Pre-Inflationary Clues from String Theory?

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\[ A_\ell \sim \ell (\ell + 1) \int \frac{dk}{k} P_R(k) \, j_\ell (k \Delta \eta)^2 \sim P_R \left( k = \frac{\ell}{\Delta \eta} \right) \]

- Cosmic Variance
\[ A_\ell \sim \ell (\ell + 1) \int \frac{dk}{k} P_R(k) j_\ell (k \Delta \eta)^2 \sim P_R \left( k = \frac{\ell}{\Delta \eta} \right) \]

- Cosmic Variance

One usually \textbf{ASSUMES} that the inflaton “starts out” high up

\textbf{String Theory (+ high-scale SUSY breaking)} \rightarrow \text{a way to bring it there}

[with potential lessons for low CMB multipoles]
\[ A_\ell \sim \ell(\ell + 1) \int \frac{dk}{k} P_R(k) j_\ell(k\Delta \eta)^2 \sim P_R \left( k = \frac{\ell}{\Delta \eta} \right) \]

- Cosmic Variance

**Hard bounce:** new class of "features"

[with potential lessons for low CMB multipoles]
a) In bulk:

b) With branes:

c) Brane SUSY breaking (B.S.B.):
  - [O-plane charges → antibranes]
  - [classically STABLE: NO Tachyons]
**SUSY Breaking in String Theory**

a) In bulk:

b) With branes:

c) Brane SUSY breaking (B.S.B.):

- [O-plane charges → antibranes]

[Classically STABLE: NO Tachyons]

**Vacuum redefinitions!**

\[
S_{10} = \frac{1}{2k_{10}^2} \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} \left( -R + 4(\partial\phi)^2 \right) - Te^{-\phi} + \ldots \right\}
\]
One–Scalar (Exp–)Cosmologies

• Consider the action for gravity and a scalar $\phi$:

\[ S_{10} = \frac{1}{2k_D^2} \int d^Dx \sqrt{-g} \left\{ -R - \frac{1}{2} (\partial \phi)^2 - V(\phi) + \ldots \right\} \]

• Look for **cosmological solutions** of the type

\[ ds^2 = -e^{2B(t)} dt^2 + e^{2A(t)} dx_i \cdot dx_i \]

• Convenient gauge choice \((V \neq 0)\)

\[ V(\phi) e^{2B} = M^2 \]

• In expanding phase:

\[ \ddot{\varphi} + \dot{\varphi} \sqrt{1 + \dot{\varphi}^2} + (1 + \dot{\varphi}^2) \frac{1}{2V} \frac{\partial V}{\partial \varphi} = 0 \]

• **OUR CASE**:

\[ V = \exp(2\gamma \varphi) \rightarrow \frac{1}{2V} \frac{\partial V}{\partial \varphi} = \gamma \]
$V = e^{2\gamma \varphi}$: Climbing & Descending Scalars

- $\gamma < 1$? Both signs of speed
  a. "Climbing" solution ($\varphi$ climbs, then descends):

  $$\varphi = \frac{1}{2} \left[ \sqrt{\frac{1-\gamma}{1+\gamma}} \coth \left( \frac{\tau}{2} \sqrt{1-\gamma^2} \right) - \sqrt{\frac{1+\gamma}{1-\gamma}} \tanh \left( \frac{\tau}{2} \sqrt{1-\gamma^2} \right) \right]$$

  b. "Descending" solution ($\varphi$ only descends):

  $$\varphi = \frac{1}{2} \left[ \sqrt{\frac{1-\gamma}{1+\gamma}} \tanh \left( \frac{\tau}{2} \sqrt{1-\gamma^2} \right) - \sqrt{\frac{1+\gamma}{1-\gamma}} \coth \left( \frac{\tau}{2} \sqrt{1-\gamma^2} \right) \right]$$

Limiting $\tau$-speed (LM attractor):
(Lucchin and Matarrese, 1985)

$$v_{lim} = -\frac{\gamma}{\sqrt{1-\gamma^2}}$$

$\gamma = 1$ is "critical": LM attractor & descending solution disappear there and beyond

CLIMBING: in ALL asymptotically exponential potentials with $\gamma \geq 1$!

