Recent Developments in Wakefield and Impedance Computation

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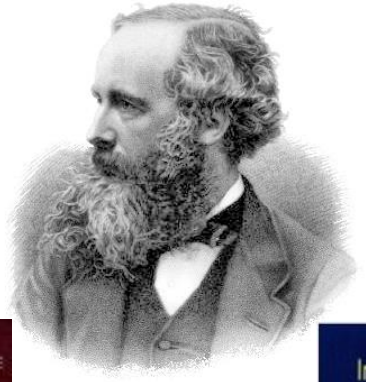
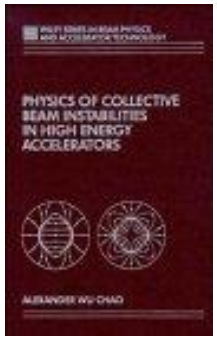
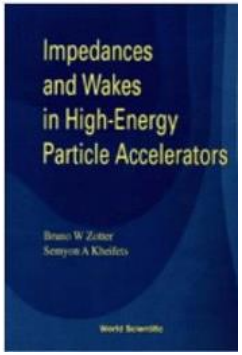
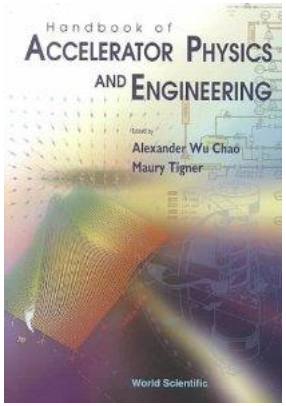
Motivation: Could There Be Anything New in Wakefields & Impedances?

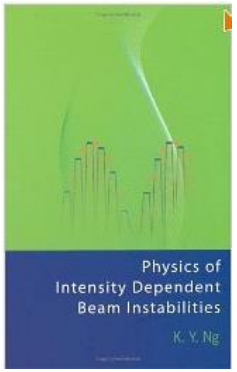
- It's a pretty well-studied subject
- Maxwell's equations are 150 years old
- Many textbooks and hundreds of papers exit
- Codes are getting ever more powerful

