Interplay between deconfinement and O(4) chiral crossover in QCD medium

- Chiral transition and O(4) pseudo-critical line in LGT
- Particle freezeout in HIC and QCD phase boundary: from HIC $\leftrightarrow$ LGT
- Confinement in SU(N) pure gauge theory and in QCD from LGT
- Probing O(4) chiral criticality and confinement properties in HIC
  - fluctuations of net baryon number
  - production of quarkonia states

1st principle calculations:
- $\mu, T \ll \Lambda_{QCD}$: $\chi$ - perturbation theory
- $\mu, T \gg \Lambda_{QCD}$: pQCD
- $\mu_q < T$: LGT

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In QCD the quark masses are finite: the diagram has to be modified.

Expected phase diagram in the chiral limit, for massless u and d quarks:

TCP: Rajagopal, Shuryak, Stephanov
Y. Hatta & Y. Ikeda
The phase diagram at finite quark masses

- The $u,d$ quark masses are small
- Is there a remnant of the O(4) criticality at the QCD crossover line?

At the CP:
Divergence of Fluctuations, Correlation Length and Specific Heat

Asakawa-Yazaki
Stephanov et al., Hatta & Ikeda

Ch. Schmidt
The phase diagram at finite quark masses

Can the QCD crossover line appear in the O(4) critical region?

YES, It has been confirmed in LQCD calculations

LQCD results: BNL-Bielefeld
O(4) scaling and magnetic equation of state

\[ P = P_R(T, \mu_q, \mu_I) + b^{-1} P_S(b^{(2-\alpha)^{-1}} t, b^{\beta \delta / \nu} h) \]

- Phase transition encoded in the magnetic equation of state

\[ \langle qq \rangle = -\frac{\partial P}{\partial m} \Rightarrow \text{pseudo-critical line} \]

\[ \frac{\langle qq \rangle}{m^{1/\delta}} = f_z(z), \quad z = tm^{-1/\beta \delta} \]

QCD chiral crossover transition in the critical region of the O(4) 2\text{nd} order

universal scaling function common for all models belonging to the O(4) universality class: known from spin models

J. Engels & F. Karsch (2012)
Chemical freezout defines a lower bound for the QCD phase boundary $T_{pc} = 154 \pm 9 \text{MeV}$.
Excellent description of QCD equation of states by the Hadron Resonance Gas

A. Bazavov et al. HotQCD Coll.

[hep-lat] 23 Jul 2014
The thermodynamics of strongly interacting medium is well described by the HRG due to confinement properties. It is valid for different pion masses and in the strong coupling limit.
Polyakov loop on the lattice needs renormalization

- Introduce Polyakov loop:
  \[ L_{\text{bare}} = \frac{1}{N_c} \text{Tr} \prod_{\tau=1}^{N_\tau} U(\vec{x},\tau), \]
  \[ L_{\text{bare}} = \frac{1}{N_c^3} \sum_{\vec{x}} L_{\text{bare}} \]

- Renormalized ultraviolet divergence
  \[ L_{\text{ren}} = (Z(g^2))^{N_\tau} L_{\text{bare}} \]

- Usually one takes \( \langle |L_{\text{ren}}|^2 \rangle \) as an order parameter

\[ L \rightarrow c_N L \quad c_N = e^{2\pi i k/N} \in Z(N) \]

\[ \langle L_{\text{ren}} \rangle \rightarrow \left\{ \begin{array}{l}
\neq 0 \iff \text{deconfined } T > T_c \\
0 \iff \text{confined } T < T_c
\end{array} \right. \]
To probe deconfinement: consider fluctuations

\[ T^3 \chi_A = \frac{N_c^3}{N_f^3} \left( \langle |L^{\text{ren}}|^2 \rangle - \langle |L^{\text{ren}}| \rangle^2 \right) \]

- the Polyakov loop
  \[ L = L_R + iL_I \]
- Consider fluctuations of real \( \chi_L = \chi_R \)
- modulus \( \chi_A = \chi_{|L|} \)
- imaginary \( \chi_T = \chi_I \)

and take their ratios:

\[ \chi_A < \chi_L = \chi_T \]
Ratios of the Polyakov loop fluctuations as an excellent probe for deconfinement