10D STRING THEORY HAS PRECISELY $\gamma = 1$

- $\gamma = 1$:

  $$\varphi(\tau) = \varphi_0 + \frac{1}{2} \left[ \log |\tau - \tau_0| - \frac{1}{2} (\tau - \tau_0)^2 \right]$$

  $$A(\tau) = A_0 + \frac{1}{2} \left[ \log |\tau - \tau_0| + \frac{1}{2} (\tau - \tau_0)^2 \right]$$
Climbing can inject Inflation

a. “Hard” exponential of Brane SUSY Breaking
b. “Soft” exponential ($\gamma < 1/\sqrt{3}$):

$$V(\varphi) = V_0 \left( e^{2\varphi} - e^{2\gamma \varphi} + V'(\varphi) \right)$$

For $n_s \approx 0.96$

- BSB “Hard exponential” $\Rightarrow$ makes initial **climbing** phase inevitable
- “Soft exponential” (or other $V'$) $\Rightarrow$ drives **inflation** in subsequent descent

Local signature of “bounce”

E.g.: in Starobinsky-like potentials

$\varphi_0$: “intensity” of kick!
With BSB in String Theory

\[ V(\varphi) = V_0 \left( e^{2\varphi} + e^{2\gamma \varphi} + V'(\varphi) \right) \]

- **D = 10**: 
  - \( \gamma = 1 \rightarrow \text{Climbing!} \)
  - **Natural**: weak string coupling
  - **Surprise**: [NON singular in "string frame"]

- **D < 10**: 
  - \((\varphi, \sigma) \rightarrow (\Phi_s, \Phi_t) \) & \( \Phi_t \text{ HAS } \gamma = 1 \)
  - **Climbing** if \( \Phi_s \) is stabilized (!)

- **Branes with** \( T \sim e^{-\alpha \Phi} \) (if \( \Phi_s \) is stabilized):

  \[ \gamma = \frac{1}{12} (p + 9 - 6 \alpha) \]

- **Briefly**:

  \[
  S_D = \frac{1}{2k_D^2} \int d^D x \sqrt{-g} \left\{ -R - \frac{1}{2} (\partial \phi)^2 - \frac{2(10-D)}{D-2} (\partial \sigma)^2 - Te^{\frac{3}{2} \phi - \frac{10-D}{D-2} \sigma} + \ldots \right\}
  \]

  \[
  \Delta = \sqrt{\frac{2(D-1)}{(D-2)}}
  \]

- **Two scalar combinations** \((\phi \text{ and } \sigma \rightarrow \Phi_s \text{ and } \Phi_t)\). Focus on \( \Phi_t \):

  \[
  S_D = \frac{1}{2k_D^2} \int d^D x \sqrt{-g} \left\{ -R - \frac{1}{2} (\partial \Phi_s)^2 - \frac{1}{2} (\partial \Phi_t)^2 - Te^{\Delta \Phi_t} + \ldots \right\}
  \]

  \[ \gamma_9 = 1 \quad \forall D \leq 10 ! \]

(Dudas, AS, Kitazawa, 2010)
(AS, 2013)
(Fré, A.S., Sorin, 2013)

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**MS equation:** Schroedinger-like (**INITIAL VALUE**, scalar or tensor fluctuations)

\[
\frac{d^2 v_k(\eta)}{d\eta^2} + \left[ k^2 - W_s(\eta) \right] v_k(\eta) = 0
\]

