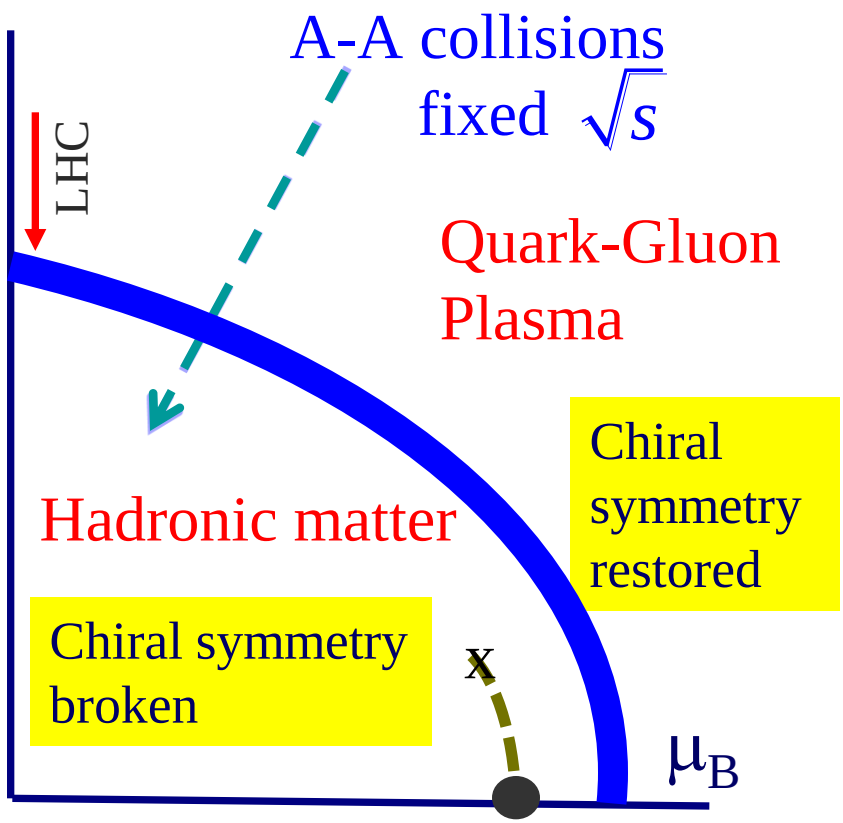
Interplay between deconfinement and O(4) chiral crossover in QCD medium

Krzysztof Redlich University of Wroclaw

- Chiral transition and O(4) pseudo-critical line in LGT
- Particle freezout in HIC and QCD phase boundary: from HIC ⟷ LGT
- Confinement in SU(N) pure gauge theory and in QCD from LGT
- Probing O(4) chiral criticality and confinement properties in HIC
  - fluctuations of net baryon number
  - production of quarkonia states



1st principle calculations:

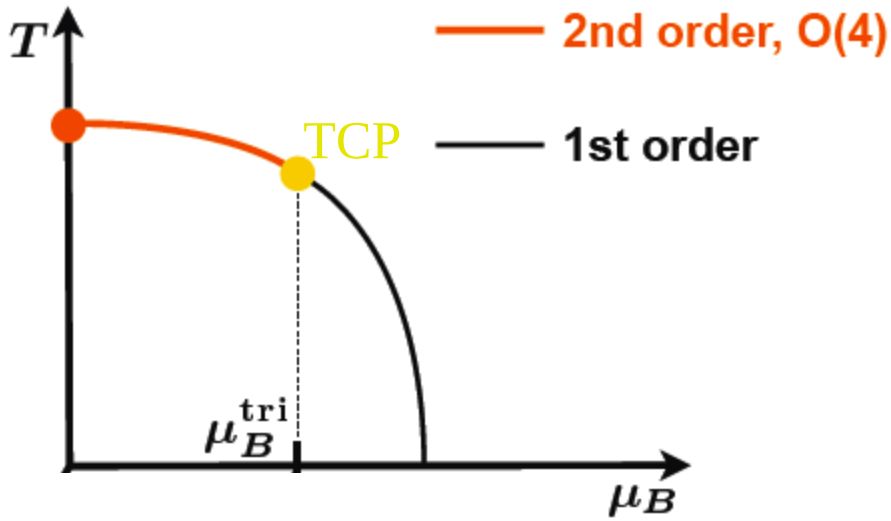
$\mu, T << \Lambda_{QCD}$: χ – perturbation theory

$\mu, T >> \Lambda_{QCD}$: pQCD

$\mu_q < T$: LGT

QCD phase diagram and the O(4) criticality

Pisarki & Wilczek conjecture

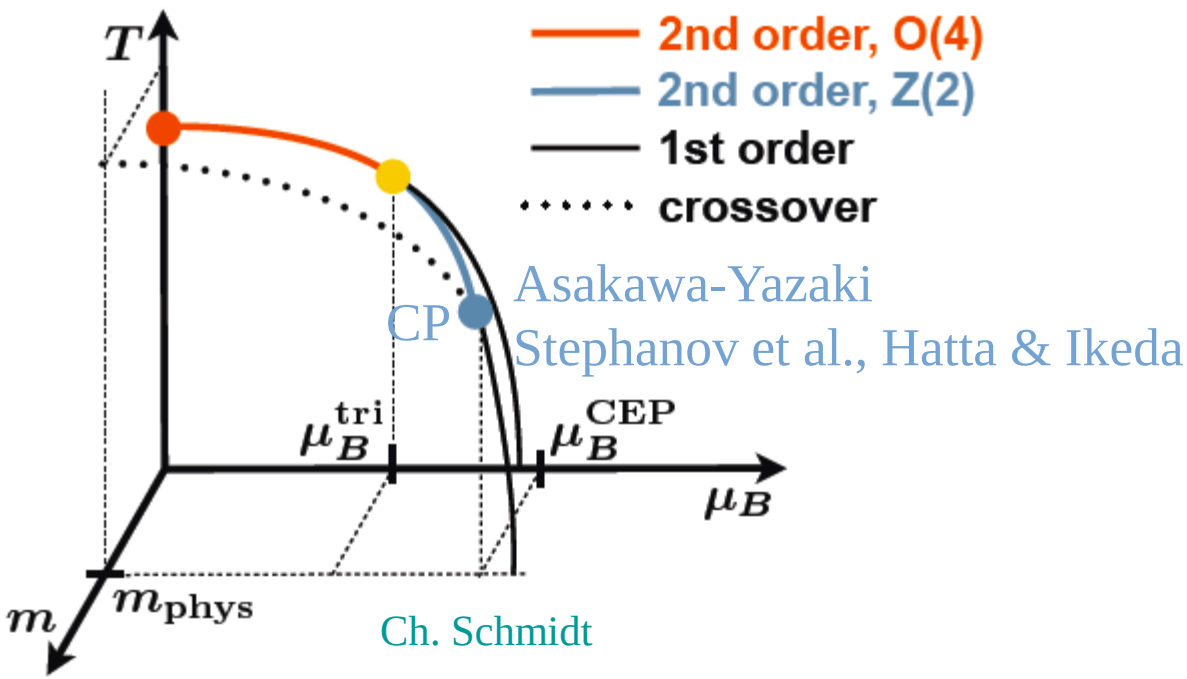


Expected phase diagram in the chiral limit, for massless u and d quarks:

TCP: Rajagopal, Shuryak, Stephanov
Y. Hatta & Y. Ikeda

- In QCD the quark masses are finite: the diagram has to be modified

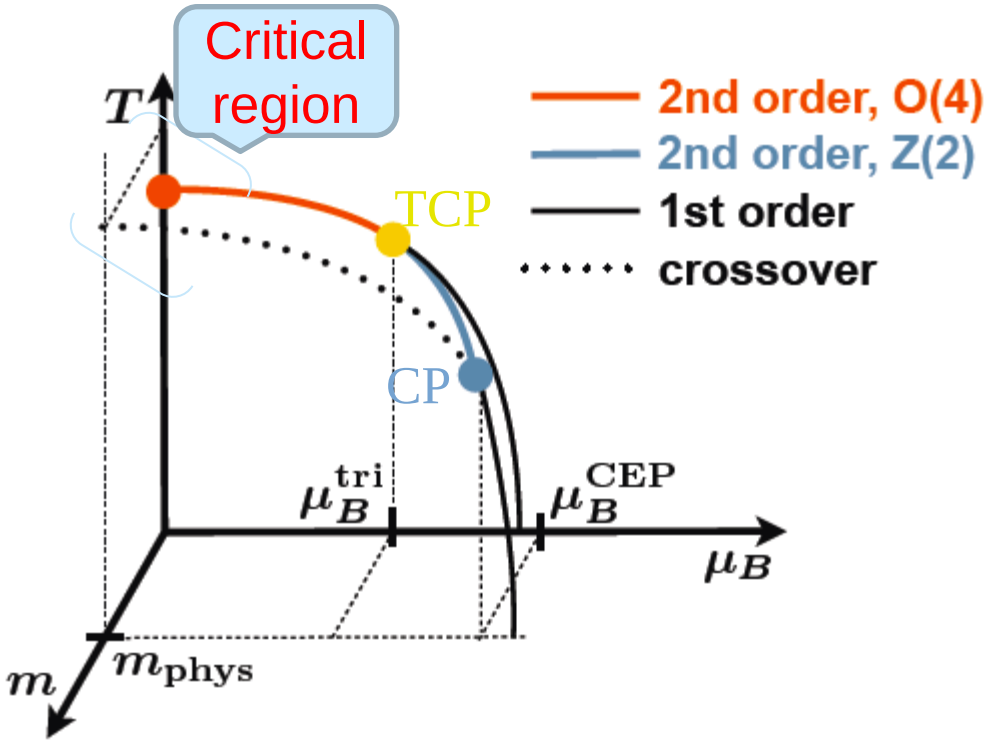
The phase diagram at finite quark masses



At the CP:
Divergence of Fluctuations, Correlation Length and Specific Heat

- The u,d quark masses are small
- Is there a remnant of the O(4) criticality at the QCD crossover line?

The phase diagram at finite quark masses



- Can the QCD crossover line appear in the O(4) critical region?

YES, It has been confirmed in LQCD calculations

LQCD results: BNL-Bielefeld

Phys. Rev. D83, 014504 (2011)

Phys. Rev. D80, 094505 (2009)

O(4) scaling and magnetic equation of state

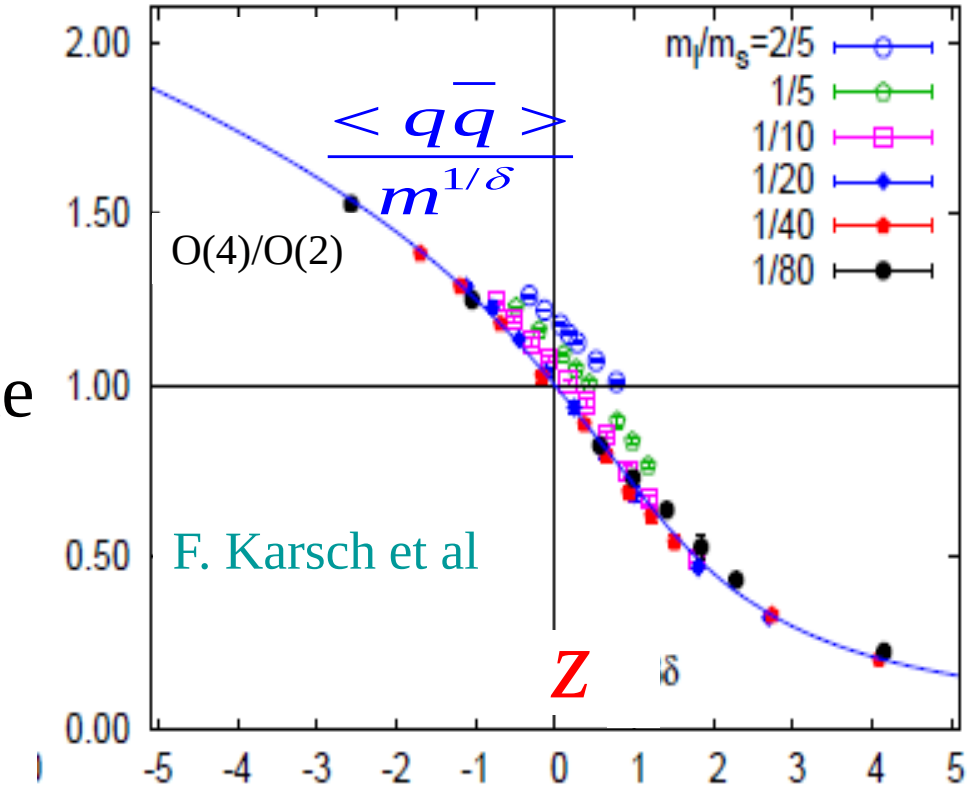
$$P = P_R(T,\mu_q,\mu_I) + b^{-1} P_S(b^{(2-\alpha)^{-1}} t, b^{\beta\delta/\nu} h)$$