- In the deconfined phase $R_A \approx 1$

Indeed, in the real sector of $Z(3)$

$L_R \approx L_0 + \delta L_R$ with $L_0 = \langle L_R \rangle$

$L_I \approx L_0^I + \delta L_I$ with $L_0^I = 0$, thus

$\chi^L_R = V \langle (\delta L_R)^2 \rangle$, $\chi^L_I = V \langle (\delta L_I)^2 \rangle$

Expand the modulus,

$\left| L \right| = \sqrt{|L_R|^2 + |L_I|^2} \approx L_0 (1 + \frac{\delta L_R}{L_0} + \frac{(\delta L_I)^2}{2L_0^2})$

get in the leading order

$\langle |L|^2 \rangle - \langle |L| \rangle^2 \approx \langle (\delta L_R)^2 \rangle$

thus

$\chi_A \approx \chi_R$
In the confined phase for any symmetry breaking operator its average vanishes, thus

\[ \chi_{LL} = \langle L^2 \rangle - \langle L \rangle^2 = 0 \]

and

\[ \chi_{LL} = \chi_R - \chi_I \]

thus \[ \chi_R = \chi_I \]

In deconfined phase the ratio of \[ \chi_I / \chi_R \neq 0 \] and its value is model dependent

In the confined phase

\[ \chi_{R,(I)} = 4\pi \int dr \ r^2 C_{R,(I)}(r) \]

\[ C_{R,(I)}(r) = \langle L_{R,(I)}(r)L_{R,(I)}(0) \rangle_c \]

WHOT QCD Coll. (Y. Maezawa et al.)

\[ C_{R,(I)}(r) \to r \to \infty \to \gamma_{R,(I)}(T) \frac{e^{-M_{R(I)}r}}{rT} \]

and WHOT-coll. identified \( M_{R(I)} \) as the magnetic and electric mass:

\[ \chi_I \propto 1/m_E^2, \quad \chi_R \propto 1/m_M^2 \]

Since

\[ m_E^2 >> m_M^2 \Rightarrow \chi_I << \chi_R \]
String tension from the PL susceptibilities

Pok Man Lo, et al. (in preparation)

- $T < T_c \implies \chi_I = \chi_R$

- Common mass scale for $C_{R,(I)}(r)$

\[ C_{R,(I)}(r) \approx \frac{e^{-Mr}}{4\pi rT} \]

- In confined phase a natural choice for $M$

\[ M = \frac{b}{T} \]

- String tension

\[ b(T) / T_c^2 \approx (T / T_c)^2 (T^3 \chi_{R,(I)})^{-1/2} \]
Deriving partition function from YM Lagrangian [Gross-Pisarski-Yaffe (81)]

\[ Z = \int D A_\mu D C D \bar{C} \exp \left[ i \int d^4 x \mathcal{L} \right], \quad \mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}} \]

1. employ background field method.
\[ A_\mu = \bar{A}_\mu + g \tilde{A}_\mu \]

- thermodynamic potential (gluon part)

\[ \Omega_g = 2T \int \frac{d^3 p}{(2\pi)^3} \text{tr} \ln \left( 1 - \hat{L}_A e^{-|\vec{p}|/T} \right) \]

traced Polyakov loops \( \Phi = \text{tr} \hat{L}_F / N_c, \Phi = \text{tr} \hat{L}_F / N_c \) (gauge invariant)

full thermodynamics potential: \( \Omega = \Omega_g + \Omega_{\text{Haar}} \quad \text{C. Sasaki, et al. PRD (13)} \)

\[ \Omega_g = 2T \int \frac{d^3 p}{(2\pi)^3} \ln \left( 1 + \sum_{n=1}^{7} C_n e^{-n|\vec{p}|/T} + e^{-8|\vec{p}|/T} \right), \]

\[ \Omega_{\text{Haar}} = -a_0 T \ln \left[ 1 - 6\Phi \Phi + 4 (\Phi^3 + \bar{\Phi}^3) - 3 (\Phi \Phi)^2 \right], \]

