Symmetry Energy: from Nuclei to Neutron Stars

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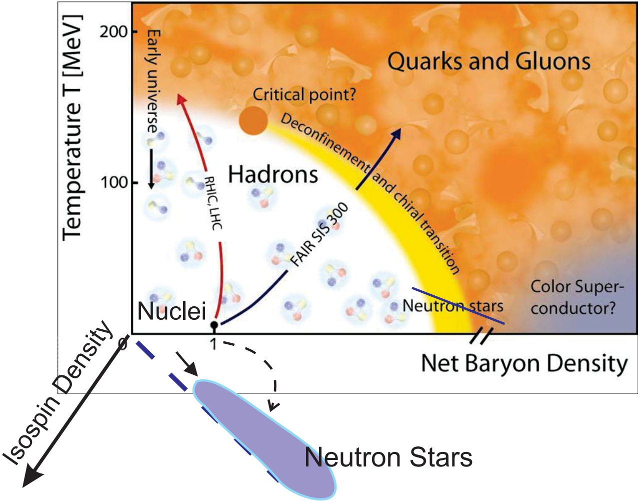
Equation of State



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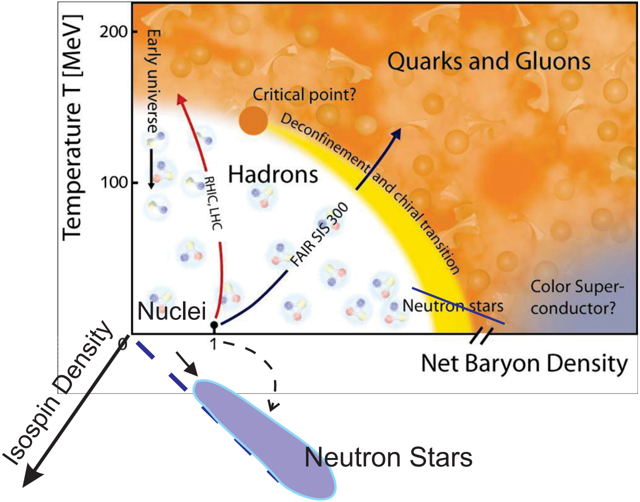

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Charge Symmetry & Charge Invariance

Charge symmetry: invariance of nuclear interactions under $n \leftrightarrow p$ interchange

An isoscalar quantity F does not change under $n \leftrightarrow p$ interchange. E.g. nuclear energy. Expansion in asymmetry $\eta = (N - Z)/A$, for smooth F, yields even terms only:

$$F(\eta) = F_0 + F_2\,\eta^2 + F_4\,\eta^4 + \dots$$

An isovector quantity G changes sign. Example: $\rho_{np}(r) = \rho_n(r) - \rho_p(r)$. Expansion with odd terms only:

$$G(\eta) = G_1\,\eta + G_3\,\eta^3 + \dots$$

Note: $G/\eta = G_1 + G_3\,\eta^2 + \dots$.

In nuclear practice, analyticity requires shell-effect averaging!

Charge invariance: invariance of nuclear interactions under rotations in n-p space

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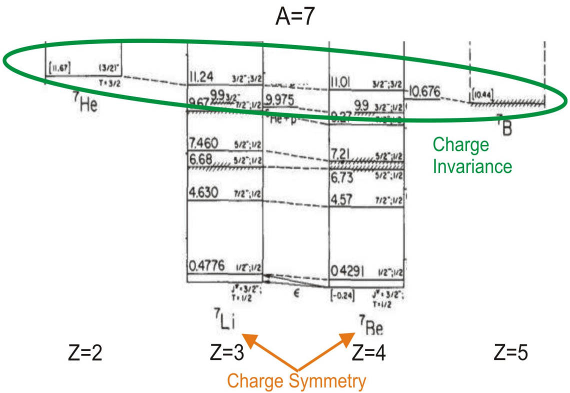
Isobars: Nuclei with the same A



Isospin doublets

$p : (\tau, \tau_z) \quad = (\frac{1}{2}, \frac{1}{2})$

$n : (\tau, \tau_z) \quad = (\frac{1}{2}, -\frac{1}{2})$

Net isospin

$$\vec{T} = \sum_{i=1}^{A} \vec{\tau}_i$$

$T = \frac{3}{2}, \ldots \qquad T = \frac{1}{2}, \frac{3}{2}, \ldots \qquad T = \frac{3}{2}, \ldots$