**"MS Potential"** : determined by the background

\[
\begin{align*}
\text{Initial Singularity : } W_s & \underset{\eta \to -\eta_0}{\to} - \frac{1}{4} \frac{1}{(\eta + \eta_0)^2} \\
\text{LM attractor : } W_s & \underset{\eta \to 0}{\to} \nu^2 - \frac{1}{4} \\
\left[ \nu = \frac{3}{2} \frac{1 - \gamma^2}{1 - 3 \gamma^2} \right]
\end{align*}
\]

\[
v_k(-\epsilon) \sim \frac{1}{\sqrt{W_s(-\epsilon) - k^2}} \exp \left( \int_{-\eta^*}^{-\epsilon} \sqrt{W_s(y) - k^2} \, dy \right)
\]

**Power Spectrum :** \( P(k) \sim k^3 \left| \frac{v_k(-\epsilon)}{z(-\epsilon)} \right|^2 \)

\[
ds^2 = a^2(\eta) \left( -d\eta^2 + dx \cdot dx \right)
\]

**Scalar :** \( z(\eta) = a^2(\eta) \frac{\phi'(\eta)}{a'(\eta)} \)

**Tensor :** \( z(\eta) = a(\eta) \)

**MS Pot :** \( W_s = \frac{1}{z} \frac{d^2 z}{d\eta^2} \)
But playing with the "kick" $\varphi_0$ ...

NOTE (D = 4): \[ \left| \frac{\Delta C_\ell}{C_\ell} \right| = \sqrt{\frac{2}{2\ell + 1}} \]
But playing with the “kick” $\phi_0$...

Qualitatively the low-$\ell$ tail

NOTE (D = 4) : 

\[
\left| \frac{\Delta C_\ell}{C_\ell} \right| = \sqrt{\frac{2}{2\ell + 1}}
\]

“Best” fit of WMAP9 raw data : $\phi_0 = -1.8$

$\chi^2_{\text{reduced}} = 0.74$, $p$-value = 0.85

[$\gamma = 0.08$, better fits with lower $\gamma$'s]

(Kitazawa, A5, 2014)
WMAP9 Best Fits & Analogies

• The key quantity is $\chi^2$/DOF
• The results are $[\gamma = 0.08, \gamma = 0.04, \gamma = 0.02]$ : [Better for lower $\gamma$ (CONCAVE Potentials )]
• Pre-inflationary peak around $\ell=5$

• Cfr. phase transitions:
  • Quadrupole dominance
  • Peak dominance

$\chi^2 = \sum_{\ell=2}^{32} \left( \frac{A_\ell - A_\ell^{WMAP9}}{\Delta A_\ell^{WMAP9}} \right)^2$

$A_\ell(\varphi_0, M, \delta) = M\ell(\ell + 1) \int_0^\infty \frac{dk}{k} \mathcal{P}_\zeta(k, \varphi_0) j_\ell^2(k, 10^\delta)$

$V(\varphi) = V_0 (e^{2\varphi} + e^{2\gamma \varphi})$

$\frac{\chi^2(\varphi_0)}{DOF}$
• The key quantity is $\chi^2$/DOF
• The results are $[\gamma = 0.08, \gamma = 0.04, \gamma = 0.02]$ : [Better for lower $\gamma$ (CONCAVE Potentials)]
• Pre-inflationary peak around $\ell=5$

• Cfr. phase transitions:
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  • Peak dominance
Low CMB Multipoles

- String Theory FRAMEWORK at face value:
  - Early Universe picture: climbing as a mechanism to start inflation
  - “Natural” quadrupole depression close to initial singularity

But:
  - Far from standard picture of inflation as a route to the present Universe
  - However: with a non-singular start go backwards with homogeneous Cosmology

- String Theory LESSON at face value:
  - “Scalar bounce” → a new class of “local features” of the power spectrum
  - Optimize the (local) comparison with low CMB multipoles?
  - YES: two-exponential systems can simulate a “locally trapped bounce”