\[ C_1 = C_7 = 1 - N_c^2 \Phi \Phi, \quad C_2 = C_6 = 1 - 3 N_c^2 \Phi \Phi + N_c^3 (\Phi^3 + \bar{\Phi}^3), \]

\[ C_3 = C_5 = -2 + 3 N_c^2 \Phi \Phi - N_c^4 (\Phi \Phi)^2, \]

\[ C_4 = 2 \left[ -1 + N_c^2 \Phi \Phi - N_c^3 (\Phi^3 + \bar{\Phi}^3) + N_c^4 (\Phi \Phi)^2 \right] \]

\( \Rightarrow \) energy distributions solely determined by group characters of SU(3)
Modelling the partition function form QCD in background field approach

\[ Z = \int dLdL^+ e^{-\beta VUL(L,L^+)} + \ln \det[Q_f] \]

\[ \ln \det[Q_f] \approx -h_{\text{eff}} L_R \]

\[ h_{\text{eff}} \approx N_f (M_q / T)^2 K_2(M_q / T) \]

Deconfinement CEP appears in effective QCD at \( M_Q \approx 1.5 \) GeV
Divergent longitudinal susceptibility at the critical point

See also LGT results for the position of CEP

Polyakov loop and fluctuations in QCD

- Smooth behavior for the Polyakov loop and fluctuations
difficult to determine where is "deconfinement"

The inflection point at \( T_{dec} \approx 0.22 \text{GeV} \)
The influence of fermions on ratios of the Polyakov loop susceptibilities

- Z(3) symmetry broken, however ratios still showing the transition
- Change of the slopes at fixed $T$

Kurtosis as an excellent probe of deconfinement

Ch. Schmidt at QM’12 (012)

- Factorization of pressure:
  \[
  P^B(T, \mu_q) = \cosh(B \mu_B / T) \sum_{i \in \text{baryons}} F(T, m_i)
  \]

- Kurtosis measures the squared of the baryon number carried by leading particles in a medium
  \[
  \chi_B^n = \frac{\partial^n (P / T^4)}{\partial (\mu_B / T)^n}
  \]

\[
\chi_B^B / \chi_2^B = \frac{\langle (\delta N_q)^4 \rangle}{\langle (\delta N_q)^2 \rangle} - 3 \frac{\langle (\delta N_q)^2 \rangle}{\langle (\delta N_q)^2 \rangle} 
\]

\[
\kappa \sigma^2 = \frac{\chi_4^B}{\chi_2^B} \approx B^2 = \begin{pmatrix}
1 & T < T_{PC} \\
\frac{1}{9} & T > T_{PC}
\end{pmatrix}
\]
The deconfinement of color, controlled by the Polyakov loop susceptibility ratios, and deconfinement of quarks, controlled by the kurtosis of the net baryon number, appears in the narrow temperature range which coincides with the chiral crossover.
Due to expected O(4) scaling in QCD the free energy:

\[ P = P_R(T, \mu_q, \mu_I) + b^{-1} P_S(b^{(2-\alpha)^{-1}} t(\mu), b^{\beta \delta / \nu} h) \]

Consider generalized susceptibilities of the net-quark number

\[ c_B^{(n)} = \frac{\partial^n (P / T^4)}{\partial (\mu_B / T)^n} = c_R^{(n)} + c_S^{(n)} \]

with

\[ c_s^{(n)} \big|_{\mu=0} = d \ h^{(2-\alpha-n/2) / \beta \delta} \ f_\pm^{(n)}(z) \]

\[ c_s^{(n)} \big|_{\mu \neq 0} = d \ h^{(2-\alpha-n) / \beta \delta} \ f_\pm^{(n)}(z) \]

Since for \( T < T_{pc} \), \( c_R^{(n)} \) are well described by the HRG

search for deviations (in particular for larger n) from HRG

to quantify the contributions of \( c_S^{(n)} \), i.e. the O(4) criticality


Modelling O(4) transition: effective Lagrangian and FRG

\[ \mathcal{L}_{QM} = \bar{q} [i \gamma_\mu \partial^\mu - g (\sigma + i \gamma_5 \vec{\tau} \cdot \vec{\pi})] \sigma + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \vec{\pi})^2 - U(\sigma, \vec{\pi}) \]

