light.

We thank Dr A. G. Gregory and Professors C. A. Hurst and J. R. Prescott for comments, Mr K. W. Morris for advice on statistical analysis, and Ms J. M. Taylor for chart reading. One of us (R.W.C.) holds a Queen Elizabeth Fellowship. The University of Calgary is thanked for the loan of equipment.

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, M. N., *Phys. Rev.*, **171**, 1357 (1968).
Yeh, N., Linsker, R., *Phys. Rev. D.*,
ch, G. R., Borenstein, S. R., Strand,
Chapman, J. W., Lys, J., *Phys. Rev.*
, *Lett. Nuovo Cim.* Ser. **2**, **1**, 908
m. Part. cosm. Ray Phys., **10**, 170
Rossi, B., *Phys. Rev.*, **92**, 441 (1953).
34, 299 (1947).
hys. Sci., **223**, 129 (1971).

Black hole explosions?

QUANTUM gravitational effects are usually ignored in calculations of the formation and evolution of black holes. The justification for this is that the radius of curvature of space-time outside the event horizon is very large compared to the Planck length $(G\hbar/c^3)^{1/2} \approx 10^{-33}$ cm, the length scale on which quantum fluctuations of the metric are expected to be of order unity. This means that the energy density of particles created by the gravitational field is small compared to the space-time curvature. Even though quantum effects may be small locally, they may still, however, add up to produce a significant effect over the lifetime of the Universe $\approx 10^{17}$ s which is very long compared to the Planck time $\approx 10^{-43}$ s. The purpose of this letter is to show that this indeed may be the case: it seems that any black hole will create and emit particles such as neutrinos or photons at just the rate that one would expect if the black hole was a body with a temperature of $(\kappa/2\pi)(\hbar/2k) \approx 10^{-6}\ (M_\odot/M)K$ where κ is the surface gravity of the black hole [1]. As a black hole emits this thermal radiation one would expect it to lose mass. This in turn would increase the surface gravity and so increase the rate of emission. The black hole would therefore have a finite life of the order of $10^{71}\ (M_\odot/M)^{-3}$ s. For a black hole of solar mass this is much longer than the age of the Universe. There might, however, be much smaller black holes which were formed by fluctuations in the early Universe[2]. Any such black hole of mass less than 10^{15} g would have evaporated by now. Near the end of its life the rate of emission would be very high and about 10^{30} erg would be released in the last 0.1 s. This is a fairly small explosion by astronomical standards but it is equivalent to about 1 million 1 Mton hydrogen bombs.

To see how this thermal emission arises, consider (for simplicity) a massless Hermitean scalar field ϕ which obeys the covariant wave equation $\phi;_{ab}g^{ab} = 0$ in an asymptotically flat space time containing a star which collapses to produce a black hole. The Heisenberg operator ϕ can be expressed as

$$\phi = \sum_i \{f_i a_i + \bar{f}_i a_i^+\}$$

where the f_i are a complete orthonormal family of complex valued solutions of the wave equation $f_{i;ab}g^{ab} = 0$ which are asymptotically ingoing and positive frequency—they contain only positive frequencies on past null infinity I^-[3,4,5]. The position-independent operators a_i and a_i^+ are interpreted as annihilation and creation operators respectively for incoming scalar particles. Thus the initial vacuum state, the state containing no incoming scalar particles, is defined by $a_i|0_-\rangle = 0$ for all i. The operator ϕ can also be expressed in terms of solutions which represent outgoing waves and waves crossing the event horizon:

$$\phi = \sum_i \{p_i b_i + \bar{p}_i b_i^+ + q_i c_i + \bar{q}_i c_i^+\}$$

where the p_i are solutions of the wave equation which are zero on the event horizon and are asymptotically outgoing positive frequency waves (positive frequency on future null infinity I^+) and the q_i are solutions which contain no outgoing component (they are zero on I^+). For the present purposes it is not necessary that the q_i are positive frequency on the horizon even if that could be defined. Because fields of zero rest mass are completely determined by their values on I^-, the p_i and the q_i can be expressed as linear combinations of the f_i and the $\bar{f}_i$:

$$p_i = \sum_j \{\alpha_{ij} f_j + \beta_{ij}\bar{f}_j\} \quad \text{and so on}$$

The β_{ij} will not be zero because the time dependence of the metric during the collapse will cause a certain amount of mixing of positive and negative frequencies. Equating the two expressions for ϕ, one finds that the b_i, which are the annihilation operators for outgoing scalar particles, can be expressed as a linear combination of the ingoing annihilation and creation operators a_i and a_i^+

$$b_i = \sum_j \{\bar{\alpha}_{ij} a_j - \bar{\beta}_{ij} a_j^+\}$$

Thus when there are no incoming particles the expectation value of the number operator $b_i^+ b_i$ of the ith outgoing state is

$$\langle 0_- | b_i^+ b_i | 0_- \rangle = \sum_j |\beta_{ij}|^2$$

The number of particles created and emitted to infinity in a gravitational collapse can therefore be determined by calculating the coefficients β_{ij}. Consider a simple example in which

the collapse is spherically symmetric. The angular dependence of the solution of the wave equation can then be expressed in terms of the spherical harmonics Y_{lm} and the dependence on retarded or advanced time u, v can be taken to have the form $\omega^{-1/2}\ exp\ (i\omega u)$ (here the continuum normalisation is used). Outgoing solutions $p_{lm\omega}$ will now be expressed as an integral over incoming fields with the same l and m:

$$p_\omega = \int \{\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'}\bar{f}_{\omega'}\}\, d\omega'$$

(The lm suffixes have been dropped.) To calculate $\alpha_{\omega\omega'}$ and $\beta_{\omega\omega'}$ consider a wave which has a positive frequency ω on I^+ propagating backwards through spacetime with nothing crossing the event horizon. Part of this wave will be scattered by the curvature of the static Schwarzschild solution outside the black hole and will end up on I^- with the same frequency ω. This will give a $\delta(\omega - \omega')$ behaviour in $\alpha_{\omega\omega'}$. Another part of the wave will propagate backwards into the star, through the origin and out again onto I^-. These waves will have a very large blue shift and will reach I^- with asymptotic form

$$C\omega^{-1/2} \exp\{-i\omega\kappa^{-1} \log(v_0 - v) + i\omega v\}\ for\ v < v_0$$

and zero for $v \geq v_0$, where v_0 is the last advanced time at which a particle can leave I^-, pass through the origin and escape to I^+. Taking Fourier transforms, one finds that for large ω', $\alpha_{\omega\omega'}$ and $\beta_{\omega\omega'}$ have the form:

$$\alpha_{\omega\omega'} \approx C \exp[i(\omega - \omega)v_0](\omega'/\omega)^{1/2} \cdot \Gamma(1 - i\omega/\kappa)[-i(\omega - \omega')]^{-1+i\omega/\kappa}$$

$$\beta_{\omega\omega'} \approx C \exp[i(\omega + \omega)v_0](\omega'/\omega)^{1/2} \cdot \Gamma(1 - i\omega/\kappa)[-i(\omega + \omega')]^{-1+i\omega/\kappa}.$$

The total number of outgoing particles created in the frequency range $\omega \to \omega + d\omega$ is $d\omega\int_0^\infty |\beta_{\omega\omega'}|^2 d\omega'$. From the above expression it can be seen that this is infinite. By considering outgoing wave packets which are peaked at a frequency ω and at late retarded times one can see that this infinite number of particles corresponds to a steady rate of emission at late retarded times. One can estimate this rate in the following way. The part of the wave from I^+ which enters the star at late retarded times is almost the same as the part that would have crossed the past event horizon of the Schwarzschild solution had it existed. The probability flux in a wave packet peaked at ω is roughly proportional to $\int_0^\infty \omega'\{|\alpha_{\omega\omega'}|^2 - |\beta_{\omega\omega'}|^2\}\, d\omega$ where $\omega_s' \gg \omega_s' \gg 0$. In the expressions given above for $\alpha_{\omega\omega'}$ and $\beta_{\omega\omega'}$ there is a logarithmic singularity in the factors $[-i(\omega - \omega')]^{-1+i\omega/\kappa}$ and $[-i(\omega + \omega')]^{-1+i\omega/\kappa}$. Value of the expressions on different sheets differ by factors of $\exp(2\pi n\omega\kappa^{-1})$. To obtain the correct ratio of $\alpha_{\omega\omega'}$ to $\beta_{\omega\omega'}$ one has to continue $[-i(\omega + \omega')]^{-1+i\omega/\kappa}$ in the upper half ω' plane round the singularity and then replace ω' by $-\omega'$. This means that, for large ω',

$$|\alpha_{\omega\omega'}| = \exp(\pi\omega/\kappa)\ |\beta_{\omega\omega'}|$$

From this it follows that the number of particles emitted in this wave packet mode is $(\exp(2\pi\omega/\kappa) - 1)^{-1}$ times the number of particles that would have been absorbed from a similar wave packet incident on the black hole from I^-. But this is just the relation between absorption and emission cross sections that one would expect from a body with a temperature in geometric units of $\kappa/2\pi$. Similar results hold for massless fields of any integer spin. For half integer spin one again gets a similar result except that the emission cross section is $(\exp(2\pi\omega/\kappa) + 1)^{-1}$ times the absorption cross section as one would expect for thermal emission of fermions. These results do not seem to depend on the assumption of exact spherical symmetry which merely simplifies the calculation.

Beckenstein[6] suggested on thermodynamic grounds that some multiple of κ should be regarded as the temperature of a black hole. He did not, however, suggest that a black hole could emit particles as well as absorb them. For this reason Bardeen, Carter and I considered that the thermodynamical similarity between κ and temperature was only an analogy. The present result seems to indicate, however, that there may be more to it than this. Of course this calculation ignores the back reaction of the particles on the metric, and quantum fluctuations on the metric. These might alter the picture.

Further details of this work will be published elsewhere. The author is very grateful to G. W. Gibbons for discussions and help.

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1 Bardeen, J. M., Carter, B., and Hawking, S. W., *Commun. math. Phys.*, **31**, 161–170 (1973).
2 Hawking, S. W., *Mon. Not. R. astr. Soc.*, **152**, 75–78 (1971).
3 Penrose, R., in *Relativity, Groups and Topology* (edit. by de Witt, C. M., and de Witt, B. S). Les Houches Summer School, 1963 (Gordon and Breach, New York, 1964).
4 Hawking, S. W., and Ellis, G. F. R., *The Large-Scale Structure of Space-Time* (Cambridge University Press, London 1973).
5 Hawking, S. W., in *Black Holes* (edit. by de Witt, C. M., and de Witt, B. S), Les Houches Summer School, 1972 (Gordon and Breach, New York, 1973).
6 Beckenstein, J. D., *Phys. Rev.*, **D7**, 2333–2346 (1973).

Absorption and emission by interstellar CH at 9 cm

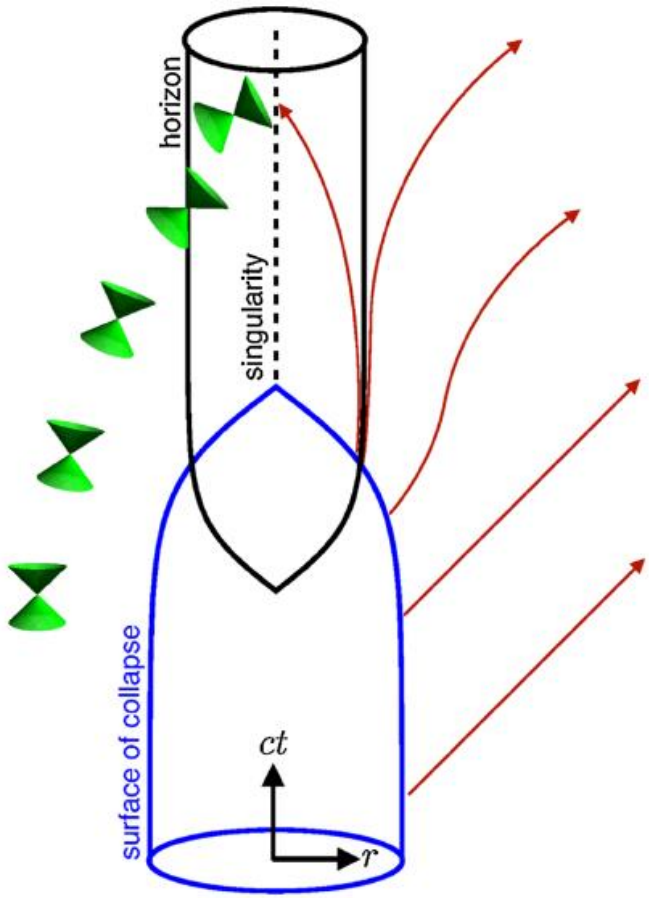
RYDBECK, Ellder and Irvine[1] have recently detected the 9-cm lines of the $^2\Pi_{1/2}$, $J = \frac{1}{2}\Delta$ doublet of interstellar CH. The $F = 1 \to 1$ transition at 3,335.475 MHz was observed in emission in a wide range of galactic sources ranging from dark clouds to the spiral arms in front of Cassiopea A. The two satellite transitions $F = 0 \to 1$ (at 3,263.788 MHz) and $F = 1 \to 0$ (at 3,349.185 MHz) were also observed in emission in several sources.