QCD chiral crossover transition in the critical region of the O(4) 2nd order

- Phase transition encoded in the magnetic equation of state

$$<q\bar{q}> = -\frac{\partial P}{\partial m} \Rightarrow \text{pseudo-critical line}$$

$$\frac{<q\bar{q}>}{m^{1/\delta}} = f_s(z) \ , \quad z = t m^{-1/\beta\delta}$$



universal scaling function common for all models belonging to the O(4) universality class: known from spin models

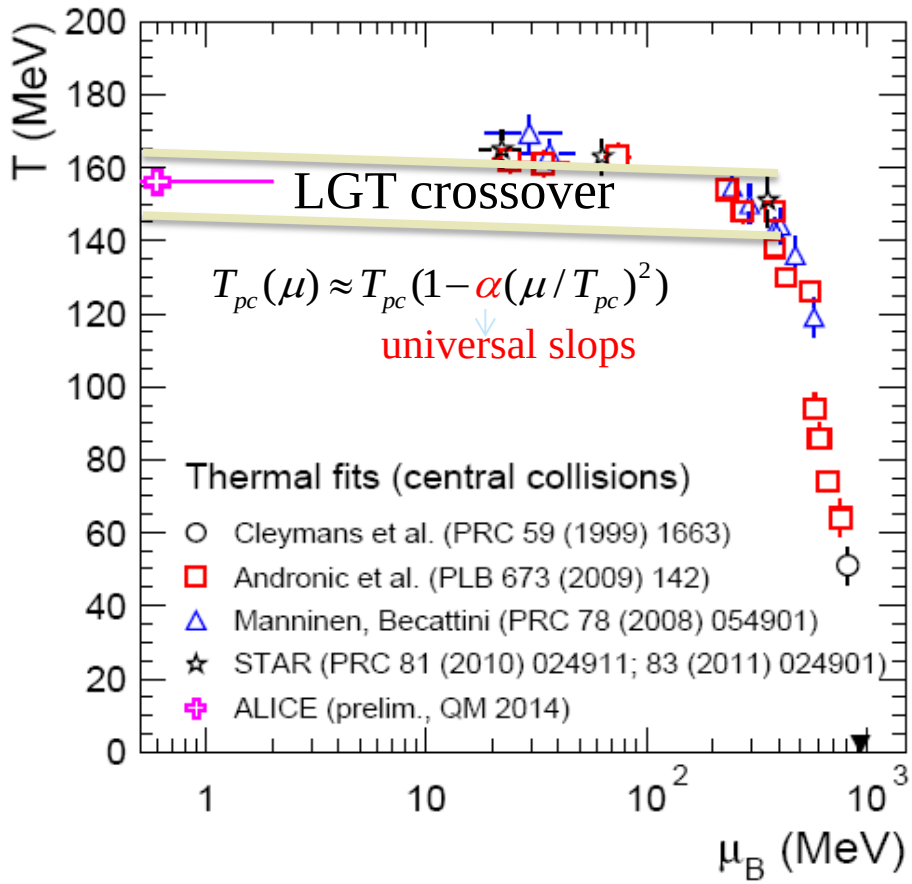
J. Engels & F. Karsch (2012)

Chemical Freezeout in HRG and LGT crossover

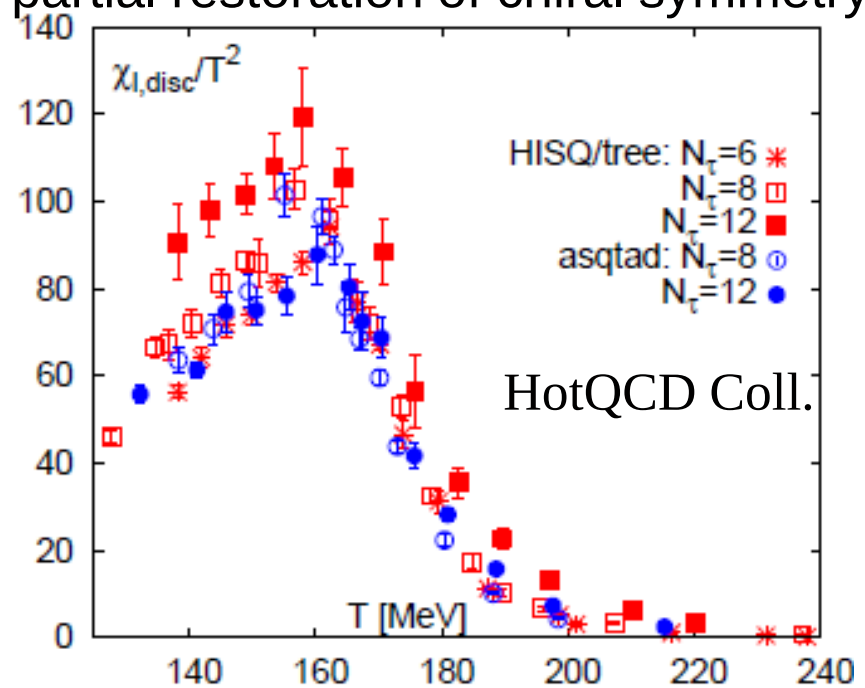
QCD Thermodynamics of hadronic phase

HIC & HRG ? LGT

A. Andronic QM 2014



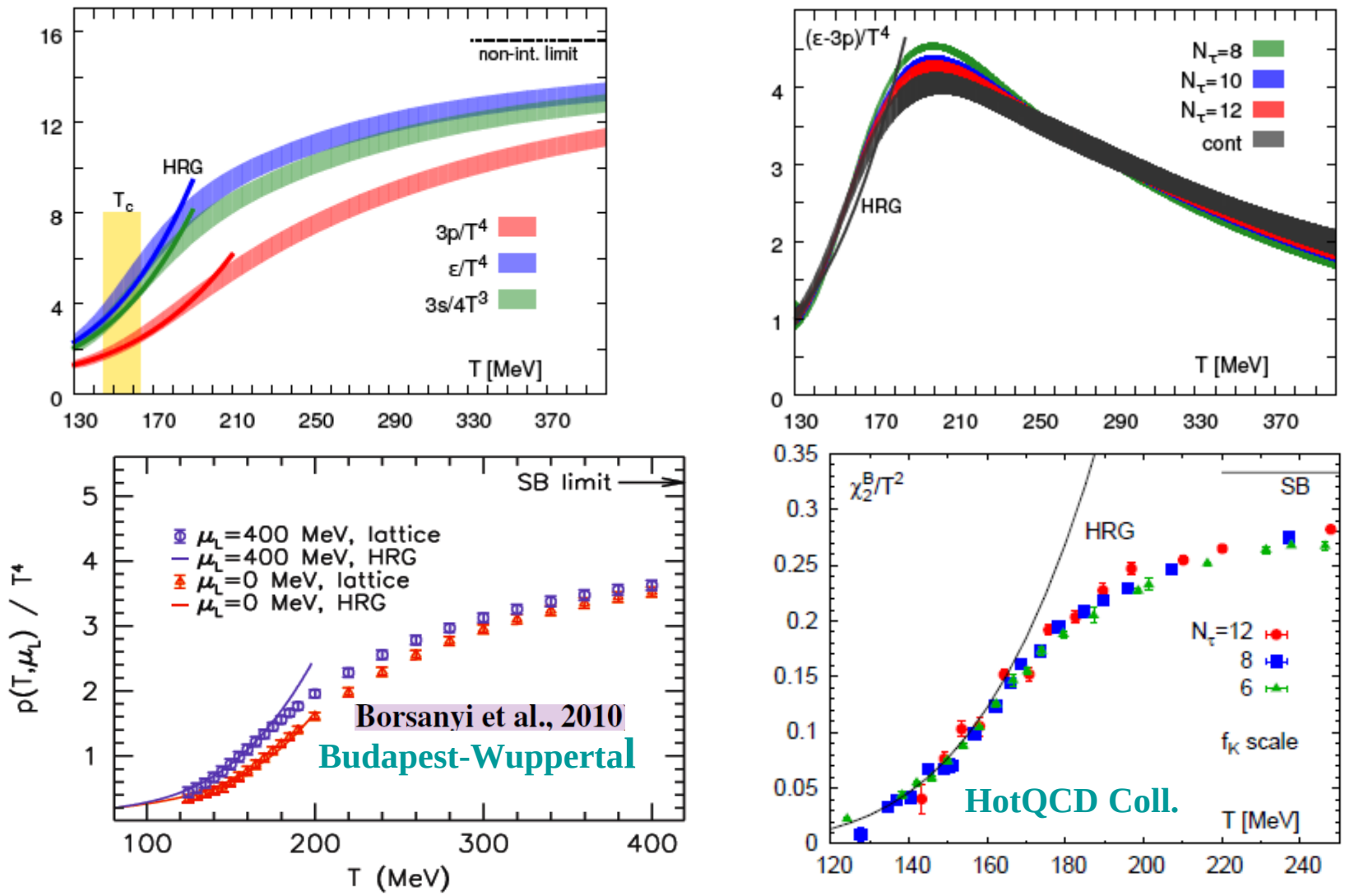
LGT crossover related with the partial restoration of chiral symmetry



Chemical freezeout defines a lower bound for the QCD phase boundary $T_{pc} = 154 \pm 9 MeV$

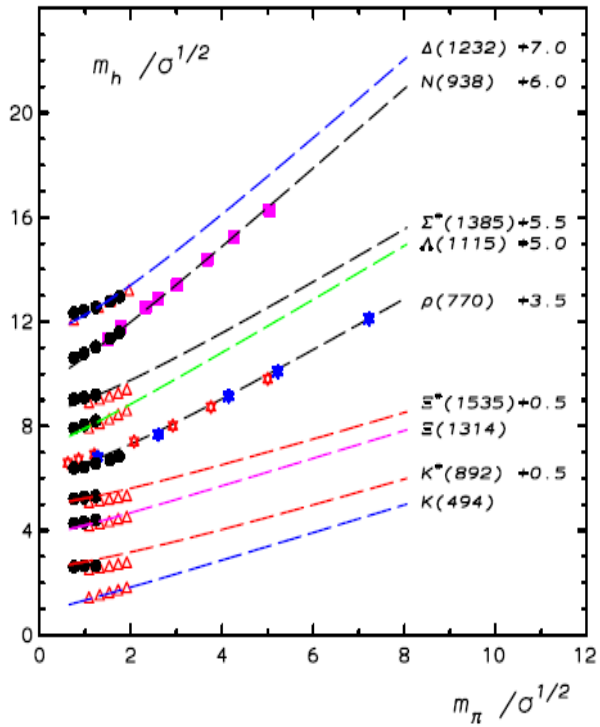
Excellent description of QCD equation of states by the Hadron Resonanse Gas

A. Bazavov et al. HotQCD Coll. [hep-lat] 23 Jul 2014

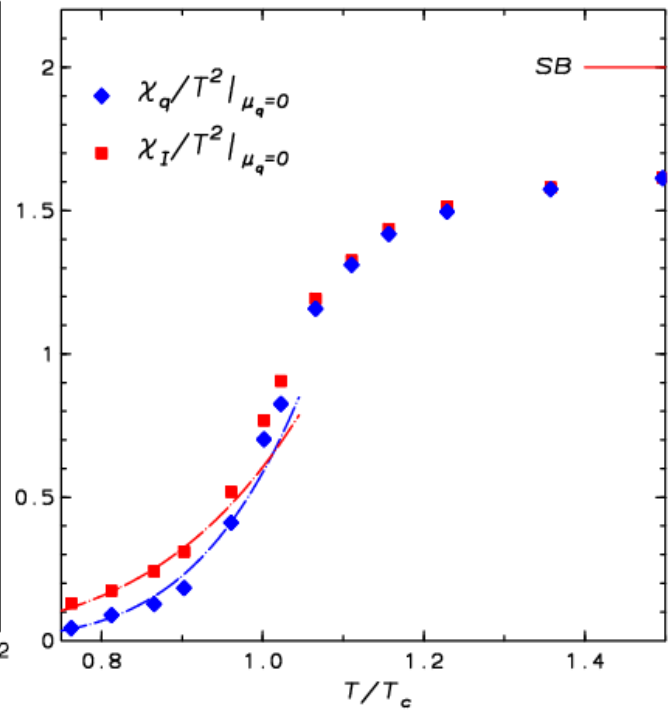


HRG and LGT with large pion mass

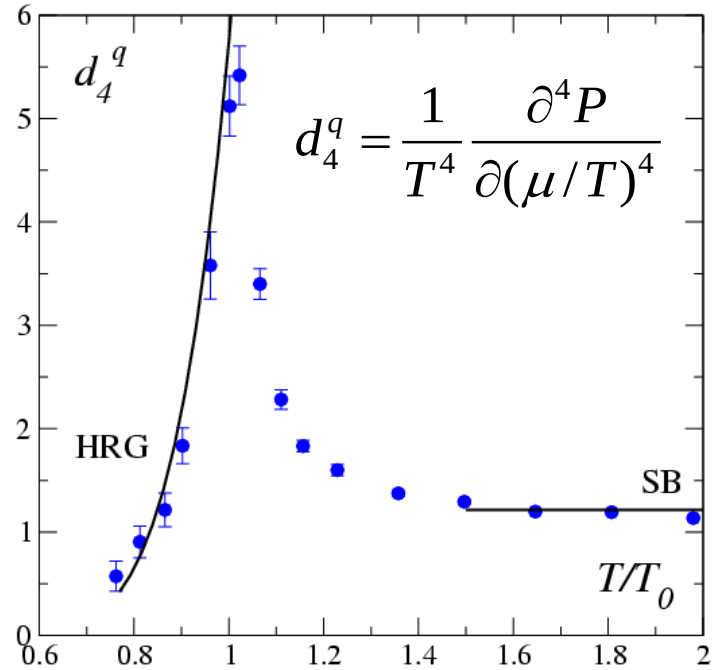
F. Karsch, A. Tawfik &K.R. (03)