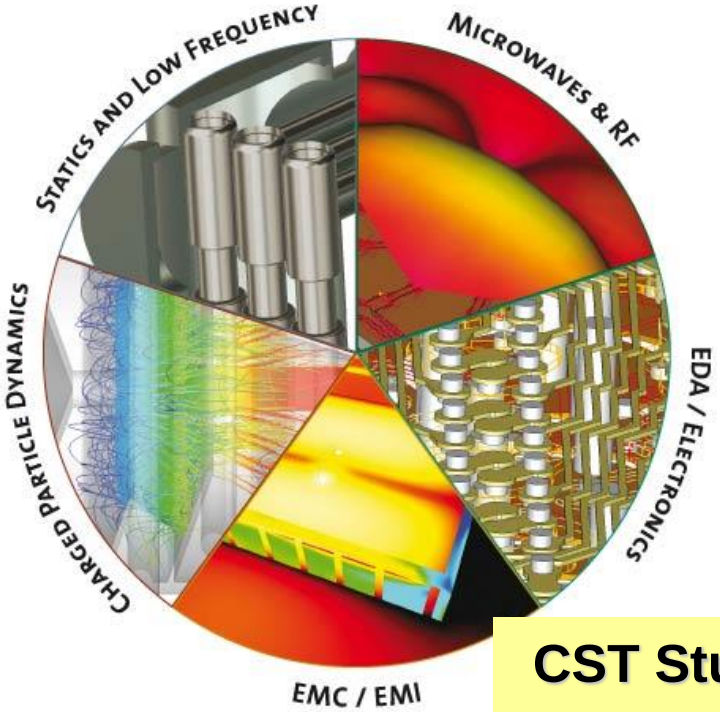








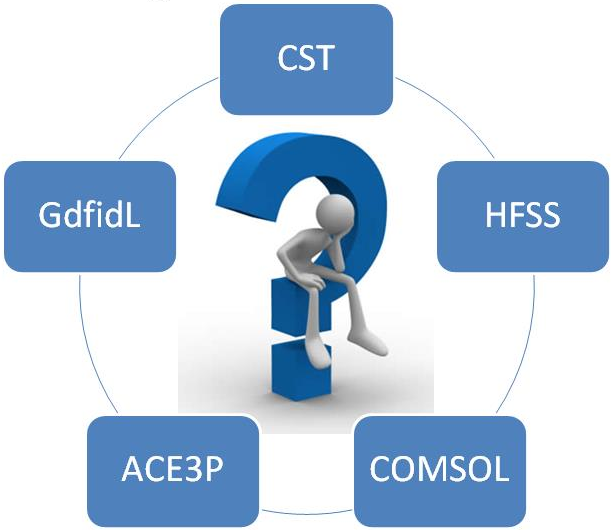




CST Studio Suite

More Powerful Codes are Available

Packages for computer simulations of electromagnetic EM fields and more

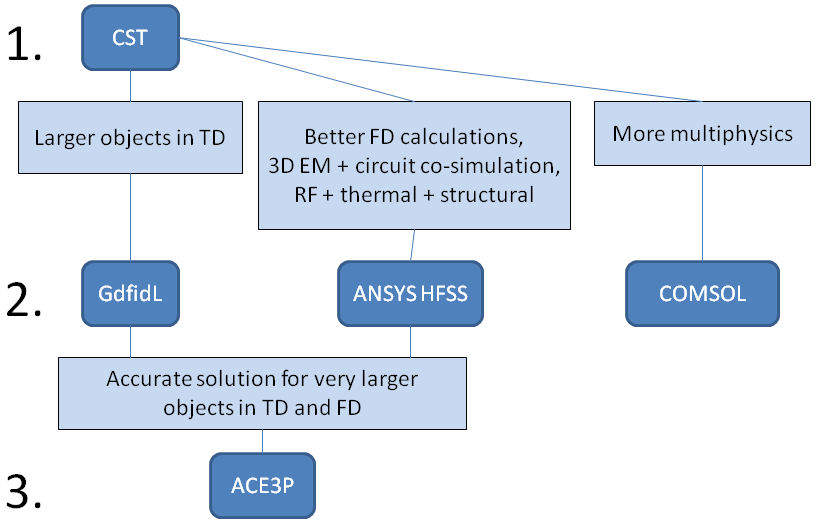


An Excellent Review of Codes was Recently Presented by Alexej Grudiev at CERN

26-27 June 2013 CERN

Electromagnetic field simulations for accelerator optimization

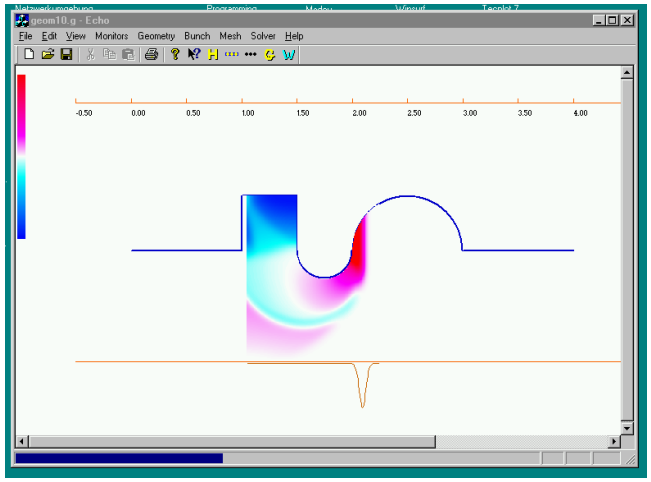
Summary



http://indico.cern.ch/contributionDisplay.py?contribId=11&confId=243336

Motivation

- Knowledge of wakefields, incl. geometric ones, is critically important for accelerator beam dynamics.
- Detailed wakefield calculations for realistic vacuum chambers are done with time domain EM solvers, which calculate the fields due to finite length bunches.
- Extremely fine meshes are needed to compute wakes at small distances, so calc's are slow and lots memory is req'd.
- This is especially true if the structures are very big (compared to bunch length) and smoothly varying.