We have observed the 3,335.475 MHz transition of CH in several southern galactic sources. In RCW38 this line is seen in absorption, while the two satellite lines are seen in emission. In several sources the distribution of CH is found to be extended.

The observations were made on December 10 and 11, 1973, with the Parkes 64-m telescope equipped with a 9-cm parametric amplifier having a noise temperature of 150 K. The telescope beam at 9 cm is 6 arc min. The receiver output was analysed by a 512-channel digital correlator producing a spectral resolution of 19.5 kHz.

The Onsala observations[1] of CH emission at 3,335.475 MHz from Cloud 2 and W12 were confirmed. In Cloud 2 we measured an antenna temperature similar to that found with the Onsala 25-m telescope. For W12 it was 0.23 K, about 50% greater than the Onsala value of 0.15 K; however, at a position 5 arc min south (where the continuum intensity had fallen to one seventh of its peak value) the line signal had decreased by only 30%. A similar situation occurred in RCW36. Thus the CH distribution is considerably more extended than the continuum for these HII regions; in Cloud 2 it must be comparable with the 16 arc min beam of the Onsala 25-m telescope.

Black holes and event horizons



$$k_B T = \frac{\hbar \alpha}{2\pi}, \quad \alpha = \frac{c^3}{4GM} = \frac{c}{2R}$$

M: mass

R: Schwarzschild Radius

2

Nation et al., *Rev. Mod. Phys*. **84**, (2012)

Hawking radiation: connections

$$k_B T = \frac{\hbar \alpha}{2\pi}, \quad \alpha = \frac{c^3}{4GM} = \frac{c}{2R}$$

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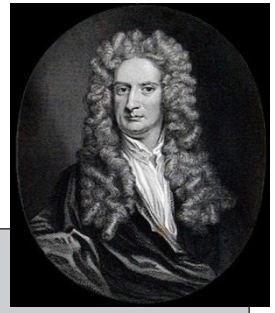
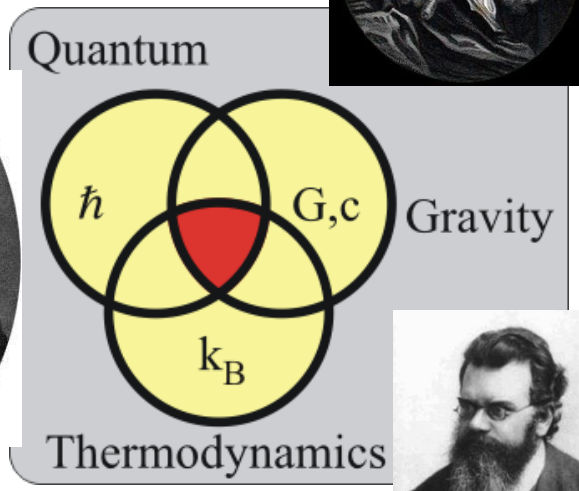
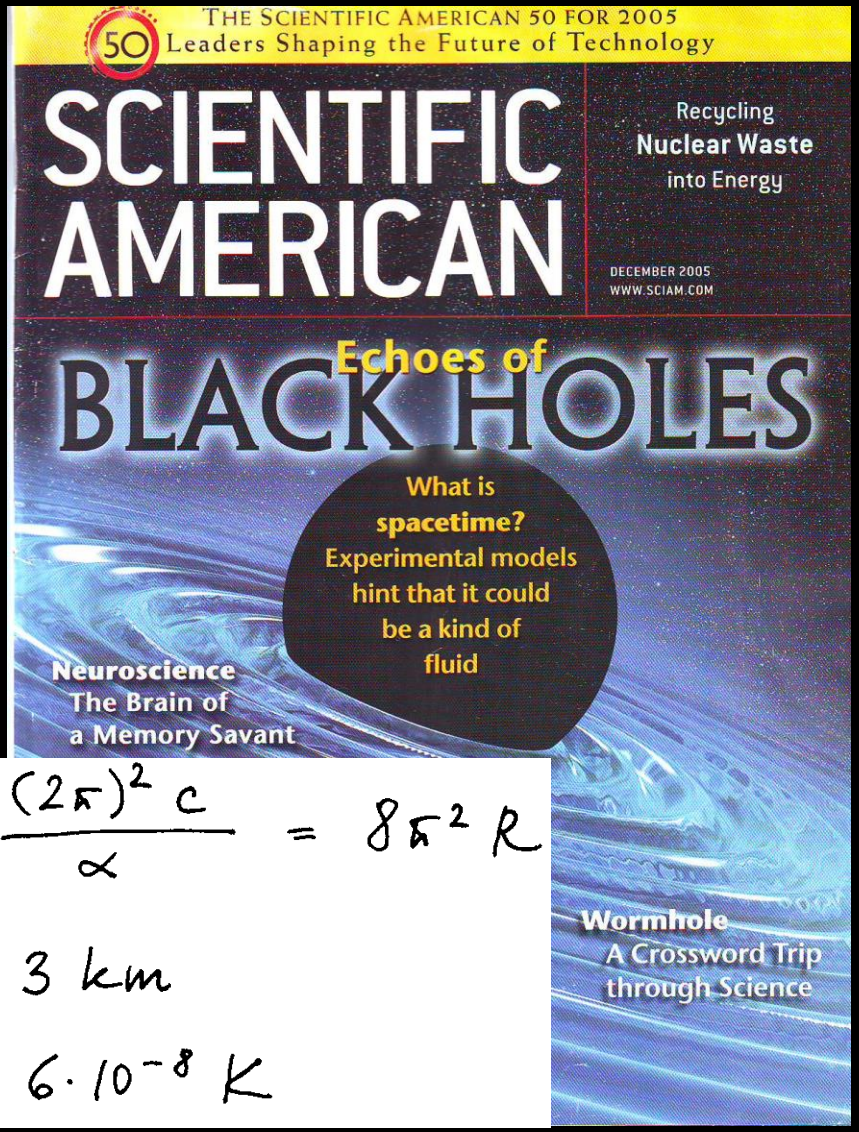




S. M. Hawking, *Nature* **248**, 30 (1974)

The Hawking effect connects the physics of everyday,
the physics of the very large, and the very small !

Hawking radiation: problems



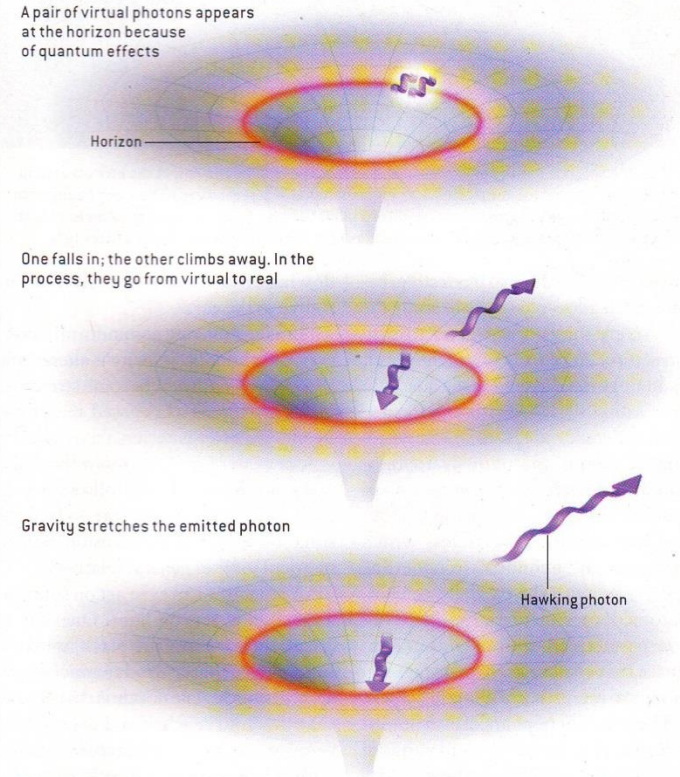
$$\lambda = \frac{(2\pi)^2 c}{\alpha} = 8\pi^2 R$$

$$R = 3\ \text{km}$$

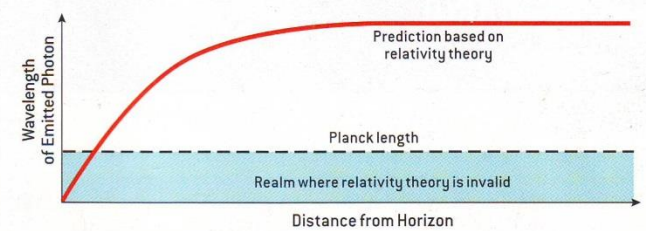
$$T = 6 \cdot 10^{-8}\ \text{K}$$

WAS HAWKING WRONG?

One of the greatest—and least recognized—mysteries of black holes concerns a flaw in Stephen W. Hawking's famous prediction that black holes emit radiation. A hole is defined by an event horizon, a one-way door: objects on the outside can fall in, but objects on the inside cannot get out. Hawking asked what happens to pairs of virtual particles (which continually appear and disappear everywhere in empty space because of quantum effects) that originate at the horizon itself.



Relativity theory predicts that a photon from the horizon gets stretched by an infinite amount (*red curve, below*). In other words, an observed photon must have originated as a virtual one with a wavelength of almost precisely zero, which is problematic because unknown quantum gravity effects take over at distances shorter than the so-called Planck length of 10^{-35} meter. This conundrum has driven physicists to design experimentally realizable analogues to black holes to see whether they indeed emit radiation and to understand how it originates.



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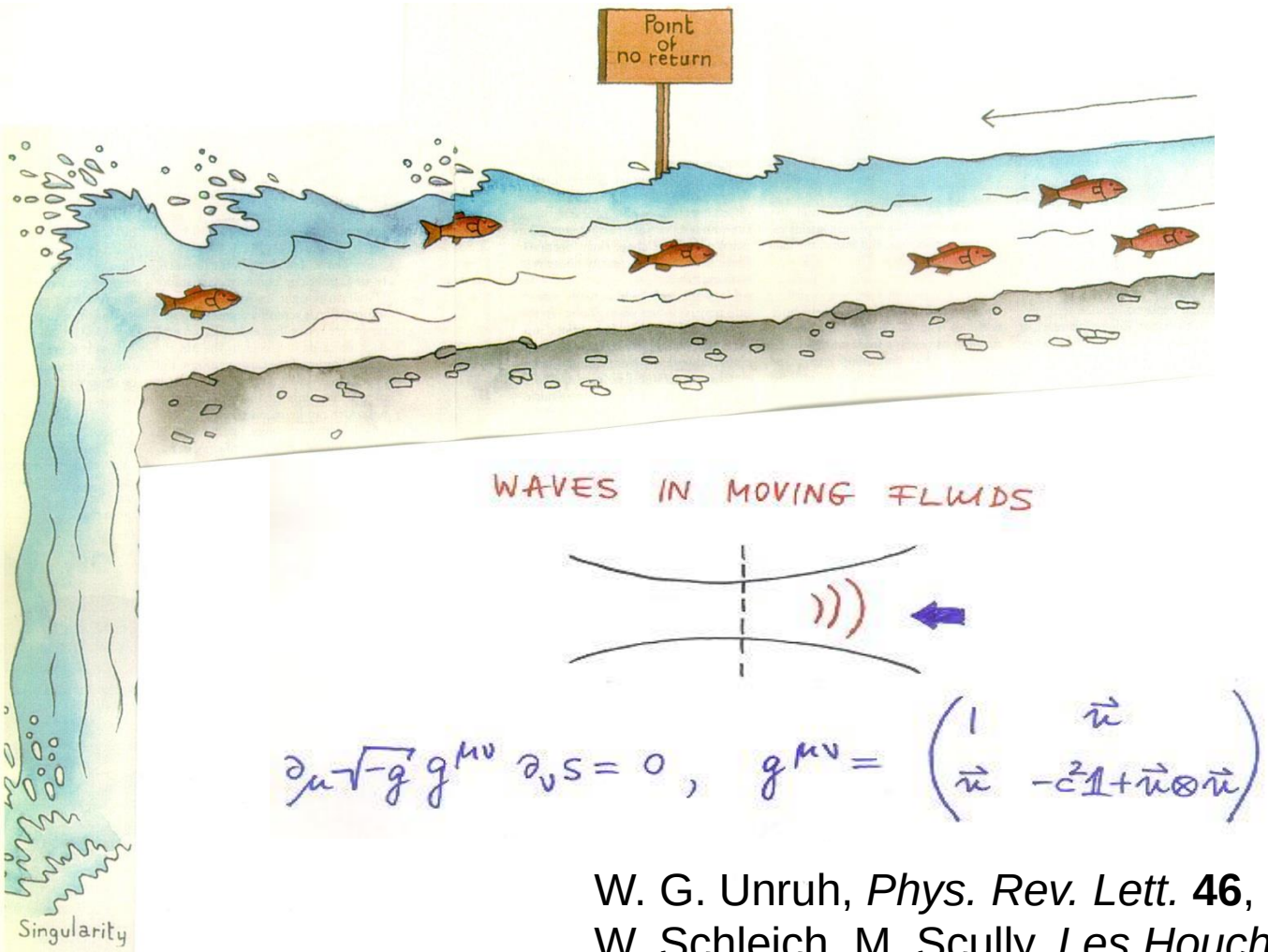


