Quantum thermometer of the Quark-Gluon Plasma

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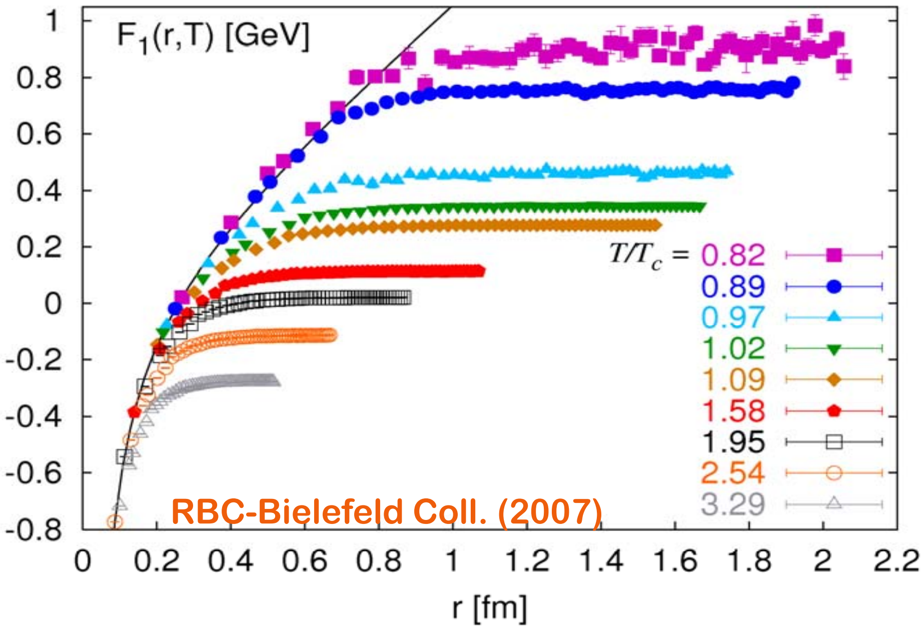
Probing deconfinement ?

How can we prove that we have really achieved a *deconfined* state of matter in ultra-relativistic heavy ions collisions ?

Challenge

"deconfinometer" ≡
- Color fluctuations
- Propagation of individual quarks over large distances

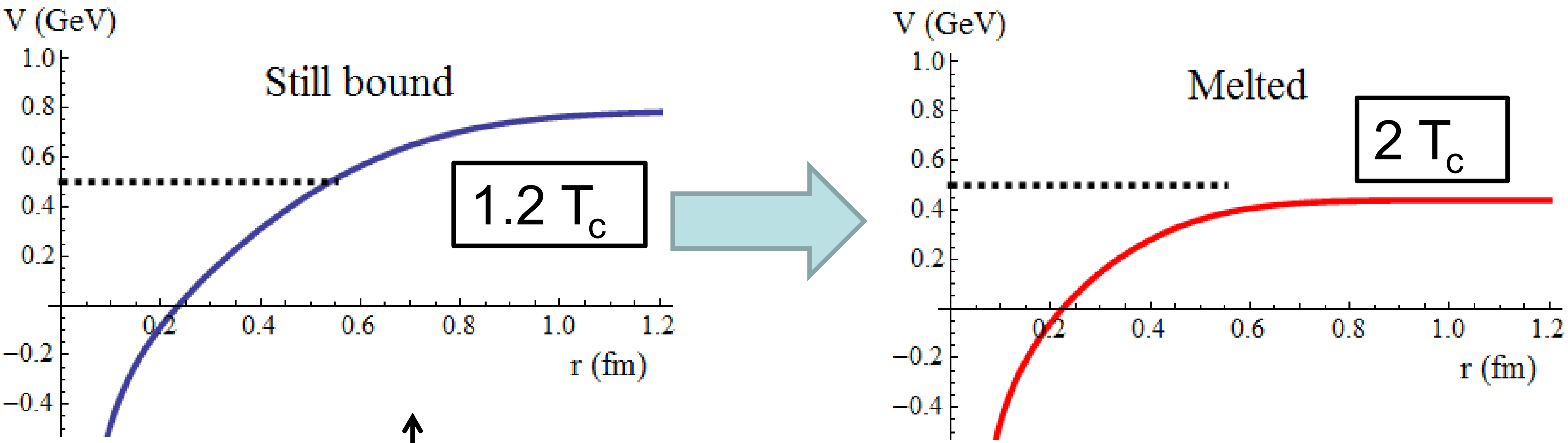
Looking at the QQbar potential on the lattice



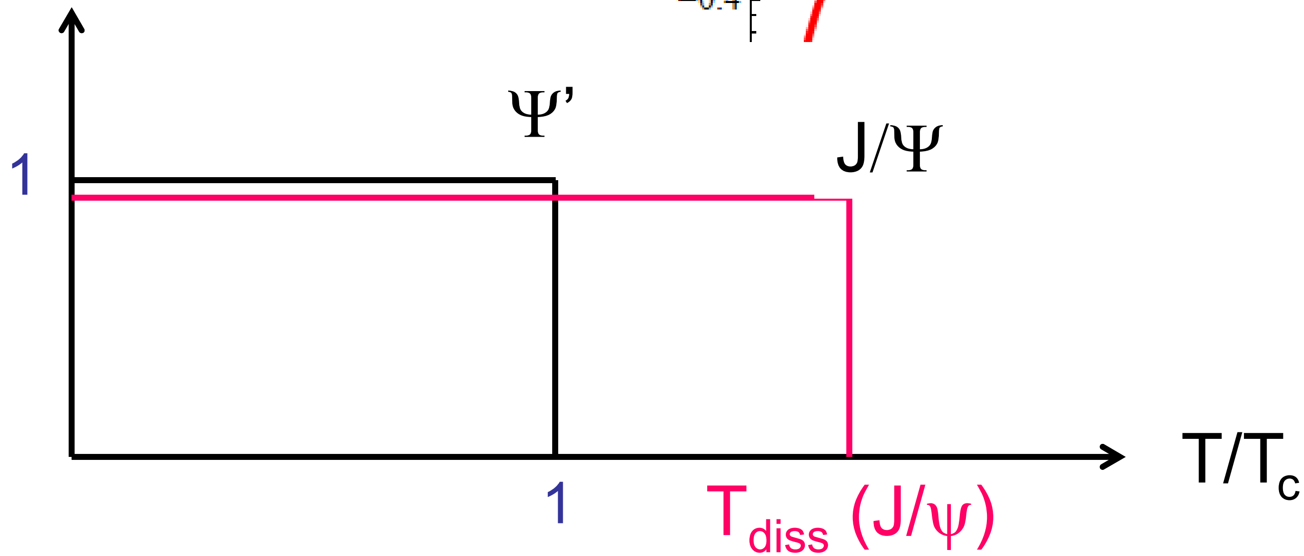
Increased screening at larger temperatures

Quarkonia in Stationary QGP

Consequence for Q-Qbar states (Q: heavy quark):



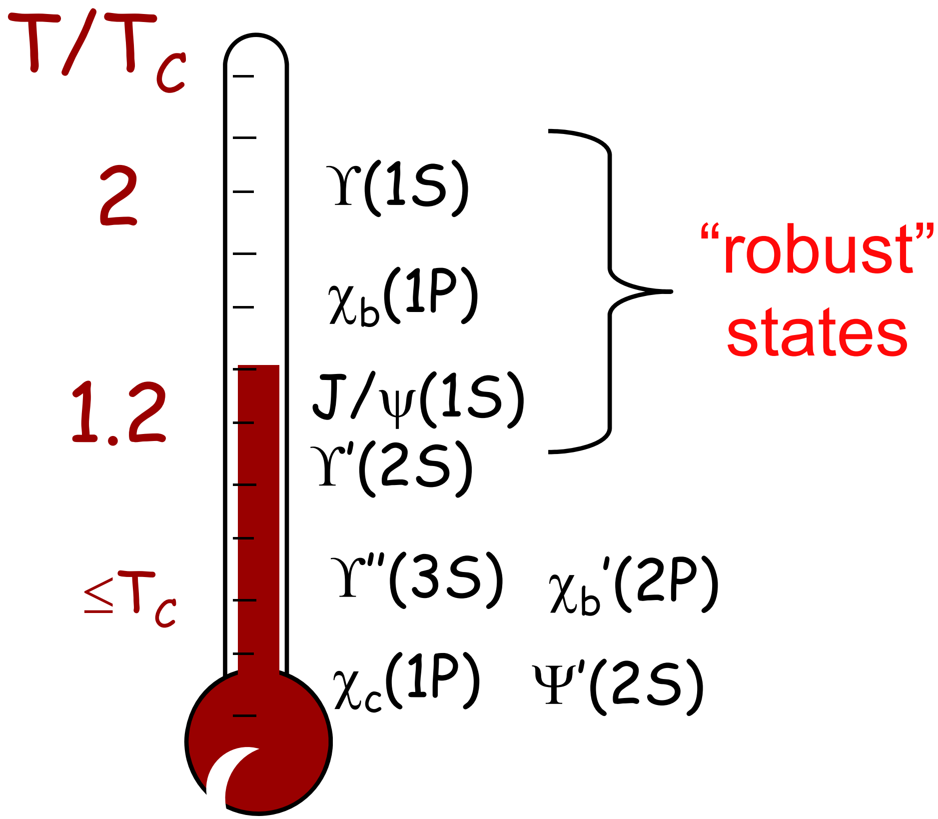
Survivance as a function of T: abrupt pattern



Best candidate: Quarkonia sequential "suppression", i.e. melting and/or dissociation (Matsui & Satz 86)

Quarkonia in Stationary QGP

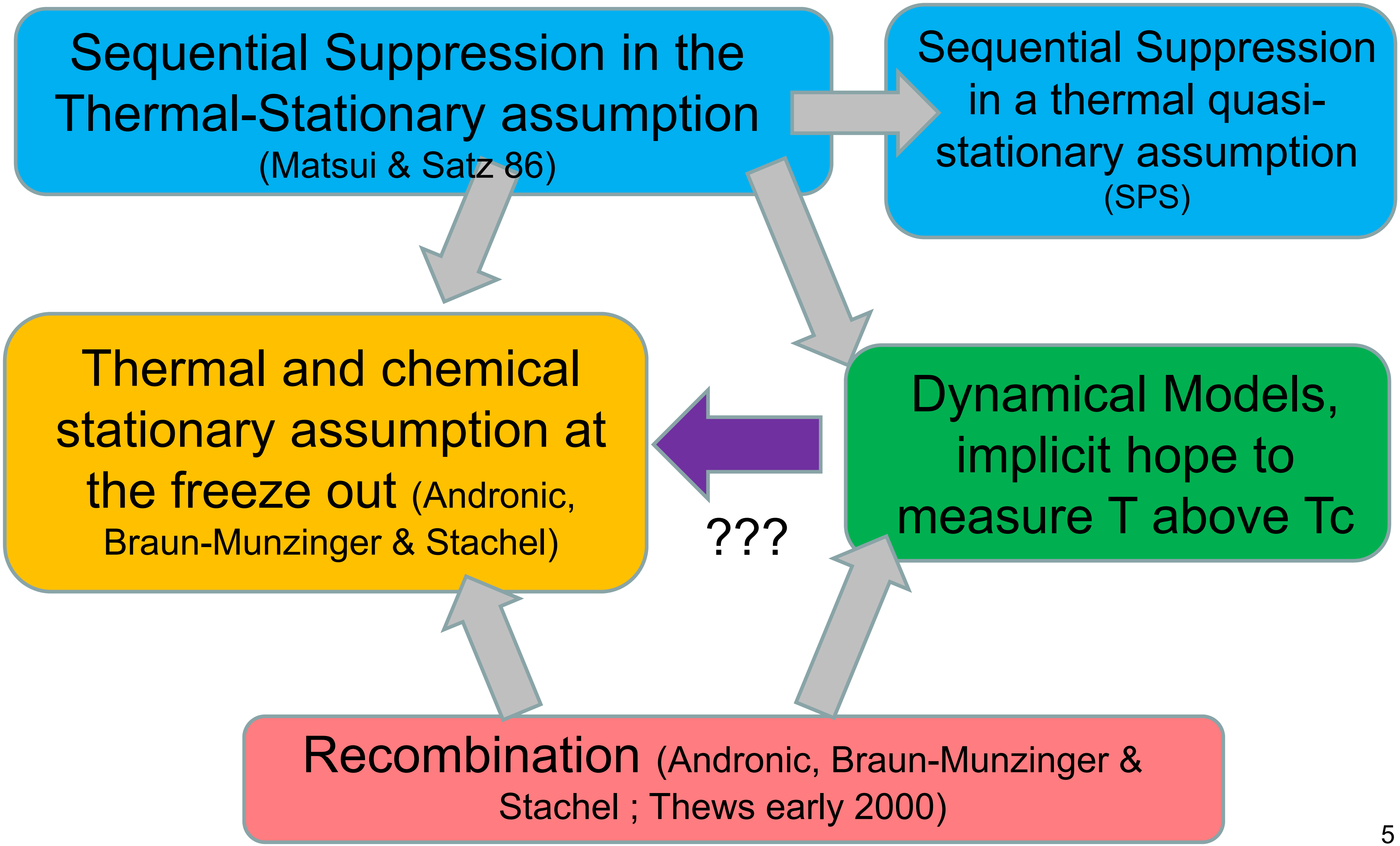
QGP
Thermometer



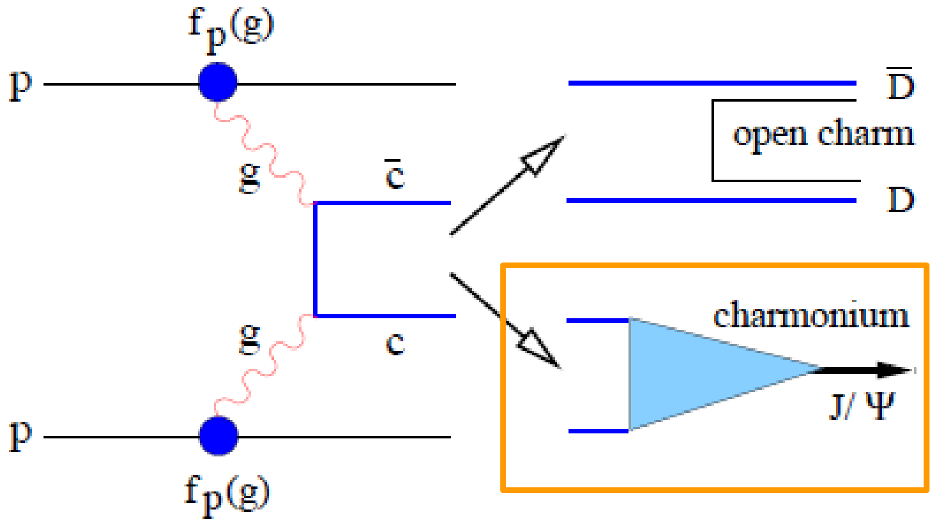
Indeed observed at SPS (CERN) and RHIC (BNL) experiments. However:

- alternative explanations, lots of unknown (also from theory side)
- less suppression at LHC

Schematic view



Dynamical version of the sequential suppression scenario



From H Satz

Formed after some "formation time"

τ_f (typically the Heisenberg time),

usually assumed to be independent

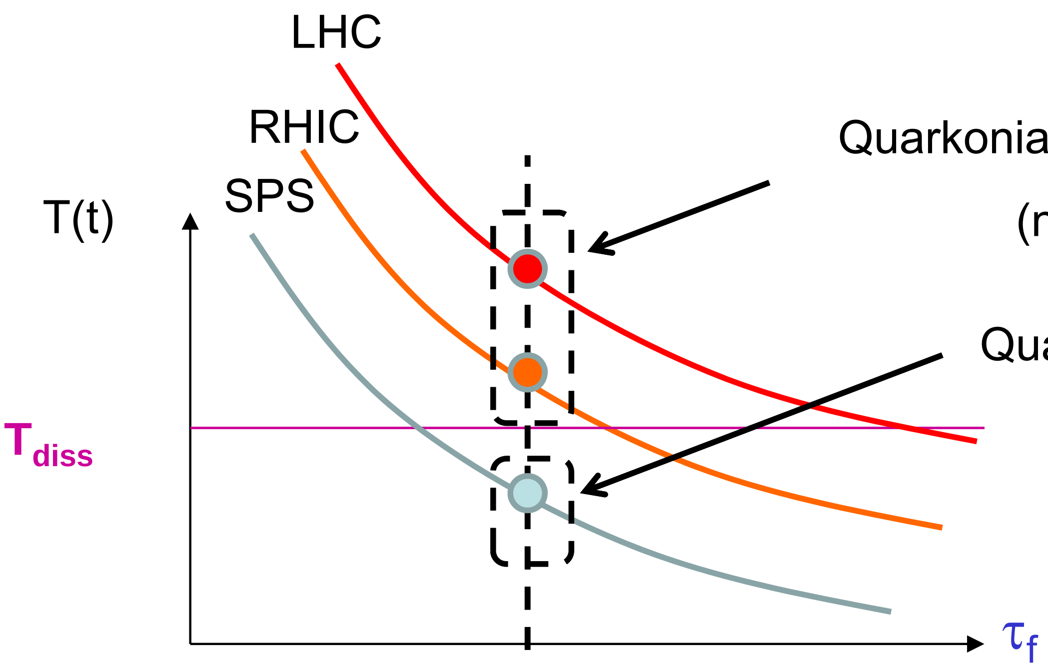
of the surrounding medium

Standard folklore of sequential suppression: The quarkonia which should be formed at (τ_f,x_0) is not if $T(\tau_f,x_0)>T_{diss}$ => Q-Qbar pair is "lost" for quarkonia formation

Pictorially

Local temperature

at formation time



Quarkonia state "suppressed"

(not formed)

Quarkonia state formed as in the

vacuum

Common ingredients in (most of the) state of the art *dynamical* models (see f.i. G. Wolschin at this conference)

Early decoupling btwn various states in the initial stage (as in H. Satz)

Mean field (screening)

- Vetoing at the time of production if $T > T_{dissoc}$
- Evaluation of the wave functions ψ_n at finite T

Fluctuations (dissociation)

- Evaluate dissociation cross sections using transition operators + ψ_n

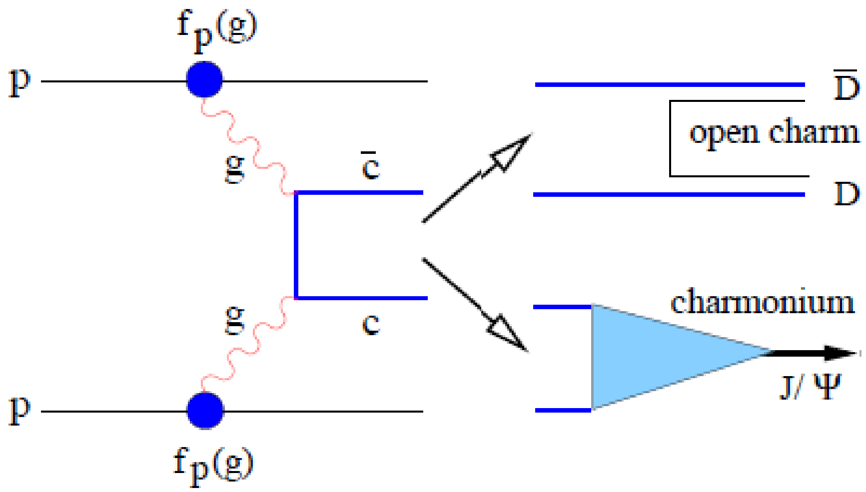
or

- Evaluation of the width Γ using some imaginary potential => survival a $\exp(-\Gamma t)$

+ recombination (using detailed balance of)

Back to the concepts

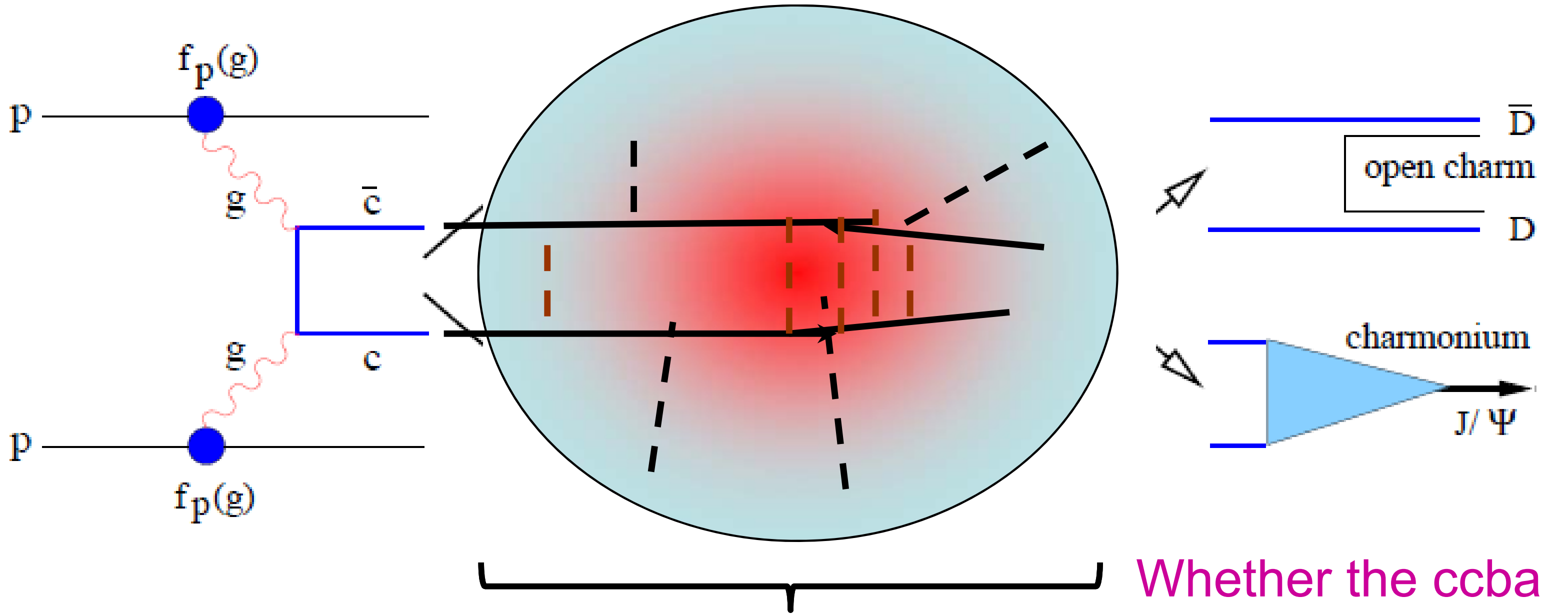
Picture



Early decoupling between various states

Back to the concepts

Reality



Very complicated QFT problem at finiteT(t) !!!