Electromagnetic
Code for
Handling
Of
Harmful
Collective
Effects

EM solvers do not calculate point-charge wakefields

Asymptotic Model for Short-Bunch Wakefields of Cavity-like Structures

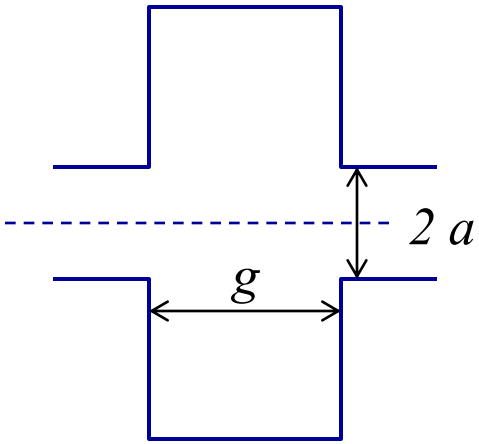
- For cavity-like structures we use the diffraction model:

$$W_d^{\delta}(z) = k_d\, z^{-1/2}, \quad z > 0$$

wake-function

$$W_d^{\sigma}(z) = \frac{k_d}{\sqrt{\sigma}}\, f\left(z/\sigma\right)$$

wake-potential



$$k_d = -\frac{Z_0 c}{\pi^2 a}\sqrt{g/2}$$

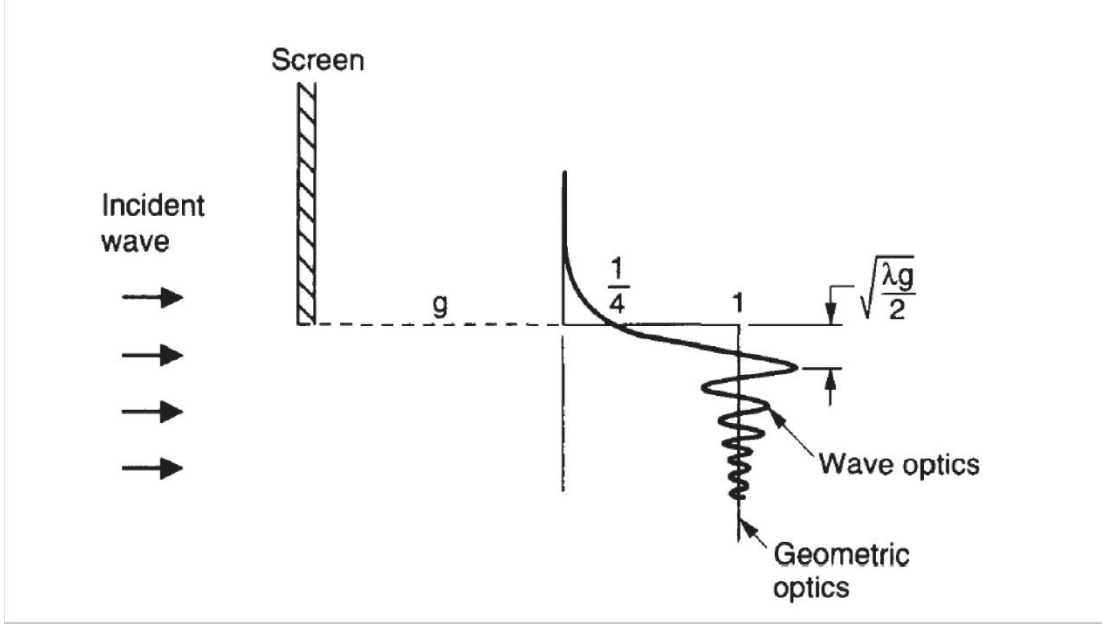
$$f(s) = e^{-s^2/4}\sqrt{\tfrac{\pi}{8}|s|}\left(\mathrm{I}(-\tfrac{1}{4},\tfrac{s^2}{4}) + \mathrm{sign}(s)\,\mathrm{I}(\tfrac{1}{4},\tfrac{s^2}{4})\right)$$