Illustration: Scientific American

W. Unruh

W. G. Unruh, *Phys. Rev. Lett.* **46**, 1351 (1981)
W. Schleich, M. Scully, *Les Houches 1982,* (1984)

Horizons in moving media

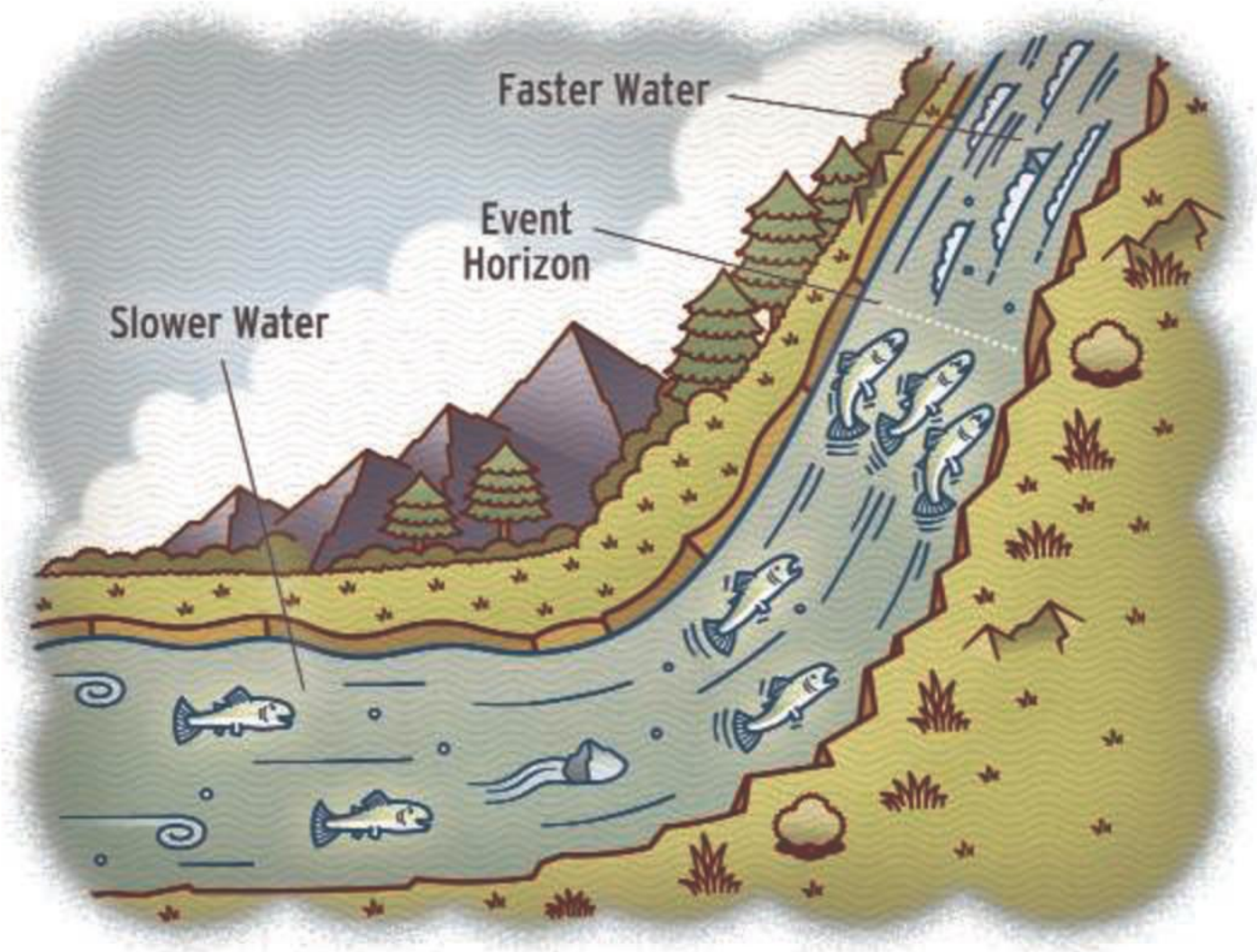


Illustration: Peter Hoey, Science **319**, 1321 (2008)

Th. Philbin et al., *Science* **319**, 1367 (2008)

Hydraulic jump

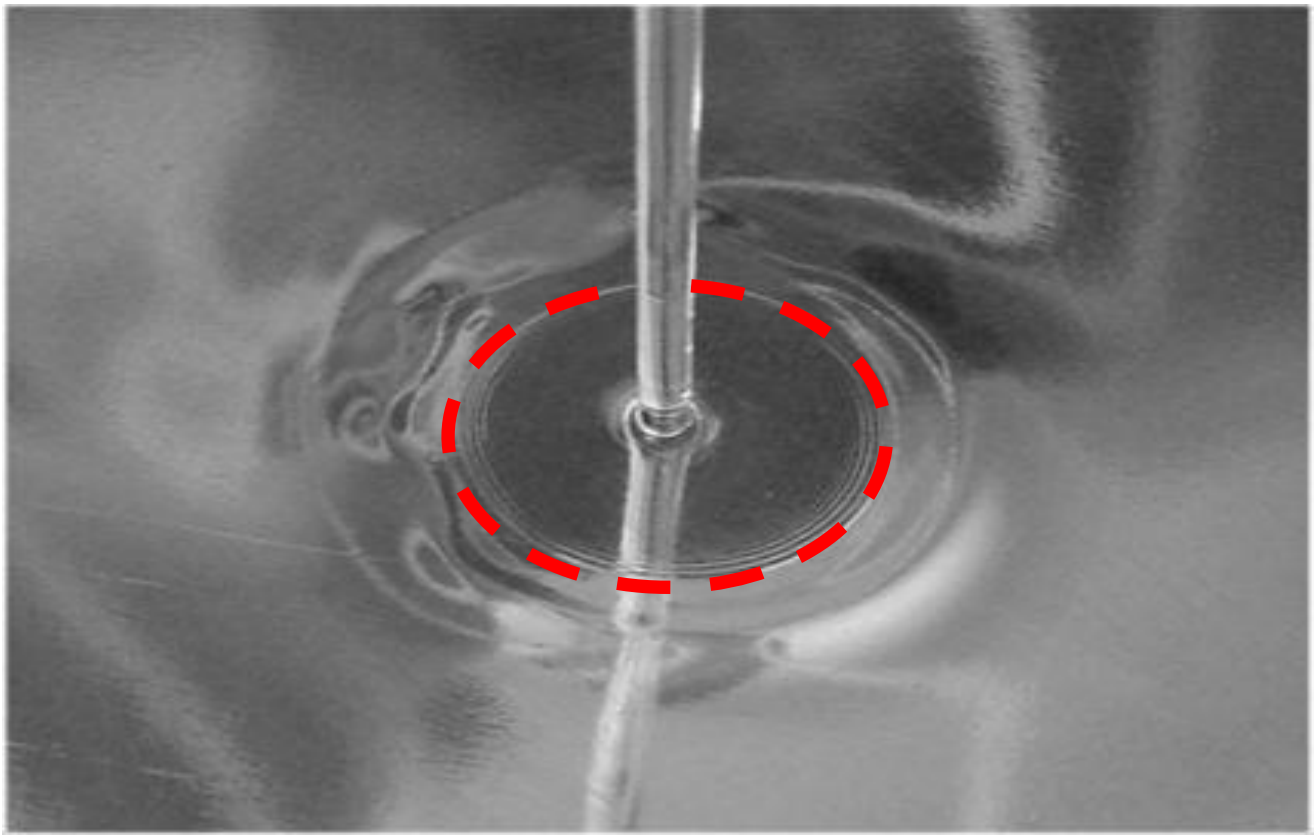
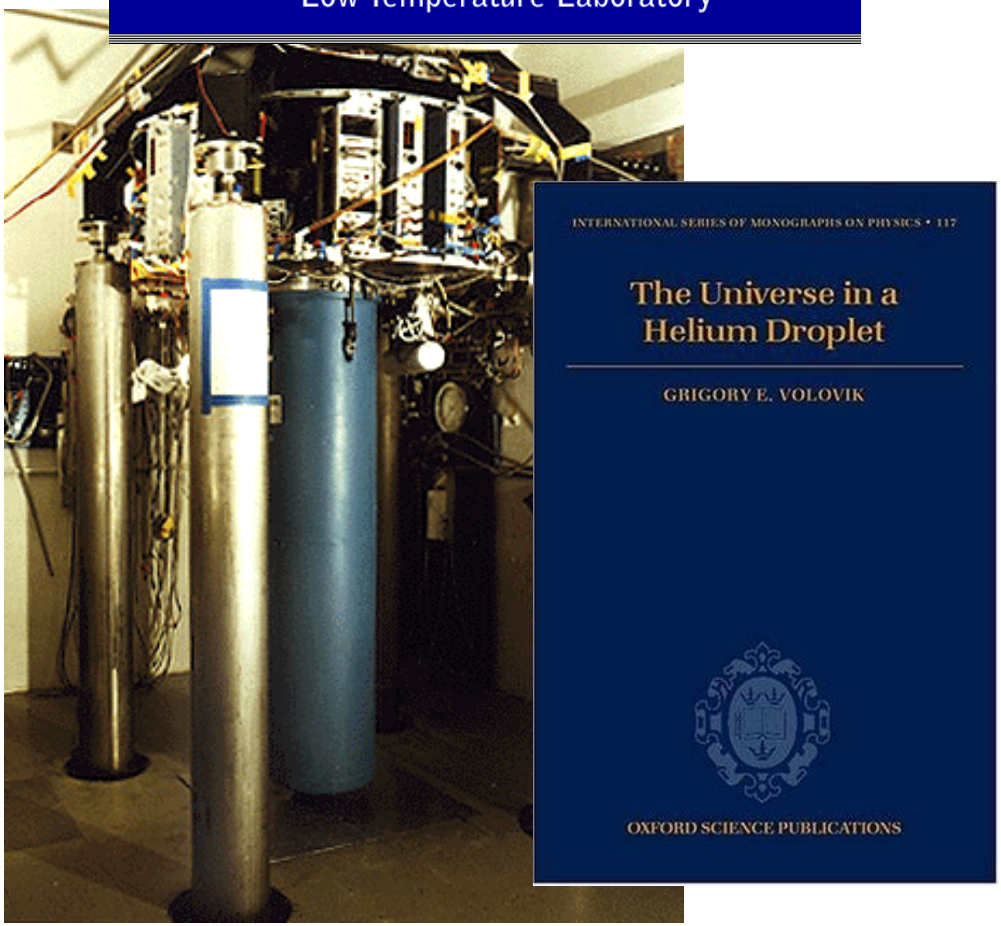
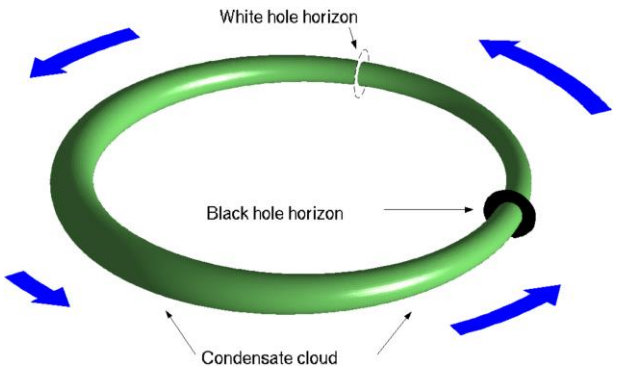