Whether the ccbar pair emerges as a bound quarkonia or as DDbar pair is only resolved at the end of the evolution

Beware of quantum coherence during the evolution

But one should aim at solving it, especially as the quarkonia content of a QQbar quantum state is at most of the order of a few % (continuous transitions under external perturbations)

Need for full quantum treatment

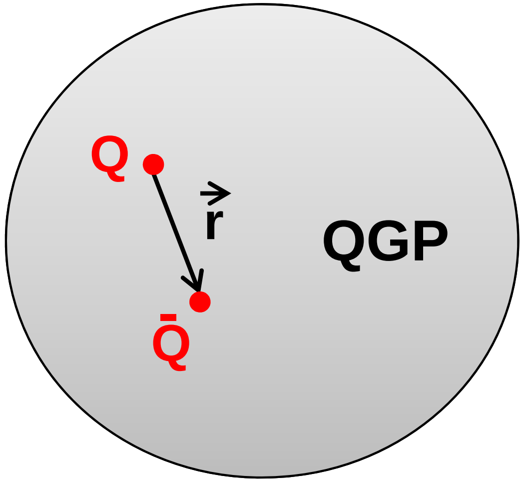
Quantum evolution in the mean field

Dating back from Blaizot & Ollitrault, Cugnon and Gossiaux; early 90's

- **Time-dependent Schrödinger equation for the $Q\bar{Q}$ pair**

Where $$\widehat{H} = 2m_q - \frac{(\hbar c)^2}{m_q}\nabla^2 + V(r, T_{\rm red})$$

$$\Psi_{Q\bar{Q}}(\mathbf{r}, t) = R_{Q\bar{Q}}(r, t) \times \cancel{Y_{Q\bar{Q}}(\theta, \phi)}$$

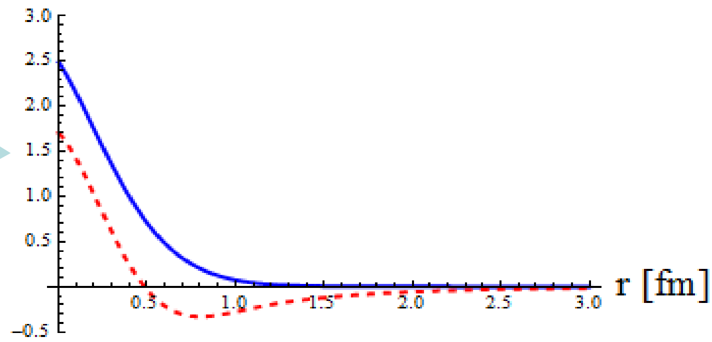


- **Projection onto the S states: the S weights**

$$W_S(t) = \left(4\pi \,\mathrm{Abs}\left[\int_0^\infty R_{Q\bar{Q}}\,(r, t, T_{red}) \times \underline{R_S(r, T_{red}^{had})}\; r^2\, dr\right]\right)^2$$

Radial eigenstates of the hamiltonian

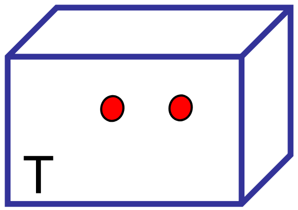
Charmonium radial S states



Additional ingredients

The color potentials V(Tred, r) binding the $Q\bar{Q}$

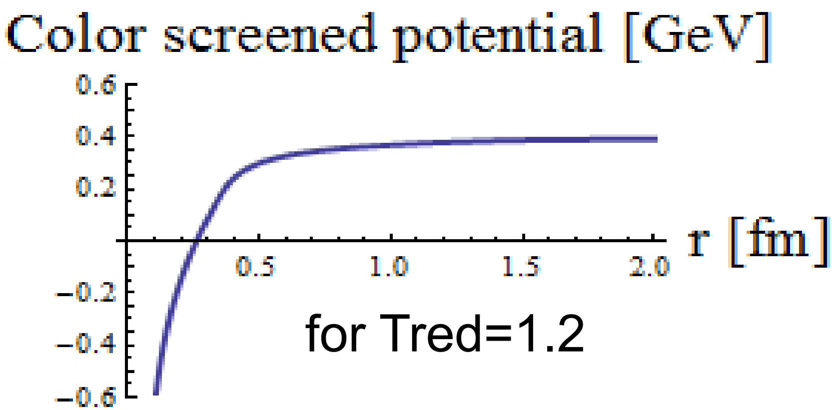
Static lQCD calculations (maximum heat exchange with the medium):

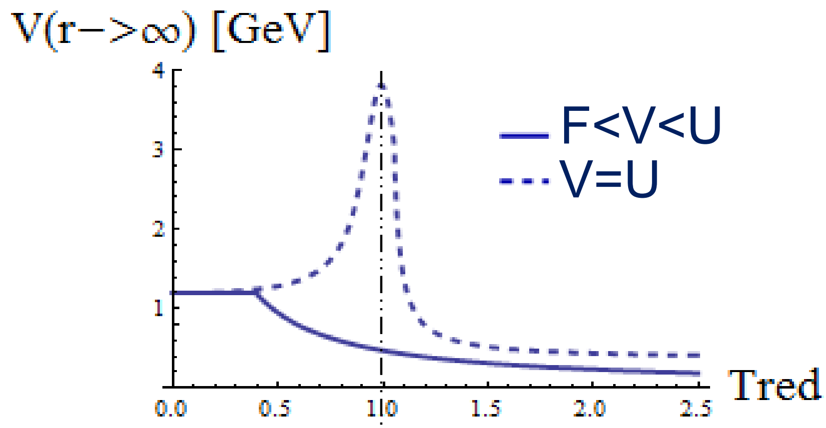


{ F : free energy
S : entropy

U=F+TS : internal energy
(no heat exchange)

- "Weak potential" F<V<U * => some heat exchange
- "Strong potential" V=U ** => adiabatic evolution



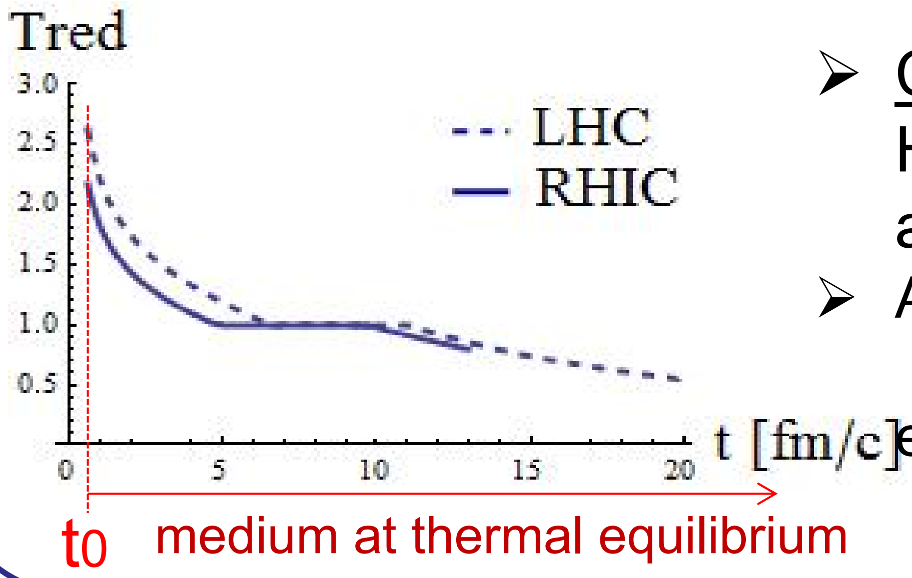


Evaluated by Mócsy & Petreczky* and Kaczmarek & Zantow** from lQCD results

* Phys.Rev.D77:014501,2008 **arXiv:hep-lat/0512031v1

Additional ingredients

The QGP homogeneous temperature scenarios



- Cooling over time by Kolb and Heinz* (hydrodynamic evolution and entropy conservation)
- At LHC ($\sqrt{s_{NN}} = 2.76$ TeV) and RHIC ($\sqrt{s_{NN}} = 200$ GeV) energies

Initial $Q\bar{Q}$ pair radial wavefunction

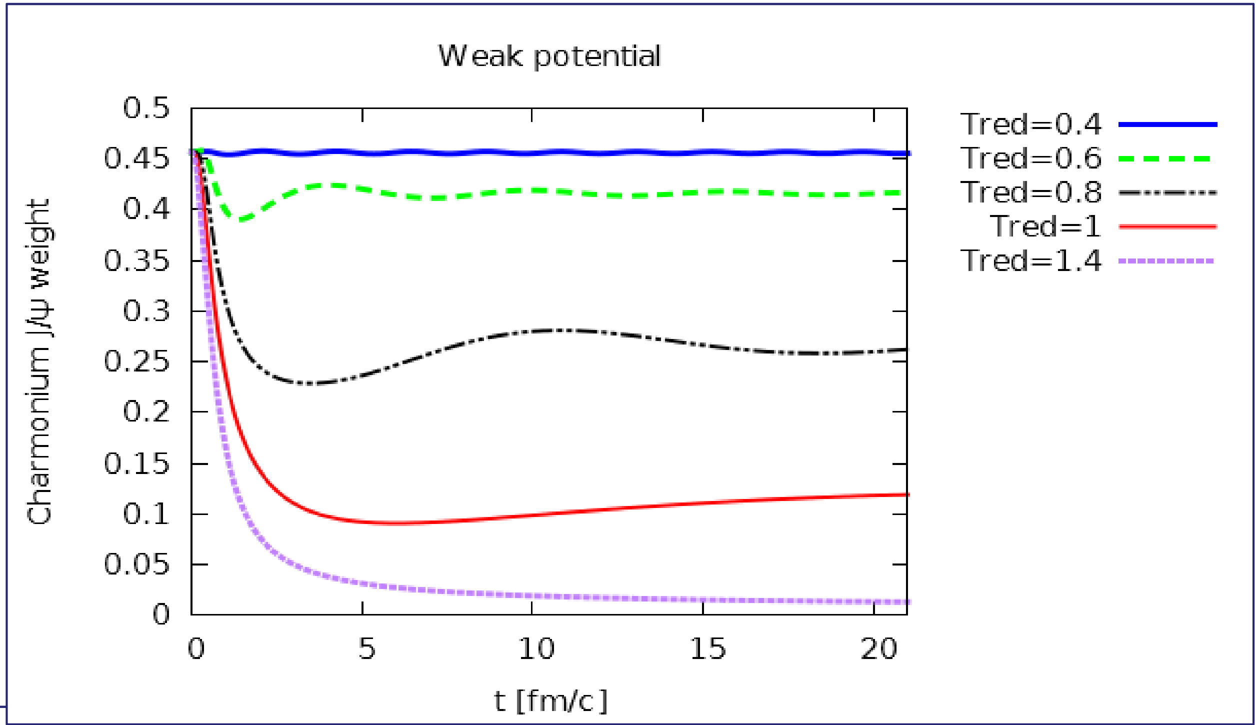
- Assumption: QQ pair created at t_0 in the QGP core
- Gaussian shape with parameters (Heisenberg principle):

$$a_{c\bar{c}} = 0.165 \text{ fm} \qquad a_{b\bar{b}} = 0.045 \text{ fm}$$

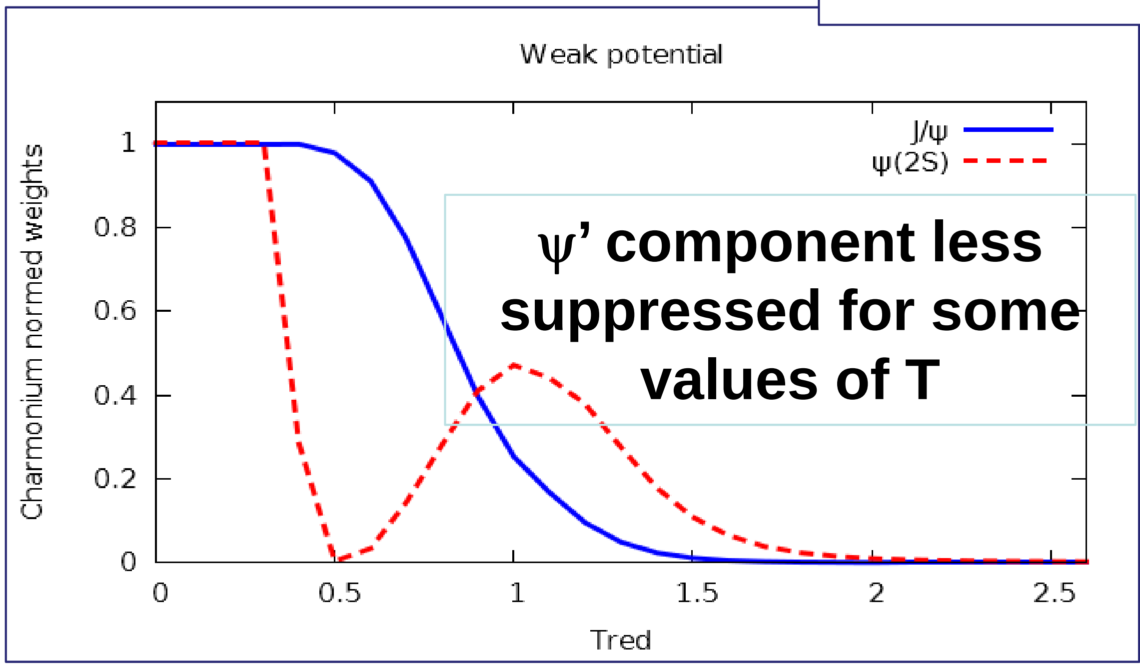
* arXiv:nucl-th/0305084v2

Evolution at fixed temperature

Charmonia and weak color potential (F<V<U)



The normed weights at t->∞ function of the temperature



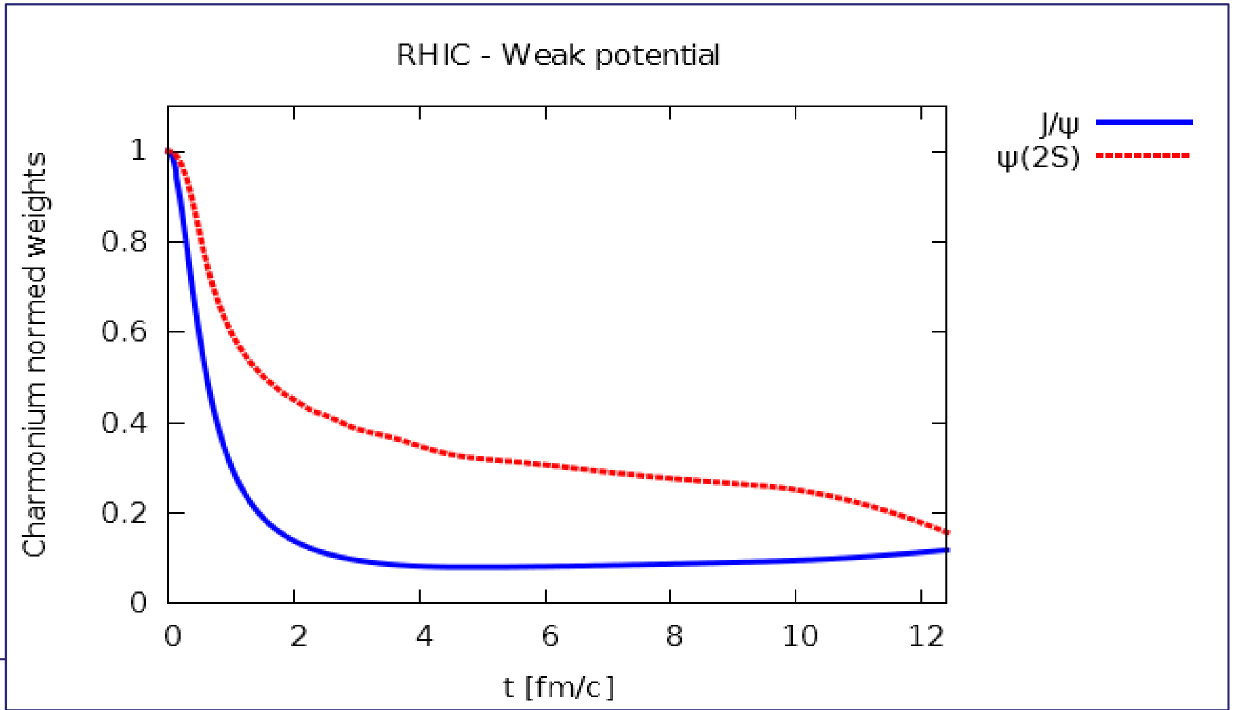
ψ' component less suppressed for some values of T

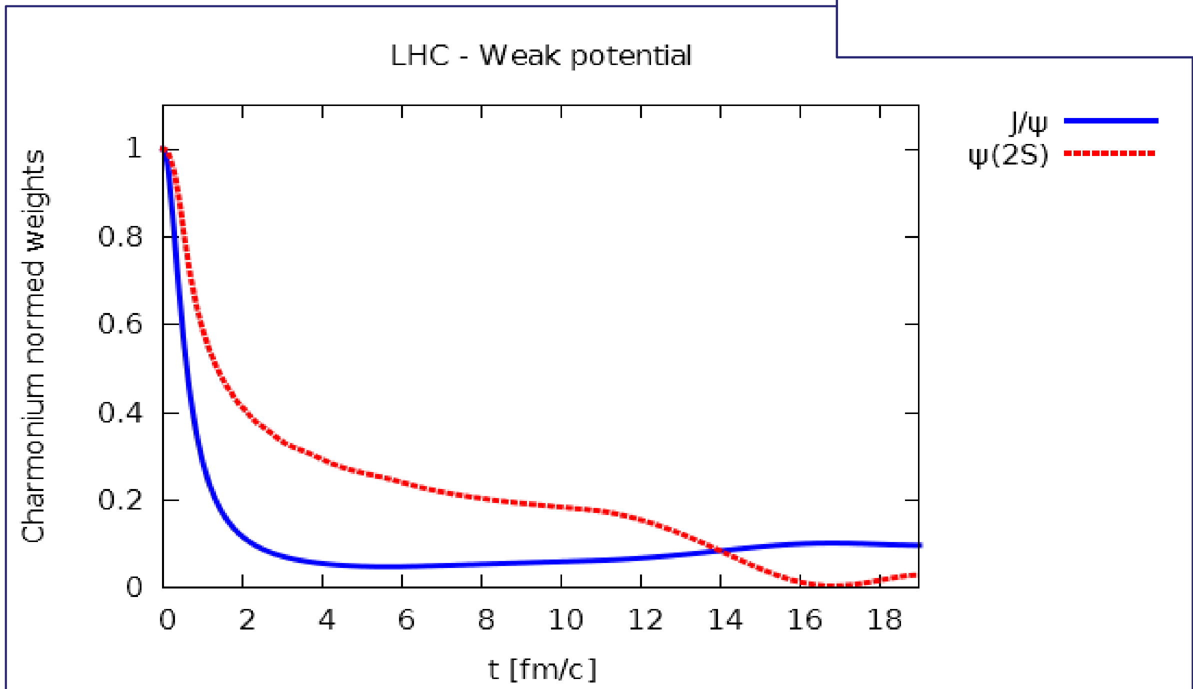
Smooth evolution and no discontinuity in the parameter space

Evolution in realistic T scenarios

Charmonia and weak color potential (F<V<U)