Nuclear states: $\quad (T, T_z), \quad T \geq |T_z| = \frac{1}{2}|N - Z|$

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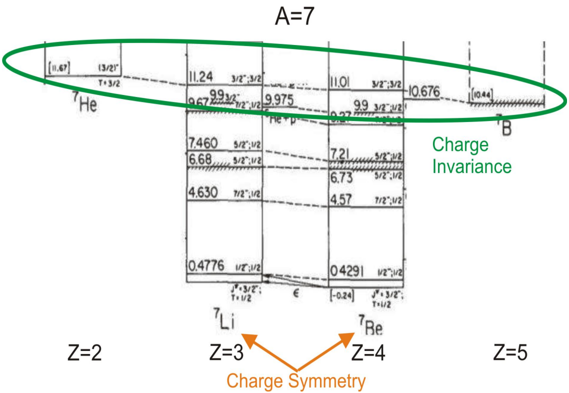

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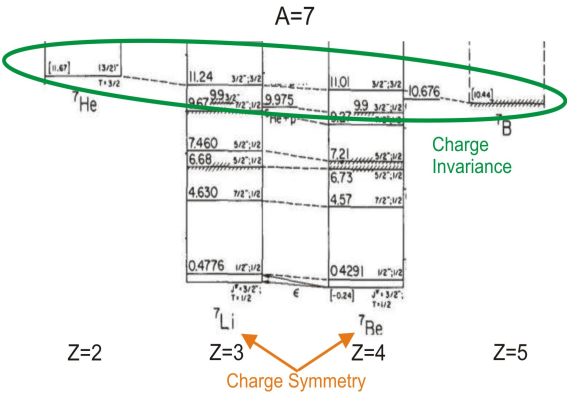

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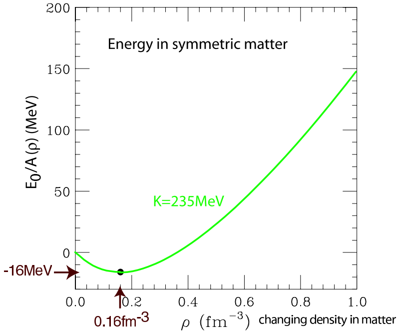
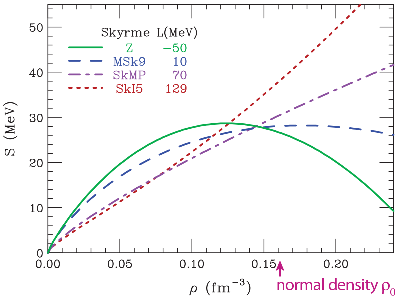
Energy in Uniform Matter

$$\frac{E}{A}(\rho_n, \rho_p) = \frac{E_0}{A}(\rho) + S(\rho)\left(\frac{\rho_n - \rho_p}{\rho}\right)^2 + \mathcal{O}(\ldots^4)$$

symmetric matter (a)symmetry energy $\rho = \rho_n + \rho_p$

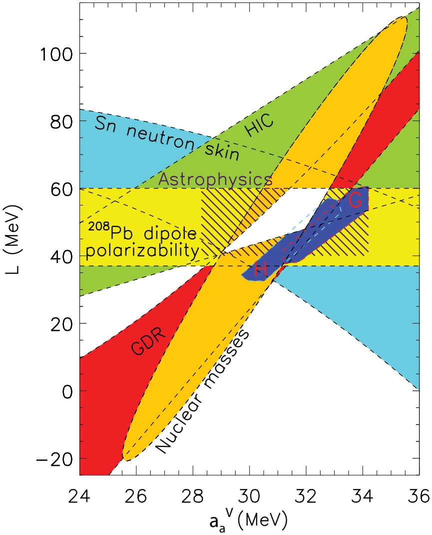




$$\frac{E_0}{A}(\rho) = -a_V + \frac{K}{18}\left(\frac{\rho - \rho_0}{\rho_0}\right)^2 + \ldots \qquad S(\rho) = -a_a^V + \frac{L}{3}\frac{\rho - \rho_0}{\rho_0} + \ldots$$

Known: $a_a \approx 16\,\text{MeV}$ $K \sim 235\,\text{MeV}$ Unknown: a_a^V ? L ?