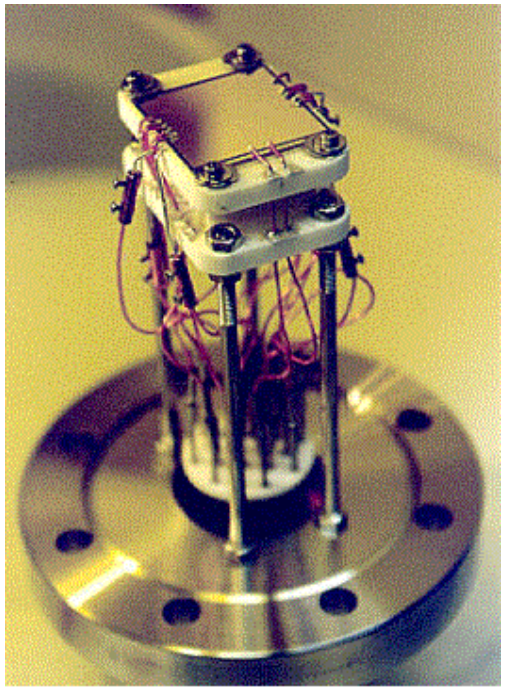
Photo: Piotr Pieranski

Artificial event horizons in superfluid Helium-3 and Bose Einstein Condensates

Kylmälaboratorio
Lågtemperaturlaboratoriet
Low Temperature Laboratory







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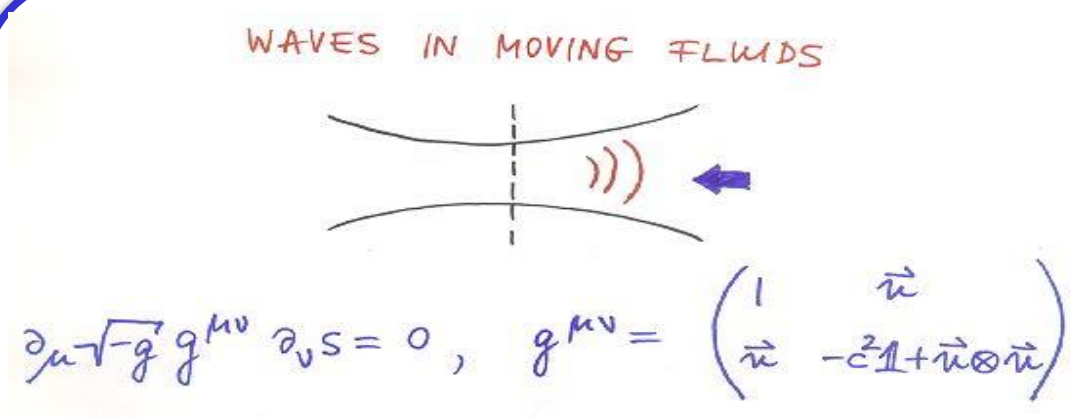
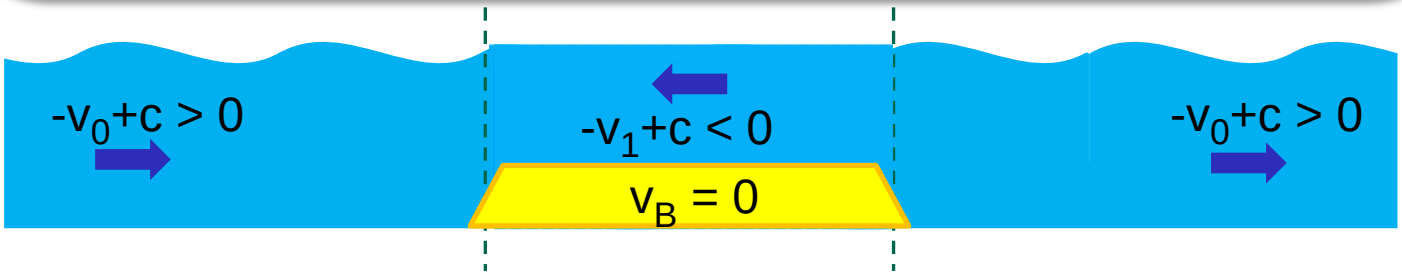
Waves and Barriers



W. G. Unruh, *Phys. Rev. Lett.* **46**, 1351 (1981)
W. Schleich, M. Scully, *Les Houches 1982,* (1984)

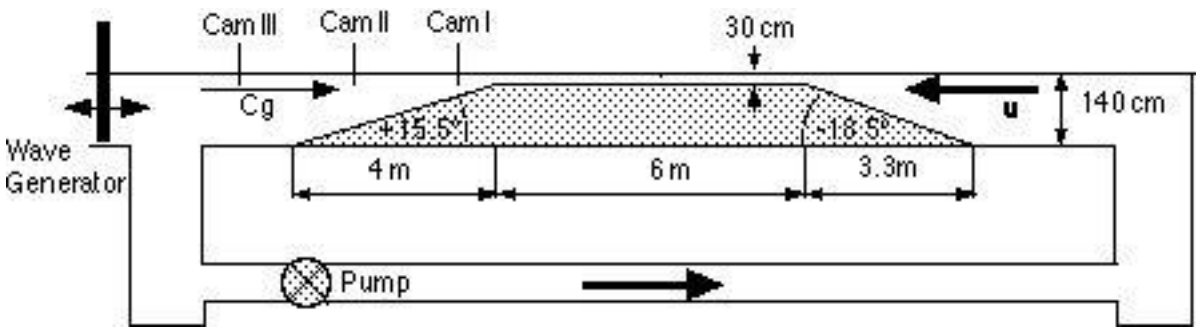


white hole
horizon WH

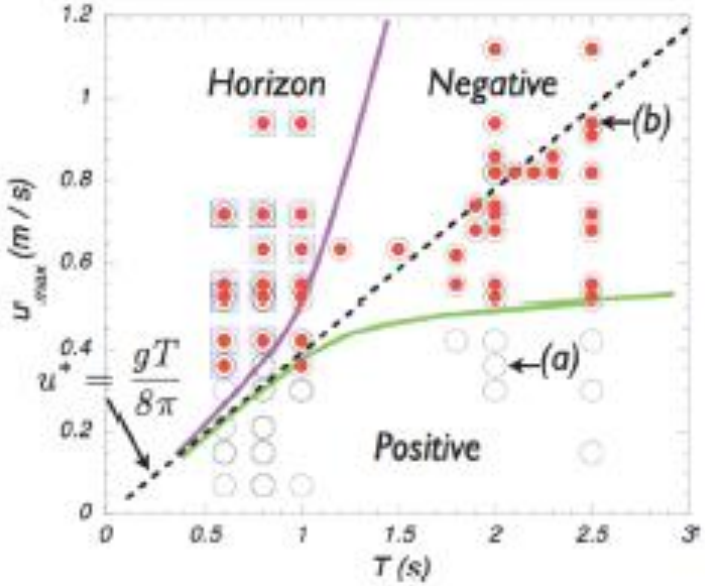
black hole
horizon BH

subluminal | superluminal | subluminal

Water waves



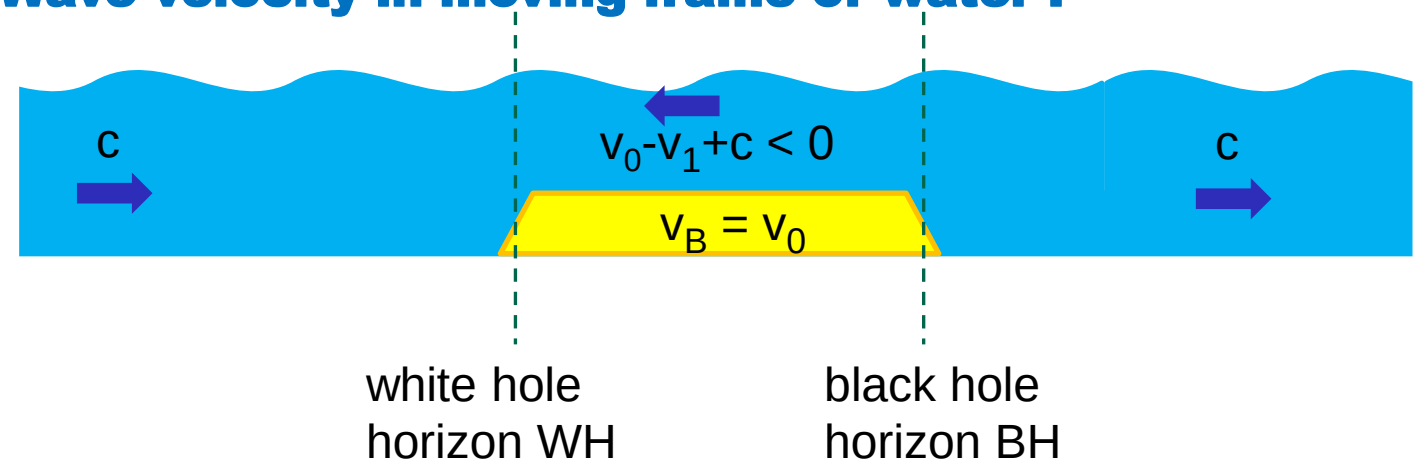
Germain Rousseaux



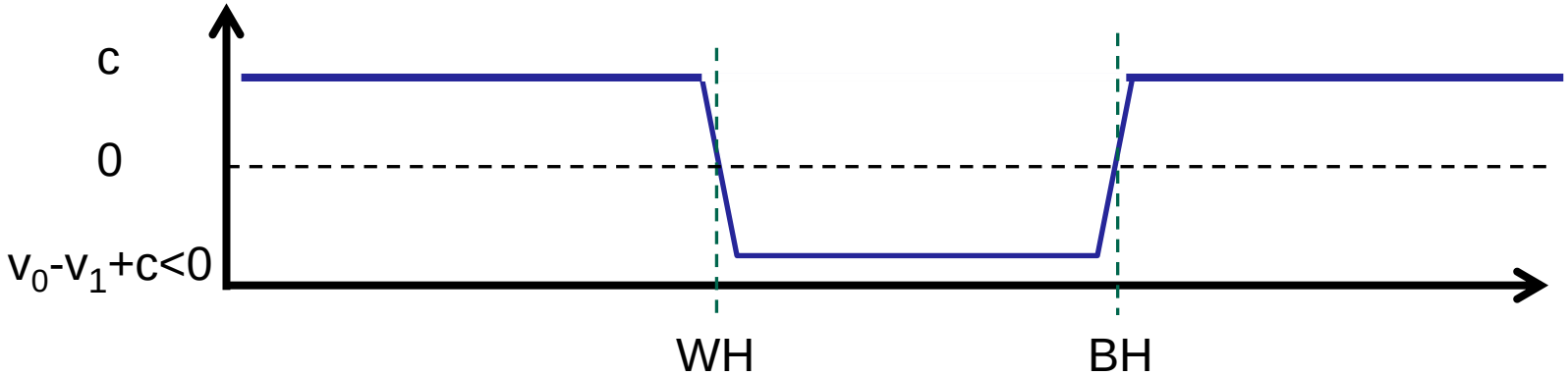
Rousseaux, G; Mathis, C; Maissa, P, et al. NEW JOURNAL OF PHYSICS **10**, 053015 (2008)

Waves and Barriers

Wave velocity in moving frame of water :

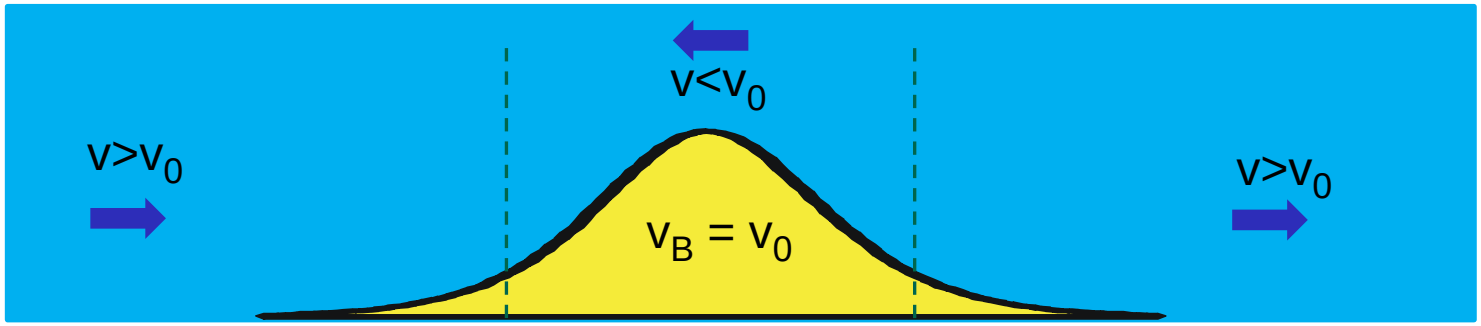


Velocity profile:

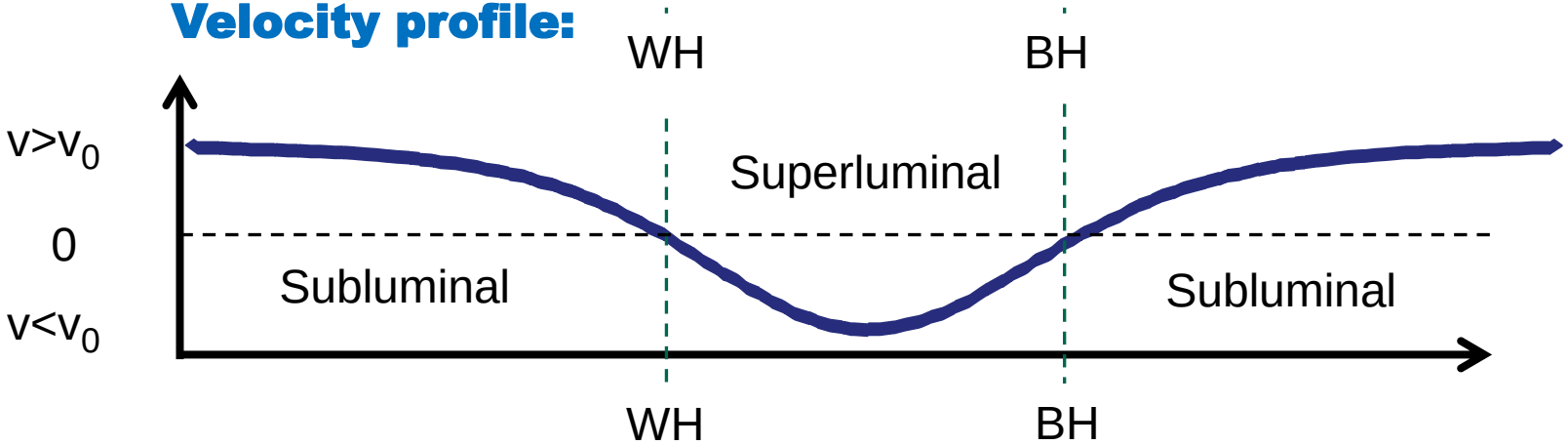


Optical horizons in media at rest?