RHIC temperature scenario

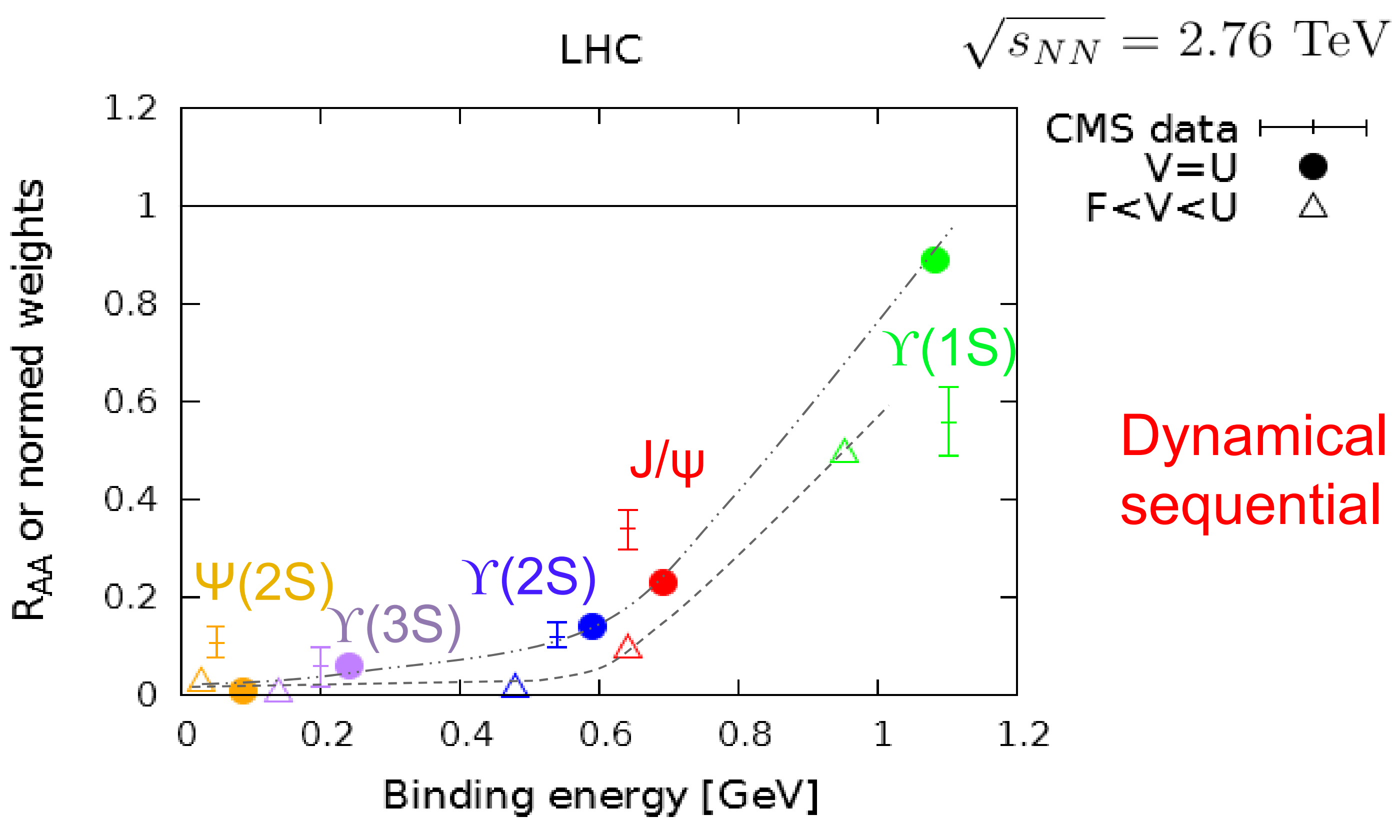




LHC temperature scenario

Inversion of the ψ' vs ψ suppression pattern at longer time

Sum up of LHC results

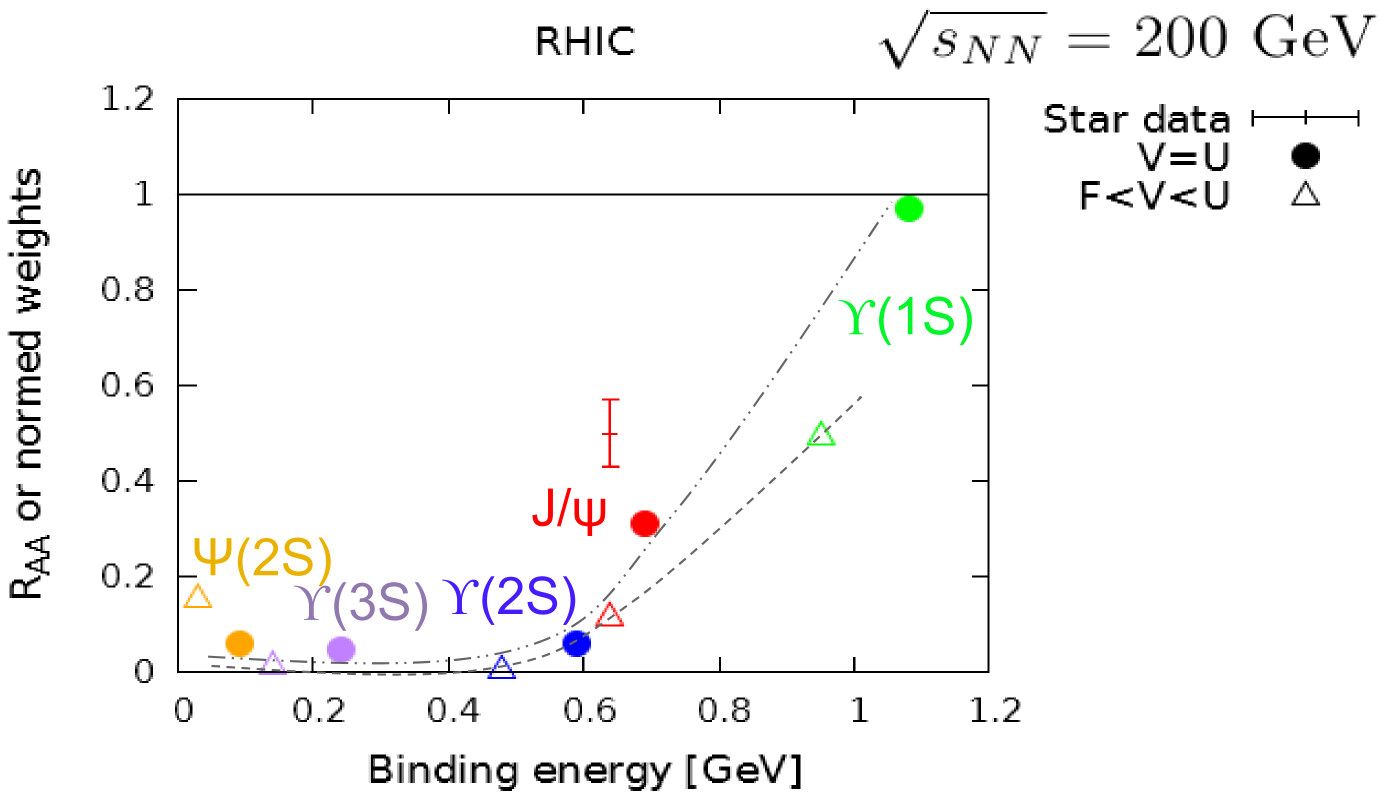


(High pT and most central)

Dynamical quarkonia sequential suppression

- The results are quite encouraging for such a simple scenario !
- J/ψ and ψ(2S) are underestimated (room for regeneration) and $\Upsilon(1S)$ overestimated
- Feed downs from exited states and CNM to be implemented

Sum up of RHIC results



- Similar suppression trends obtained for both RHIC and LHC.
- Less J/ψ suppression at RHIC than at LHC.
- Υ(1S+2S+3S) suppression can be estimated with Star data to ~ 0.55±0.10, we obtain ~ 0.48 for V=U and ~ 0.24 for F<V<U.

Common ingredients in (most of the) state of the art dynamical models

Early decoupling between various states in the initial stage

Mean field (screening)

- Vetoing at the time of production if $T>T_{dissoc}$
- Evaluation of the wave functions ψ_n at finite T

Fluctuations (dissociation)

- Evaluate dissociation cross sections using transition operators + ψ_n

or

- Evaluation of the width Γ using some imaginary potential => survival a $\exp(-\Gamma t)$

Should be treated consistently with quantum properties of the QQbar subsystem

A taste of quantum thermalisation

Background?

- RHIC and LHC experimental results => quarkonia thermalise partially in the QGP
- But how to thermalise our wavefunction ? Quantum friction/stochastic effects have been a long standing problem because of their irreversible nature ("old frontier in Φ")

The open quantum approach: ✗
Considering the whole system, quarkonia and environment, the latter being finally integrating out

Y. Akamatsu [arXiv:1209.5068]
Laine et al. JHEP 0703 (2007) 054

2nd possible approach: ✓
Mock the open quantum approach by using a stochastic operator and a dissipative non-linear potential

A. Rothkopf et al. Phys. Rev. D 85, 105011 (2012)
N. Borghini et al. Eur. Phys. J. C 72 (2012)
S. Garashchuk et al. Jou. of Chem. Phys. 138, 054107 (2013)

- **Stochastic Schrödinger equation**

$$i\hbar\frac{\partial\Psi_{Q\bar{Q}}(\mathbf{r},t)}{\partial t} = \Big(\underbrace{\widehat{H}(\mathbf{r})}_{\text{MF}} - \underbrace{\mathbf{F}(t).\mathbf{r}}_{\text{Fluctuation}} + \underbrace{A\big(S(\mathbf{r},t) - \langle S(\mathbf{r},t)\rangle_{\mathbf{r}}\big)}_{\text{Friction}}\Big)\Psi_{Q\bar{Q}}(\mathbf{r},t)$$

Derived from the Heisenberg-Langevin equation*, in Bohmian mechanics** …

* Kostin The J. of Chem. Phys. 57(9):3589–3590, (1972)
** Garashchuk et al. J. of Chem. Phys. 138, 054107 (2013)

Schrödinger-Langevin (SL) equation

$$i\hbar\frac{\partial\Psi_{Q\bar{Q}}(\mathbf{r},t)}{\partial t} = \left(\widehat{H}(\mathbf{r}) - \mathbf{F}(t).\mathbf{r} + \boxed{A\big(S(\mathbf{r},t) - \langle S(\mathbf{r},t)\rangle_{\mathbf{r}}\big)}\right)\Psi_{Q\bar{Q}}(\mathbf{r},t)$$

dissipative non-linear potential
(wavefunction dependent

where $S(\mathbf{r},t) = \arg\big(\Psi_{Q\bar{Q}}(\mathbf{r},t)\big)$

- ✓ Brings the QQ to the lowest state (0 node)
- ✓ Friction (assumed to be local in time)

- Solution for V=0 (free wave packet): $\psi(\vec{x},t) \propto e^{i\vec{p}_{\mathrm{cl}}(t)\cdot\vec{r}+i\alpha(t)(\vec{r}-\vec{r}_{\mathrm{cl}}(t))^2-i\varphi(t)}$

where $\vec{p}_{\mathrm{cl}}(t)$ and $\vec{x}_{\mathrm{cl}}(t)$ satisfy the classical laws of motion

- $\vec{p}_{\mathrm{cl}}(t) = \vec{p}_{\mathrm{cl}}(0)e^{-At} \Rightarrow$ A is the drag coefficient (inverse relaxation time)

A can be fixed through the modelling of single heavy quarks observables and comparison with the data OR using lattice QCD calculations

Schrödinger-Langevin (SL) equation

$$i\hbar\frac{\partial\Psi_{Q\bar{Q}}(\mathbf{r},t)}{\partial t} = \left(\widehat{H}(\mathbf{r}) - \mathbf{F}(t).\mathbf{r} + \boxed{A\left(S(\mathbf{r},t) - \langle S(\mathbf{r},t)\rangle_{\mathbf{r}}\right)}\right)\Psi_{Q\bar{Q}}(\mathbf{r},t)$$

dissipative non-linear potential
(wavefunction dependent

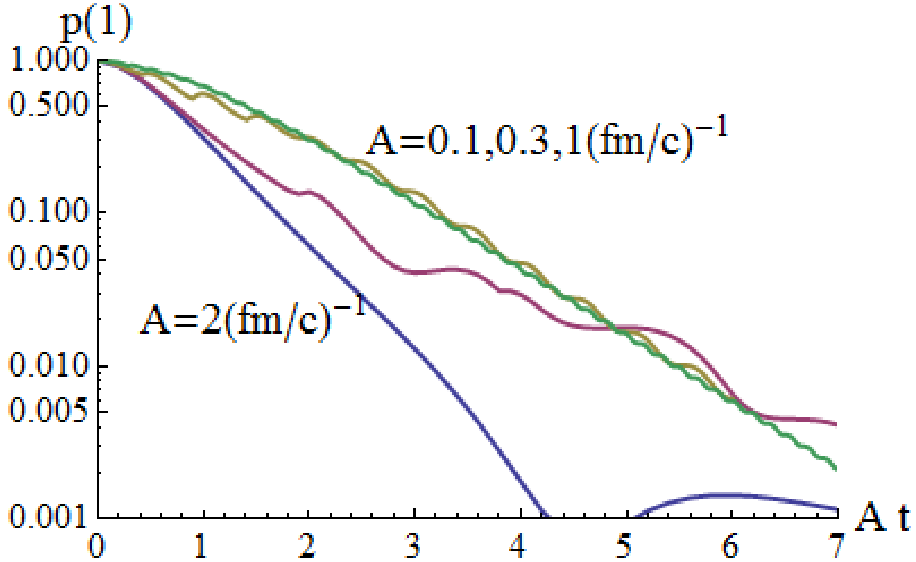
where $S(\mathbf{r},t) = \arg\left(\Psi_{Q\bar{Q}}(\mathbf{r},t)\right)$

✓ Brings the QQ to the lowest state (0 node)
✓ Friction (assumed to be local in time)

➢ Solution for harmonic potential as well: $\psi(\vec{x},t) \propto e^{i\vec{p}_{\rm cl}(t)\cdot\vec{r}+i\alpha(t)(\vec{r}-\vec{r}_{\rm cl})^2-i\varphi(t)}$

Illustration: probability of finding the first excited state in a 1D-harmonic potential, as function of time, for various values of A …

Scaling relation found for A<ω



Schrödinger-Langevin (SL) equation

$$i\hbar \frac{\partial \Psi_{Q\bar{Q}}(\mathbf{r},t)}{\partial t} = \left(\hat{H}(\mathbf{r}) - \boxed{\mathbf{F}(t).\mathbf{r}} + A\big(S(\mathbf{r},t) - \langle S(\mathbf{r},t)\rangle_{\mathbf{r}}\big) \right) \Psi_{Q\bar{Q}}(\mathbf{r},t)$$

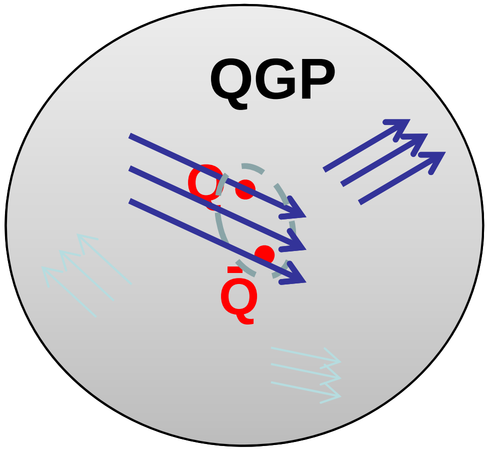
Stochastic operator; “warming”

$\langle \mathbf{F}(t) \rangle = 0\,,\ \langle \mathbf{F}(t)\mathbf{F}(t') \rangle = \Gamma(t,t')$?

Brownian hierarchy: $m \gg T \ \Rightarrow \ \sigma \ll \tau_{\text{relax}}$

- ✓ σ = autocorrelation time of the gluonic fields
- ✓ τ_{relax} = quarkonia relaxation time

$\Gamma(t,t')$: gaussian correlation of parameter σ and norm B



<u>3 parameters</u>: A (the drag coef), B (the diffusion coef) and σ (autocorrelation time)

Properties of the SL equation

- Unitarity (no decay of the norm as with imaginary potential)

- Heisenberg principle satisfied at any T

- Non linear => Violation of the superposition principle (=> decoherence)

- Evolution from pure to mixed states

- Mixed state observables:

$$\Big\langle \langle \psi_{\mathrm{S}}(t)|\hat{A}|\psi_{\mathrm{S}}(t)\rangle \Big\rangle_{\mathrm{stat}} = \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \langle \psi_{\mathrm{S}}^{(r)}(t)|\hat{A}|\psi_{\mathrm{S}}^{(r)}(t)\rangle$$

- Easy to implement numerically (especially in Monte-Carlo generator)

- For an harmonic potential:
  - Asymptotic distribution of states proven to be $\propto e^{-\frac{E_n}{kT}}$
  - Fluctuation dissipation theorem:

$$\frac{B}{2m} = A\int_{-\infty}^{+\infty} \frac{(\nabla S)^2}{m}|\psi|^2 dr \Rightarrow B = m\hbar\omega\left(\coth\left(\frac{\hbar\omega}{2kT}\right) - 1\right)A \xrightarrow{kT \gg \hbar\omega} 2mkTA$$

Classical Einstein law

NB: for quantum noise acting on operators in the Heisenberg representation

$$B = m\hbar\omega\left\{\underbrace{\left[\coth\left(\frac{\hbar\omega}{2kT}\right) -1\right]}_{\text{Same as in SL}} \underbrace{+1}_{\text{Ground state energy... included in the width of the wave packet in the Schroedinger representation}}\right\}$$

Same as in SL

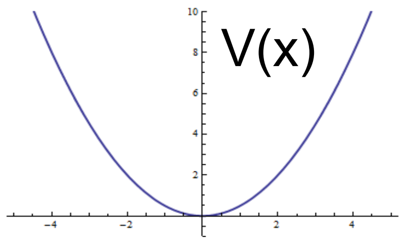
Ground state energy… included in the width of the wave packet in the Schroedinger representation

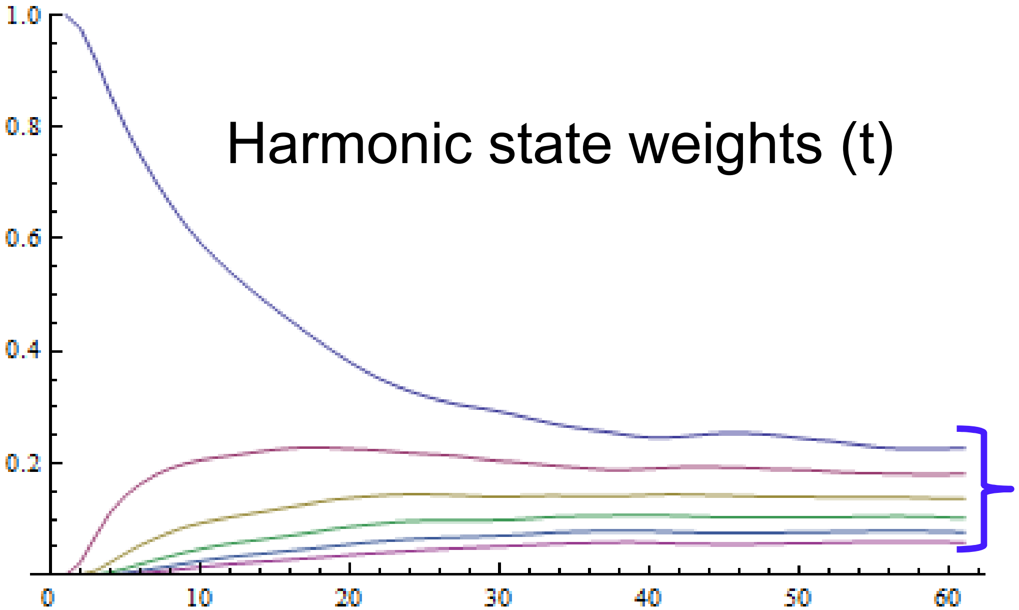
- Asymptotic convergence shown for a wide class of potentials, but distribution of states less understood **=> numerical approach instead !**

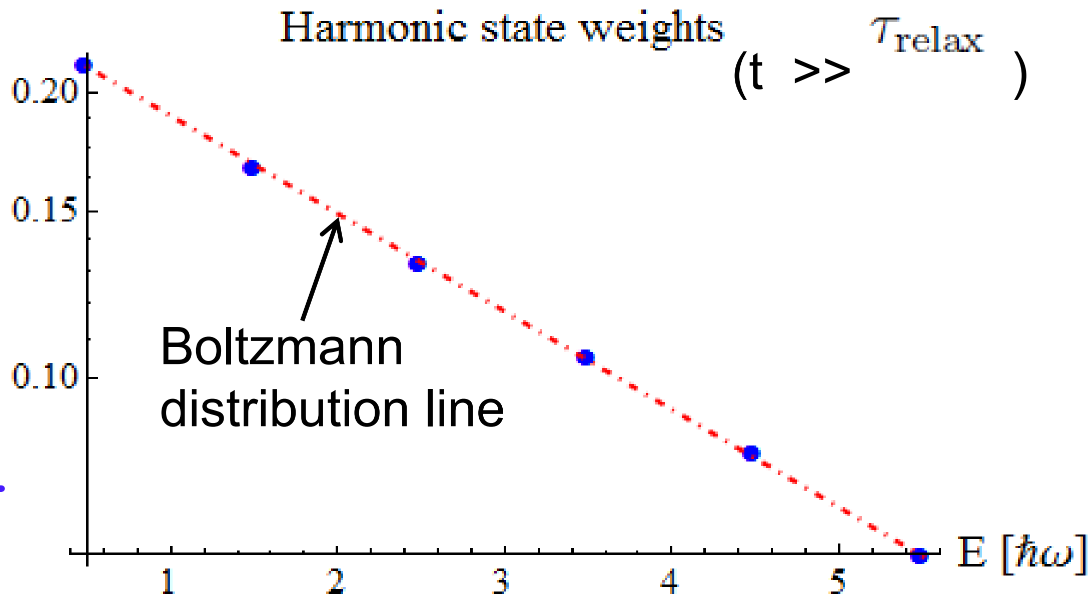
numerical tests of thermalization

Harmonic potential

Asymptotic thermal equilibrium
for any (A,B,σ) and from any initial state





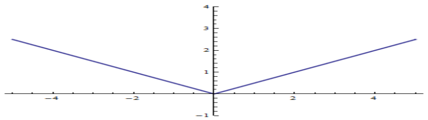


numerical tests of thermalization

Other potentials

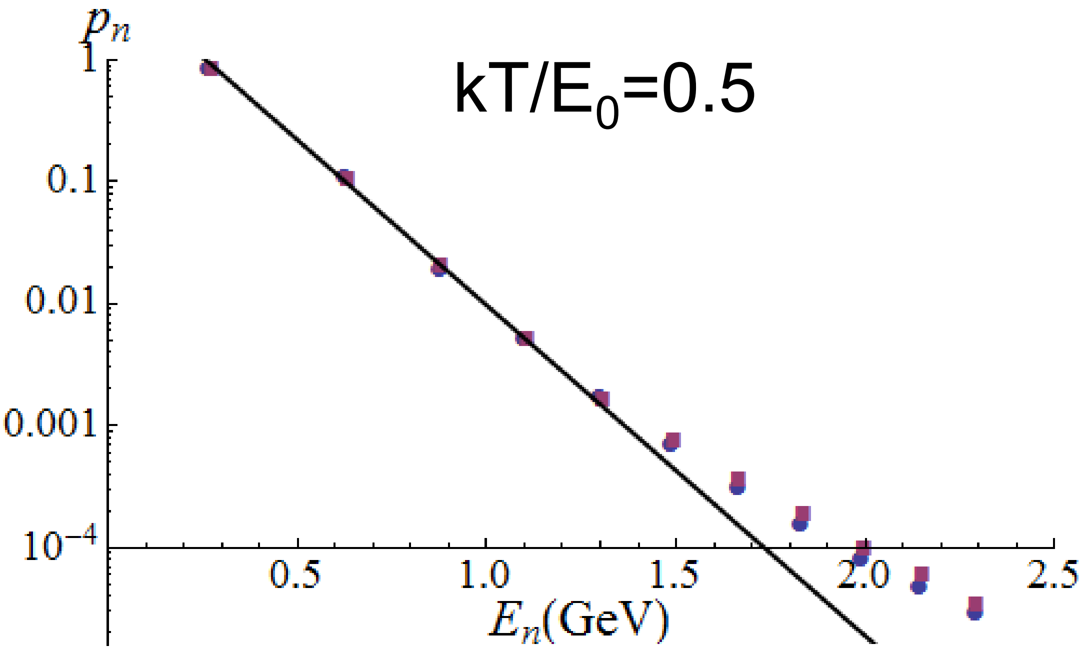
Linear Abs[x]