C. Allton et al.,



S. Ejiri, F. Karsch & K.R.



- The thermodynamics of strongly interacting medium is well described by the HRG due to confinement properties. It is valid for different pion masses and in the strong coupling limit

Polyakov loop on the lattice needs renormalization

- Introduce Polyakov loop:

$$L \Rightarrow c_N L \qquad c_N = e^{2\pi i k/N} \in Z(N)$$

$$L_{\vec{x}}^{\text{bare}} = \frac{1}{N_c} Tr \prod_{\tau=1}^{N_\tau} U_{(\vec{x},\tau),4}$$

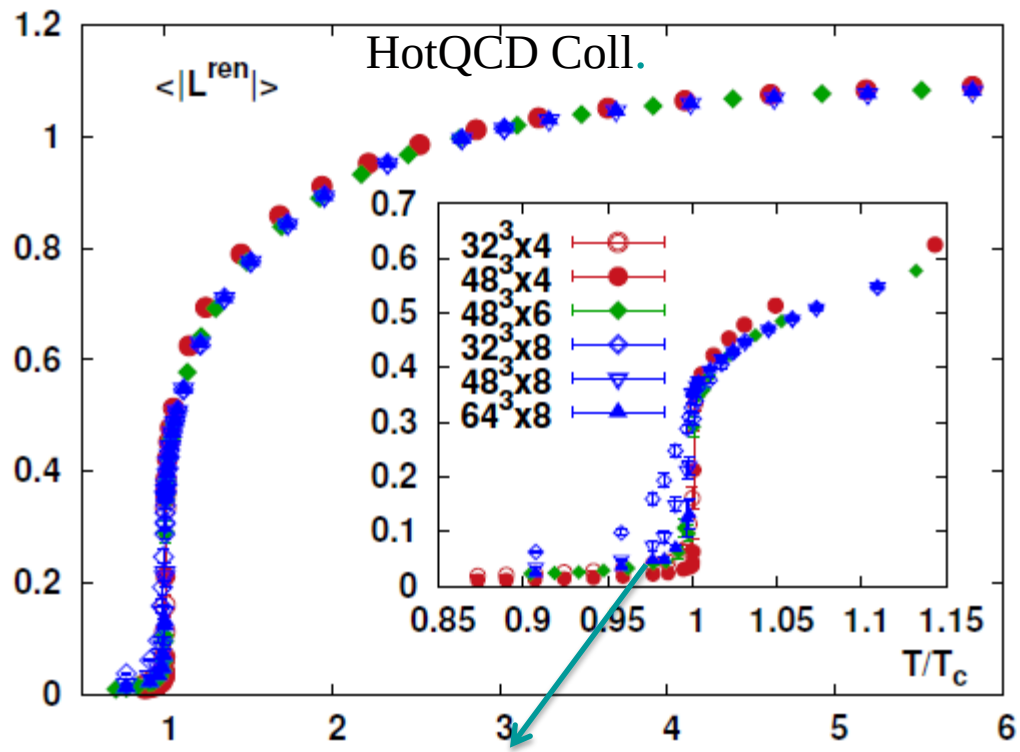
$$L^{\text{bare}} = \frac{1}{N_\sigma^3} \sum_{\vec{x}} L_{\vec{x}}^{\text{bare}}$$

$$<| L^{\text{ren}} |> = e^{-\beta F_q^{\text{ren}}} \rightarrow \begin{cases} \neq 0 \Leftrightarrow \text{deconfined} \quad T > T_c \\ 0 \Leftrightarrow \text{confined} \quad T < T_c \end{cases}$$

- Renormalized ultraviolet divergence

$$L^{\text{ren}} = \left(Z(g^2)\right)^{N_\tau} L^{\text{bare}}$$

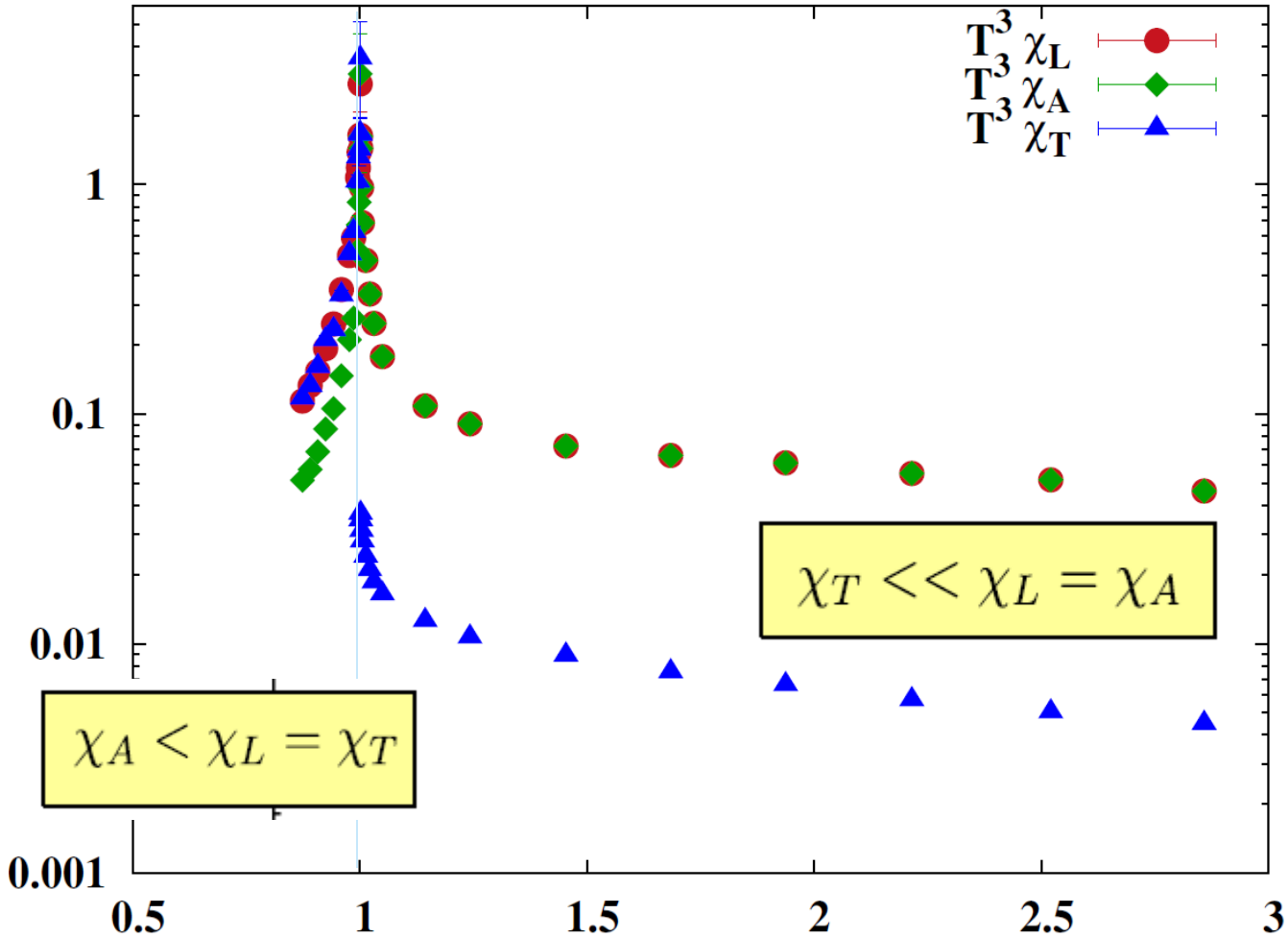
- Usually one takes $\langle |L^{\text{ren}}| \rangle$ as an order parameter



$1/\sqrt{V}$

To probe deconfinement : consider fluctuations

$$T^3\chi_A = \frac{N_\sigma^3}{N_\tau^3}\left(\langle|L^{\rm ren}|^2\rangle - \langle|L^{\rm ren}|\rangle^2\right)$$



- the Polyakov loop

$$L = L_R + iL_I$$

- Consider fluctuations of

real $\chi_L = \chi_R$

modulus $\chi_A = \chi_{|L|}$

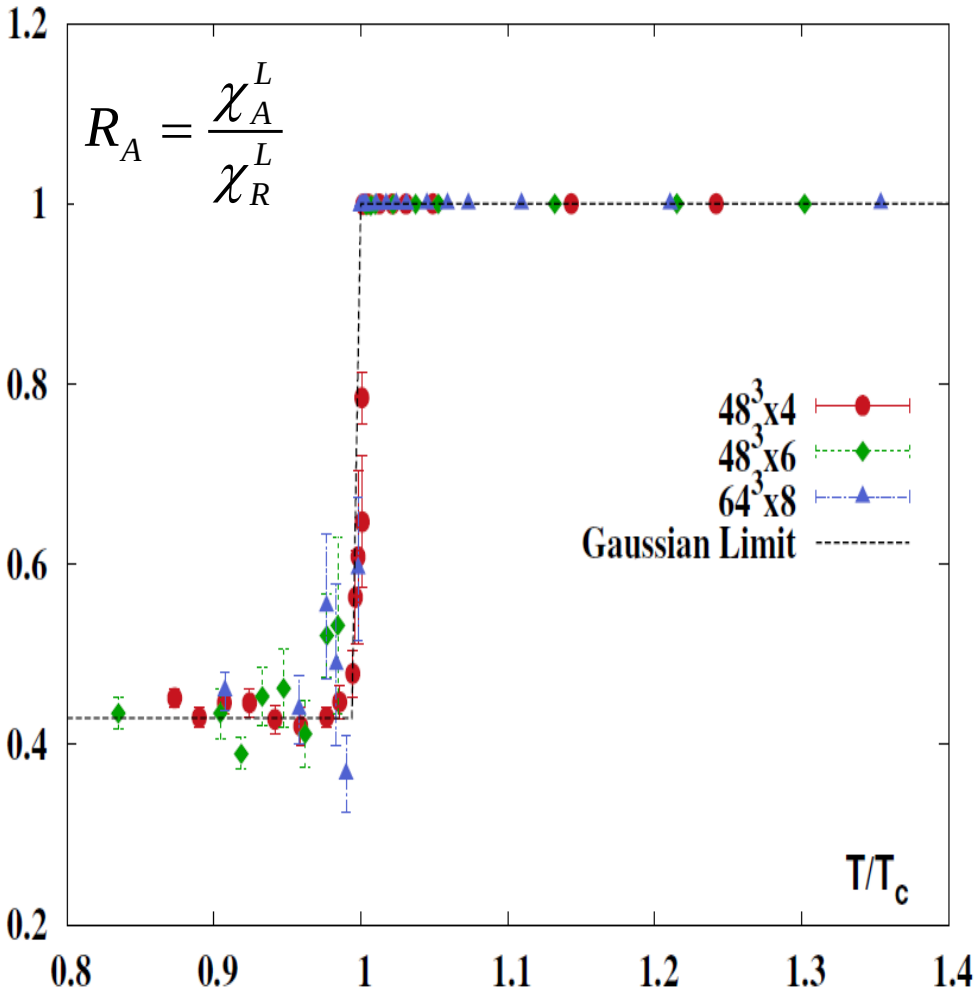
imaginary $\chi_T = \chi_I$

and take their ratios:

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)

Ratios of the Polyakov loop fluctuations as an excellent probe for deconfinement

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)



- In the deconfined phase $R_A \approx 1$

Indeed, in the real sector of Z(3)

$$L_R \approx L_0 + \delta L_R \quad \text{with} \quad L_0 = < L_R >$$

$$L_I \approx L_0^I + \delta L_I \quad \text{with} \quad L_0^I = 0, \quad \text{thus}$$

$$\chi_R^L = V < (\delta L_R)^2 >, \quad \chi_I^L = V < (\delta L_I)^2 >$$

Expand the modulus,

$$|L| = \sqrt{L_R^{\,2} + L_I^{\,2}} \approx L_0 (1 + \frac{\delta L_R}{L_0} + \frac{(\delta L_I)^2}{2 L_0^{\,2}})$$

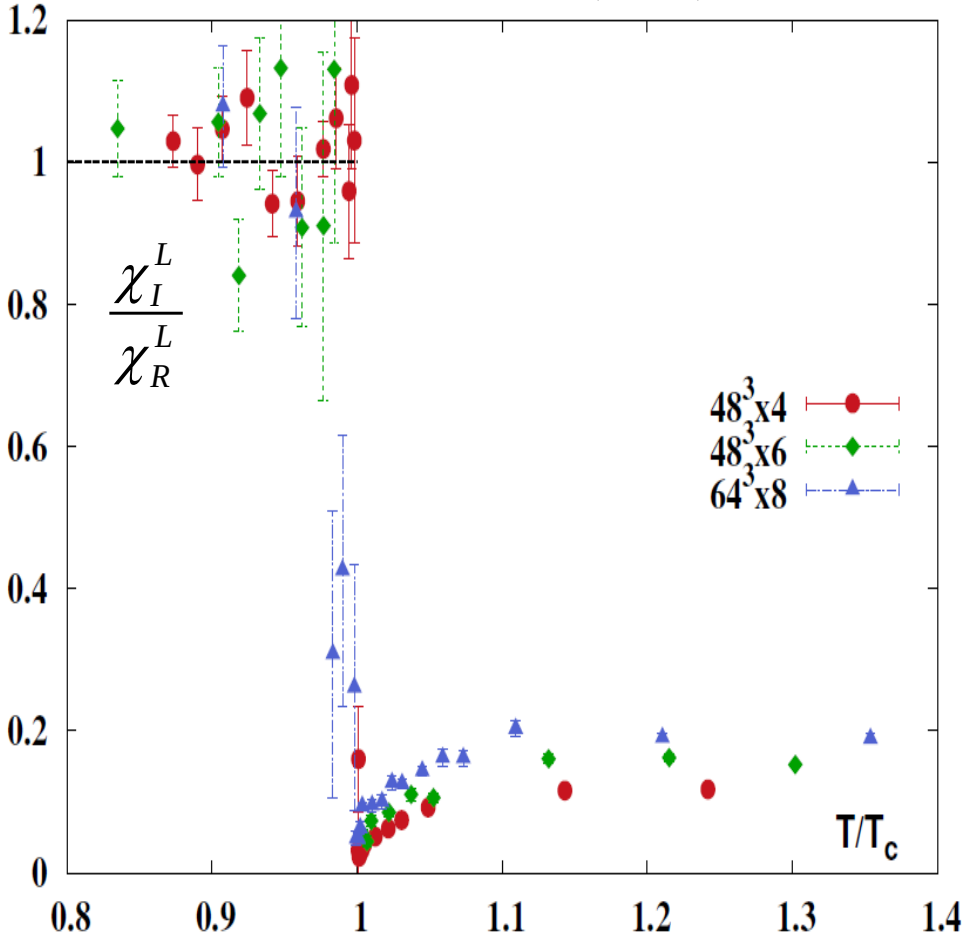
get in the leading order

$$\boxed{<|L|^2> - <|L|>^2 \approx <(\delta L_R)^2>}$$

thus $\chi_A \approx \chi_R$

Ratio Imaginary/Real of Polyakov loop fluctuations

Pok Man Lo, B. Friman, O. Kaczmarek,
C. Sasaki & K.R. , PRD (2013)



- In the confined phase for any symmetry breaking operator its average vanishes, thus

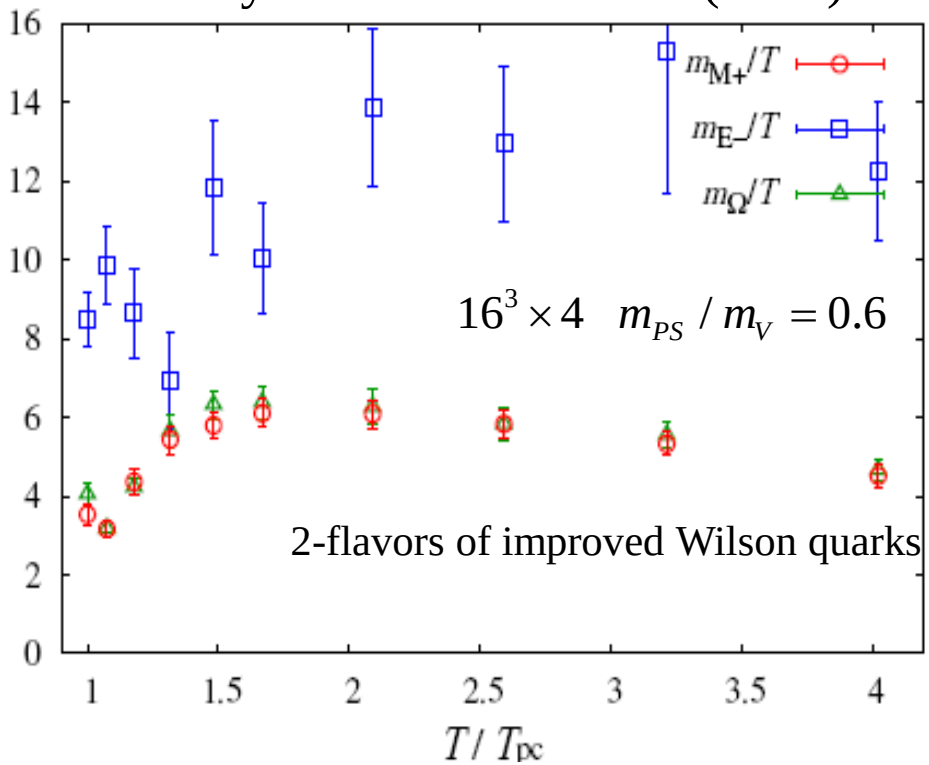
$$\chi_{LL} = \langle L^2 \rangle - \langle L \rangle^2 = 0 \quad \text{and}$$

$$\chi_{LL} = \chi_R - \chi_I \quad \text{thus} \quad \boxed{\chi_R = \chi_I}$$

- In deconfined phase the ratio of $\chi_I / \chi_R \neq 0$ and its value is model dependent

Ratio Imaginary/Real and gluon screening

WHOT QCD Coll:
Y. Maezawa[1], S. Aoki[2], S. Ejiri[3], T. Hatsuda[4],
N. Ishii[4], K. Kanaya[2], N. Ukita[5] and T. Umeda[6]
Phys. Rev. D81 091501 (2010)



- In the confined phase

$$\chi_{R,(I)} = 4\pi \int dr\ r^2 C_{R,(I)}(r)$$

$$C_{R,(I)}(r) = < L_{R,(I)}(r) L_{R,(I)}(0) >_c$$

- WHOT QCD Coll. (Y. Maezawa et al.)

$$C_{R,(I)}(r)_{r\to\infty} \to \gamma_{R,(I)}(T) \frac{e^{-M_{R,(I)} r}}{rT}$$

and WHOT-coll. identified $M_{R(I)}$ as the magnetic and electric mass:

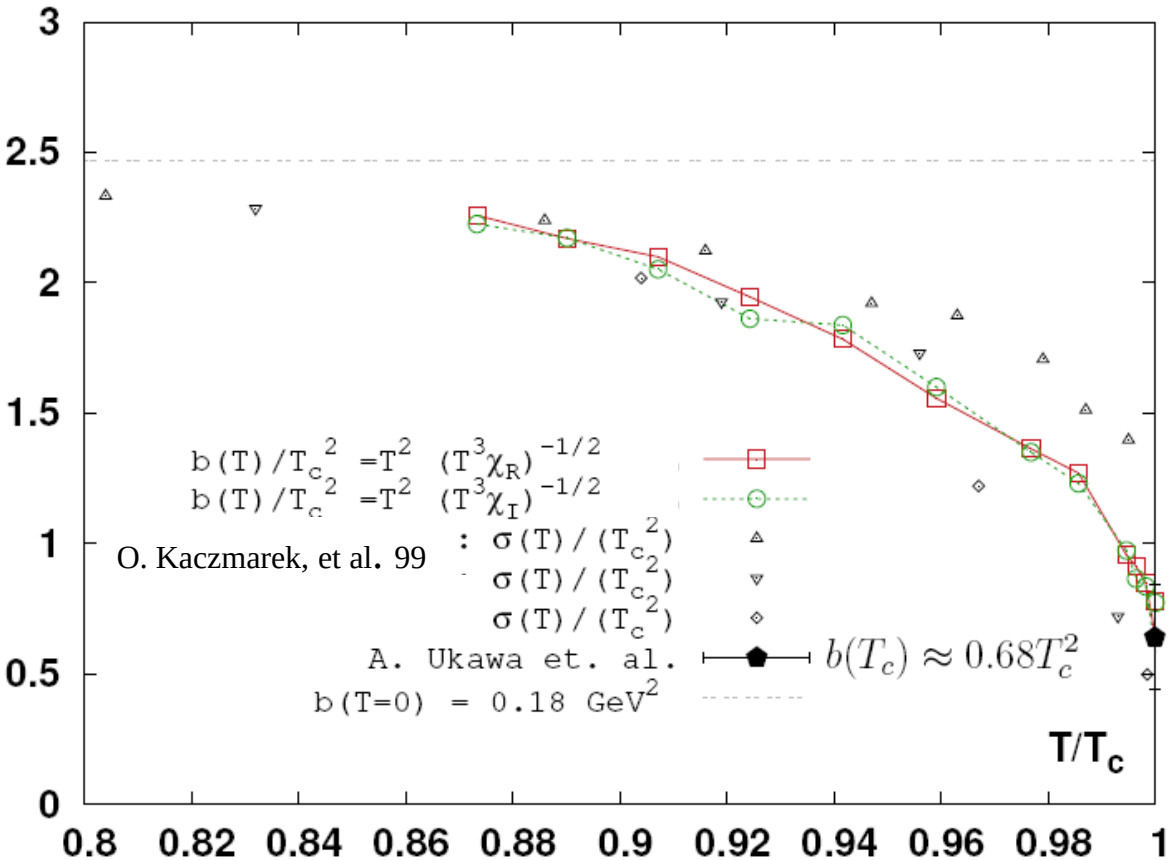
$$\chi_I \propto 1/m_E^2, \quad \chi_R \propto 1/m_M^2$$

Since

$$\boxed{m_E^2 >> m_M^2 \Rightarrow \chi_I << \chi_R}$$

String tension from the PL susceptibilities

Pok Man Lo, et al. (in preparation)



- $T < T_c \Rightarrow \chi_I = \chi_R$

$$\chi_{R,(I)} = 4\pi \int dr\ r^2 C_{R,(I)}(r)$$

- Common mass scale for $C_{R,(I)}(r)$

$$C_{R,(I)}(r) \approx \frac{e^{-M\ r}}{4\pi r T}$$

- In confined phase a natural choice for M

$$M = b / T$$

string tension

$$b(T)/T_c^2 \approx (T/T_c)^2 (T^3 \chi_{R,(I)})^{-1/2}$$

Deriving partition function from YM Lagrangian [Gross-Pisarski-Yaffe (81)]

$$Z = \int \mathcal{D}A_\mu \mathcal{D}C\mathcal{D}\bar{C} \exp\left[i\int d^4x \mathcal{L}\right] , \quad \mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}$$

1. employ background field method.