Wave velocity in moving frame: optical pulse in Kerr medium



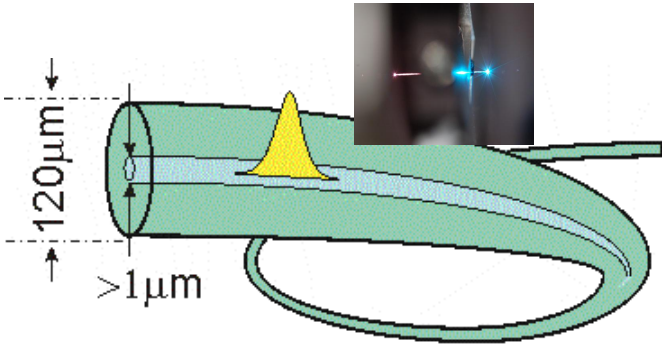
Velocity profile:



A pulse moves at velocity v_0 and locally modifies the refractive index (i.e. wave velocity) by the cross-Kerr effect.

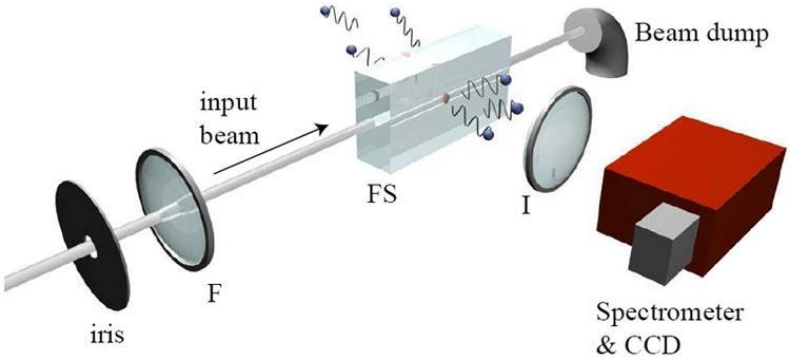
Optical event horizons

Optical fibers



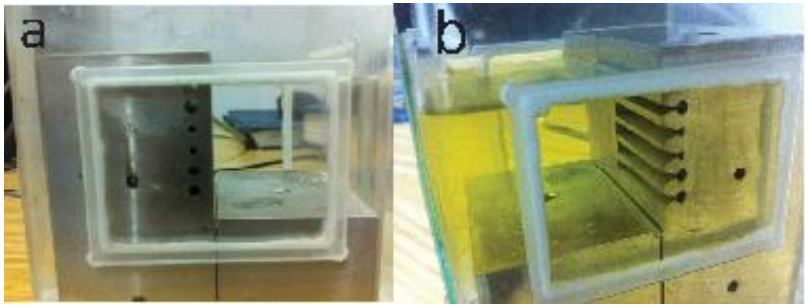
Philbin et. al. *Science* **319**, 5868 (2008)

Filamentation



Belgiorno et al., PRL **105**, 203901 (2010).

Laval nozzle

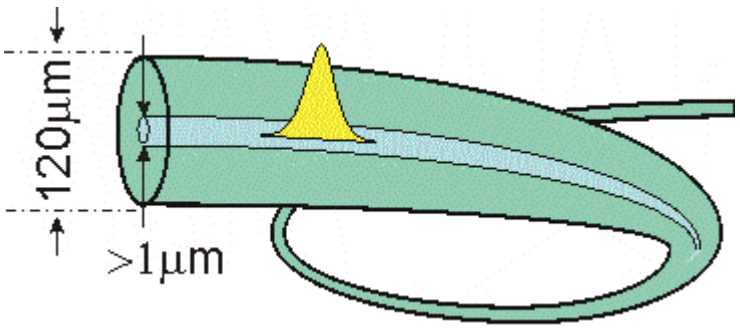


Elazar et al. Phys. Rev. A **86**, 063821(2012)

Conditions: Stationarity

Stationarity:

Soliton pulses in a 1-D fiber waveguide



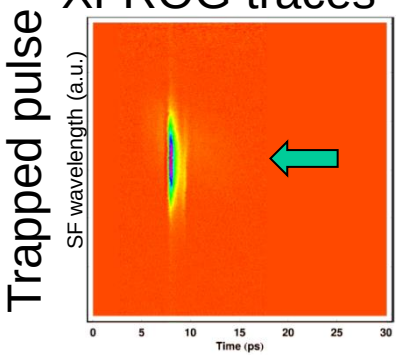
Filaments: F. Belgiorno et al., PRL **105**, 203901 (2010).

Negligible backaction: weak probe wave

Nonstationary: Pulse trapping:

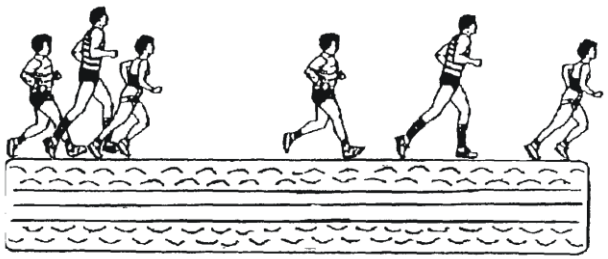
N. Nishizawa et al., *Opt. Lett*. **27**, 152 (2002)
A. V. Gorbach et al., *Nature Photonics* **1**, 653 (2007)
S. Hill et al., *Opt. Express* **17**, 13588 (2009)

Trapped pulse:
XFROG traces



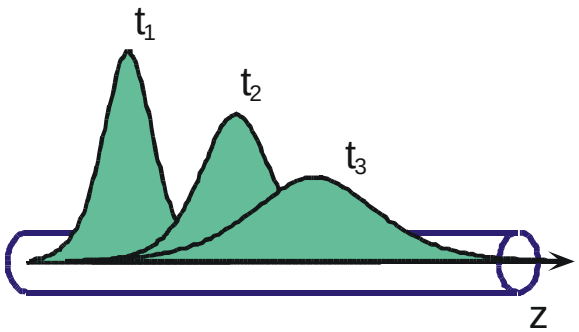
Soliton formation

"runners«wave packet" *



dispersion

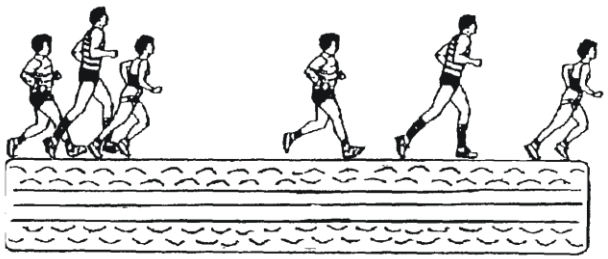
ultrashort pulses in optical fibers



Soliton formation

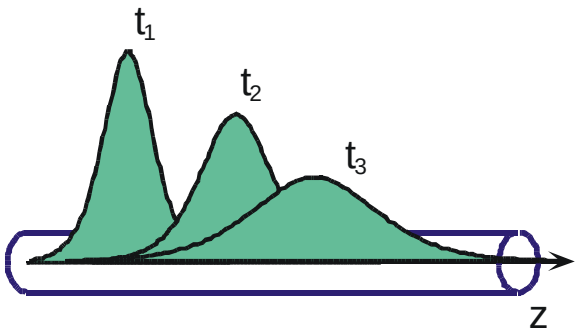
"runners«wave packet" *

ultrashort pulses in optical fibers



dispersion

+



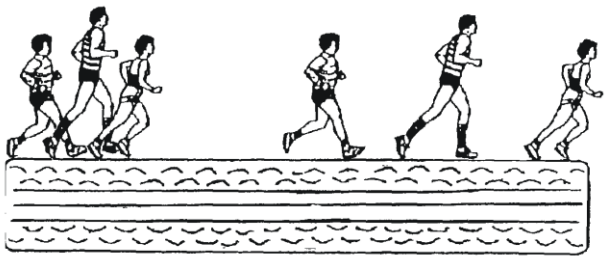
"soft mat"

nonlinearity

Kerr-effekt

Soliton formation

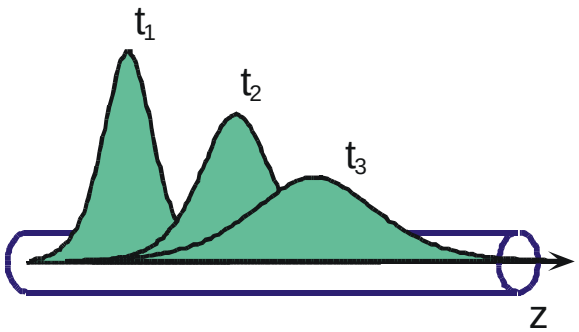
"runners«wave packet" *



dispersion

+

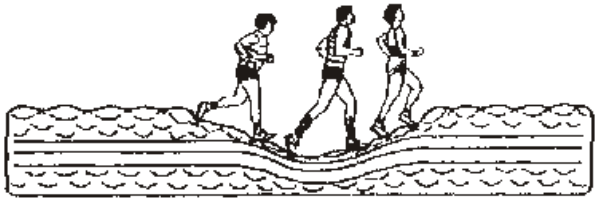
ultrashort pulses in optical fibers



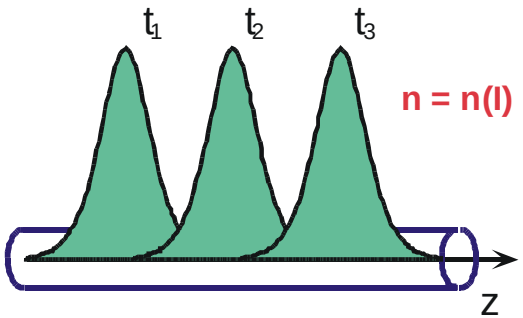
"soft mat"

nonlinearity

Kerr-effekt

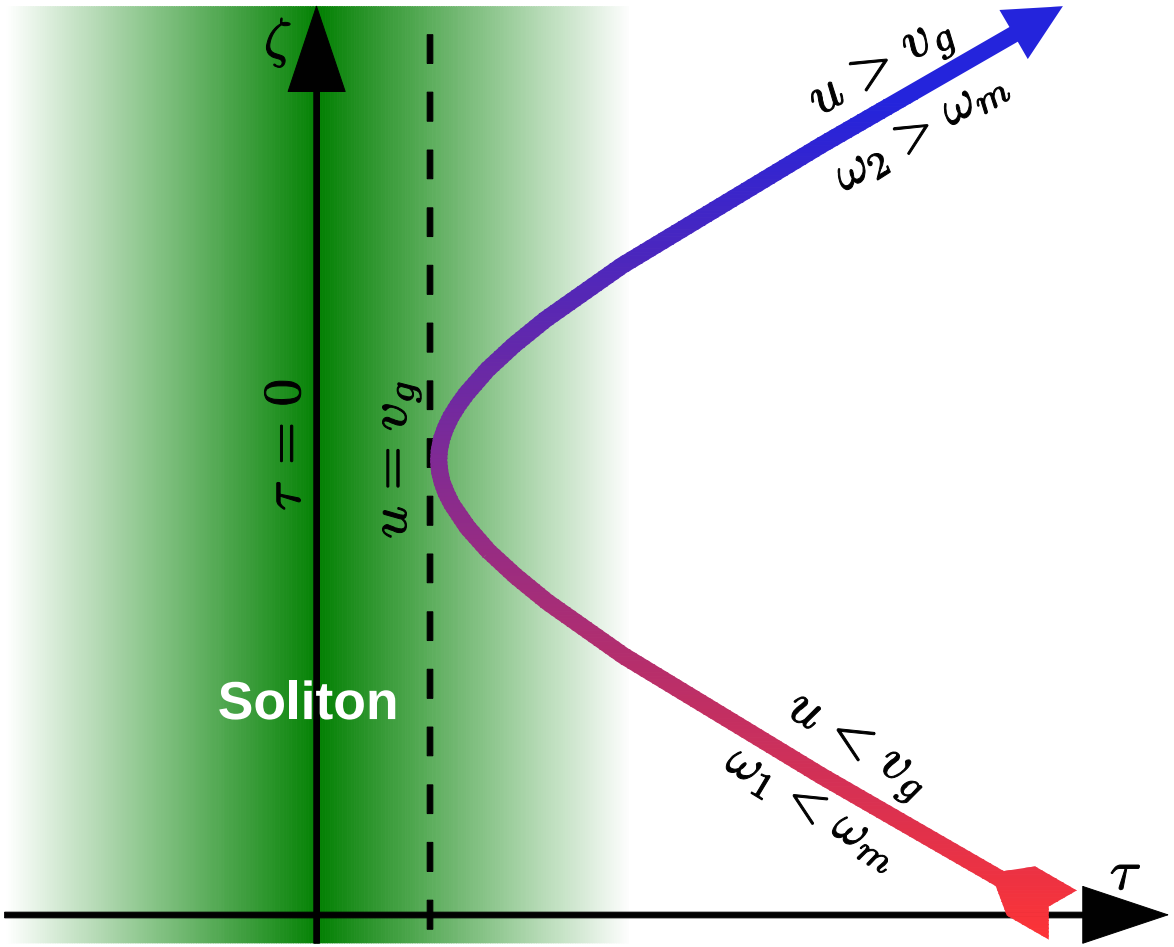


soliton



(* Mollenauer)