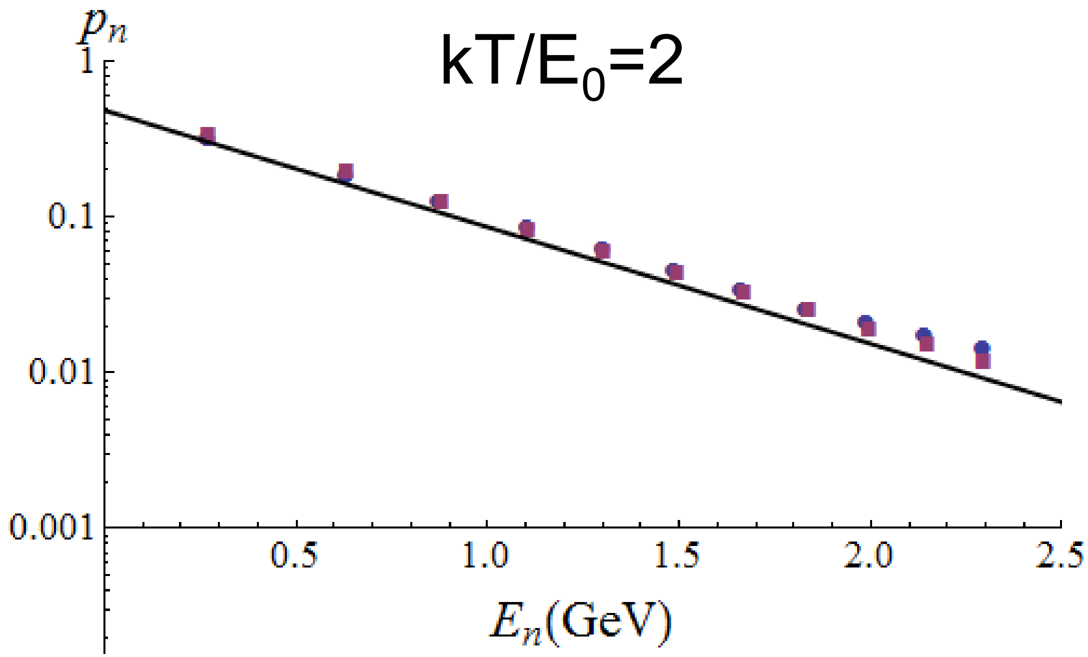
V(x)



Asymptotic Boltzmann distributions ?

Yes; deviations from Boltzmann seen for higher states for $kT \ll E_0$



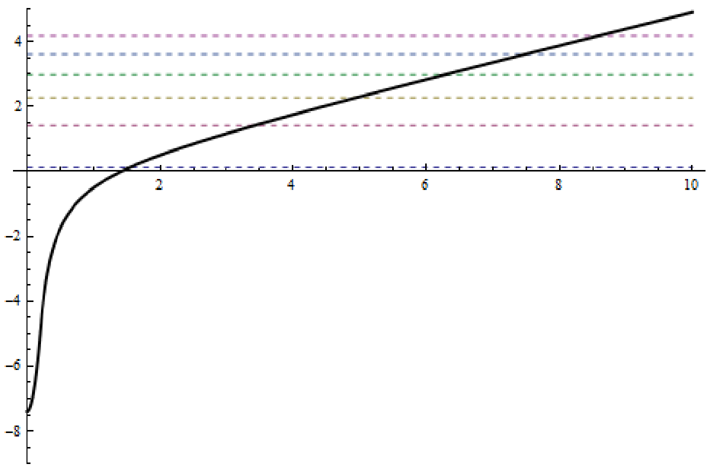


numerical tests of thermalization

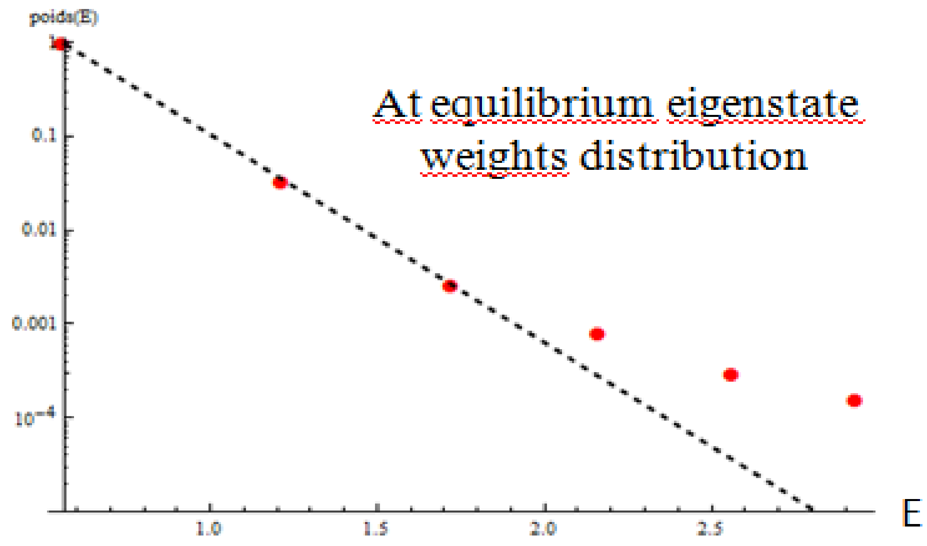
Other potentials

Quarkonia approx

V(x)



Asymptotic Boltzmann distributions ?

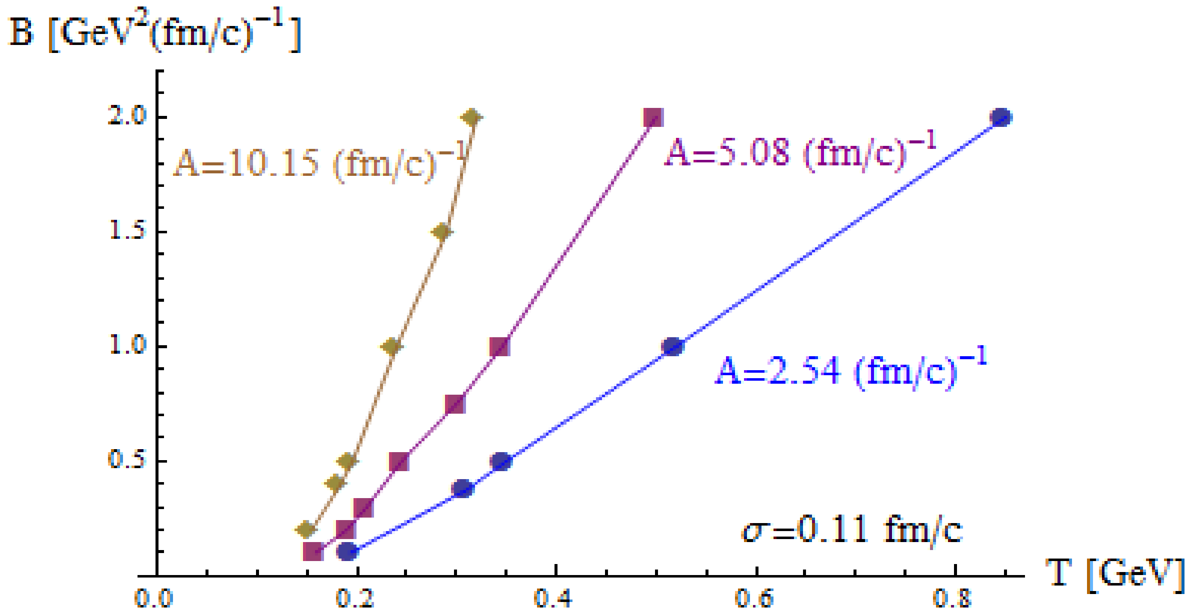


Yes; Light discrepancies from 3rd excited states for states at small T of the order of Tc

Tsallis distribution ?

Properties of the SL equation

Mastering numerically the fluctuation-dissipation relation for the Quarkonia approximated potential:



B univoquely extracted from (A,T) (as in usual quantum noise)

➢ Reducable to a small number of properties encoding the interactions with the heat bath:

- ❑ Temperature T
- ❑ Drag coefficient A
- ❑ Autocorrelation time σ

Dynamics of QQbar with SL equation

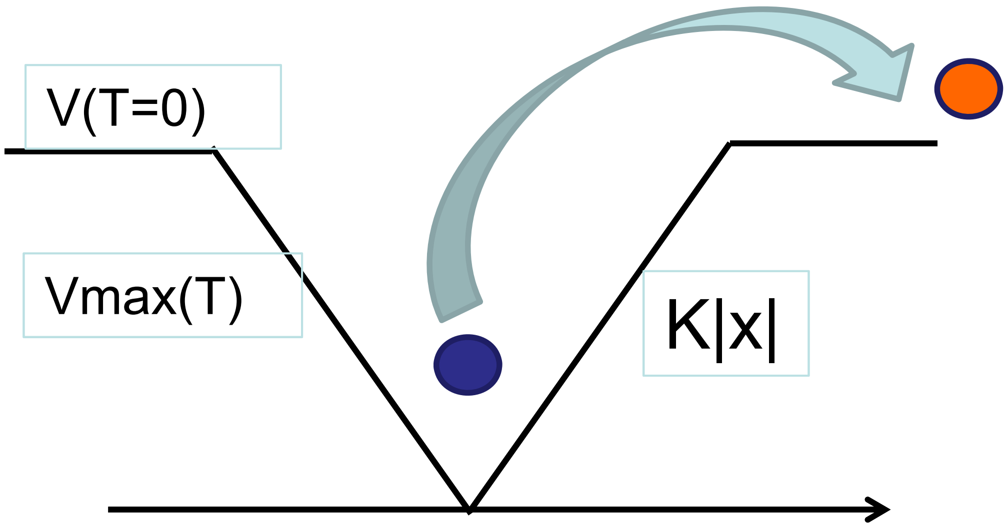
Aimed as a proof of principle => simplifying assumptions

- 3D -> 1D
- Drag coeff. for c quarks: $A(T)[(\mathrm{fm/c})^{-1}] \cong 3T[\mathrm{GeV}] + 2.5T^2$

  Typically T ∈ [0.1 ; 0.43] GeV => A ∈ [0.32 ; 1.75] $(\mathrm{fm/c})^{-1}$
- σ=0

First, considering the effect of the fluctuations-dissipation alone (neglecting the mean field contribution):

- Potential:

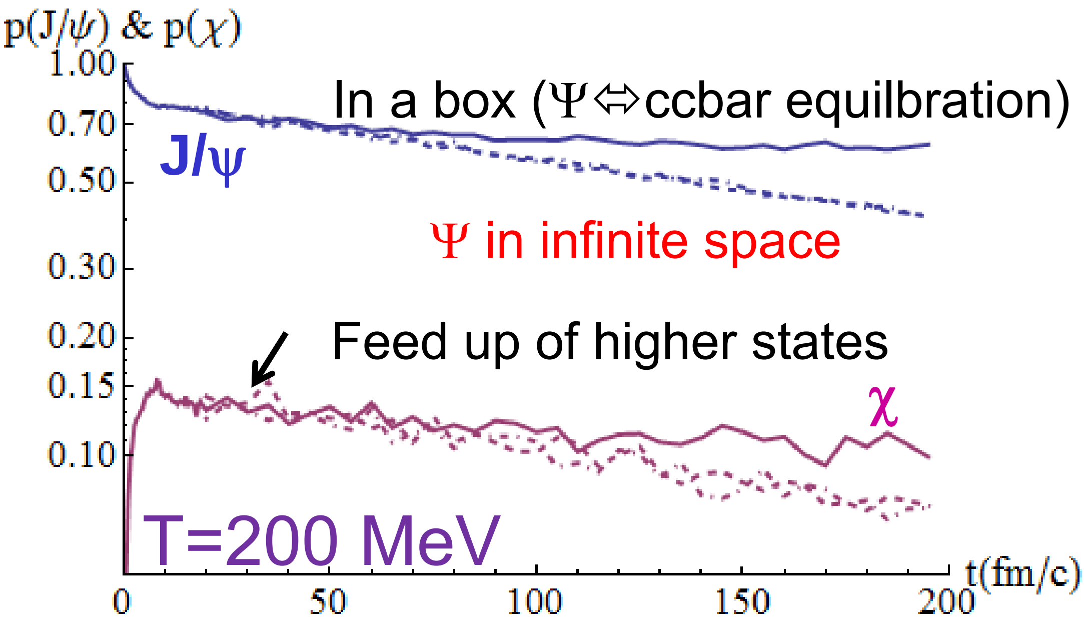


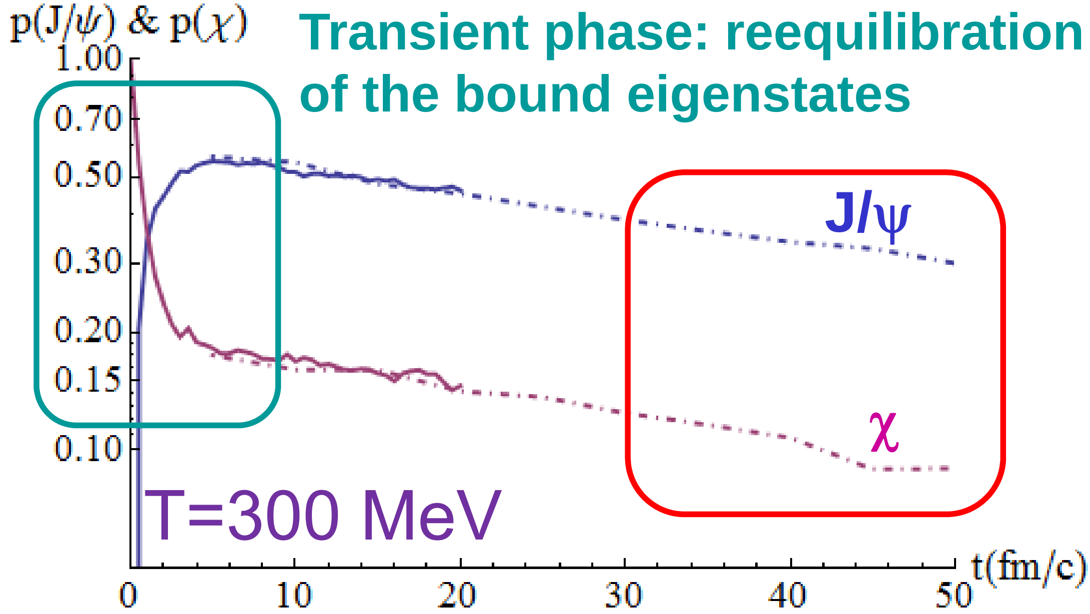
Stochastic forces => leakage

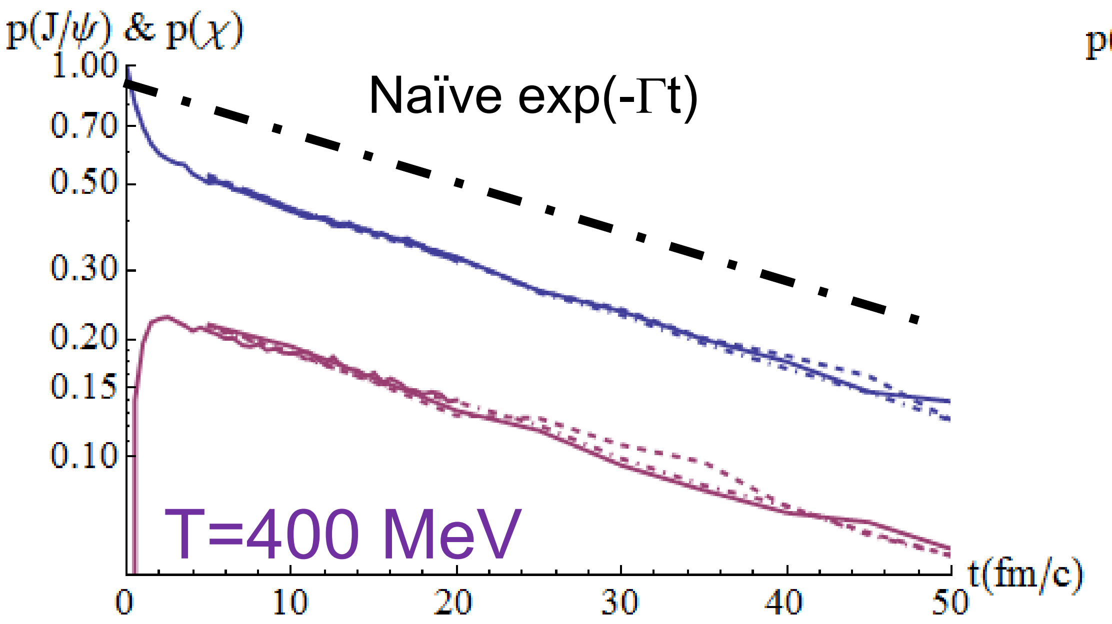
K chosen such that
E2-E0=E(ψ')- E(J/ψ))=600MeV

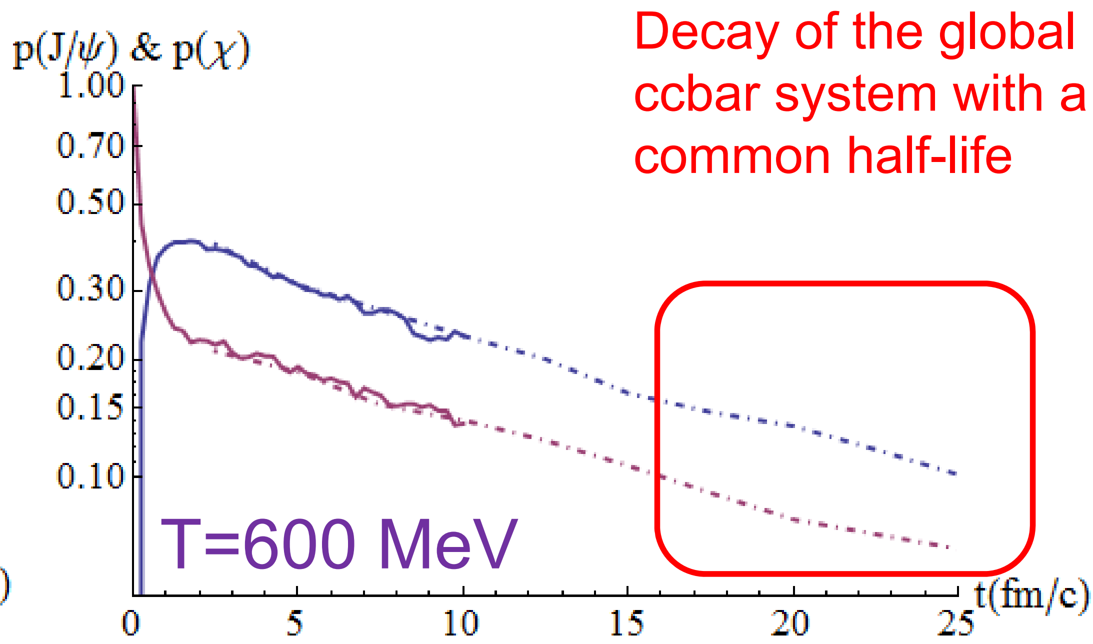
4 bound eigen-states

Evolution of eigenstates with V=V(T=0)