I(…) are Bessel functions

- Wake-potentials for all cavity shapes (tapered or not, deep or shallow, etc.) converge to this model for short enough bunches and distances.
- Model is easily expandable to 3D geometries.

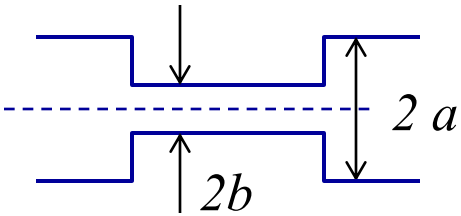
Diffraction Model

- Fresnel diffraction (from A. Chao's book).
- Wakefield is found from energy losses of an ultra-relativistic beam to diffracted waves.
- Plane waves diffracted by a straight-edge plane screen is an adequate approximation for beam in cylindrical geometry.



Asymptotic Model for Short-Bunch Wakefields of Collimator-like Structures

- For collimator-like structures use the optical model:



$$W_{opt}^{\delta}(z) = k_{opt}\delta(z)$$

wake-function

$$W_{opt}^{\sigma}(z) = k_{opt}(2\pi)^{-1/2}\sigma^{-1}e^{-\frac{z^2}{2\sigma^2}}$$

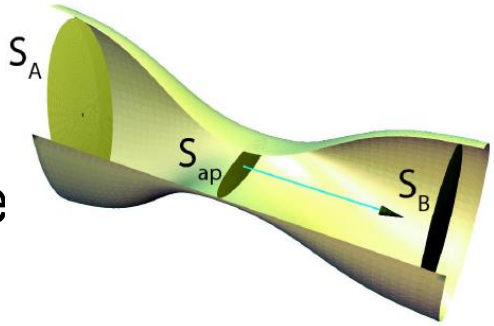
wake-potential

$$W_{opt}^{\delta}(z \to 0) = \infty$$

$$W_{opt}^{\sigma \to 0}(z) = \infty$$

$$k_{opt} = -Z_0 c\, Ln(a/b)/\pi$$

- Turns out this model describes all collimator-like structures, including 3D; A recipe to calculate geometry-dependent k_{opt} exists [see Stupakov, Bane Zagorodnov, PRST-AB 10, 054401 (2007)]



Asymptotic Short-Bunch Wakes are Known, so What's Missing?

- Asymptotic wakefields are approximate expressions (strongest singularity) at z=0.
- They fall down very sharply with z, and one cannot say where they get overtaken by other (non-singular) parts of the wake.
- They don't give us complete picture, i.e. $W^{\delta}(z)$ for any z.

Can We Approach Point-Charge Wakes through Geometry Scaling?

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **14**, 014402 (2011)

Impedance scaling for small angle transitions

G. Stupakov and K. L. F. Bane
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I. Zagorodnov
Deutsches Elektronen-Synchrotron, Notkestrasse 85, 22603 Hamburg, Germany
(Received 24 September 2010; published 10 January 2011)

Scaling Laws

- Stupakov, Bane, and Zagorodnov show that the following longitudinal impedance and wakefield scaling holds true for
  1) structures of general shape at high frequencies (i.e. >> pipe cutoff)
  2) small angle transitions at all frequencies

$$Z(\omega;\lambda) = R(\tfrac{\omega}{\lambda}) \qquad (1)$$

$$W^{\delta}(z;\lambda) = \lambda u(z\lambda). \qquad (2)$$

Here R and u are functions of one variable, and λ is the longitudinal scale factor between the geometries.