Horizons in moving media: trajectories



WH

Horizons in moving media: trajectories

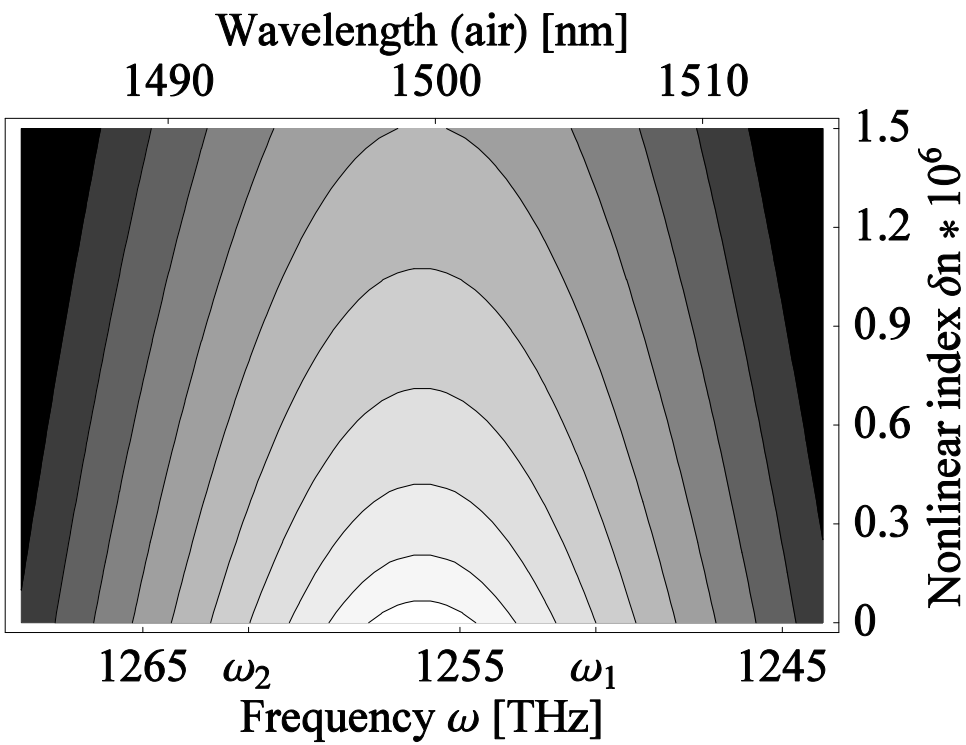
$A = \mathcal{A} e^{i\varphi}$ $\quad \varphi = \int (k\,dz - \omega\,dt)$ $\quad$ where $k = \frac{n\omega}{c}$, $\quad n = n_0(\omega) + \delta n$

$$\left.\begin{aligned} \tau &= t - \frac{z}{v} \\ \xi &= \frac{z}{u} \end{aligned}\right\} \text{ moving frame}$$

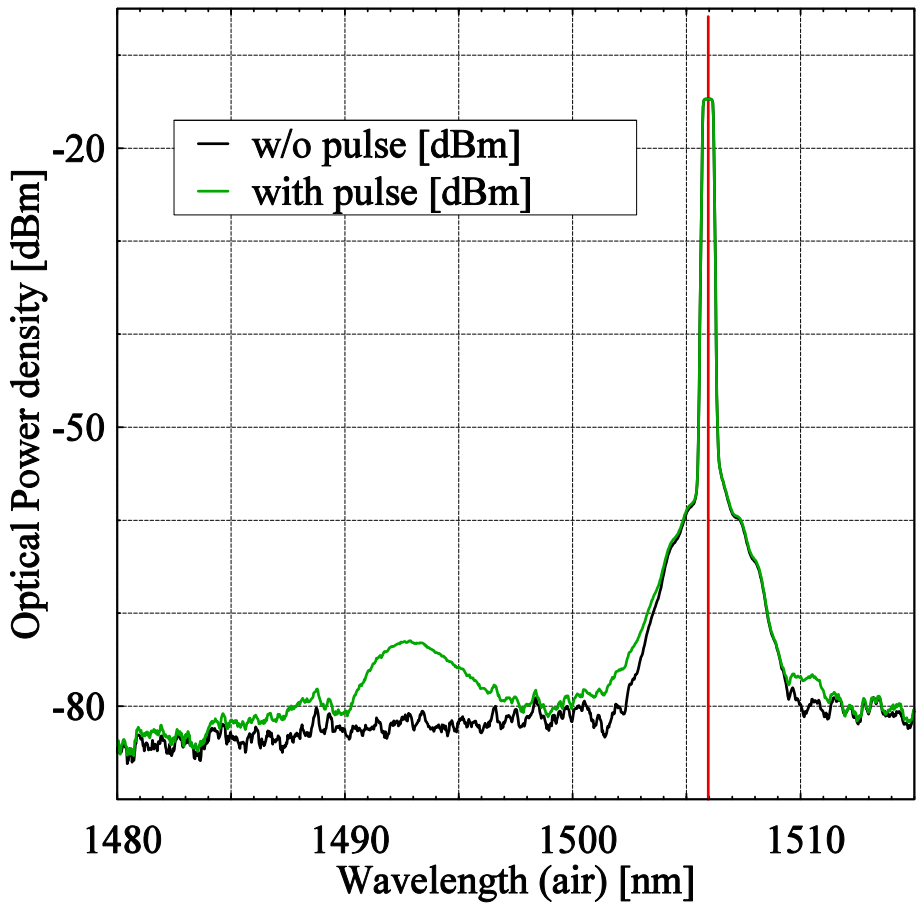
$$\rightarrow \varphi = -\int \omega\,d\tau - \omega'\xi$$

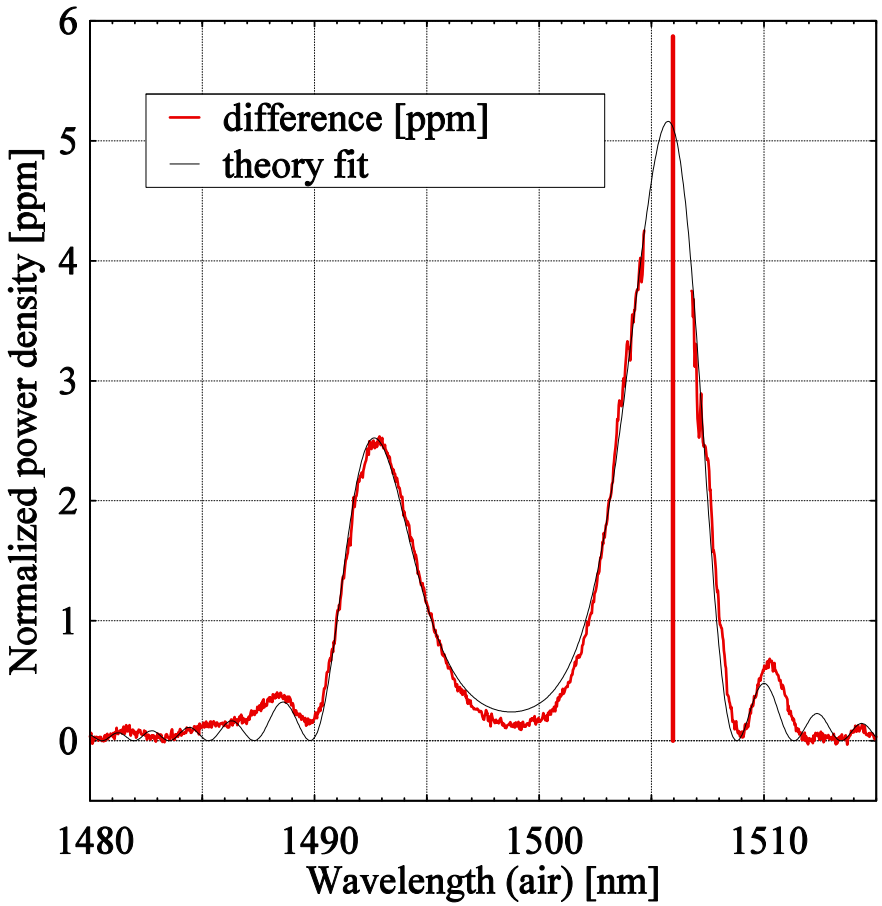
$$\omega' = \omega - vk$$

$$\boxed{\omega' = \left(1 - \frac{vn}{c}\right)\omega} \quad \text{Doppler}$$



Blue shifting at the white hole



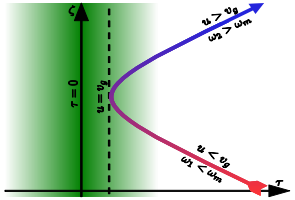


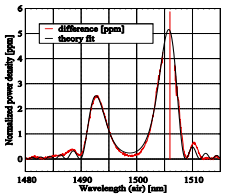
Th. Philbin, Ch. Kuklewicz, S. Robertson, S. Hill, F. König, U. Leonhardt
Science **319**, 5868, pp. 1367 – 1370 (2008)

As part of supercontinuum: A. Efimov et al., *Phys. Rev. Lett.*, **95** ,213902, (2005)

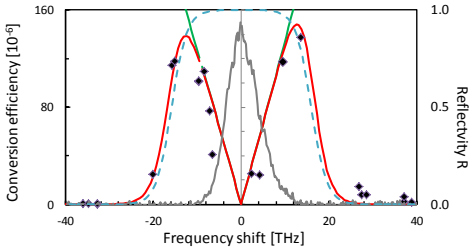
Optical horizon physics

- Characteristic and efficient mode conversion can be observed
  Th. Philbin et al., *Science* **319**, 1367 (2008)

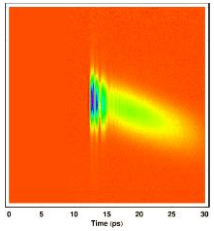




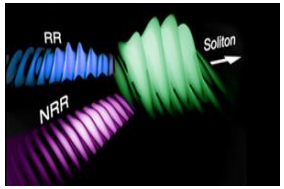
- Fast waves can tunnel through the horizon
  Choudhary and Koenig, Opt. Expr. **20**, 5538 (2012)



- Light can be trapped and released behind horizons
  S. Hill et al., *Opt. Express* **17**, 13588 (2009)



- Generation of waves with *negative frequency*
  E. Rubino et al., PRL **108**, 253901 (2012)

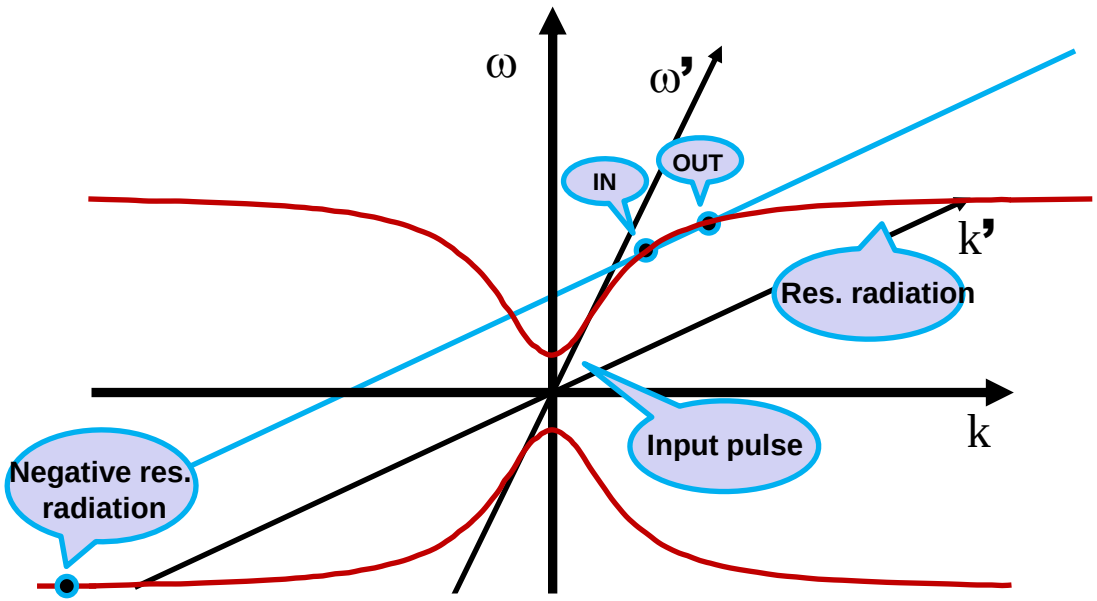


Horizons and modes in the dispersion relation

Transformation to the frame moving with the pulse:

$$\omega' = \gamma(\omega - vk)$$

$$k' = \gamma(k - v/c^2(\omega)$$



$$\beta(\omega) - k - \beta_0 - (\omega - \omega_0)/v = 0 \quad \Longleftrightarrow \quad \omega_0' - \omega' = 0$$

Relative wave vector:

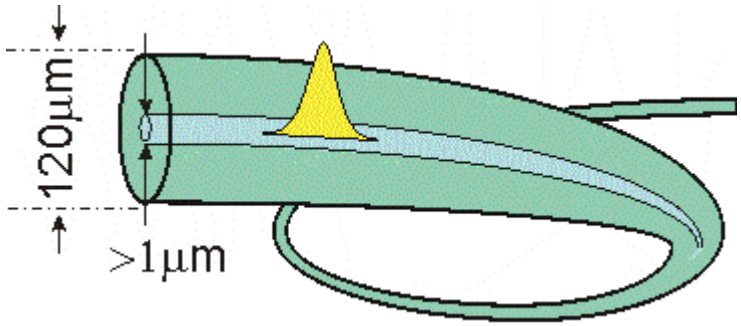
$$\beta(\omega) = k + \beta_0 + (\omega - \omega_0)/v$$

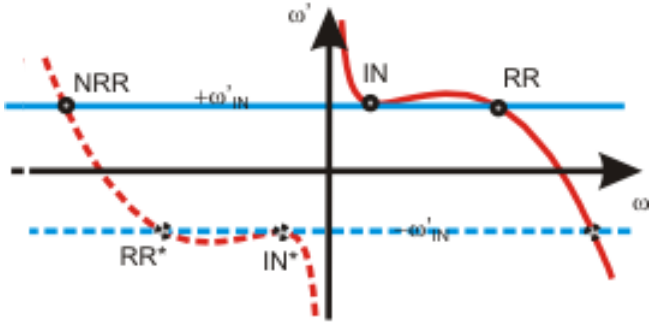
Moving frame frequency:

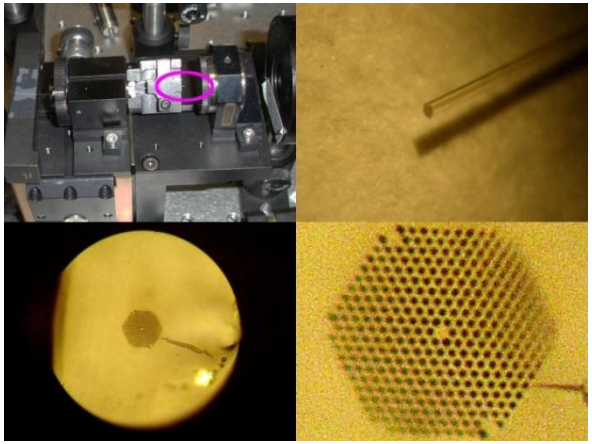
$$\omega' = \omega - v\,\beta(\omega)\,(Doppler\,relation)$$

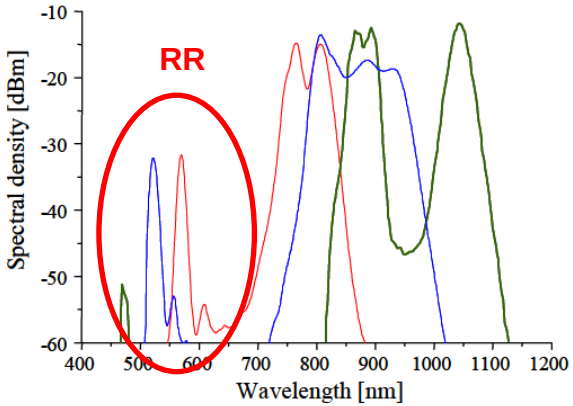
Conservation of <u>momentum</u> in the lab frame ⟷ Conservation of <u>energy</u> in the moving frame

Excitation of the RR mode in fibers









Degiorgio et. al., Opt. Expr. **12**, 124 (2004)

Resonant radiation from solitons:

- Wai et. al. Optics Lett. **11,** 464 (1986)
- Karpman, PRE, **47,** 2073, (1993)
- Akhmediev and Karlsson, PRA, **51**, 3 (1995)
- Tartara et. al. Appl. Phys. B, **77**, 307 (2003)
- Skryabin et. al., Science, **301,** 1705 (2003)
- Chang et al., Opt. Express **19**, 6635 (2011)
- …

Negative frequency ?

Light always oscillates with positive and negative frequencies:

$$A(t) = \int_{-\infty}^{\infty} \tilde{A}(\omega)\, e^{-i\,\omega t}\, d\omega$$

For real fields A therefore: $\tilde{A}(-\omega) = \tilde{A}^*(\omega)$

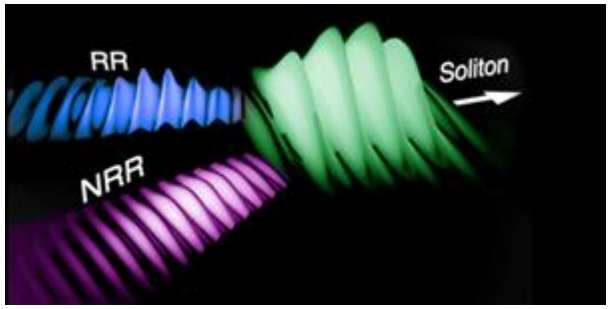


Image: APS/Alan Stonebraker

Negative frequencies are redundant !?

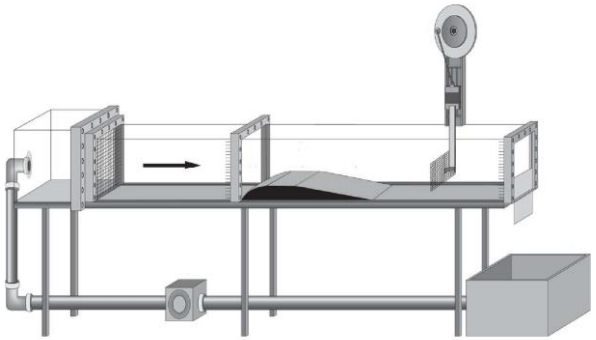
Quantum fields:

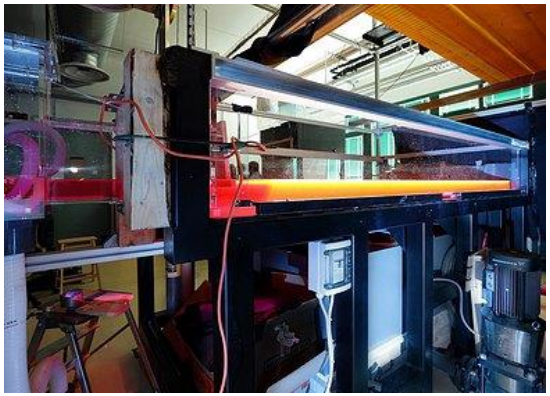
$$\hat{A} = \sum_k \left(A_k \hat{a}_k + A_k^* \hat{a}_k^\dagger \right)$$