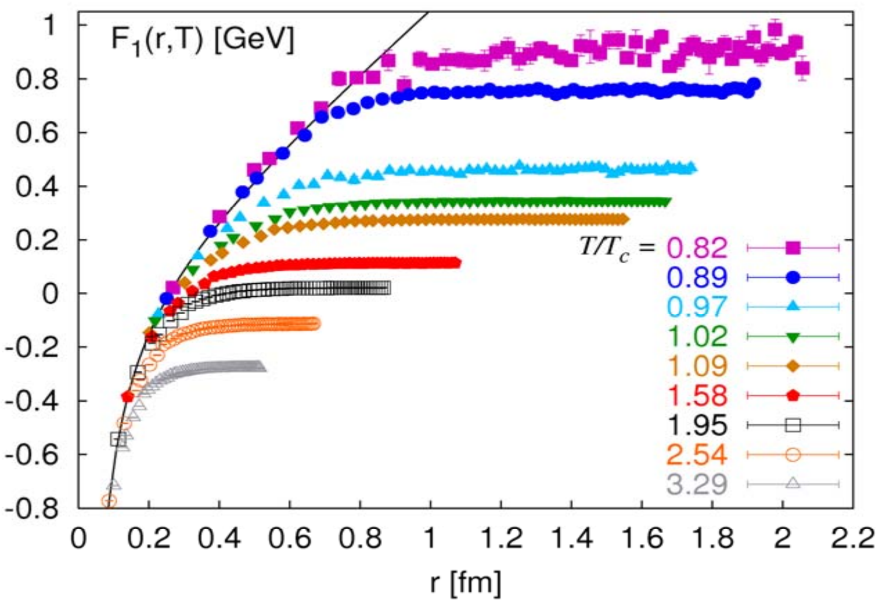


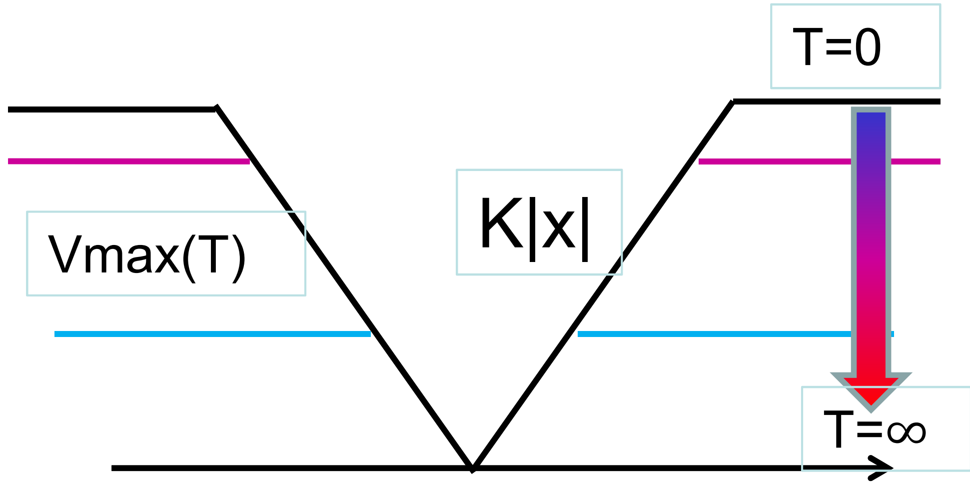


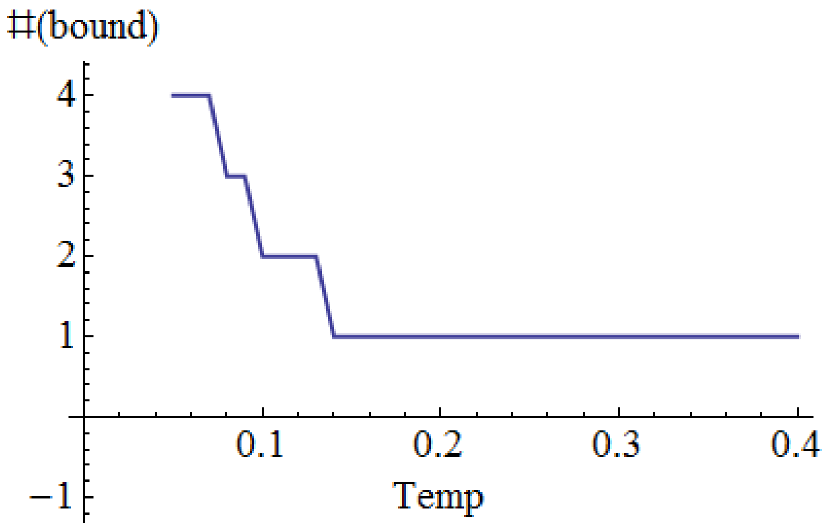
Dynamics of QQbar with SL equation

Now considering the effect of the fluctuations-dissipation combined with the mean field contribution:

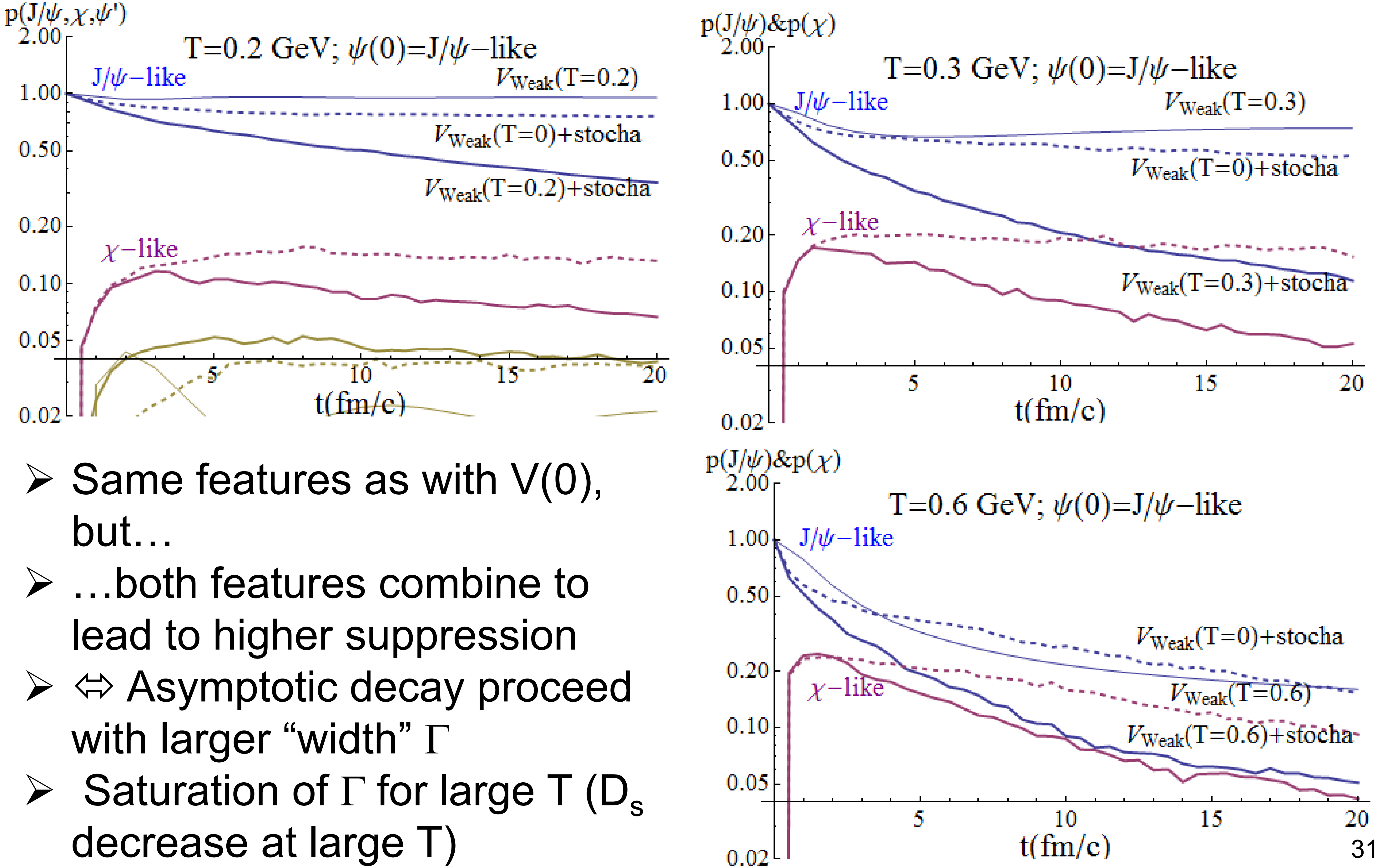
- Potential:





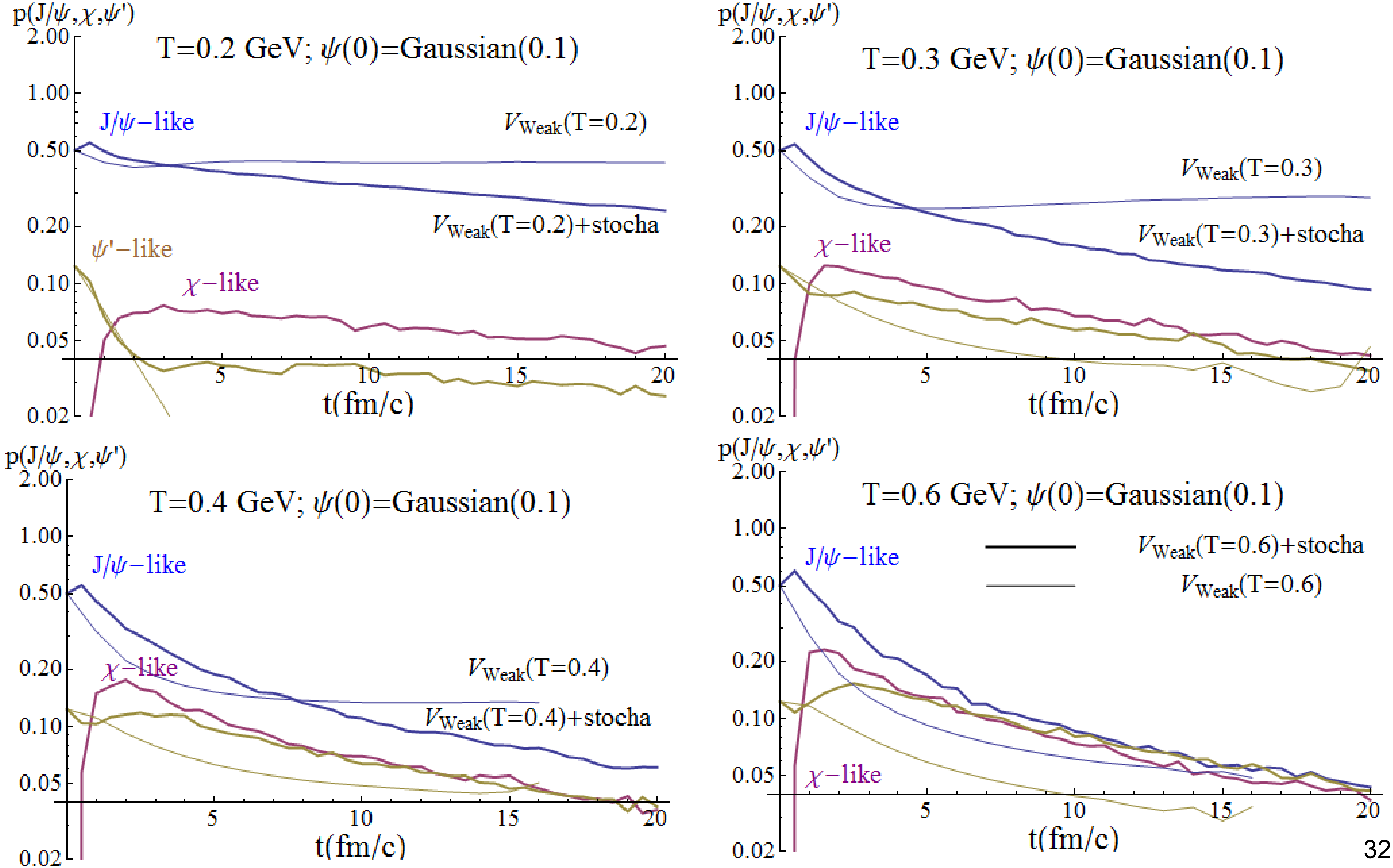


Evolution of eigenstates with V=V(T) !!! 1 bound state !!!

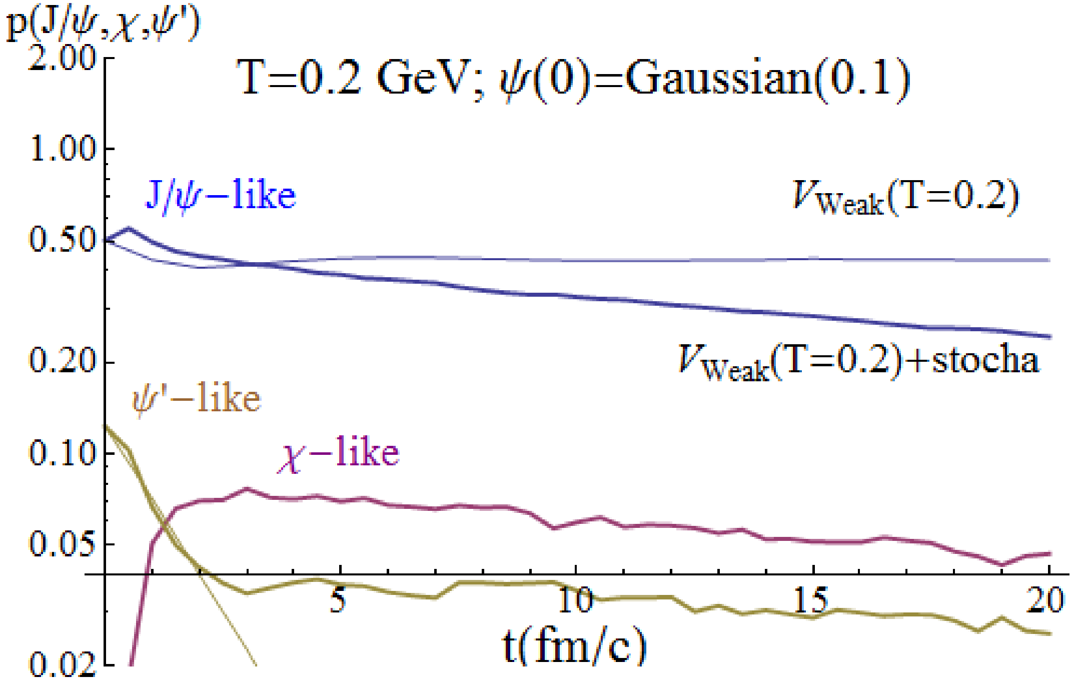


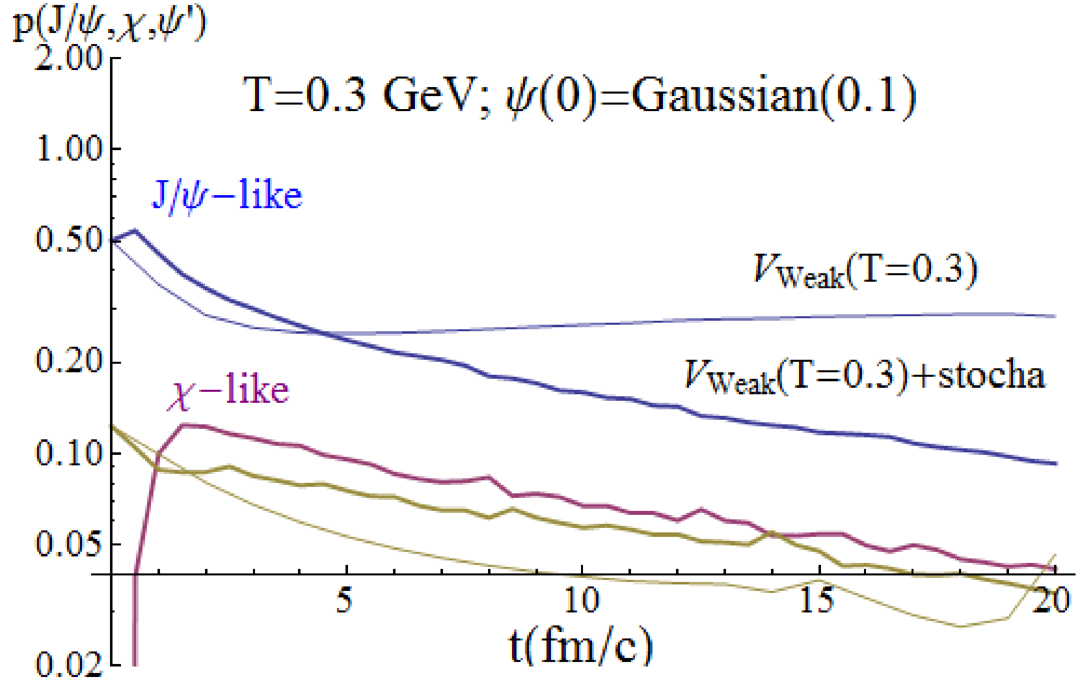
- Same features as with V(0), but…
- …both features combine to lead to higher suppression
- ⇔ Asymptotic decay proceed with larger "width" Γ
- Saturation of Γ for large T (D_s decrease at large T)

Evolution of more realistic initial state with V=V(T)

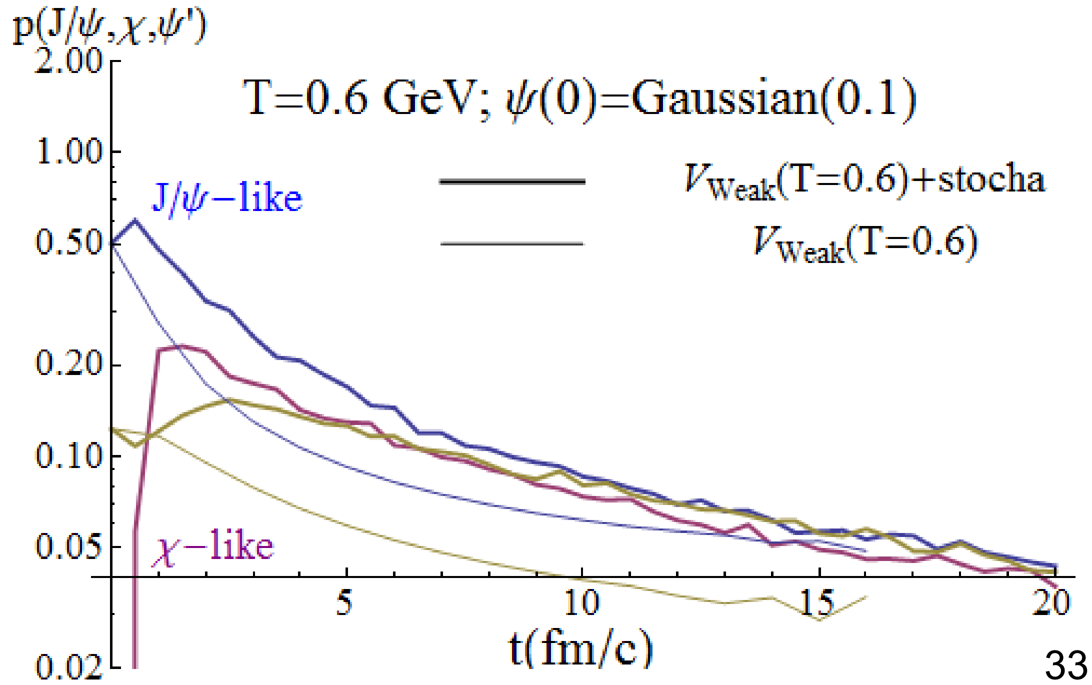


Evolution of more realistic initial state with V=V(T)



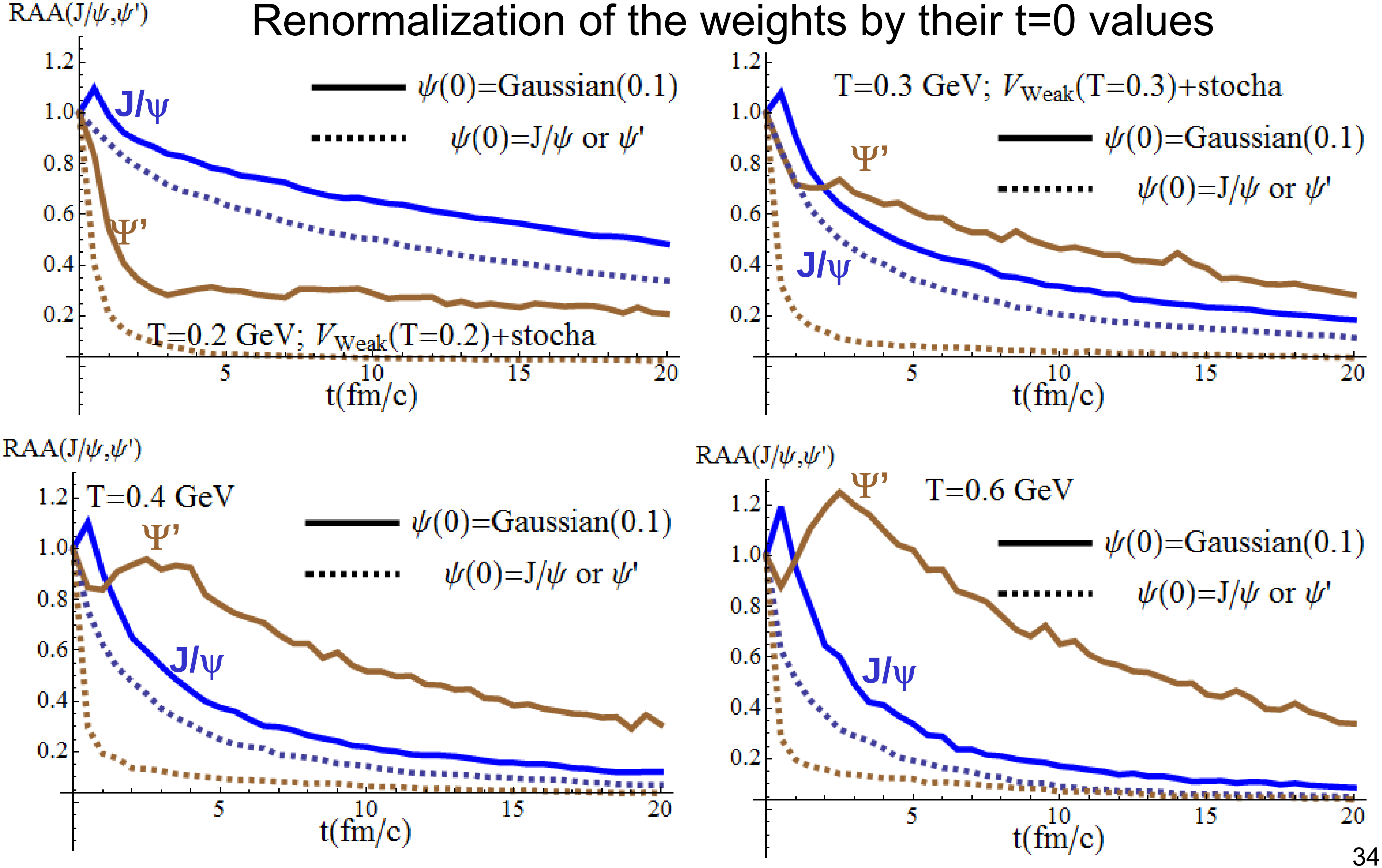


- As compared to the pure mean-field, the thermal forces can lead to an overpopulation of the initial J/ψ component at intermediate times (also true for other components)
- Universal long-time decay