Importance of Slope



Lattimer&Lim ApJ771(2013)51

$$\frac{E}{A} = \frac{E_0}{A}(\rho) + S(\rho)\left(\frac{\rho_n - \rho_p}{\rho}\right)^2$$

$$S \simeq -a_a^V + \frac{L}{3}\frac{\rho - \rho_0}{\rho_0}$$

In neutron matter: $\rho_p \approx 0$ & $\rho_n \approx \rho$.

Then, $\frac{E}{A}(\rho) \approx \frac{E_0}{A}(\rho) + S(\rho)$

Pressure:

$$P = \rho^2 \frac{\mathrm{d}}{\mathrm{d}\rho}\frac{E}{A} \simeq \rho^2 \frac{\mathrm{d}S}{\mathrm{d}\rho} \simeq \frac{L}{3\rho_0}\rho^2$$

Symmetry-Energy Connections

Symmetry energy ties research efforts in nuclear physics & astrophysics:

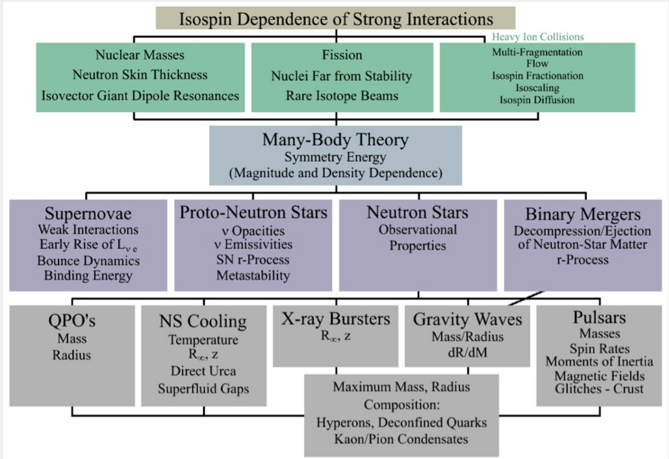


diagram by Andrew Steiner

Symmetry Energy in Nuclear Mass Formula

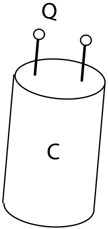
Textbook Bethe-Weizsäcker formula:

$$E = -a_V\, A + a_S\, A^{2/3} + a_C\, \frac{Z^2}{A^{1/3}} + a_a\, \frac{(N-Z)^2}{A} + E_{\text{mic}}$$

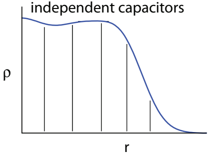
Symmetry energy: charge $n \leftrightarrow p$ symmetry of interactions

Analogy with capacitor:

$$E_a = a_a\, \frac{(N-Z)^2}{A} \equiv \frac{(N-Z)^2}{\frac{A}{a_a}} \Leftrightarrow E = \frac{Q^2}{2C}$$



?Volume Capacitance? $\quad E_a = \frac{(N-Z)^2}{\frac{A}{a_a}} \rightarrow \frac{(N-Z)^2}{\frac{A}{a_a^V} + \frac{A^{2/3}}{a_a^S}}$



Thomas-Fermi (local density) approximation:

$$'C' = \frac{A}{a_a(A)} = \int \frac{\rho\, \mathrm{d}\mathbf{r}}{S(\rho)} = \frac{A}{a_a^V}, \quad \text{for} \quad S(\rho) \equiv a_a^V$$

TF breaks in nuclear surface at $\rho < \rho_0/4$ PD&Lee NPA818(2009)36

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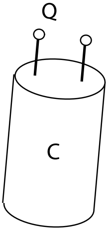

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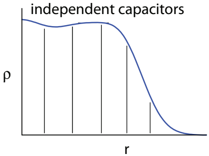

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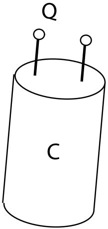

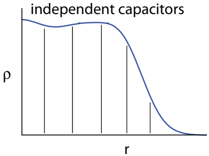

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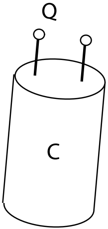

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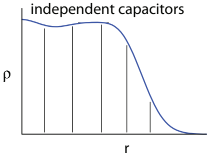

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Mass Formula & Charge Invariance

Symmetry-energy details in a mass-formula are intertwined with details of other terms: Coulomb, Wigner & pairing + even those asymmetry-independent, due to $(N-Z)/A$ - A correlations along stability line (PD)!