TWO INSTRUCTIVE EXAMPLES:

\[ V = e^2 \cdot \varphi + e^2 \cdot 0.08 \cdot \varphi \]
\[ \varphi_0 = -1.8 \]
\[ \chi^2 = 21.4 \]
\[ [\chi^2_{attr} = 25.5] \]

\[ V = e^2 \cdot \varphi + e^2 \cdot (-0.125) \cdot \varphi \]
\[ \varphi_0 = -2.2 \]
\[ \chi^2 = 17.5 \]
\[ [\chi^2_{attr} = 25.5] \]
More in Detail…

\[ \gamma = 0.08, \varphi_0 = -4.0 \]
\[ \chi^2 = 23.8 \]
\[ \chi^2_{\text{attr}} = 25.5 \]

\[ \gamma = 0.08, \varphi_0 = -1.8 \]
\[ \chi^2 = 21.3 \]
\[ \chi^2_{\text{attr}} = 25.5 \]

\[ \gamma = 0.08, \varphi_0 = -1.0 \]
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\[ \chi^2_{\text{attr}} = 25.5 \]

\[ \gamma = -1.25, \varphi_0 = -2.2 \]
\[ \chi^2 = 17.5 \]
\[ \chi^2_{\text{attr}} = 25.5 \]

\[ \gamma = -1.20, \varphi_0 = -2.5 \]
\[ \chi^2 = 20.7 \rightarrow 13.7 \]
\[ \chi^2_{\text{attr}} = 25.5 \]
Tensor & Scalar Power Spectra

WKB:
- area below $W_{S,T}(\eta)$ determines the power spectra

$$v_k(-\epsilon) \sim \frac{1}{\sqrt{|W_s(-\epsilon) - k^2|}} \exp \left( \int_{-\eta}^{-\epsilon} \sqrt{|W_s(y) - k^2|} \, dy \right)$$

- **Scalar Power Spectra:** BELOW attractor $W$
- **Tensor Power Spectra:** ABOVE

- **INDEED:** moving slightly away from the attractor trajectory (here the LM attractor) enhances the ratio $P_T / P_S$

$$V = V_0 \left( e^2 \varphi + e^2 \gamma \varphi \right) \quad \left( \gamma < \frac{1}{\sqrt{3}} \right)$$

$$\frac{W_S}{W_T} \approx 1 - 18 \frac{(1 - \gamma^2)^4}{(2 - 3 \gamma^2)} \left[ \frac{d\varphi}{d\tau} + \frac{\gamma}{\sqrt{1 - \gamma^2}} \right]^2$$

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Summary & Outlook

- **BRANE SUSY BREAKING** \((d \leq 10)\) : “hard” (critical) exponential potentials
  - **Climbing**: \(\gamma=1\) from D9 \(\rightarrow\) **Mechanism to START INFLATION**
  - **Power Spectra**: (wide) IR depression & pre-inflationary peaks
    - Naturally weak string coupling
    - [Non-singular “string-frame metric” in D=10]
    - [[Early higher-dimensional evolution: estimates of cosmic variance?]]

- **SCALAR BOUNCE OBSERVABLE?** IF we “were seeing” via the CMB the onset of inflation
  - **OR**: associated to a *later transition*, even after several e-folds

Pre-inflationary peak: signals an *incomplete transition* to slow roll

E.g. : in a Starobinsky – like model:

![Graphs showing inflationary potential and scalar field evolution](image-url)
Summary & Outlook

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- **SCALAR BOUNCE OBSERVABLE?** If we “were seeing” via CMB the **onset of inflation**
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Pre-inflationary peak: general signature of an **incomplete transition** to slow roll

**Related Work**

- **COSMIC MASK & QUADRUPOLE REDUCTION**
  - More recent work on models with low-l depression:** BISPECTRUM?
    - (Gruppuso, Natoli, Pacci, Finelli, Molinari, De Rosa, Mandolesi, 2013)
    - (Destri, De Vega, Sanchez, 2010)
    - (Cicoli, Downes, Dutta, 2013)
    - (Pedro, Westphal, 2013)
    - (Bousso, Harlow, Senatore, 2013)
    - (Liu, Guo, Piao, 2013)
    - ..........