Suppression of states as a function of time ($1c\bar{c}$ in the HB)

Renormalization of the weights by their t=0 values



Suppression of states as a function of time ($1c\bar{c}$ in the HB)

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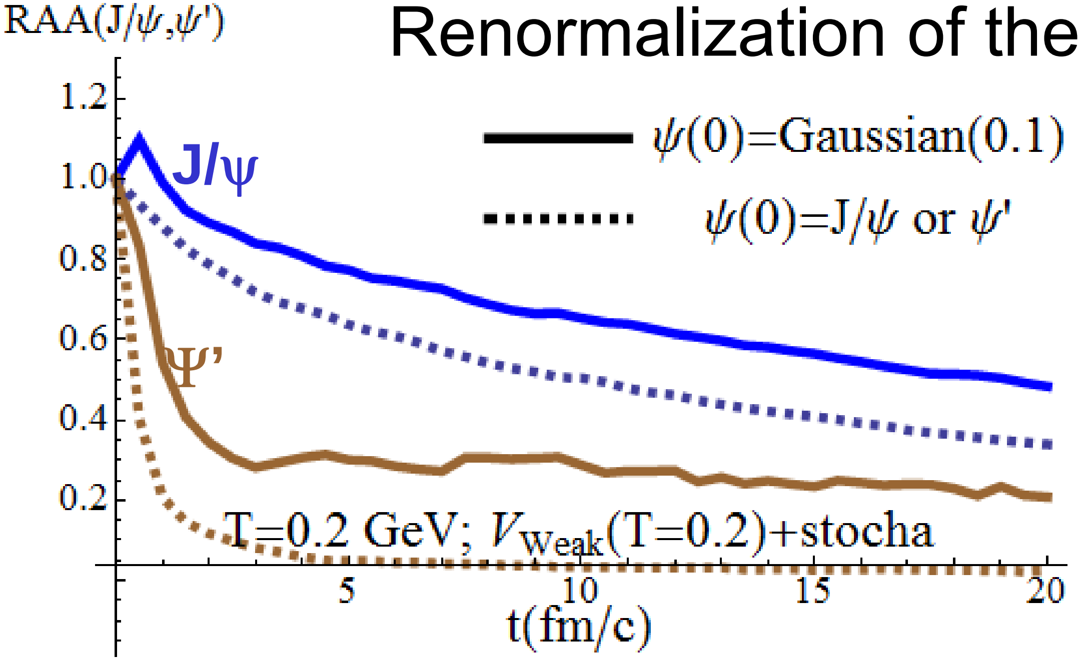
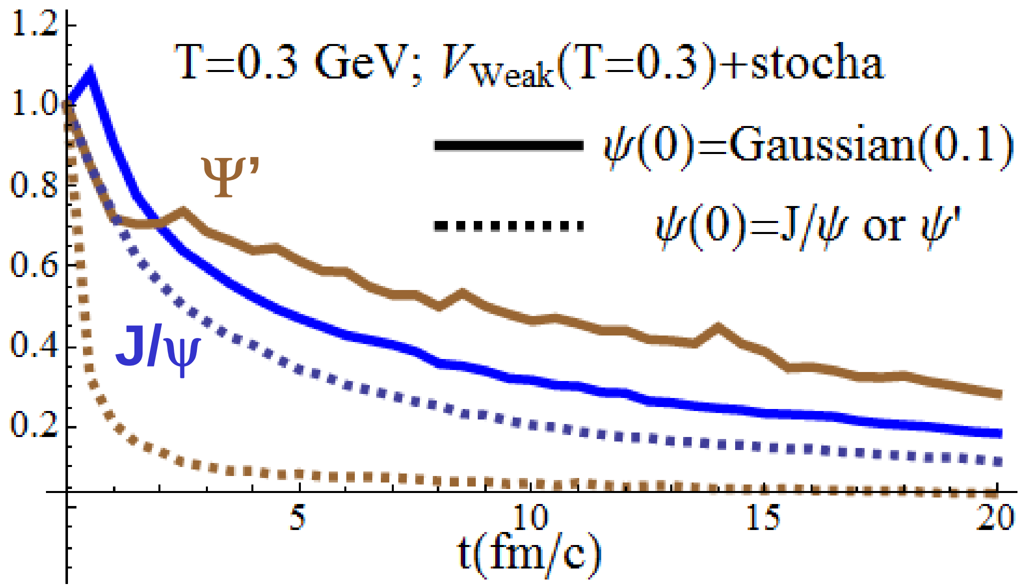
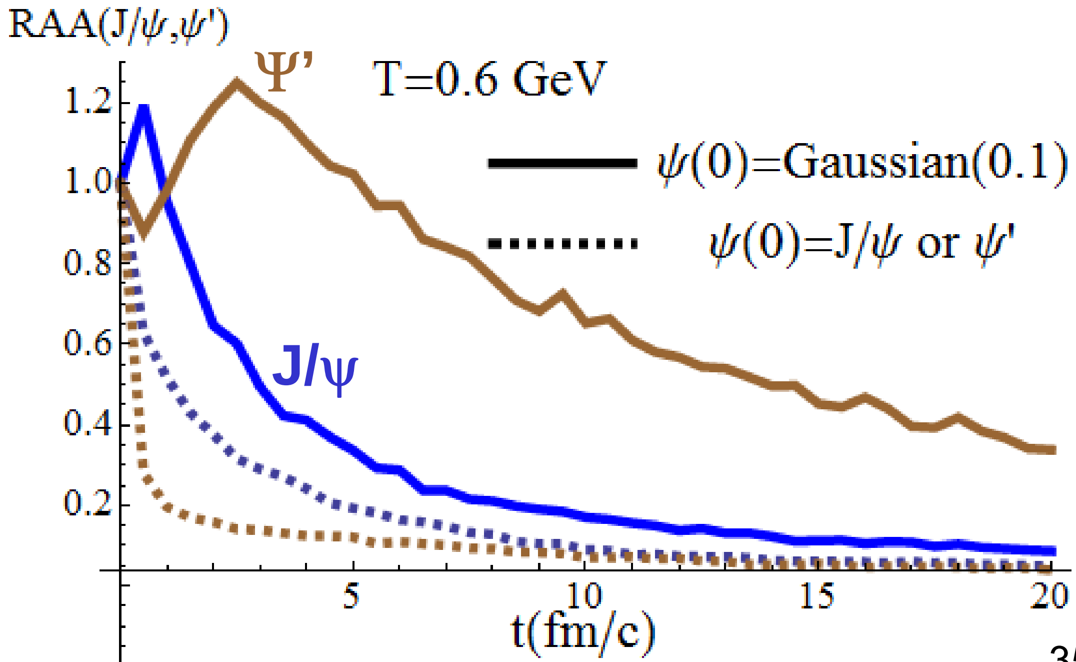




- Definitively kills the (unjustified assumption of quantum decoherence at t=0



Conclusions and Future

- Framework satisfying all the fundamental properties of quantum evolution in contact with a heat bath, easy to implement numerically
- First tests passed with success
- Rich suppression pattern found in a stationary environment
- Assumption of early decoherence ruled out.

- Future:
  - Identify the limiting cases and make contact with the other models (a possible link between statistical hadronization and dynamical models)
  - Implementation in evolution scenario of the QGP
  - …

Back up

Hard Probing QGP with heavy flavors

The Trilogy:

Hidden c & b

≡ thermometer

Quarkonia suppression and Dimuons production

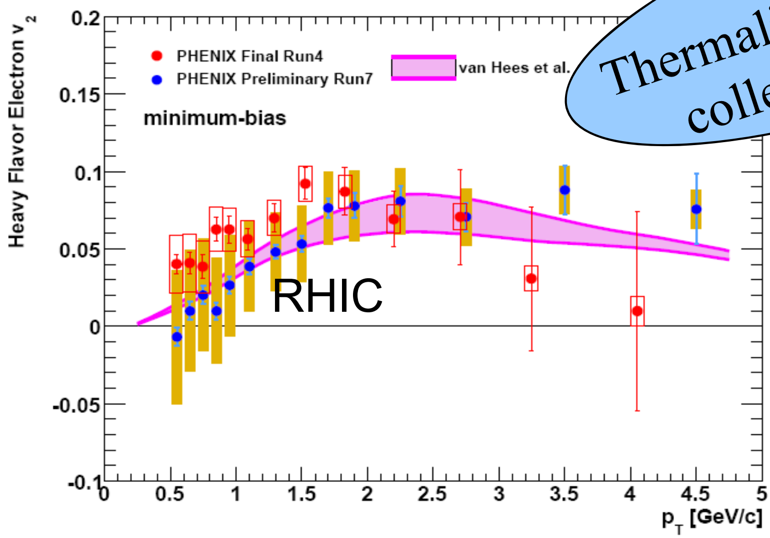
HQ

Barometer ≡

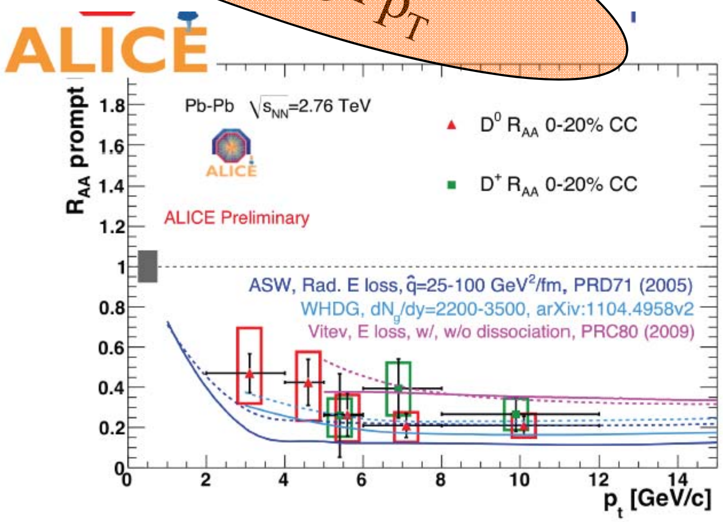
Thermalisation & collectivity

Quenching of leading hadron at high p_T

≡ "densimeter"



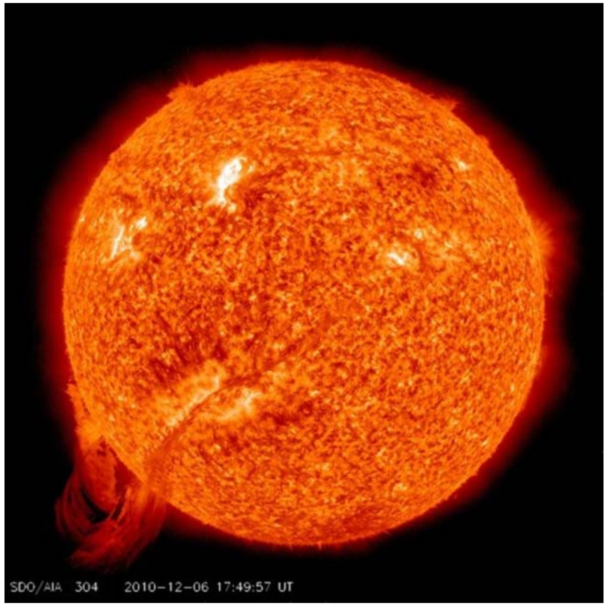
HQ gain elliptic flow from the surrounding medium… with some time delay (inertia)



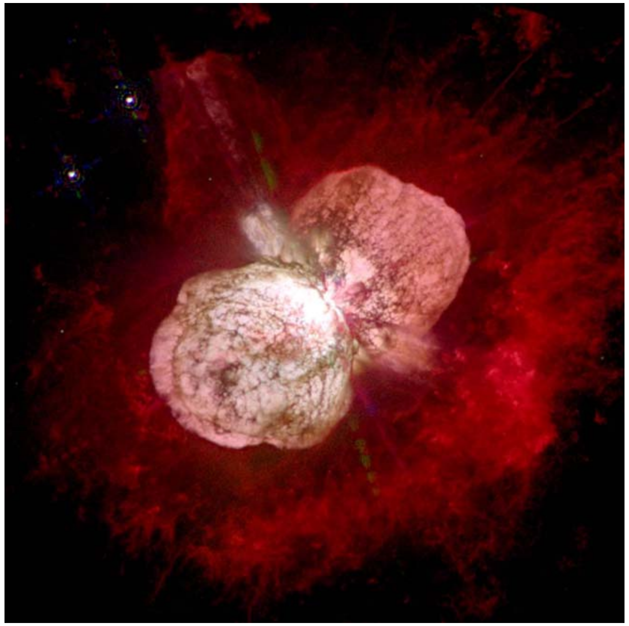
Nuclear modification factor (R_{AA}) of D mesons probes c-quark energy loss in QGP (not seen in pA)

Caviats & Uncertainties

What does the sequential suppression in a stationary QGP has to do with reality anyhow ?



Picture

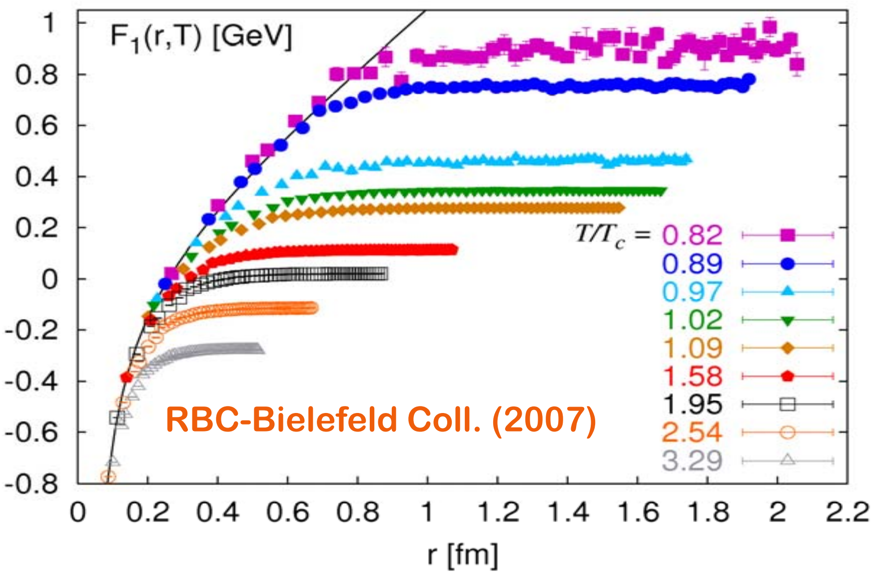


Reality

Need for a genuine time-dependent scenario

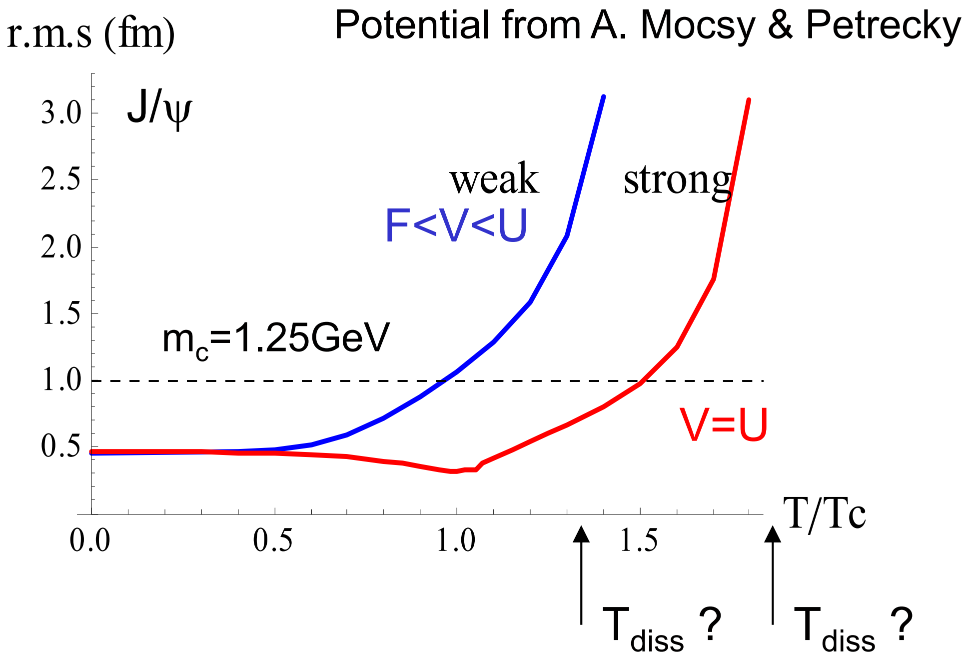
Caviats & Uncertainties

I. Quarkonia in *stationnary* medium are not well understood from the fundamental finite-T LQCD



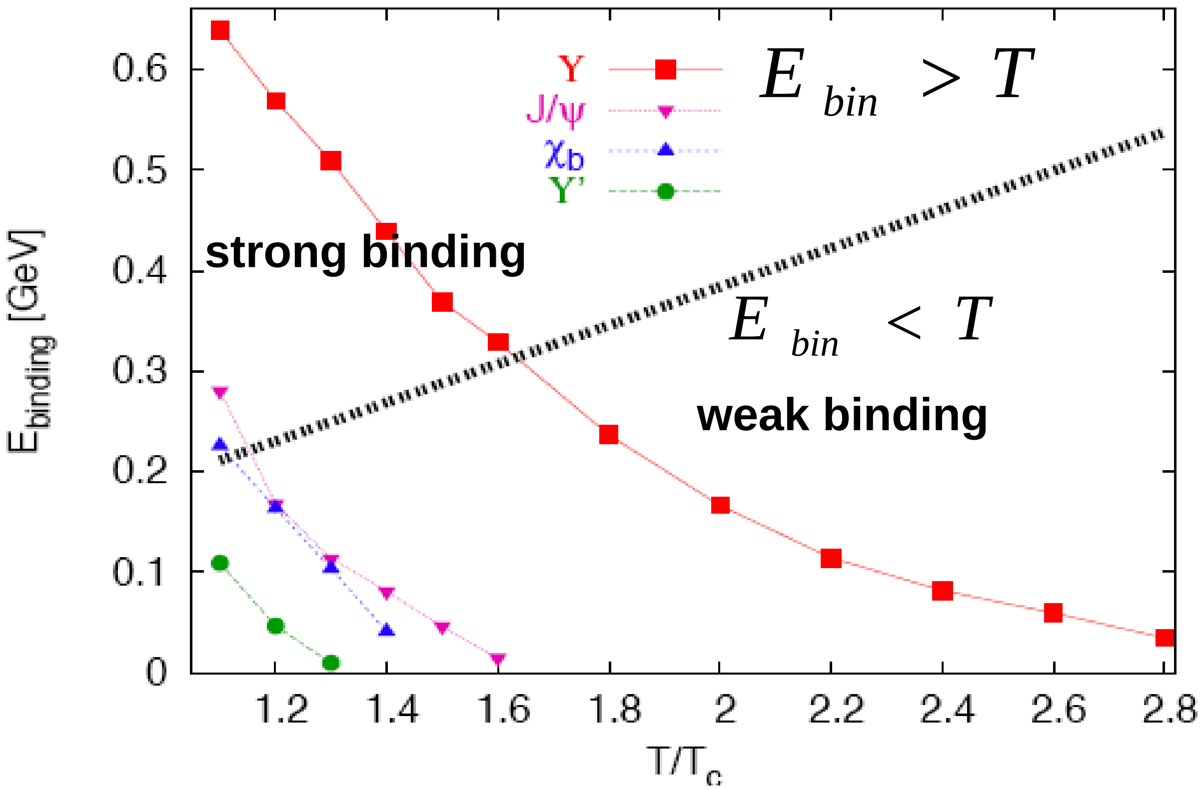
From free energy $\Rightarrow$ V(r,T) ?