$$A_\mu = \bar{A}_\mu + g\check{A}_\mu$$

- thermodynamic potential (gluon part)

$$\boxed{\Omega_g = 2T\int \frac{d^3p}{(2\pi)^3} \mathrm{tr}\ln\left(1 - \hat{L}_A e^{-|\vec{p}|/T}\right)}$$

traced Polyakov loops $\Phi = \mathrm{tr}\hat{L}_F/N_c$, $\bar{\Phi} = \mathrm{tr}\hat{L}_F^\dagger/N_c$ (gauge invariant)

full thermodynamics potential: $\Omega = \Omega_g + \Omega_{\rm Haar}$ C. Sasaki, et al. PRD (13)

$$\begin{aligned}
\Omega_g &= 2T\int \frac{d^3p}{(2\pi)^3} \ln\left(1 + \sum_{n=1}^{7} C_n e^{-n|\vec{p}|/T} + e^{-8|\vec{p}|/T}\right) , \\
\Omega_{\rm Haar} &= -a_0 T \ln\left[1 - 6\bar{\Phi}\Phi + 4\left(\Phi^3 + \bar{\Phi}^3\right) - 3\left(\bar{\Phi}\Phi\right)^2\right] , \\
C_1 &= C_7 = 1 - N_c^2\bar{\Phi}\Phi , \quad C_2 = C_6 = 1 - 3N_c^2\bar{\Phi}\Phi + N_c^3\left(\bar{\Phi}^3 + \Phi^3\right) , \\
C_3 &= C_5 = -2 + 3N_c^2\bar{\Phi}\Phi - N_c^4\left(\bar{\Phi}\Phi\right)^2 , \\
C_4 &= 2\left[-1 + N_c^2\bar{\Phi}\Phi - N_c^3\left(\bar{\Phi}^3 + \Phi^3\right) + N_c^4\left(\bar{\Phi}\Phi\right)^2\right]
\end{aligned}$$

$\Rightarrow$ energy distributions solely determined by group characters of SU(3)

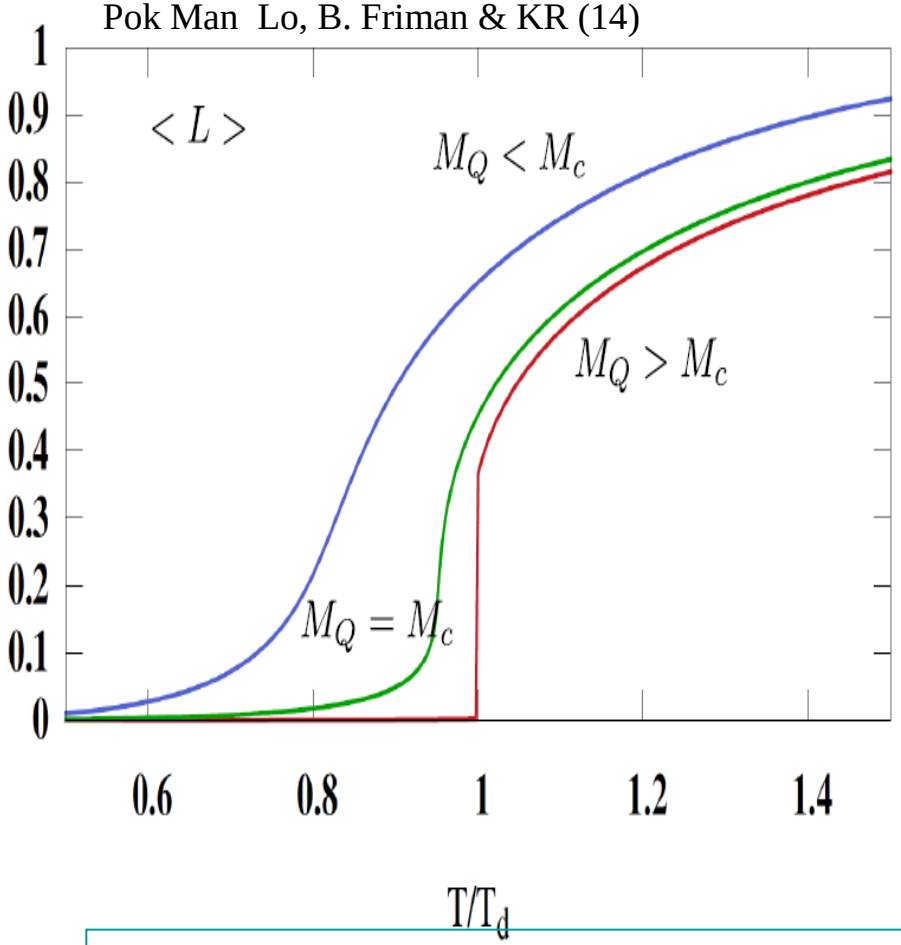
- Modelling the partition function form QCD in background field approach

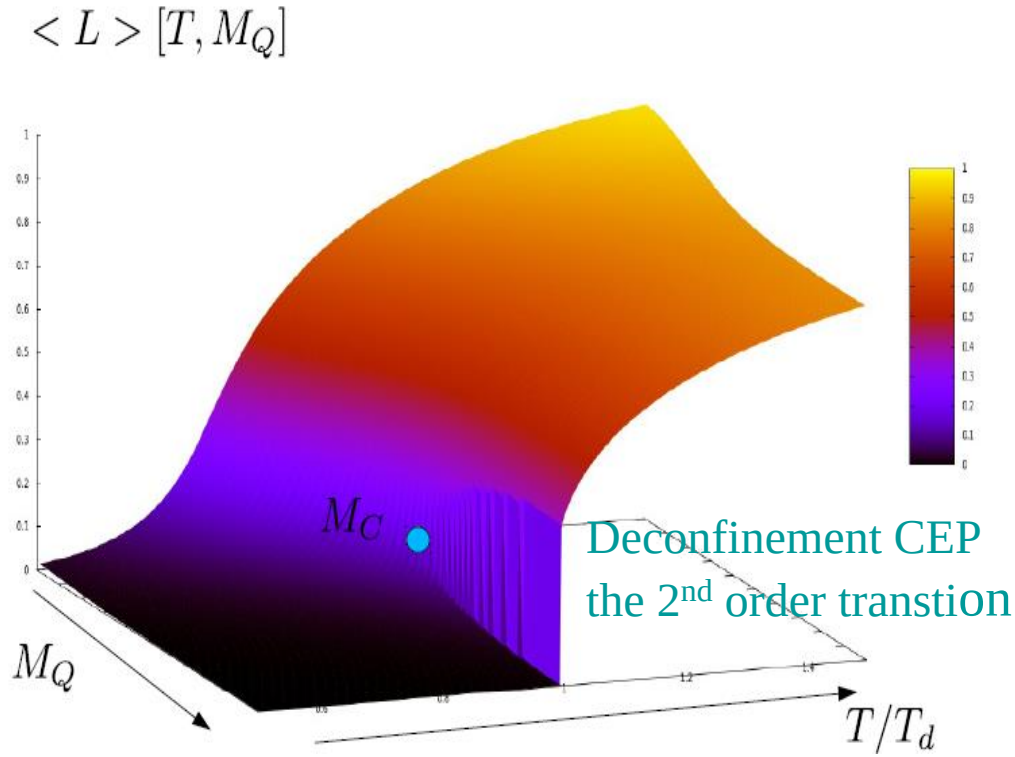
$$Z = \int dL dL^{+} e^{-\beta V U(L,L^{+}) + \ln\det[Q_f]}$$

$$\ln\det[Q_f] \approx -h_{eff} L_R$$

$$h_{eff} \approx N_f (M_q/T)^2 K_2(M_q/T)$$

Pok Man Lo, B. Friman & KR (14)



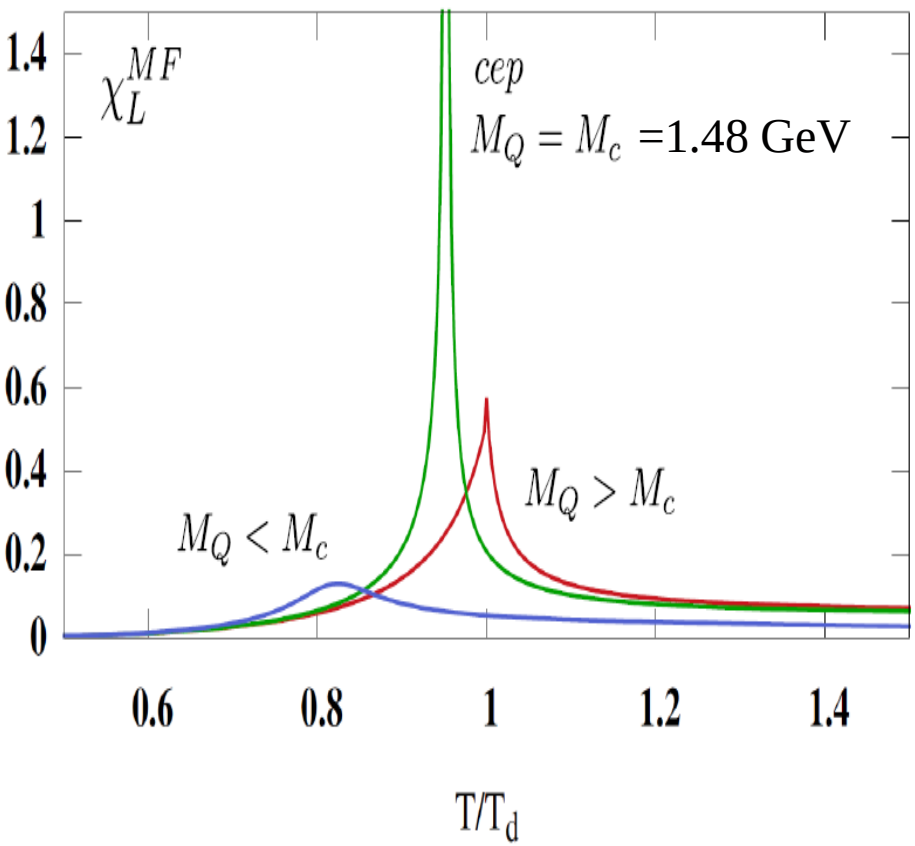


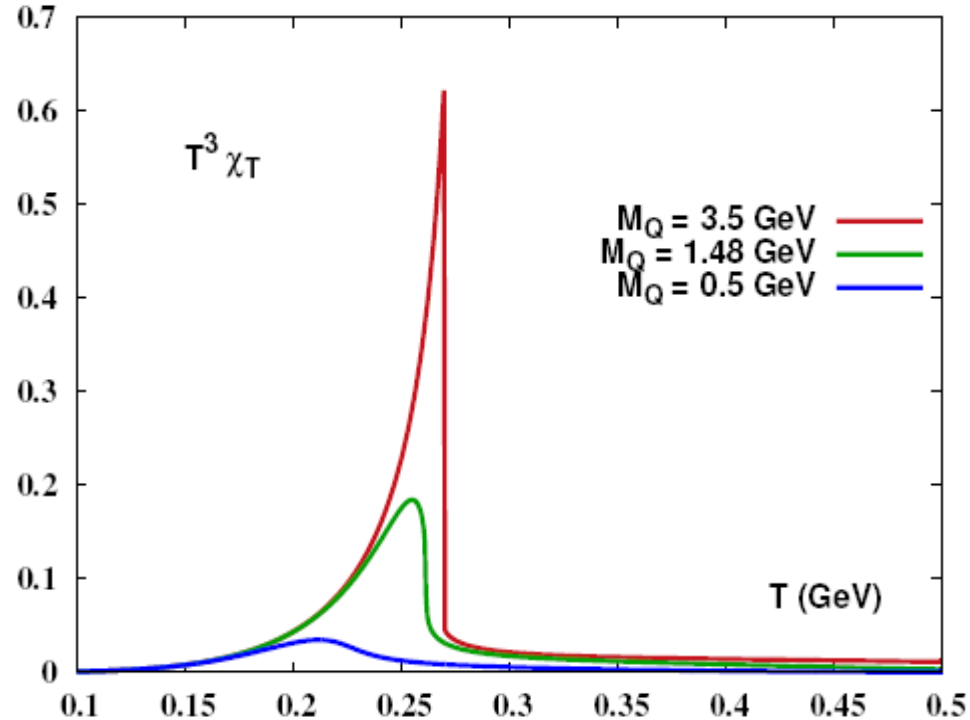
Deconfinement CEP
the 2nd order transtion

Deconfinement CEP appears in effective QCD at $M_Q \approx 1.5$ GeV

Susceptibility at the deconfinement critical endpoint

Pok Man Lo, B. Friman & KR (14)