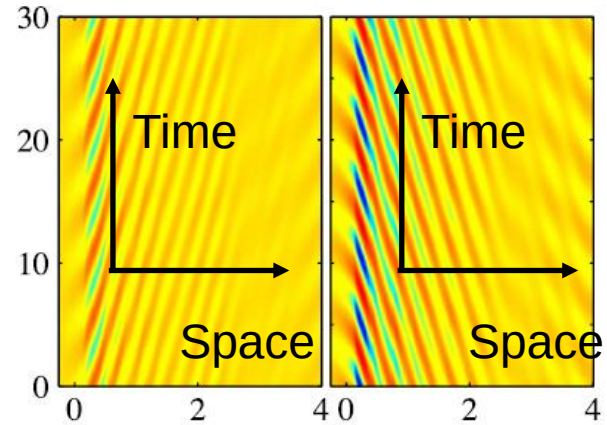
Hermitian

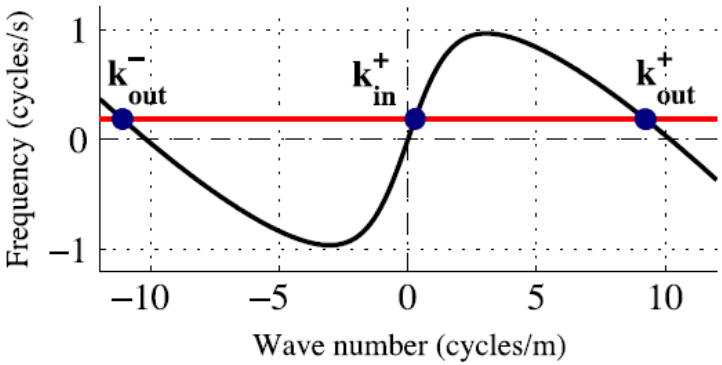
annihilation / creation amplitudes

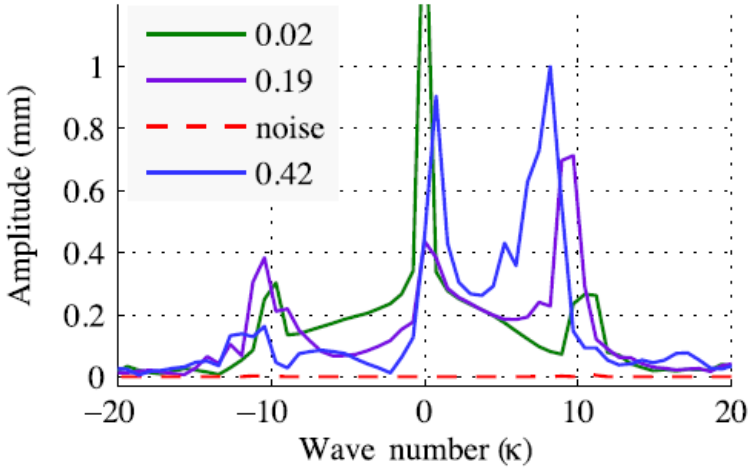
Negative frequency waves





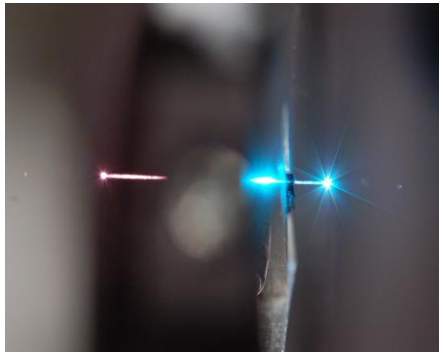


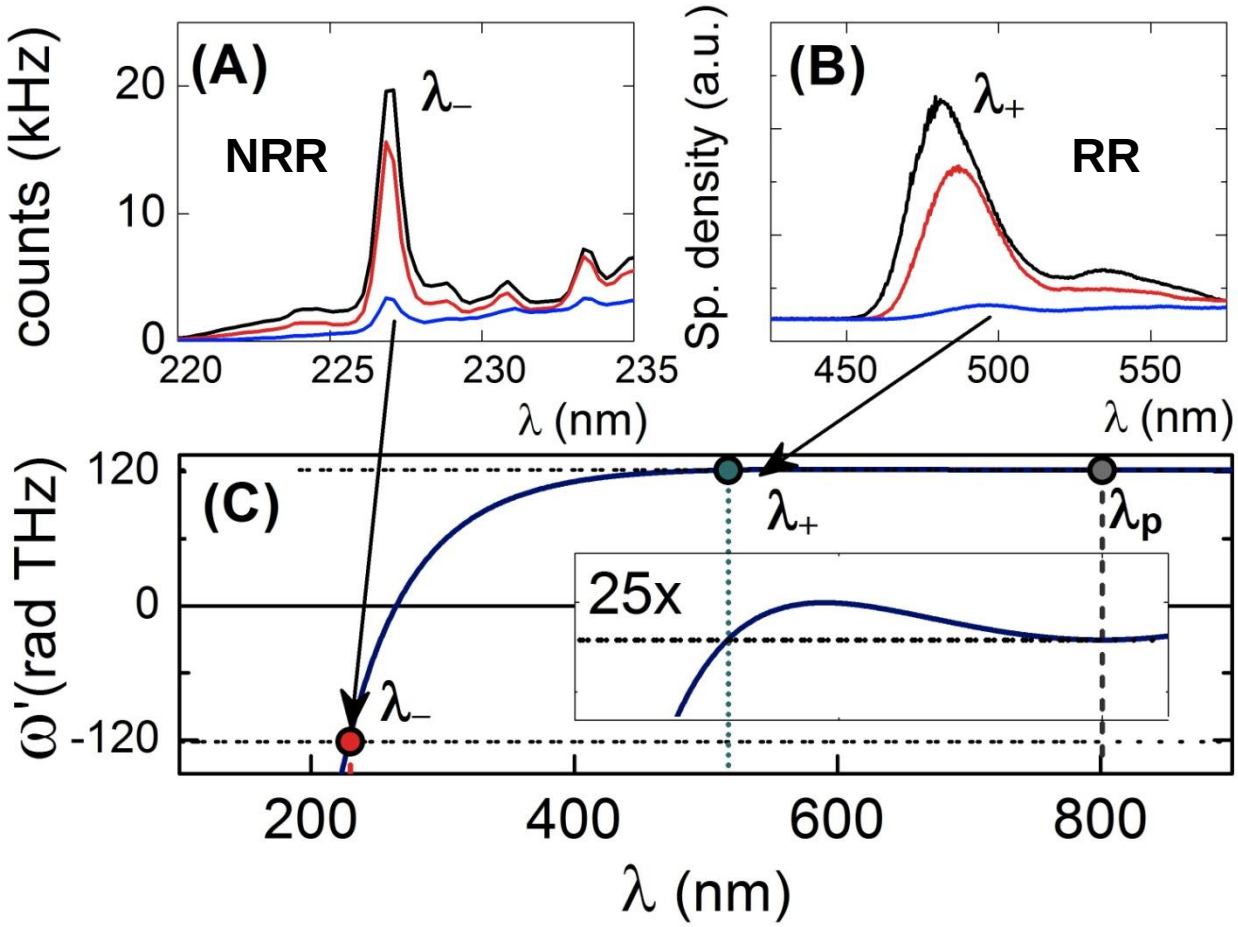




S. Weinfurtner et al., PRL **106** 021302 (2011)
Rousseaux, G., et al. New J. Phys. **10**, 053015 (2008)

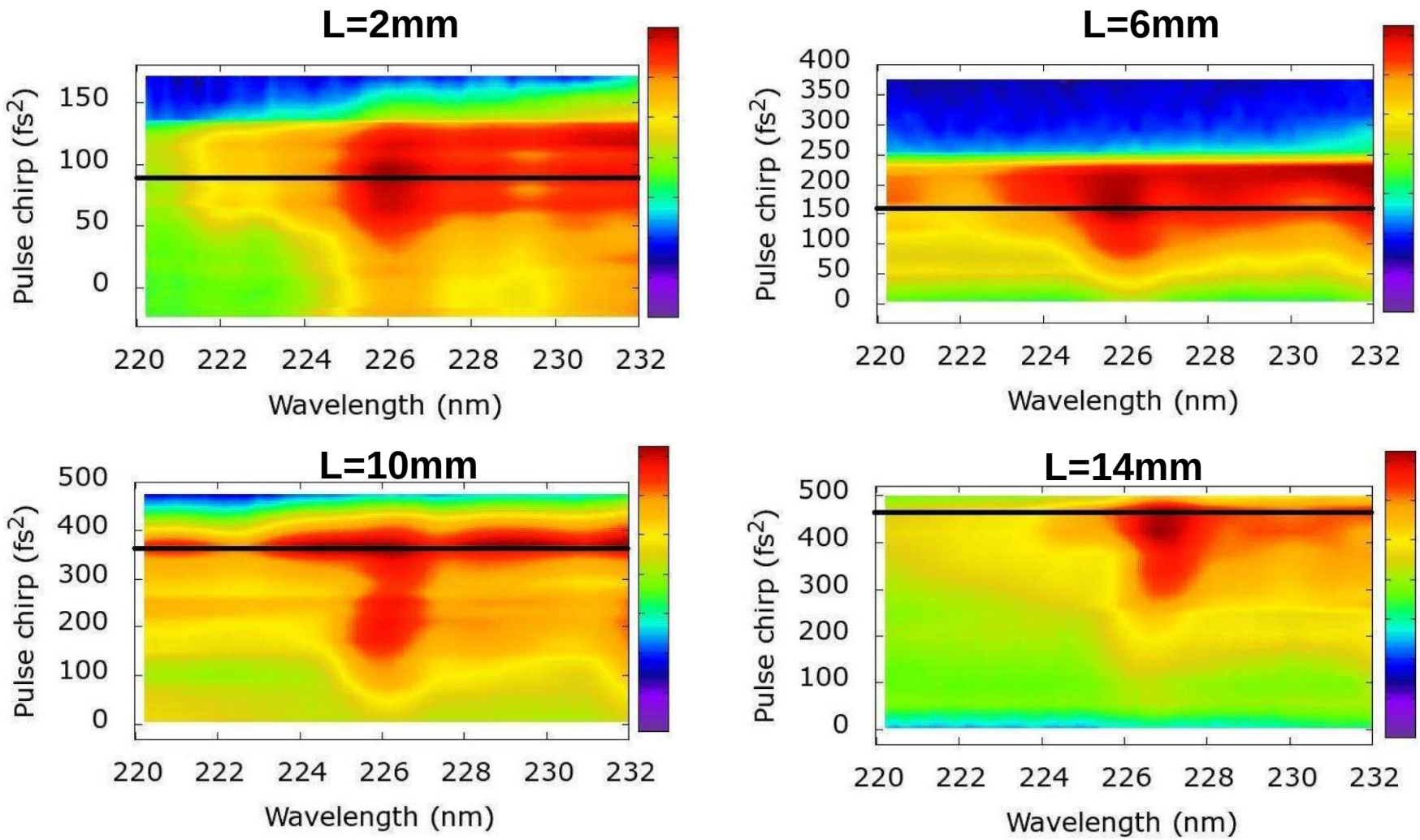
Experiments in fibers





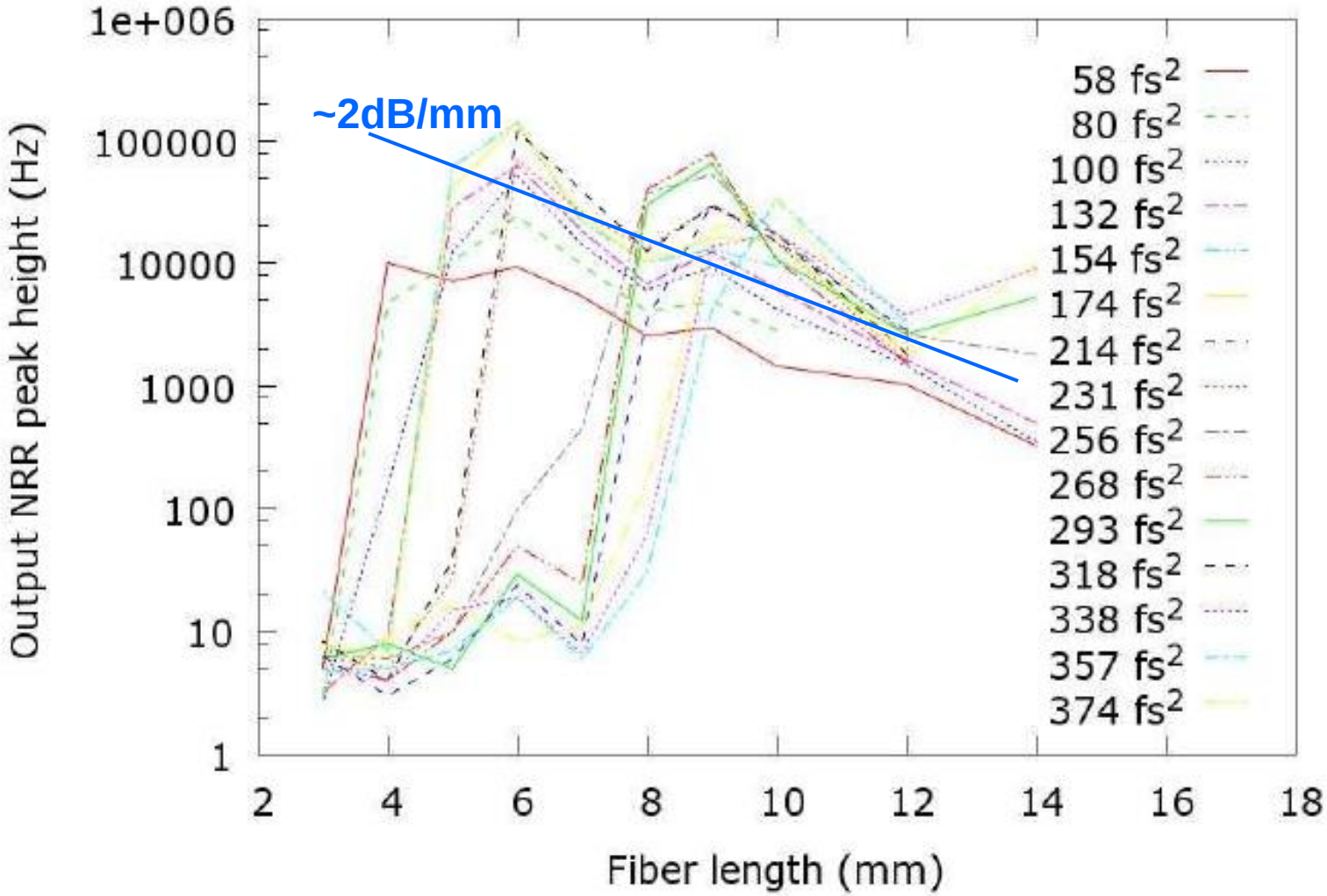
E. Rubino et al., PRL **108**, 253901 (2012)

NRR spectra



J. McLenaghan and F. Koenig New J. Phys. **16** 063017 (2014)

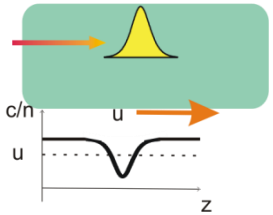
NRR spectra

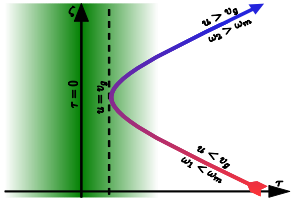


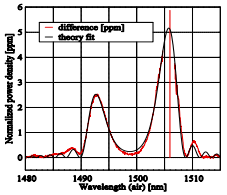
J. McLenaghan and F. Koenig New J. Phys. **16** 063017 (2014)

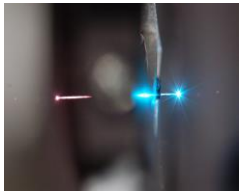
Conclusion

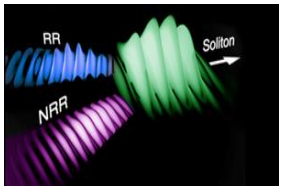
- Optical pulses in fibers form analogue black and white hole horizons
- Characteristic and efficient mode conversion can be observed
- Negative frequencies are physical because we can couple positive and negative frequency modes to generate new waves
- Conversion between negative and positive frequencies leads to photon creation/ amplification
- First steps towards astrophysical particle creation in optical analogues

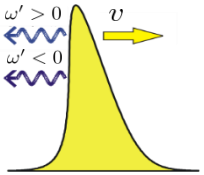












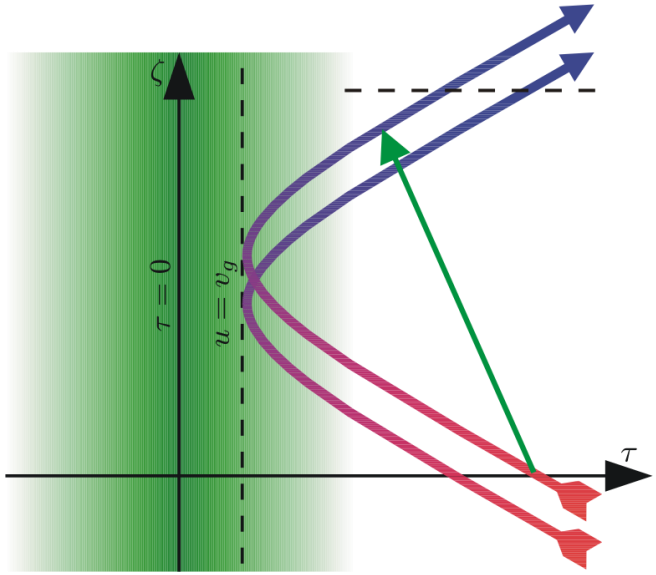
Applications of trapping and optical horizons

Frequency conversion – classical and quantum

‘Frequency conjugation’

Dispersion management

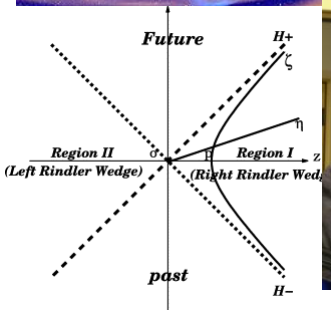
Ultrafast optical delay lines



Outlook: Mixing of A_k and A_k^*

Mixing of amplitudes creates photon pairs, even from the quantum vacuum state.
In astrophysics, the particle creation by mixing of positive and negative frequencies occurs:

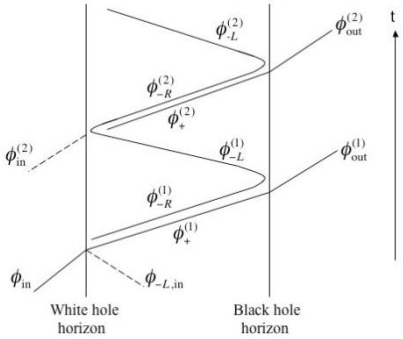
- **At the event horizon of black holes: Hawking effect**
  *(Hawking, Nature **248**, 30 (1974))*



- **In accelerated systems: Unruh effect**
  *(Unruh, Phys. Rev. D **14**, 870 (1976))*

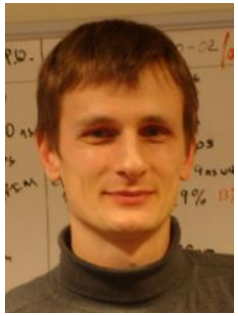
- **Novel parametric amplifier**

- **"Black hole Laser"**
  (S. Corley and T. Jacobson, PRD, 59:124011, 1999)



The horizon team

University of St Andrews



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Christopher Kuklewicz
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Scott Robertson
Sven Rohr

Thank you !

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