Several prescriptions in litterature



Caviats & Uncertainties

II. Criteria for quarkonia "existence" (as an effective degree of freedom) in *stationnary* medium is even less understood

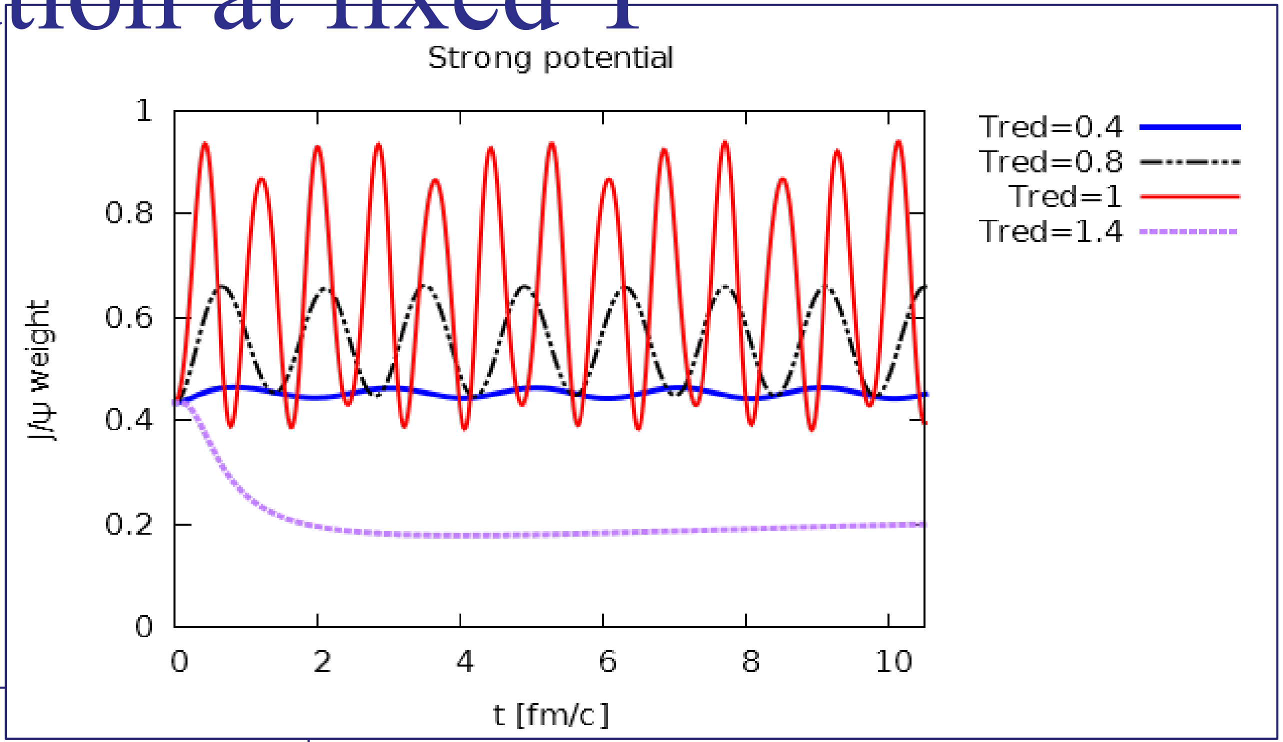


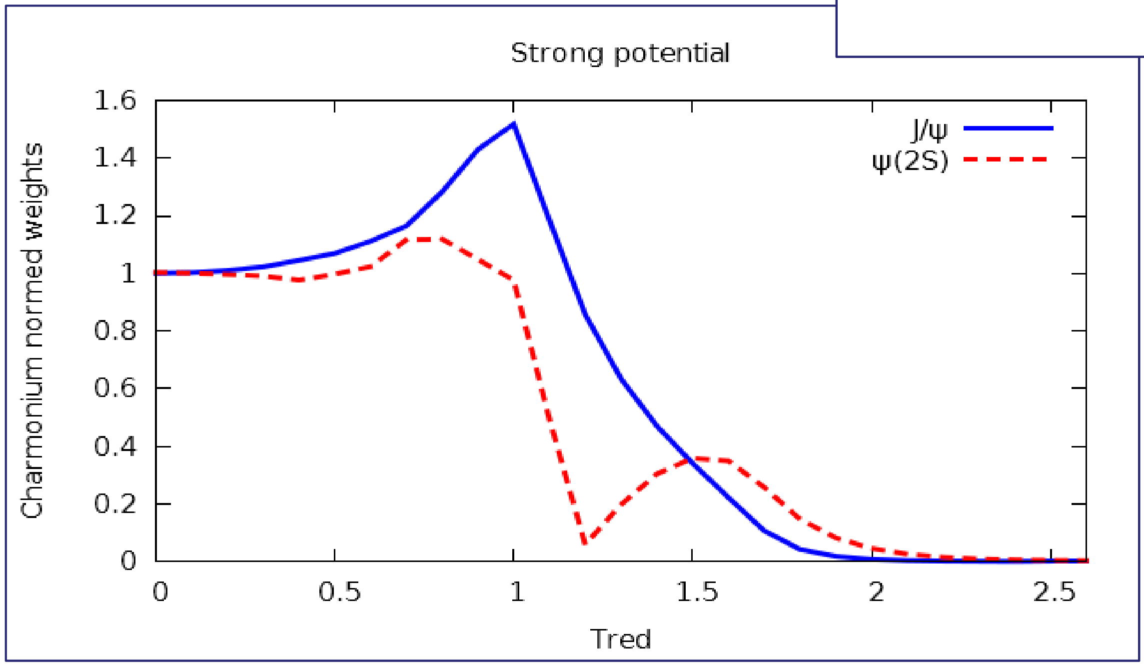
From A. Mocsy (Bad Honnef 2008)

Evolution at fixed T

Charmonia and strong color potential (V=U)

At fixed temperatures



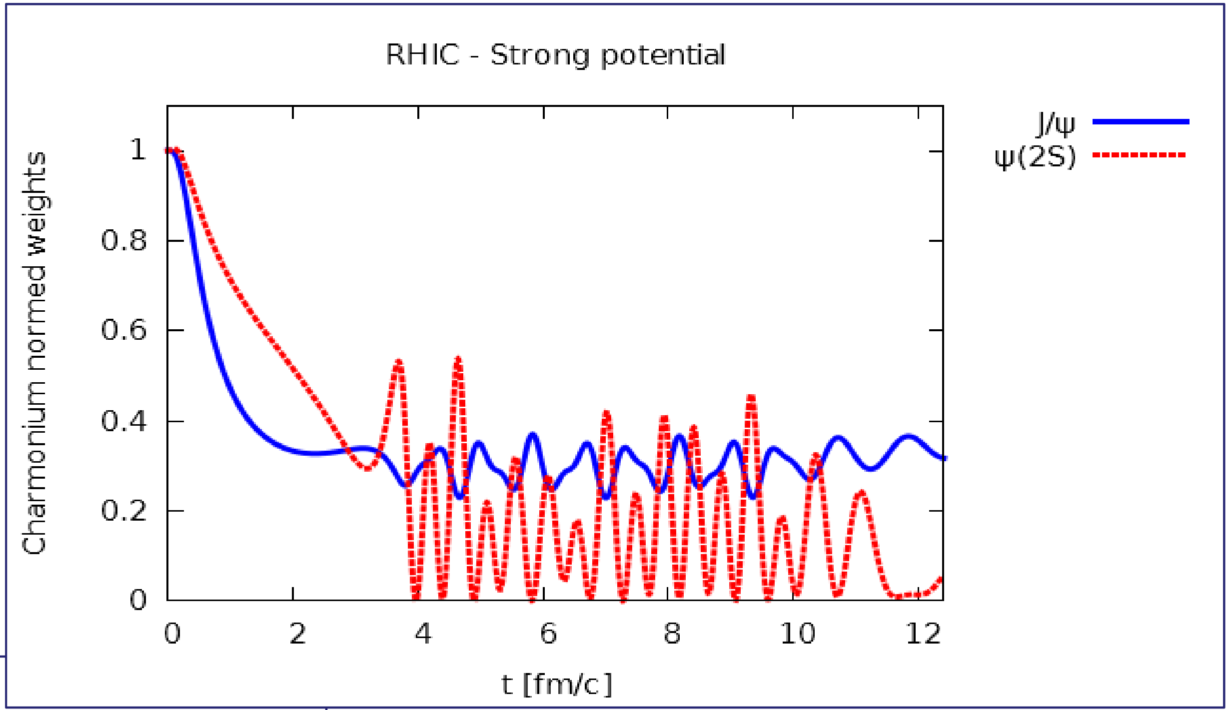


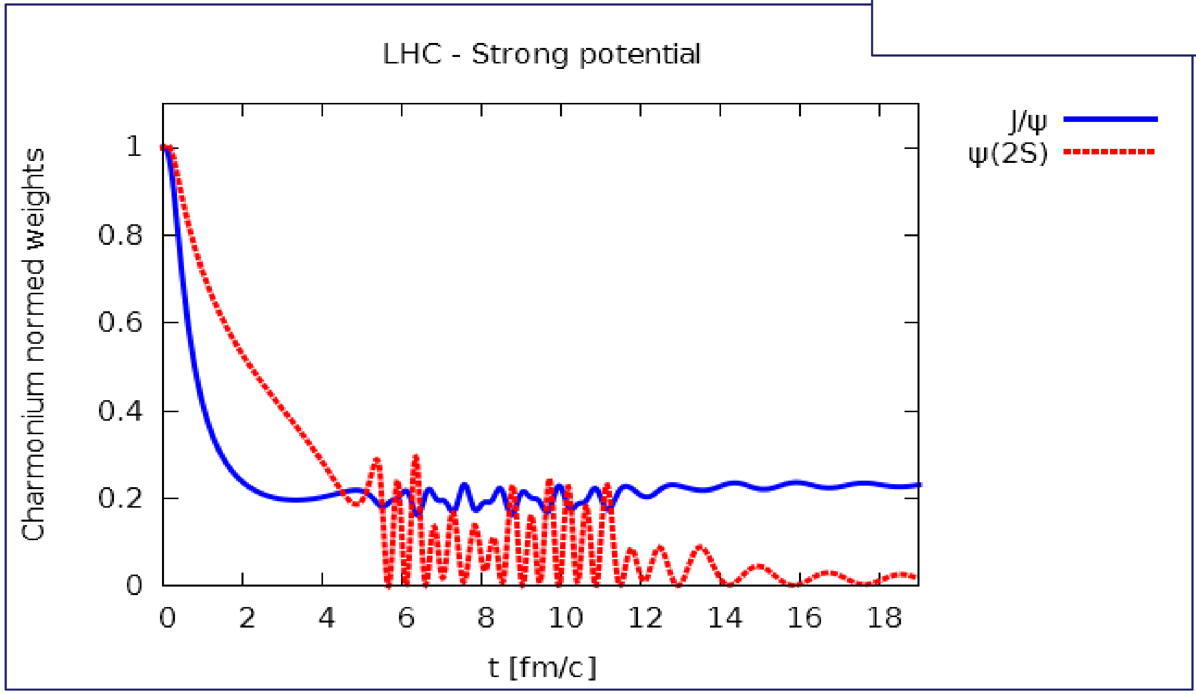
The normed weights at t->∞ function of the temperature

Evolution in realistic T scenarios

Charmonia and strong color potential (V=U)

RHIC temperature scenario



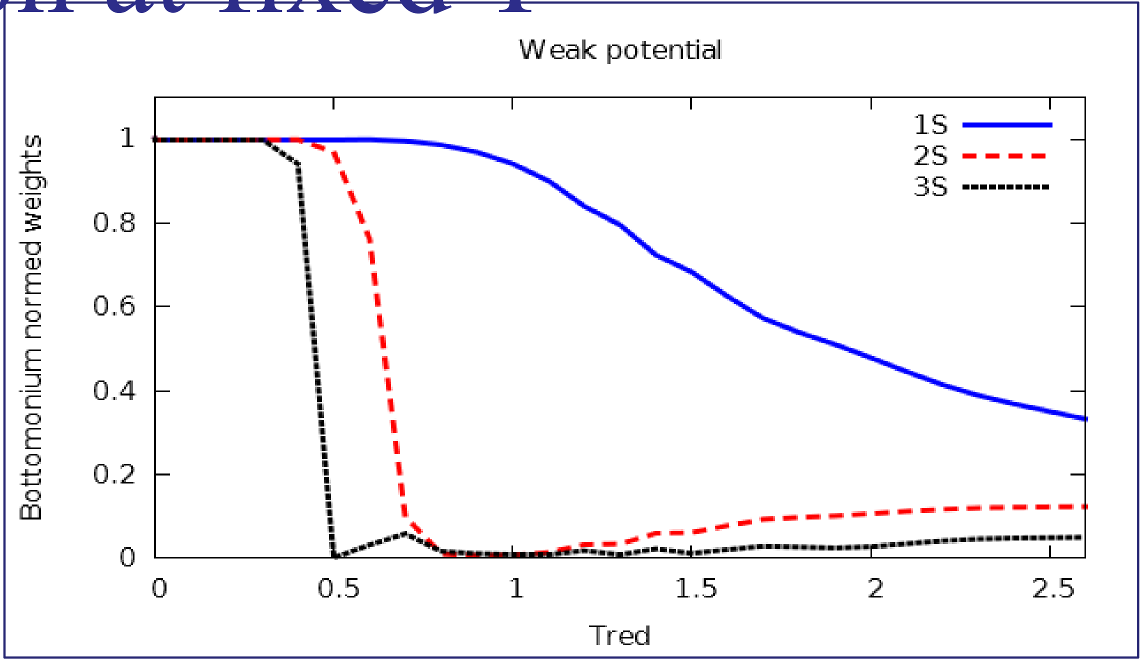


LHC temperature scenario

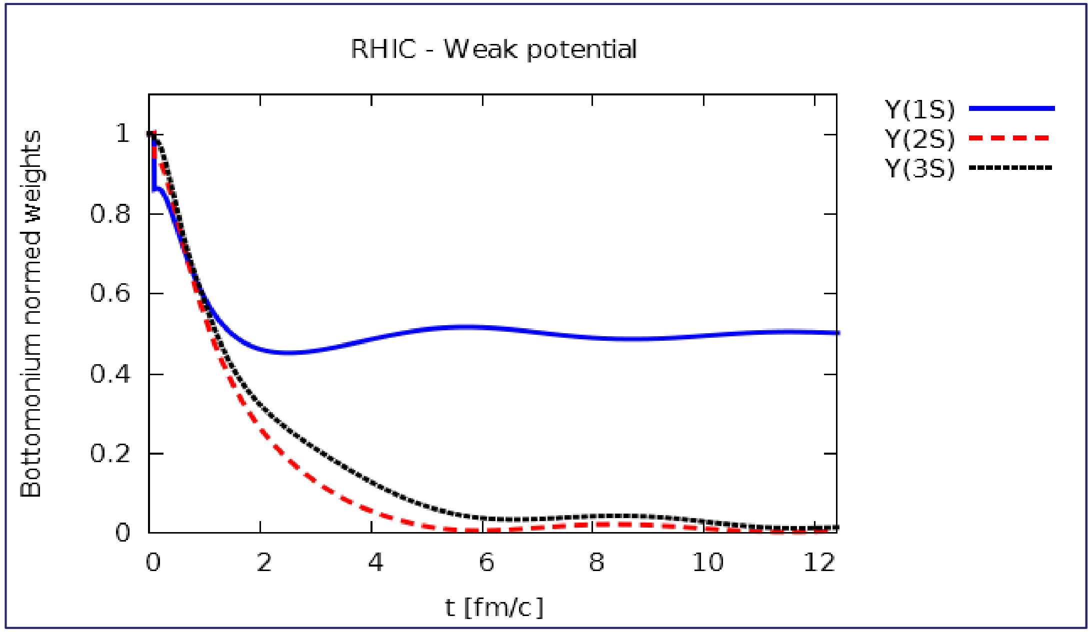
Evolution at fixed T

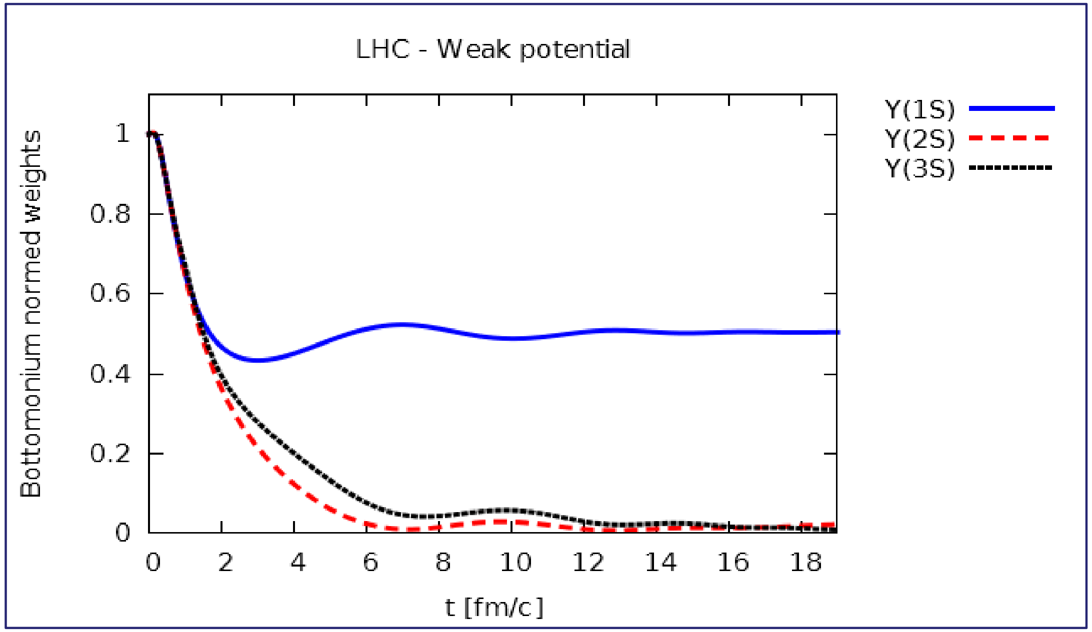
Bottomonia and weak color potential (F<V<U)

The normed weights at t->∞ function of the temperature



Temperature scenarios

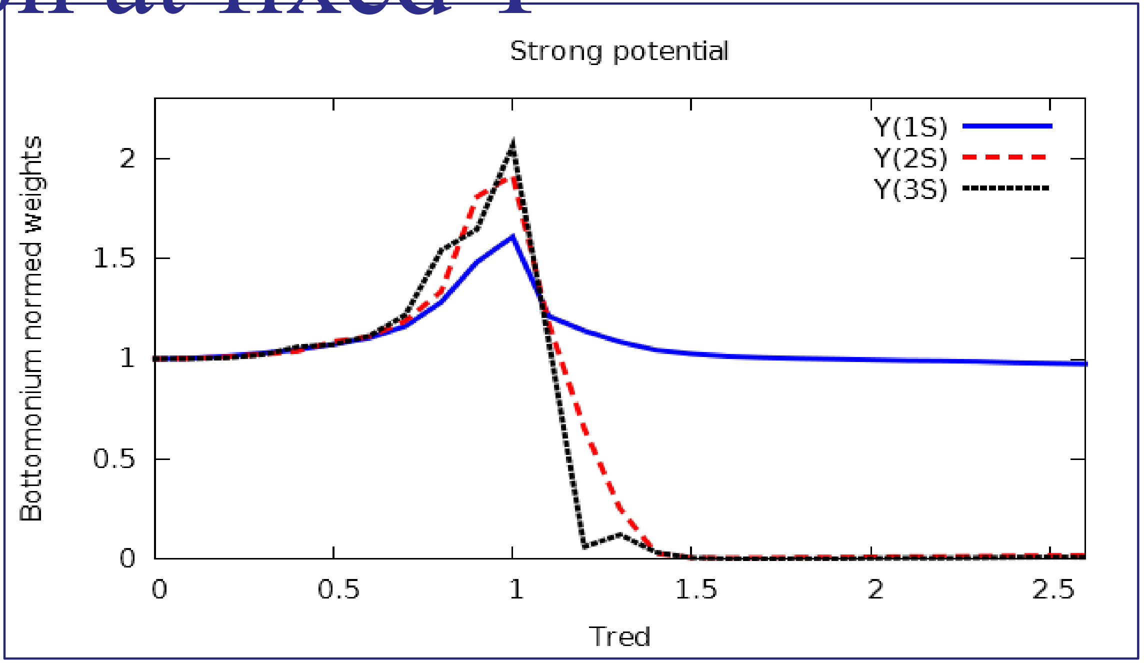




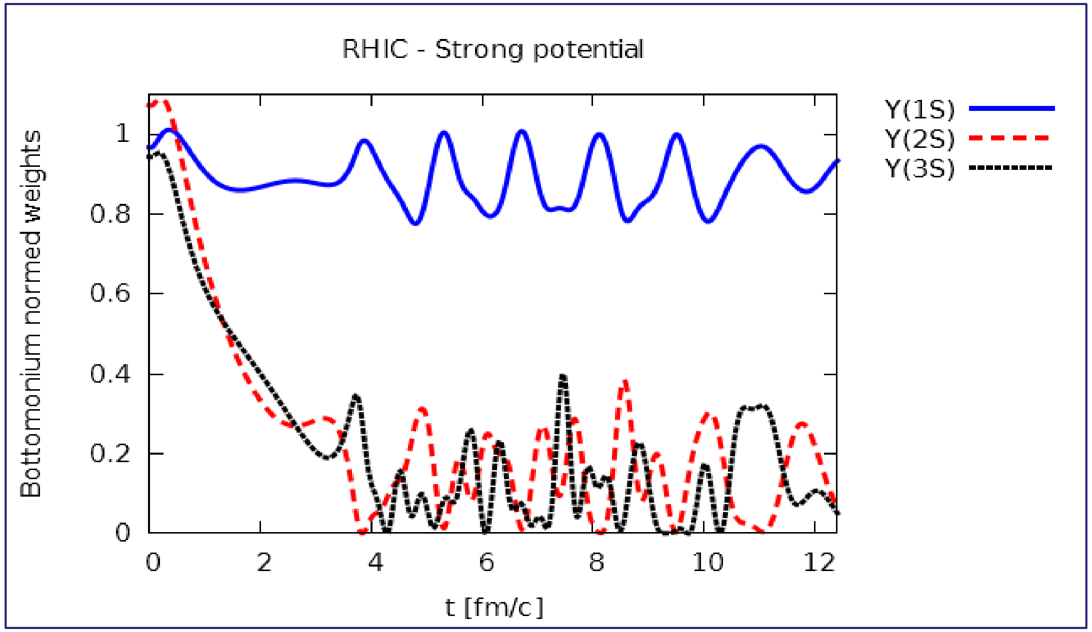
Evolution at fixed T

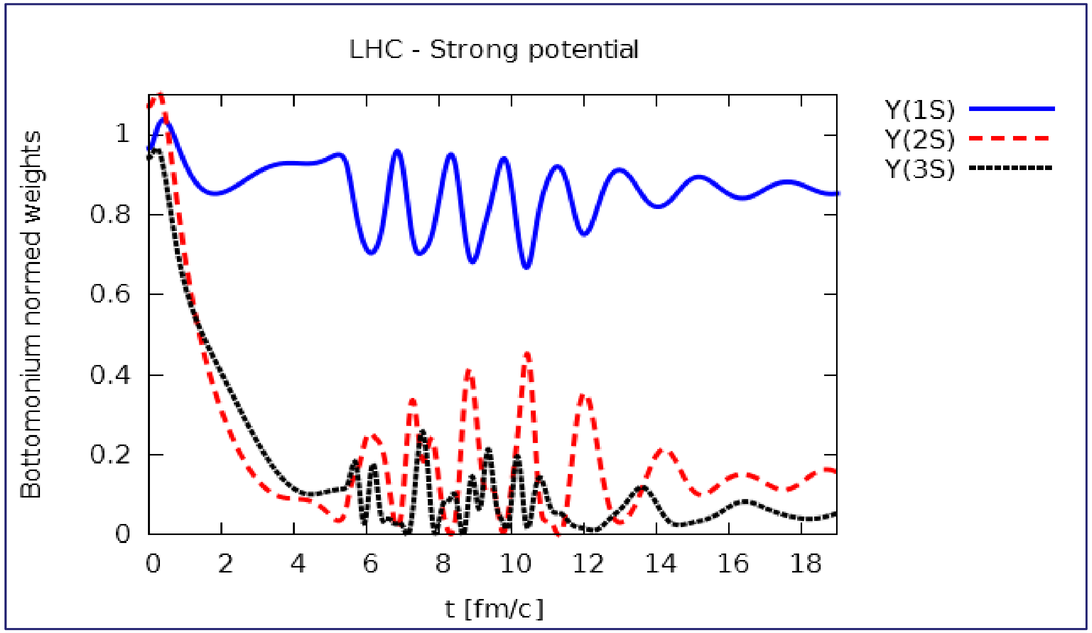
Bottomonia and strong color potential (V=U)