Best would be to study the symmetry energy in isolation from the rest of mass-formula! Absurd?!

Charge invariance to rescue: lowest nuclear states characterized by different isospin values (T, T_z), $T_z = (Z-N)/2$. Nuclear energy scalar in isospin space:

sym energy $$E_a = a_a(A)\,\frac{(N-Z)^2}{A} = 4\,a_a(A)\,\frac{T_z^2}{A}$$

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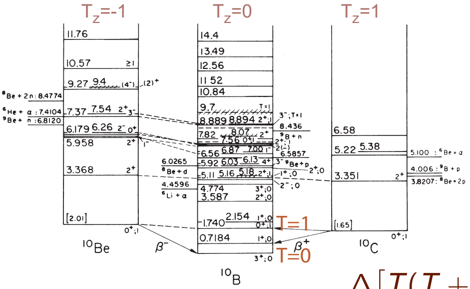
Symmetry Coefficient Nucleus-by-Nucleus

Mass formula generalized to the lowest state of a given T:

$$E(A, T, T_z) = E_0(A) + 4a_a(A)\,\frac{T(T+1)}{A} + E_{\rm mic} + E_{\rm Coul}$$

In the ground state T takes on the lowest possible value $T = |T_z| = |N - Z|/2$. Through '+1' most of the Wigner term absorbed.

?Lowest state of a given T: isobaric analogue state (IAS) of some neighboring nucleus ground-state.

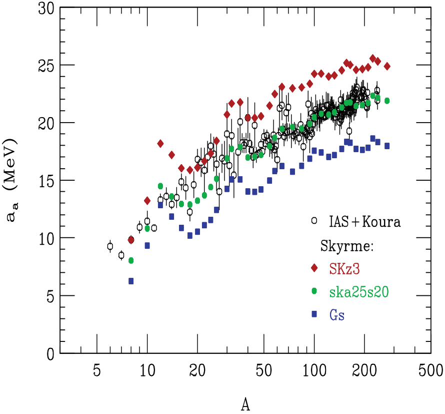


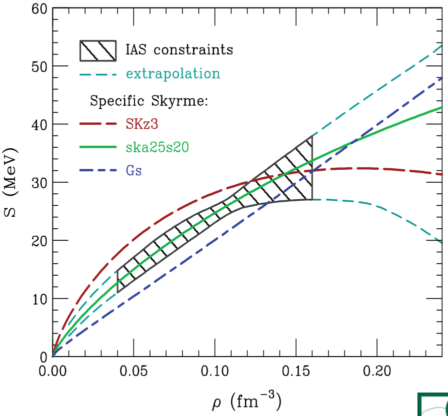
Study of changes in the symmetry term possible nucleus by nucleus

$$E^*_{\rm IAS} = \Delta E = a_a\,\frac{\Delta\left[T(T+1)\right]}{A} + \Delta E_{\rm mic}$$

From $a_a(A)$ to $S(\rho)$

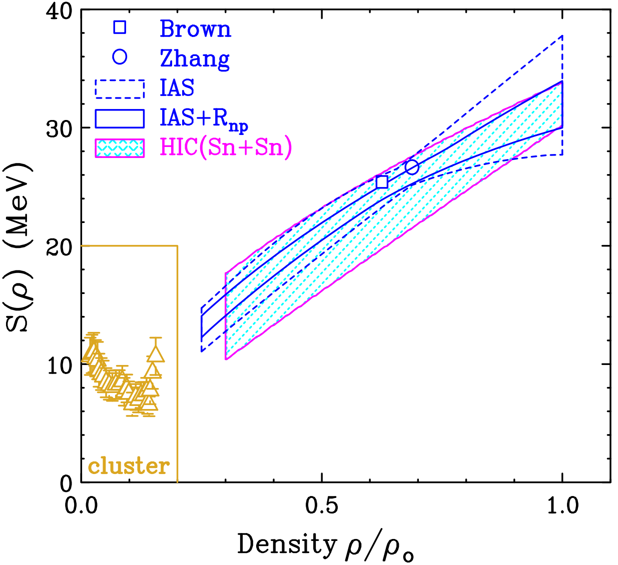
Strong $a_a(A)$ dependence [PD & Lee NPA922(14)1]:
lower $A \Rightarrow$ more surface $\Rightarrow$ lower $\rho \Rightarrow$ lower S