- Divergent longitudinal susceptibility at the critical point

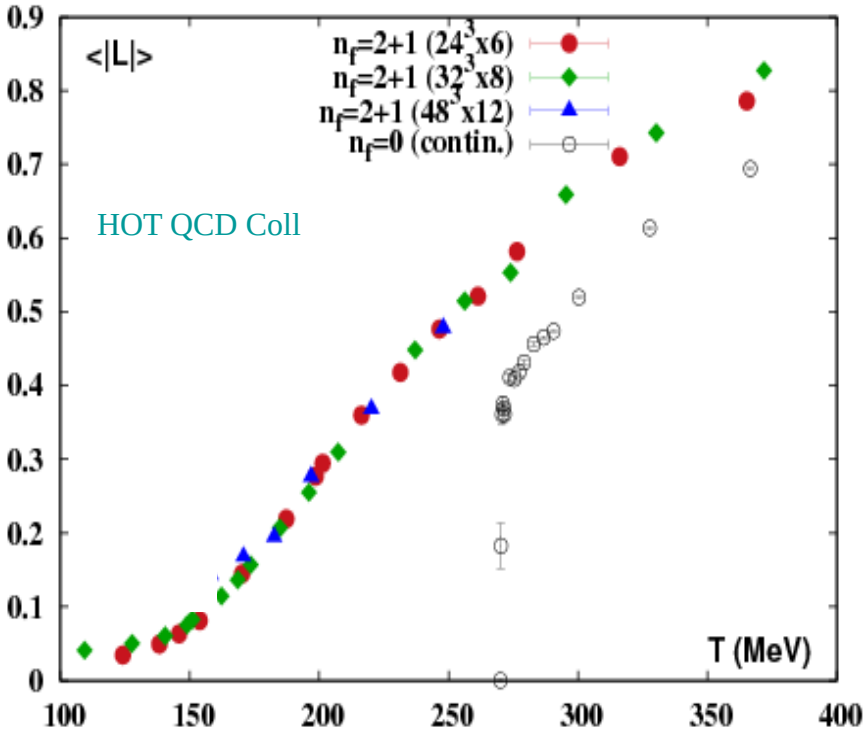
See also LGT results for the position of CEP

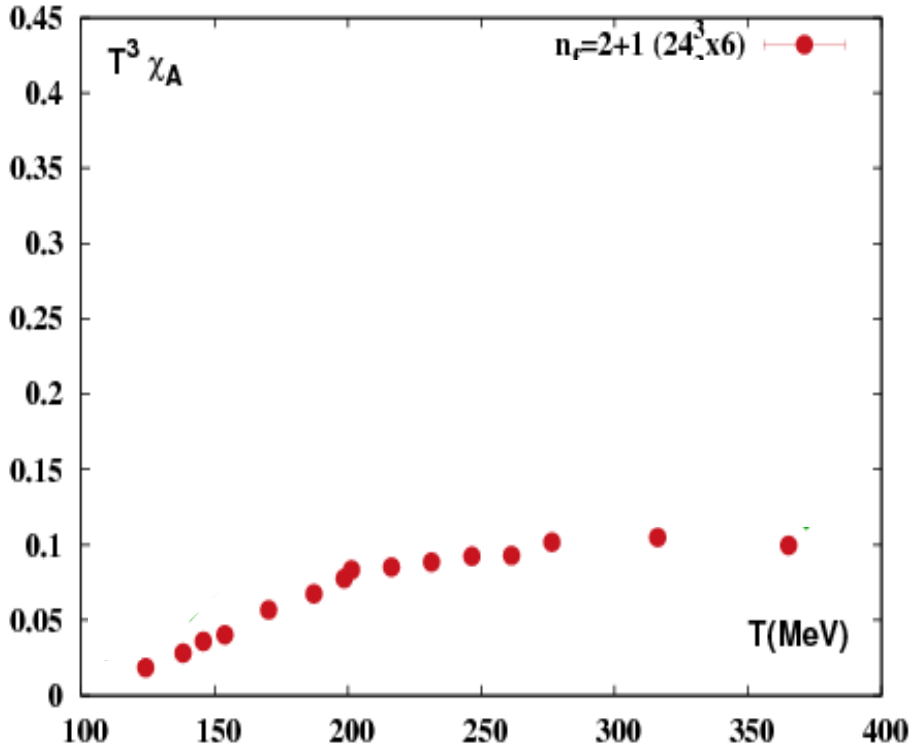
H. Saito, S. Ejiri, S. Aoki, T. Hatsuda, K. Kanaya, Y. Maezawa, H. Ohno, and T. Umeda, Phys. Rev. D **84** (2011) 054502

Polyakov loop and fluctuations in QCD

- Smooth behavior for the Polyakov loop and fluctuations

  ⇨ difficult to determine where is "deconfinement"



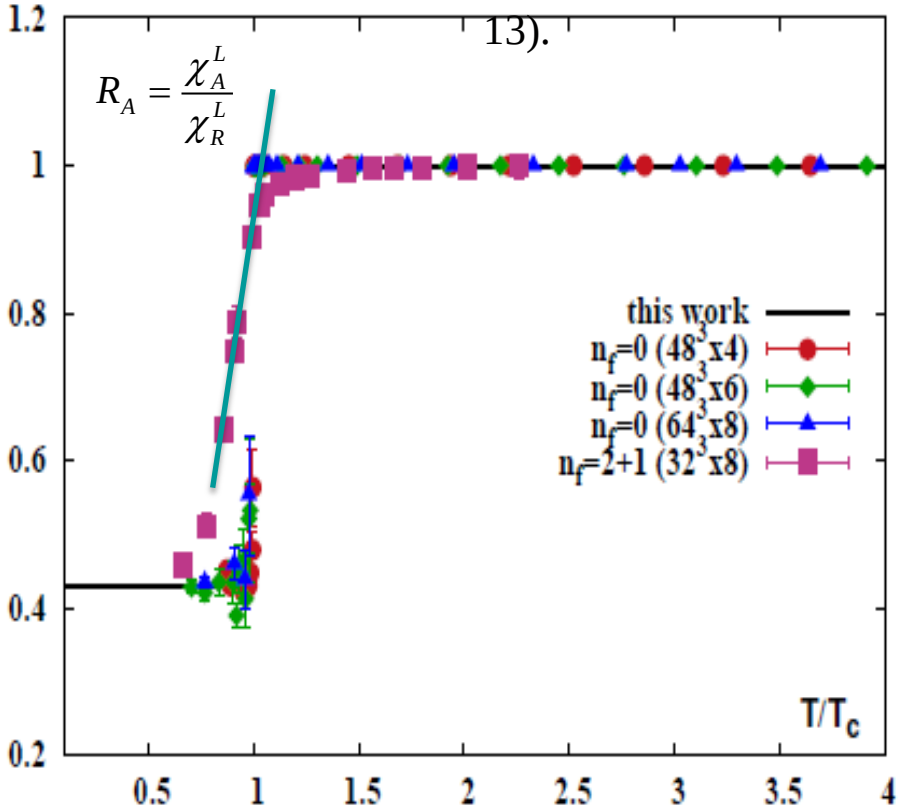


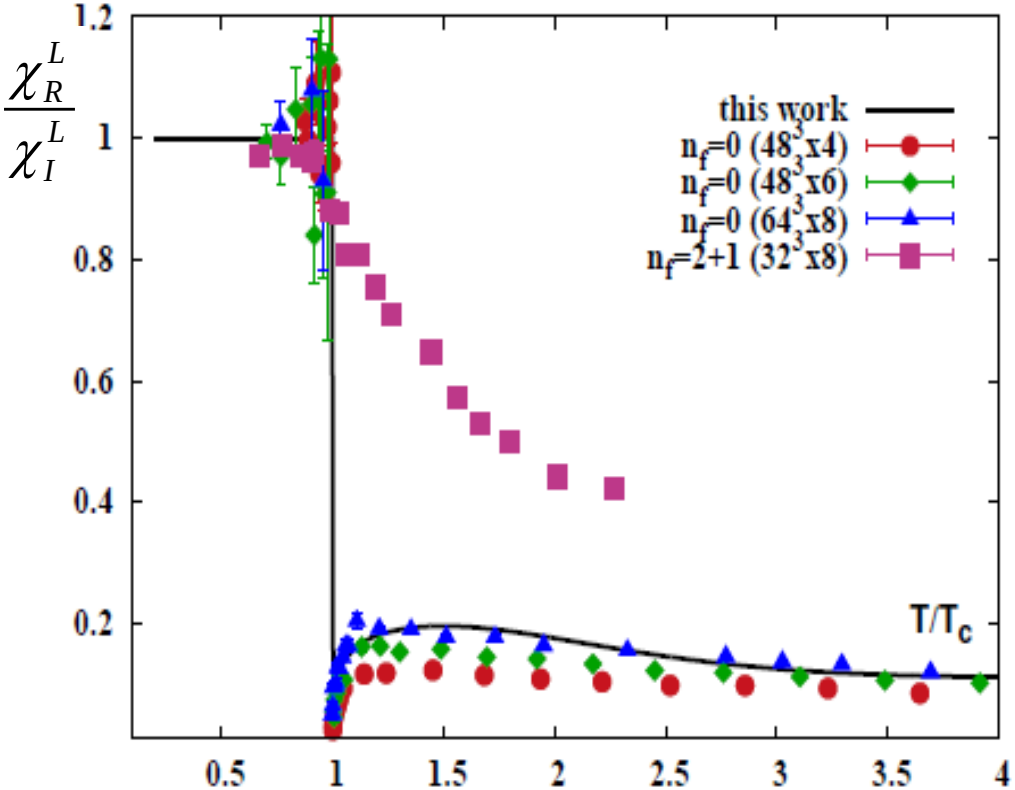
The inflection point at $T_{dec} \approx 0.22 GeV$

The influence of fermions on ratios of the Polyakov loop susceptibilities

- Z(3) symmetry broken, however ratios still showing the transition
- Change of the slopes at fixed T

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R (PRD 13).

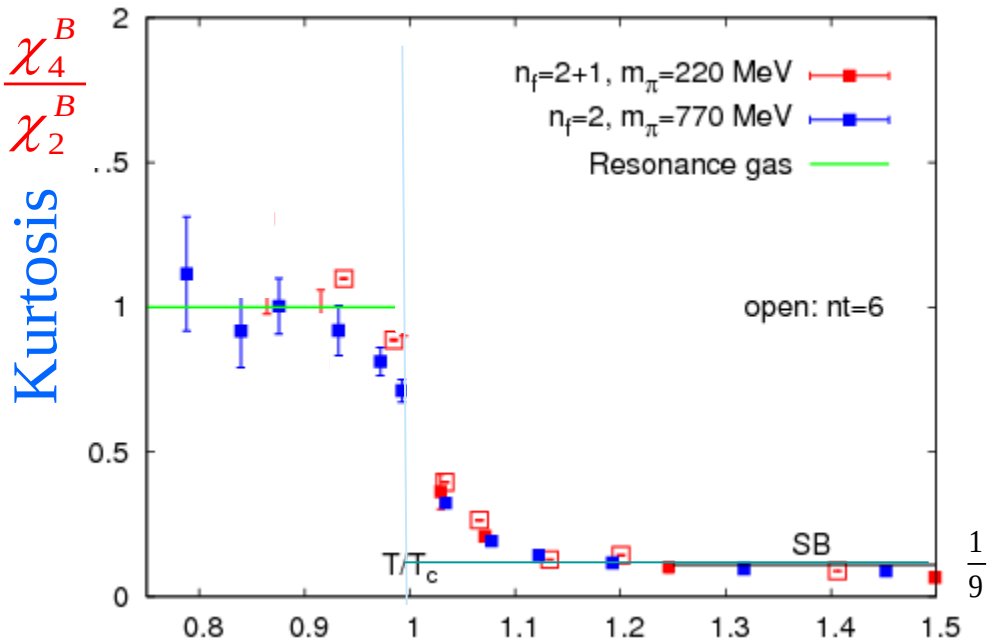




Kurtosis as an excellent probe of deconfinement

S. Ejiri, F. Karsch & K.R. Phys. Lett. B 633, 275 (2006)

Ch. Schmidt at QM'12 (012)



$$\chi_4^B / \chi_2^B = \frac{<(\delta N_q)^4>}{<(\delta N_q)^2>} - 3<(\delta N_q)^2>$$

- Factorization of pressure:

$$P^B(T,\mu_q) = \cosh(B\mu_B/T) \sum_{i \in baryons} F(T,m_i)$$

- Kurtosis measures the squared of the baryon number carried by leading particles in a medium