The normed weights at t->∞ function of the temperature



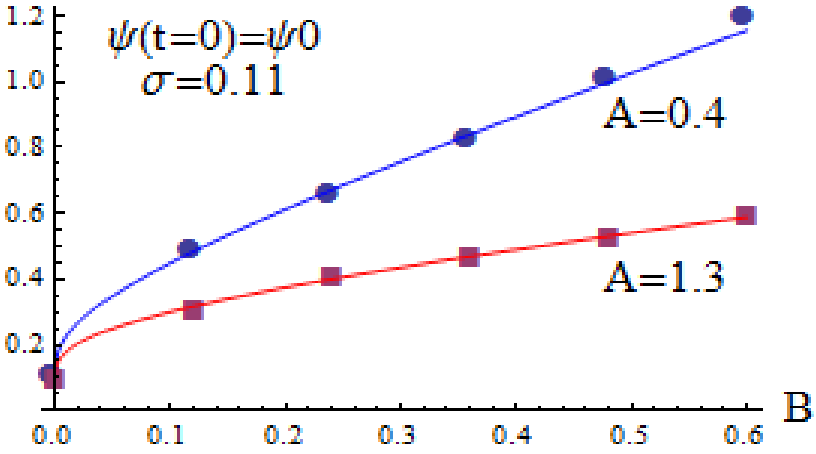
Temperature scenarios



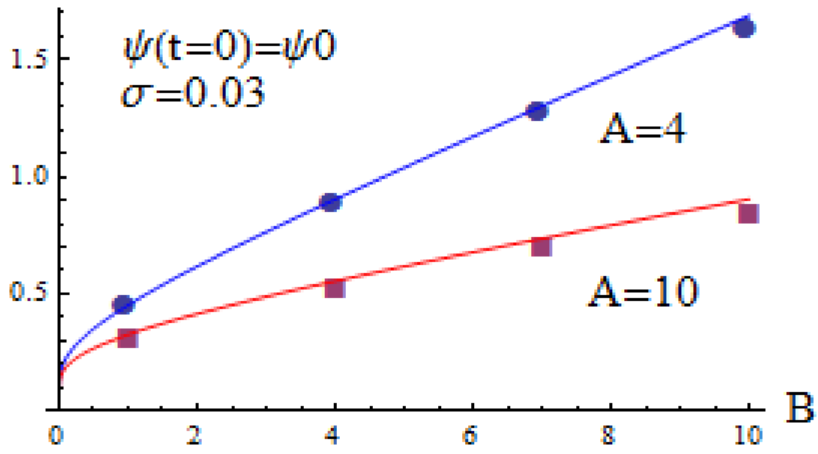


Effects of the autocorrelation

Measured temperature [ħω] at t->∞

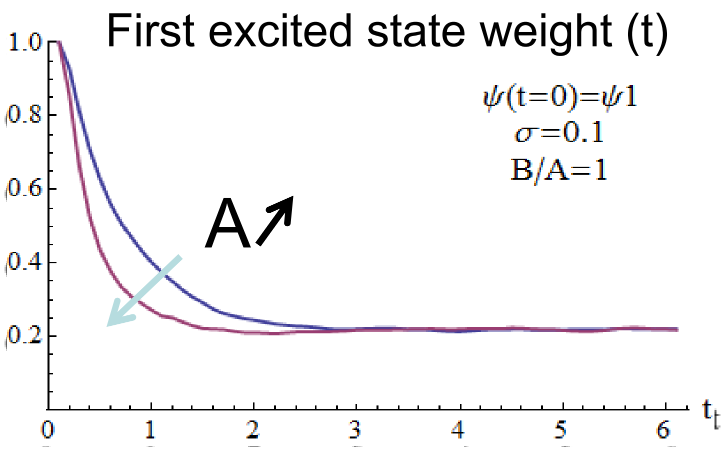


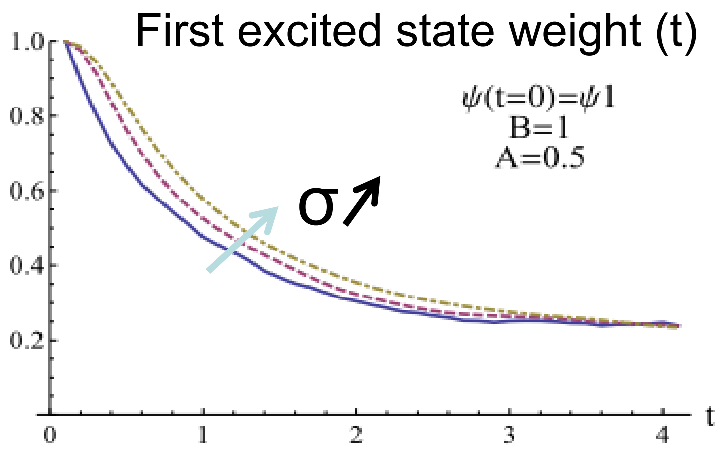
Measured temperature [ħω] at t->∞



Ok for
$\sigma \ll \tau_{\mathrm{relax}}$

Tune B/A or σ
to adjust the
relaxation time





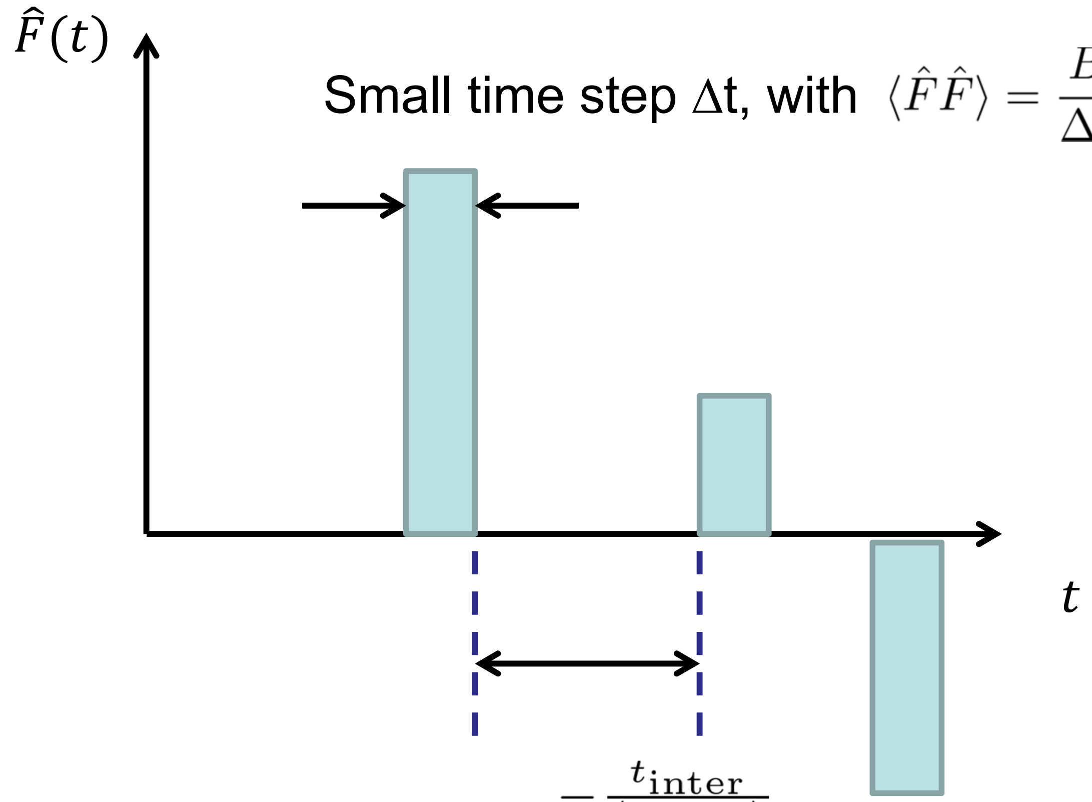
At a finite time:

high pt => high velocity => smaller σ => more excited states => more suppression

low pt => small velocity => higher σ => less excited states => less suppression <u>(=> need for regeneration ?)</u>

How legitimate (and legitimated) is it to use cross sections in *dense* medium ?

Simple toy model: Harmonic oscillator + external random forces $\hat{F}(t)$



T_{inter}, distributed like $e^{-\frac{t_{\text{inter}}}{\langle t_{\text{inter}} \rangle}}$

Starting from ground state

After 1rst interaction $\mathrm{Prob}(0 \to X) \propto \frac{B\Delta t}{m\omega}$

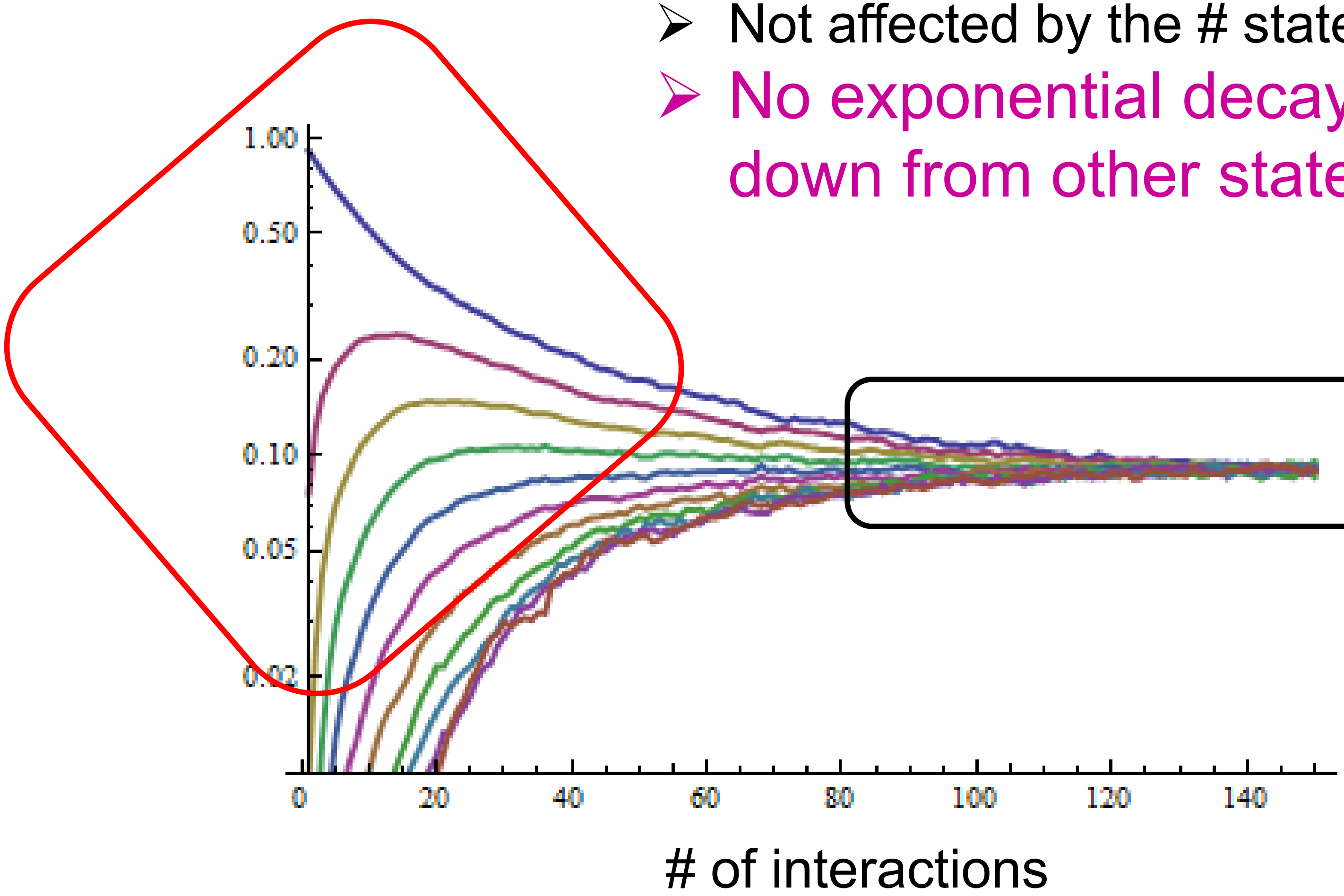
Markoffian hypothesis

After 1rst interaction $\mathrm{Prob}(0) \sim e^{-\frac{B\Delta t}{m\omega}n}$

How legitimate (and legitimated) is it to use cross sections in *dense* medium ?

Results with $\langle t_{inter} \rangle$ not << $1/\omega$: (basis of 11 lowest states)

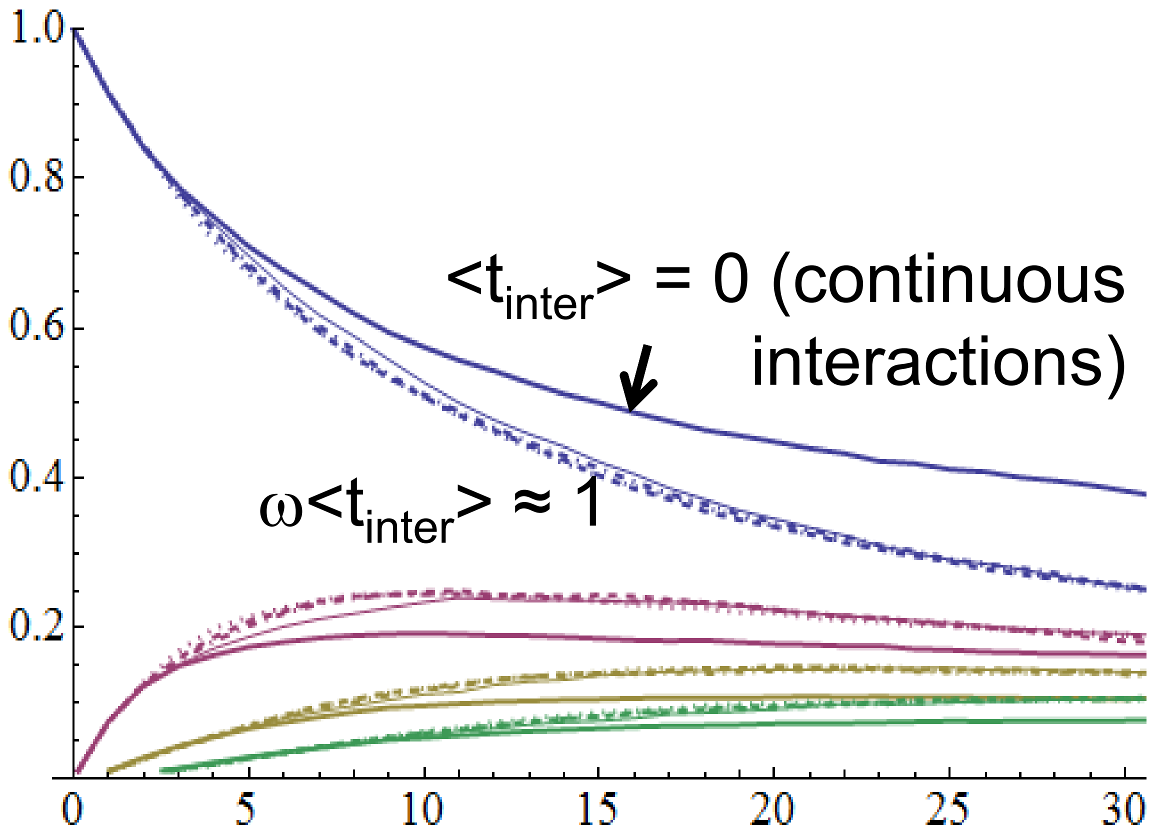
- Not affected by the # states
- No exponential decay (continuous feed down from other states)



Converges towards equilibrated distribution (no dissipation => T=∞ => all states populated with equal probability)

How legitimate (and legitimated) is it to use cross sections in *dense* medium ?

Results with for various $<t_{inter}>$, with $\psi=\psi_0$

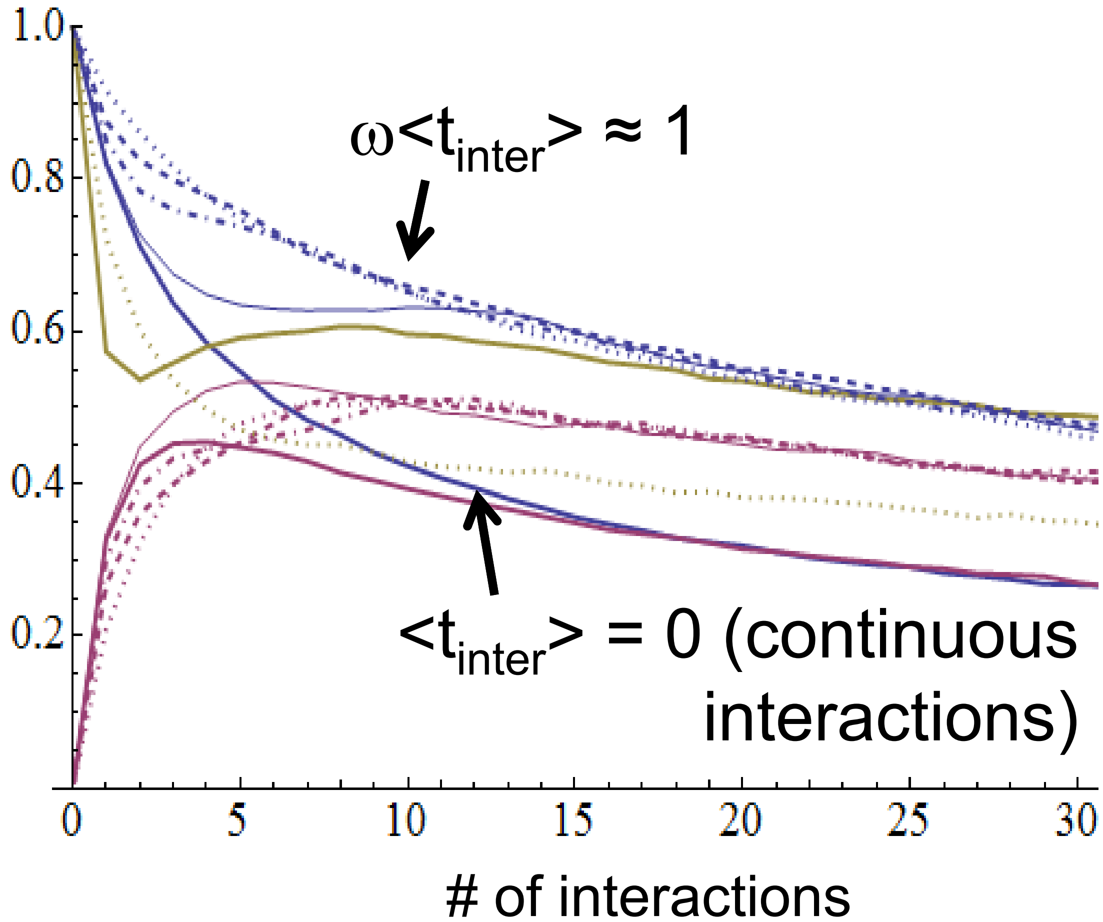


# of interactions

- The case $\omega<t_{inter}> \approx 1$ can be understood in terms of transition probabilities (master equations)
- The case $\omega<t_{inter}> << 1$ requires genuine quantum treatment

How legitimate (and legitimated) is it to use cross sections in *dense* medium ?

Results with for various $<t_{inter}>$, with even $\psi=\Sigma\, a_i\, \psi_i$



Same conclusions, larger effects