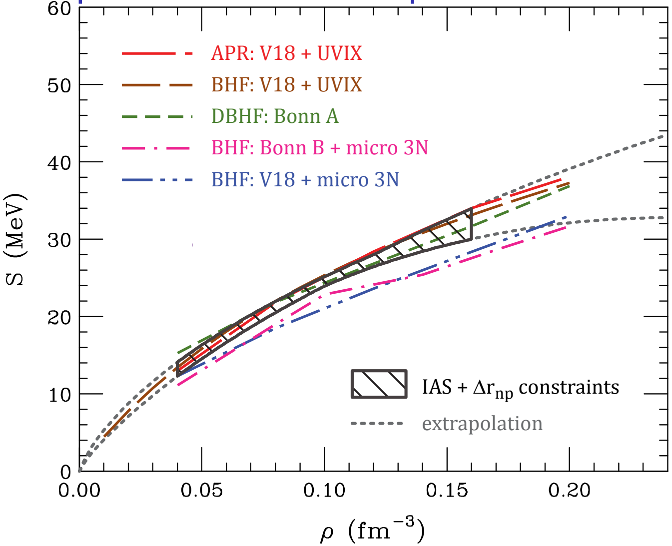


$a_a(A)$ from IAS give rise to constraints on $S(\rho)$ in Skyrme-Hartree-Fock calculations

Subnormal $S(\rho)$ from Different Data



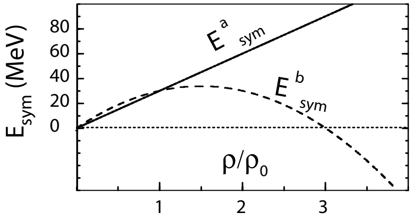
Comparison to Microscopic Calculations



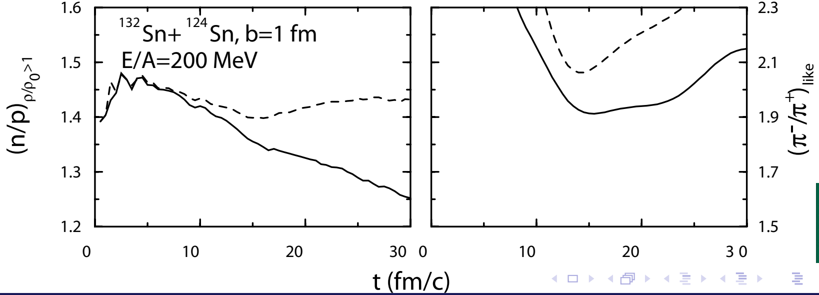
Microscopic results from Baldo *et al* PRC87(13)045803

Pions as Probe of High-ρ Symmetry Energy

B-A Li: $S(\rho > \rho_0) \Rightarrow n/p_{\rho>\rho_0} \Rightarrow \pi^-/\pi^+$



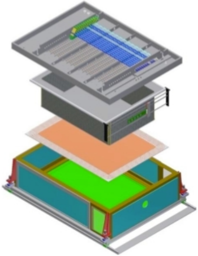
Pions originate from high ρ



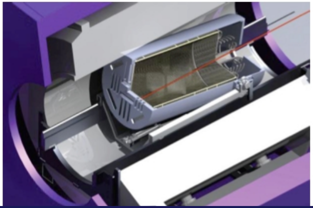
Dedicated Experimental Efforts

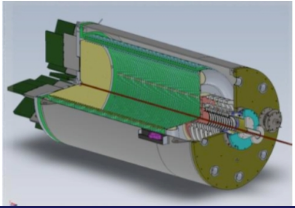
SAMURAI-TPC Collaboration (8 countries and 43 researchers): comparisons of near-threshold π^- and π^+ and also *n*-*p* spectra and flows at RIKEN, Japan.