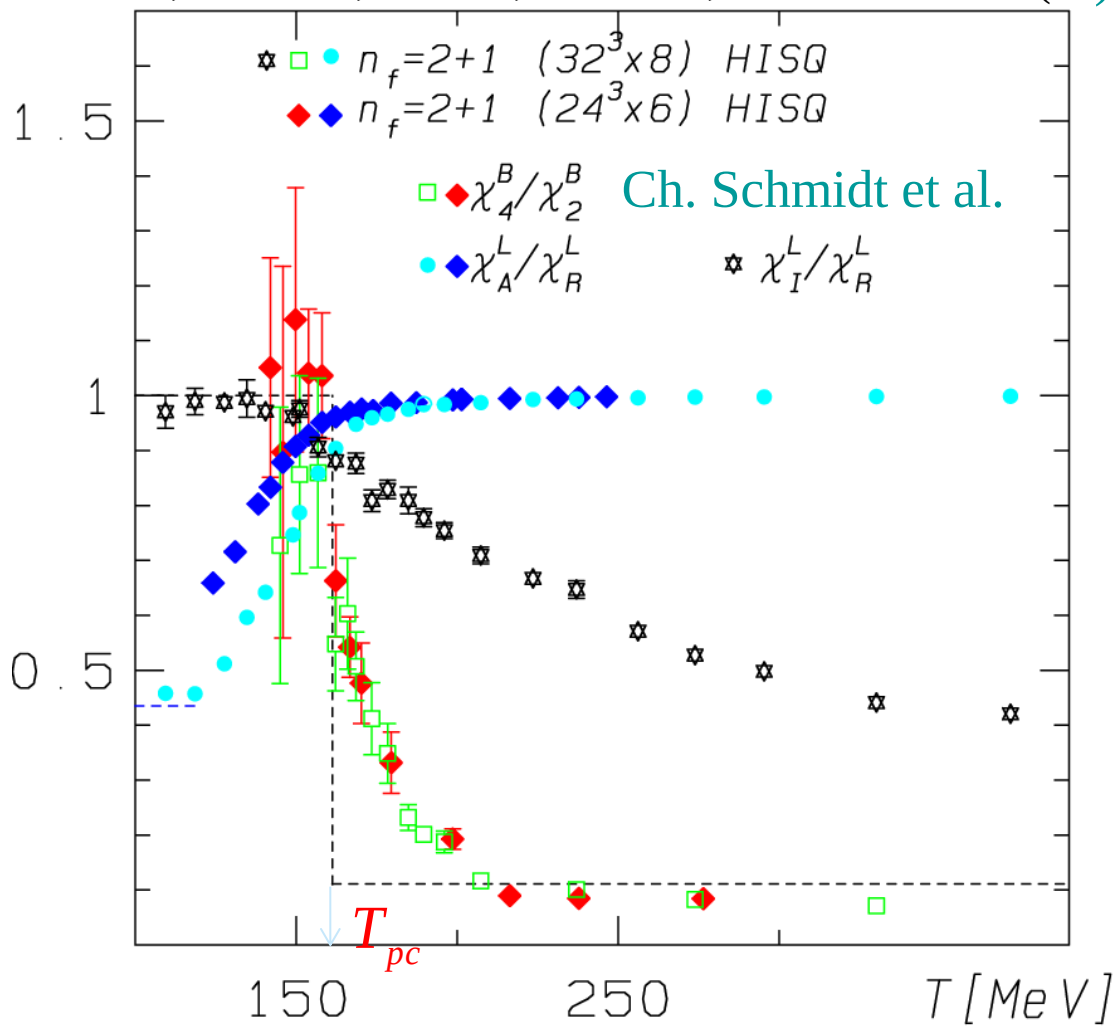
S. Ejiri, F. Karsch & K.R. (06)

$$\chi_n^B = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n}$$

$$\kappa\sigma^2 = \frac{\chi_4^B}{\chi_2^B} \approx B^2 = \begin{pmatrix} 1 & T < T_{PC} \\ \frac{1}{9} & T > T_{PC} \end{pmatrix}$$

Interplay between deconfinement and chiral crossover

O. Kaczmarek, F. Karsch, P.M. Lo, B. Friman, C. Sasaki & K.R. (14)



- The deconfinement of color, controlled by the Polyakov loop susceptibility ratios, and deconfinement of quarks, controlled by the kurtosis of the net baryon number, appears in the narrow temperature range which coincides with the chiral crossover

Probing O(4) chiral criticality with charge fluctuations

- Due to expected O(4) scaling in QCD the free energy:

$$P = P_R(T, \mu_q, \mu_I) + b^{-1} P_S(b^{(2-\alpha)^{-1}} t(\mu), b^{\beta\delta/\nu} h)$$

- Consider generalized susceptibilities of the net-quark number

$$c_B^{(n)} = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n} = c_R^{(n)} + c_S^{(n)} \quad \text{with} \quad \begin{array}{l} c_s^{(n)}\big|_{\mu=0} = d\ h^{(2-\alpha-n/2)/\beta\delta}\ f_\pm^{(n)}(z) \\ c_s^{(n)}\big|_{\mu\neq 0} = d\ h^{(2-\alpha-n)/\beta\delta}\ f_\pm^{(n)}(z) \end{array}$$

- Since for $T < T_{pc}$, $c_R^{(n)}$ are well described by the HRG

⇨ search for deviations (in particular for larger n) from HRG to quantify the contributions of $c_S^{(n)}$, i.e. the O(4) criticality

F. Karsch & K. R. Phys.Lett. B695 (2011) 136

B. Friman, et al. . Phys.Lett. B708 (2012) 179, Nucl.Phys. A880 (2012) 48

Modelling O(4) transtion: effective Lagrangian and FRG

$$\mathscr{L}_{\rm QM} = \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 - U(\sigma,\vec{\pi})$$

Effective potential is obtained by solving *the exact flow equation* (Wetterich eq.) with the approximations resulting in the O(4) critical exponents

B.J. Schaefer & J. Wambach,; B. Stokic, B. Friman & K.R.

$$\partial_k\Omega_k(\sigma) = \frac{Vk^4}{12\pi^2}\left[\sum_{i=\pi,\sigma}\frac{d_i}{E_{i,k}}[1+2n_B(E_{i,k})] - \frac{2\nu_q}{E_{q,k}}[1-n_F(E^+_{q,k})-n_F(E^-_{q,k})]\right]$$

$$E_{\pi,k} = \sqrt{k^2+\Omega'_k}$$

$$E_{\sigma,k} = \sqrt{k^2+\Omega'_k+2\rho\Omega''_k}$$

$$E_{q,k} = \sqrt{k^2+2g^2\rho}$$

$$\Omega'_k \equiv \frac{\partial\Omega_k}{\partial(\sigma^2/2)}$$

Full propagators with k < q < ☹

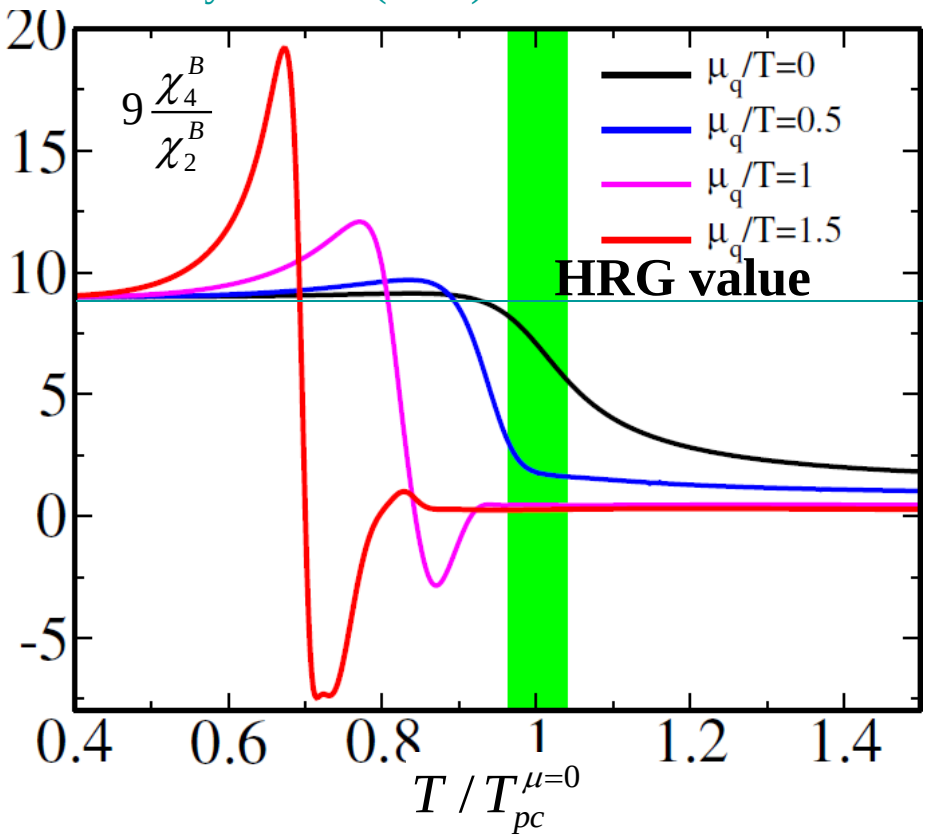
☝☹ S classical

Integrating from k=☹ to k=🗀 gives a full quantum effective potential

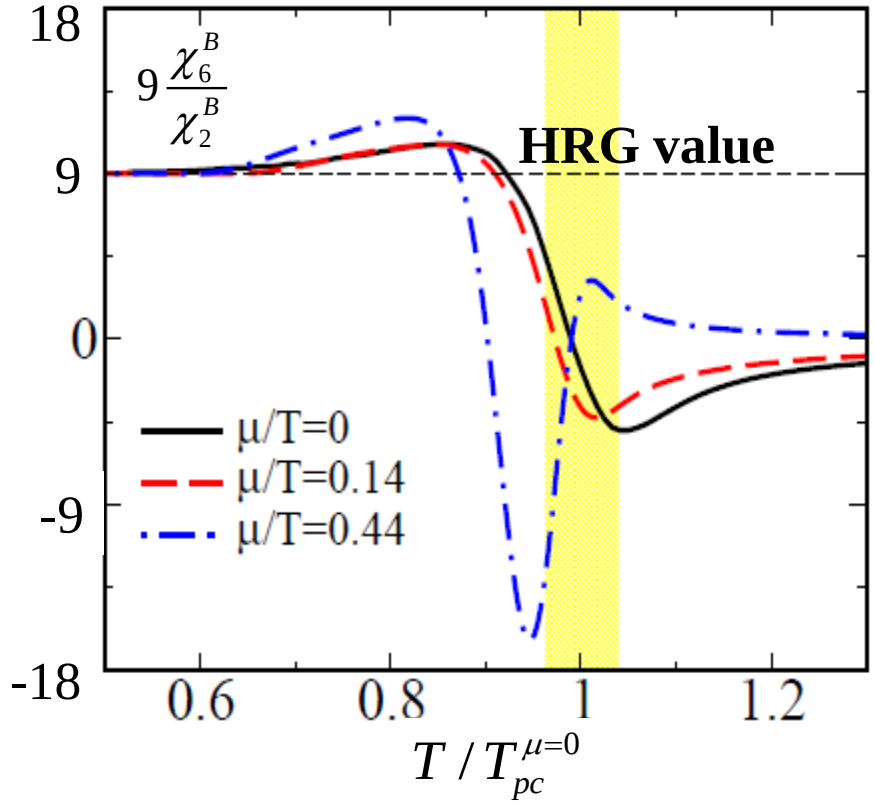
Put ⌖$_{k=0}$(♦$_{min}$) into the integral formula for P(N)