Effective potential is obtained by solving the exact flow equation (Wetterich eq.) with the approximations resulting in the O(4) critical exponents

\[ \partial_k \Omega_k (\sigma) = \frac{V k^4}{12 \pi^2} \left[ \sum_{i=\pi, \sigma} \frac{d_i}{E_{i,k}} \left[ 1 + 2 n_B (E_{i,k}) \right] - \frac{2 v_q}{E_{q,k}} \left[ 1 - n_F (E_{q,k}^+) - n_F (E_{q,k}^-) \right] \right] \]

- \[ E_{\pi,k} = \sqrt{k^2 + \Omega_k^\prime} \]
- \[ E_{\sigma,k} = \sqrt{k^2 + \Omega_k^\prime + 2 \rho \Omega_k^\prime} \]
- \[ E_{q,k} = \sqrt{k^2 + 2 q^2 \rho} \]
- \[ \Omega_k^\prime = \frac{\partial \Omega_k}{\partial (\sigma^2/2)} \]

Full propagators with \( k < q < \sigma \)

Integrating from \( k = \Sigma \) to \( k = \square \) gives a full quantum effective potential

Put \( \Phi_{k=0}(\sigma_{\min}) \) into the integral formula for \( P(N) \)
Deviations from low -T HRG values are increasing with $\mu / T$ and the cumulant order. Negative fluctuations near the chiral crossover.
STAR data on the first four moments of net baryon number

Deviations from the HRG

\[ S \sigma = \frac{\chi_{B}^{(3)}}{\chi_{B}^{(2)}} \quad \text{and} \quad \kappa \sigma^2 = \frac{\chi_{B}^{(4)}}{\chi_{B}^{(2)}} \]

\[ S \sigma |_{HRG} = \frac{N_p - N_p^{-}}{N_p + N_p^{-}} \quad \text{and} \quad \kappa \sigma |_{HRG} = 1 \]

Data qualitatively consistent with the change of these ratios due to the contribution of the O(4) singular part to the free energy
Charm produced in the “Formation phase” e.g. through: $g + g \rightarrow c + \bar{c}$

charm quarks thermalize in QGP

Quarkonium can regenerate at chemical freezeout as other hadrons but the number of $c + \bar{c}$ is exactly conserved (P. Braun-Munzinger & J. Stachel, 2000)

$$N_{cc}^{direct} = g_c N_{c+c}^{thermal} + g_c^2 N_{cc}^{thermal}$$

Obtained from pQCD or from experiment

Accounts for chemical off equilibrium

Calculated within Thermal Model
Test of hidden heavy flavor regeneration

- Yield ratios, excellent test of thermal regeneration and QCD phase boundary due to confinement

\[ \frac{Y(2s)}{Y(1s)} \approx \frac{d_2}{d_1} \left( \frac{m_2}{m_1} \right)^2 \frac{K_2(m_2 / T)}{K_2(m_1 / T)} \]

- The freezeout temperature for charmonia and botomonia ratio in Statistical Hadronisation Model (P. Braun-Munzinger & J. Stachel) is consistent, within errors, with the chiral crossover temperature from LGT
Conclusions:

- The coincident of Chiral crossover and freezeout in HIC indicates that Hadron Resonanse Gas partition function is an excellent approximation of the QCD thermodynamics in confined phase.
- Ratios of the Polyakov loop and net-charge susceptibilities are excellent probes of deconfinement and/or O(4) chiral crossover in QCD.
  
  At small baryon densities, there is a coincidence between deconfinement and the O(4) chiral criticality in LGT.
- Net baryon number fluctuations and charmonia yield ratios are ideal observable to identify the QCD phase boundary due to chiral symmetry restoration and confinement properties, respectively.
- Systematics of the net-proton fluctuations measured by STAR are qualitatively consistent with the expectations that they are influenced by the O(4) chiral criticality.