NSCL/MSU, Texas A&M U
Western Michigan U, U of Notre Dame
GSI, Daresbury Lab, INFN/LNS
U of Budapest, SUBATECH, GANIL
China IAE, Brazil, RIKEN, Rikkyo U
Tohoku U, Kyoto U



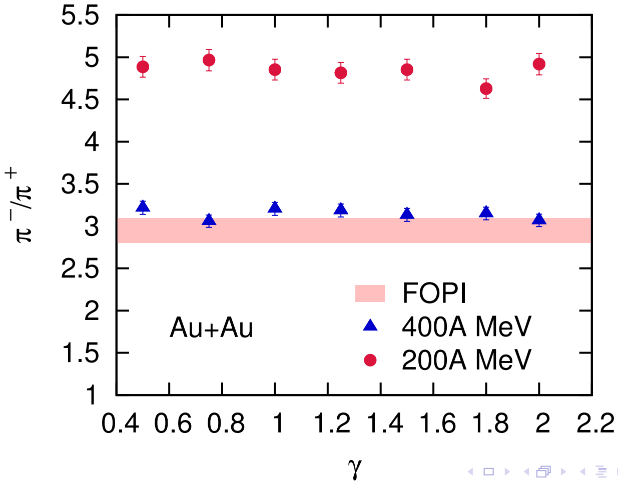
AT-TPC Collaboration (US & France)



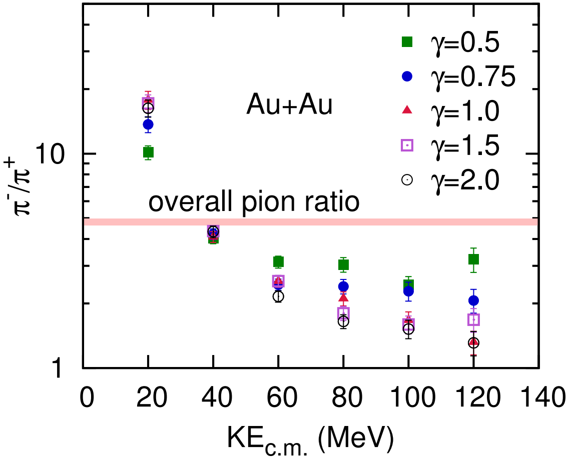


FOPI: π^-/π^+ at 400 MeV/nucl and above

Hong & PD, PRC in press: measured ratios reproduced irrespectively of $S_{\text{int}}(\rho) = S_0\,(\rho/\rho_0)^\gamma$:



Original Idea Still Correct for High-E π's



→ charge-exchange reactions blur the signal

Conclusions

- Symmetry-energy term weakens as nuclear mass number decreases: from $a_a \sim 23\,\text{Mev}$ to $a_a \sim 9\,\text{MeV}$ for $A \lesssim 8$.
- Weakening of the symmetry term can be tied to the weakening of $S(\rho)$ in uniform matter, with the fall of ρ.
- Convergence observed for conclusions on $S(\rho)$ at moderately subnormal densities, from variety of data, including isobaric analog states, and from microscopic calcs testing mostly 2-body ints.
- In the region of $\rho \gtrsim \rho_0$, $S(\rho)$ is quite uncertain. One promising observable is the high-energy charged-pion yield-ratio around NN threshold.

PD&Lee NPA922(14)1; Hong&PD arXiv:1307.7654, in press

NSF PHY-1068571 & PHY-1403906

Conclusions

- Symmetry-energy term weakens as nuclear mass number decreases: from $a_a \sim 23\,\text{Mev}$ to $a_a \sim 9\,\text{MeV}$ for $A \lesssim 8$.
- Weakening of the symmetry term can be tied to the weakening of $S(\rho)$ in uniform matter, with the fall of ρ.
- Convergence observed for conclusions on $S(\rho)$ at moderately subnormal densities, from variety of data, including isobaric analog states, and from microscopic calcs testing mostly 2-body ints.
- In the region of $\rho \gtrsim \rho_0$, $S(\rho)$ is quite uncertain. One promising observable is the high-energy charged-pion yield-ratio around NN threshold.

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