Ratios of cumulants at finite density in PQM model with FRG

B. Friman, F. Karsch, V. Skokov &K.R.
Eur.Phys.J. C71 (2011) 1694

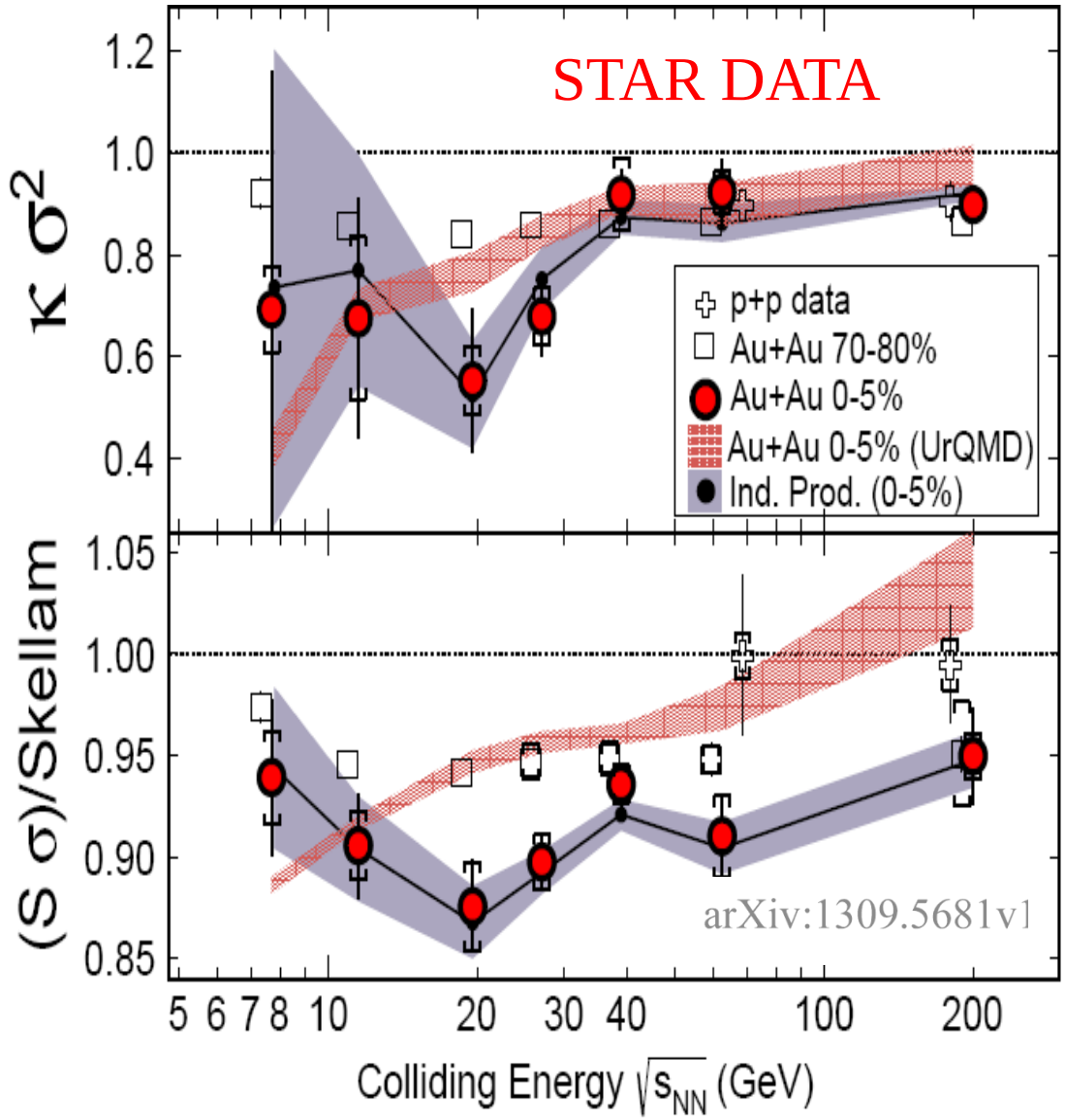


B. Friman, V. Skokov &K.R.
Phys.Rev. C83 (2011) 054904



Deviations from low -T HRG values are increasing with μ/T and the cumulant order . Negative fluctuations near the chiral crossover.

STAR data on the first four moments of net baryon number



Deviations from the HRG

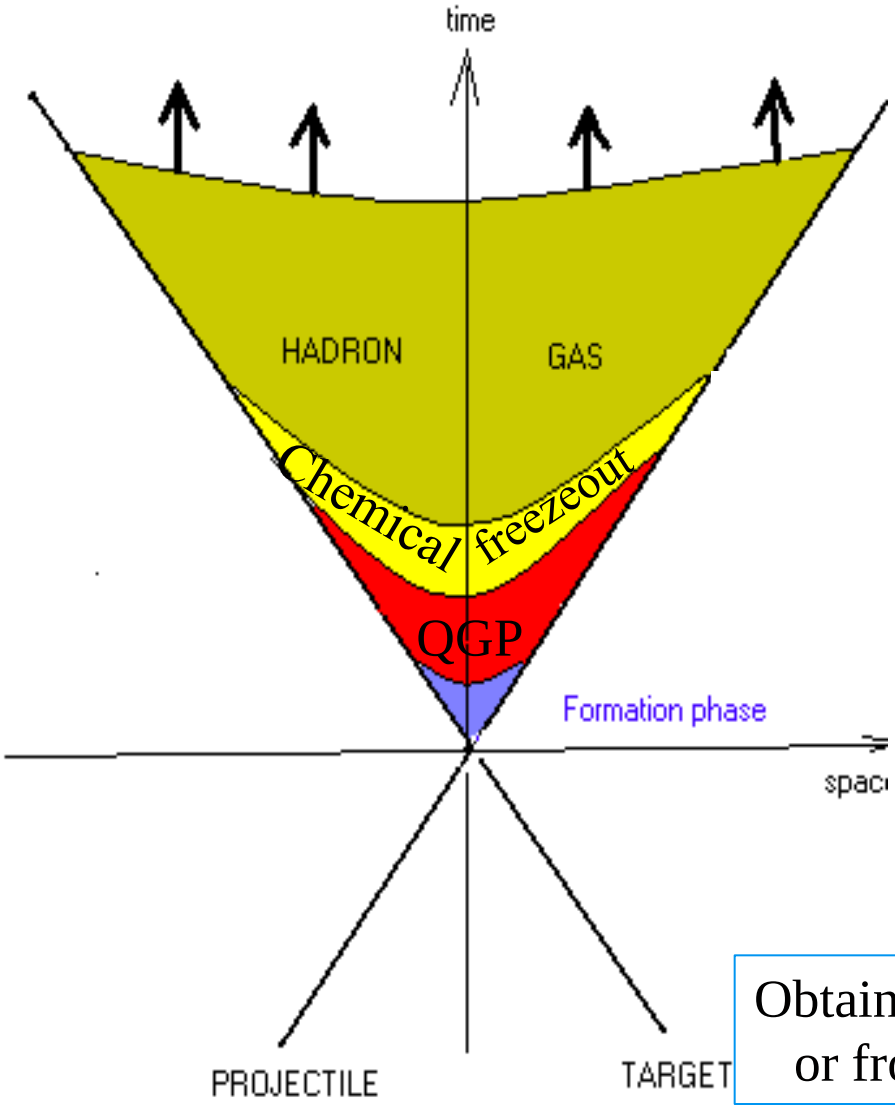
$$S\,\sigma = \frac{\chi_B^{(3)}}{\chi_B^{(2)}} \quad , \quad \kappa\,\sigma^2 = \frac{\chi_B^{(4)}}{\chi_B^{(2)}}$$

$$S\,\sigma\,|_{HRG} = \frac{N_p - N_{\bar{p}}}{N_p + N_{\bar{p}}} \quad , \kappa\sigma\,|_{HRG} = 1$$

Data qualitatively consistent with the change of these ratios due to the contribution of the O(4) singular part to the free energy

Quarkonium as a Probe for Deconfinement in HIC

The Statistical (re)Generation Picture



- Charm produced in the "Formation phase" eg. through: $g + g \rightarrow c + \bar{c}$
- charm quarks thermalize in QGP
- Quarkonium can regenerate at chemical freezeout as other hadrons but the number of $c + \bar{c}$ is exactly conserved

(P. Braun-Munzinger & J. Stachel , 2000)

$$N_{c\bar{c}}^{direct} = g_c N_{c+\bar{c}}^{thermal} + g_c^{\,2} N_{c\bar{c}}^{thermal}$$

Obtained from pQCD or from experiment

Accounts for chemical off equilibrium

Calculated within Thermal Model

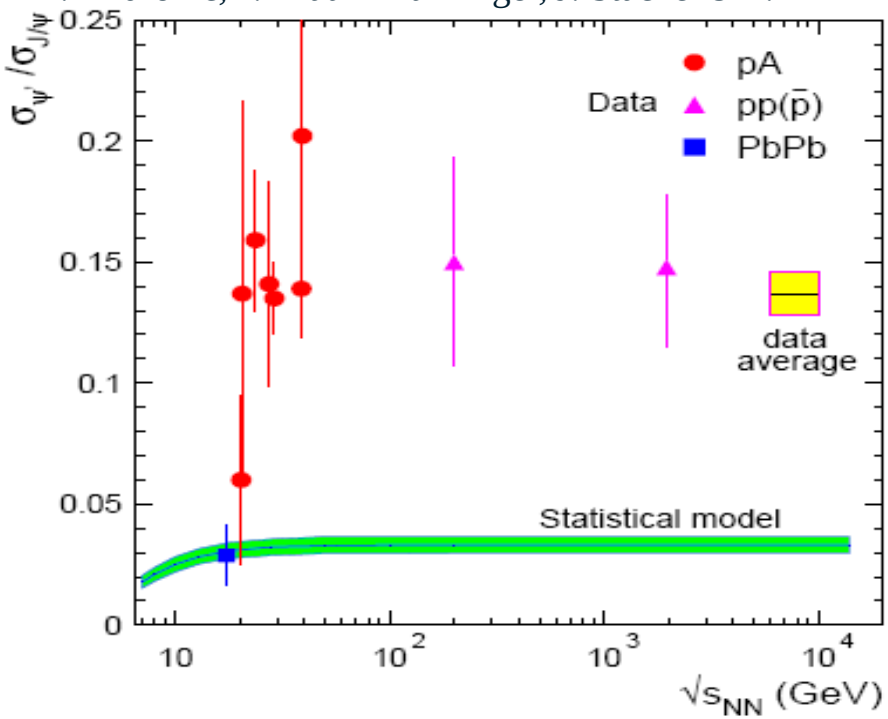
Test of hidden heavy flavor regeneration

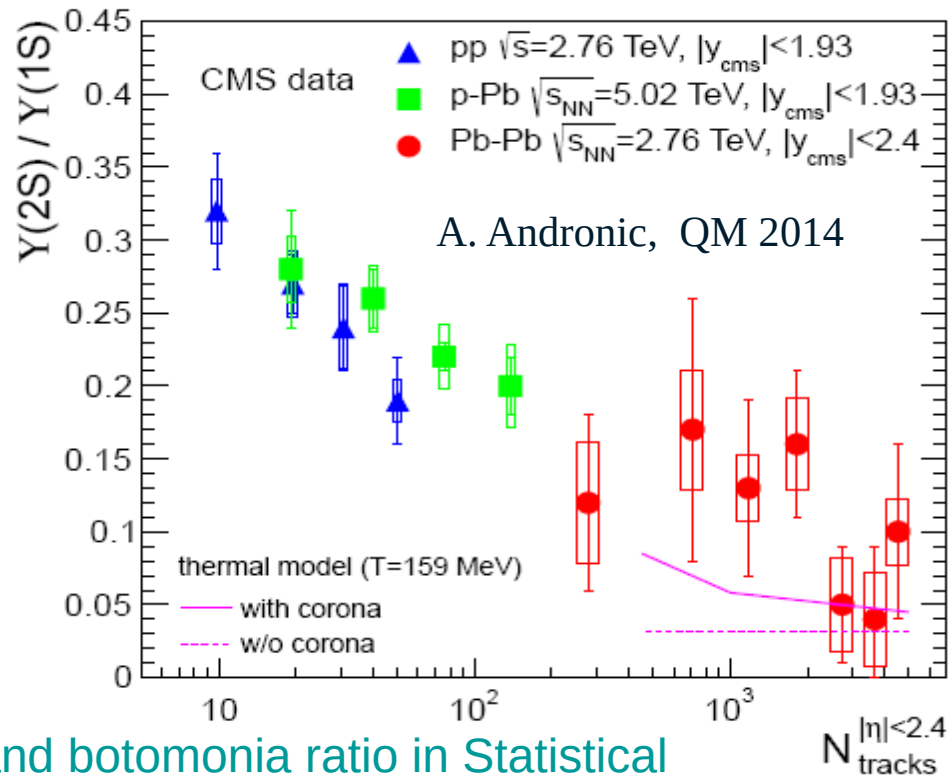
.

- Yield ratios excellent test of thermal regeneration and QCD phase boundary due to confinement

$$\frac{Y(2s)}{Y(1s)} \approx \frac{d_2}{d_1}\left(\frac{m_2}{m_1}\right)^2 \frac{K_2(m_2/T)}{K_2(m_1/T)}$$

A. Andronic, P. Braun-Munzinger, J. Stachel &K.R





- The freezeout temperature for charmonia and botomonia ratio in Statistical Hadronisation Model (P. Braun-Munzinger & J. Stachel) is consistent , within errors, with the chiral crossover temperature from LGT

Conclusions:

- The coincident of Chiral crossover and freezeout in HIC indicates that Hadron Resonanse Gas partition function is an excellent approximation of the QCD thermodynamics in confined phase
- Ratios of the Polyakov loop and net-charge susceptibilities are excellent probes of deconfinement and/or O(4) chiral crossover in QCD

  At small baryon densities, there is a coincidence between deconfinement and the O(4) chiral criticality in LGT
- Net baryon number fluctuations and charmonia yield ratios are ideal observable to identify the QCD phase boundary due to chiral symmetry restoration and confinement properties, respectively
- Systematics of the net-proton fluctuations measured by STAR are qualitatively consistent with the expectations that they are influenced by the O(4) chiral criticality.