- In terms of the wake-potential due to a finite length bunch, Eqs.(1-2) are equivalent to

$$W^{\sigma}(z) = \lambda^{-1}\underline{W}^{\sigma/\lambda}(z/\lambda). \qquad (3)$$

where W and $\underline{W}$ stand, respectively, for the wake-potentials due to the original and the scaled structures, and λ relates the longitudinal dimensions by $\underline{L}=\lambda L$.

- Similar scaling laws are derived for transverse.

Wake for a 3D Tapered Transition

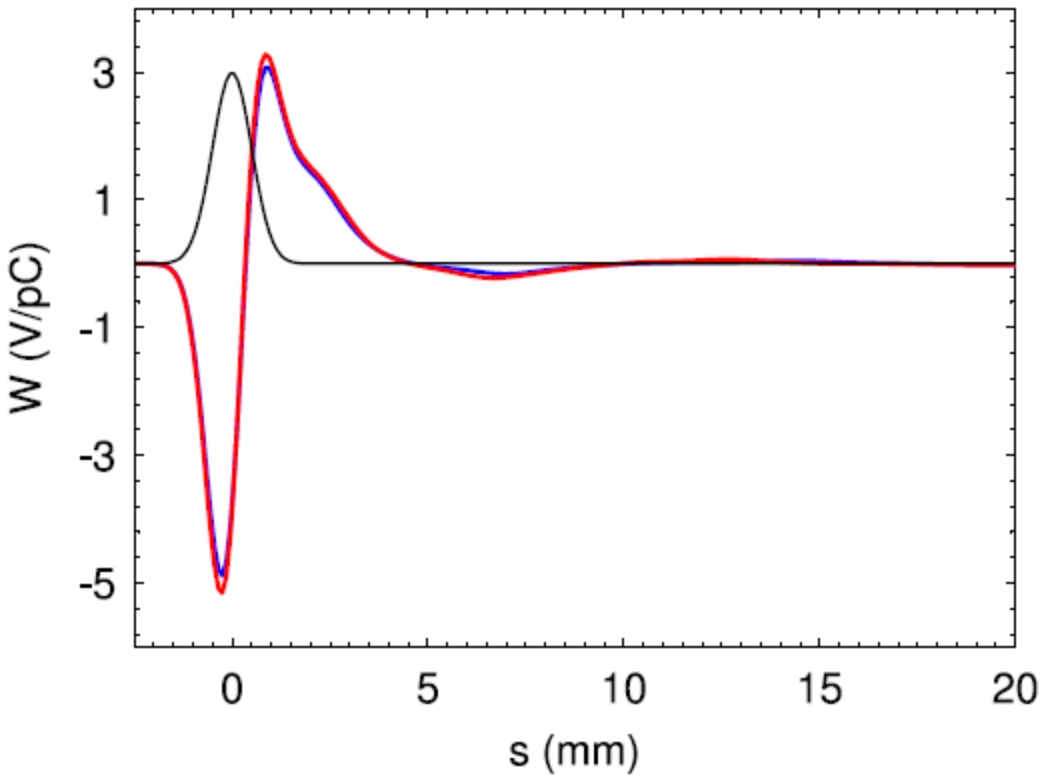


FIG. 8. Longitudinal wake for $\sigma_z = 0.5$ mm bunch in the 3D collimator (blue) and results obtained from the scaled problem (red).

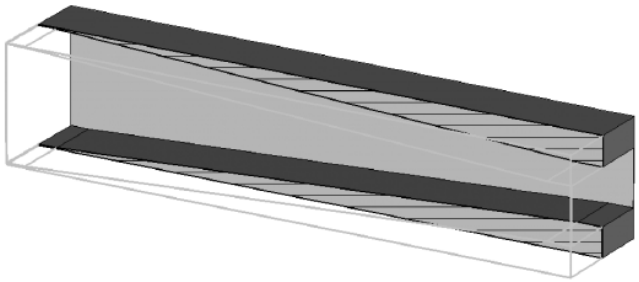
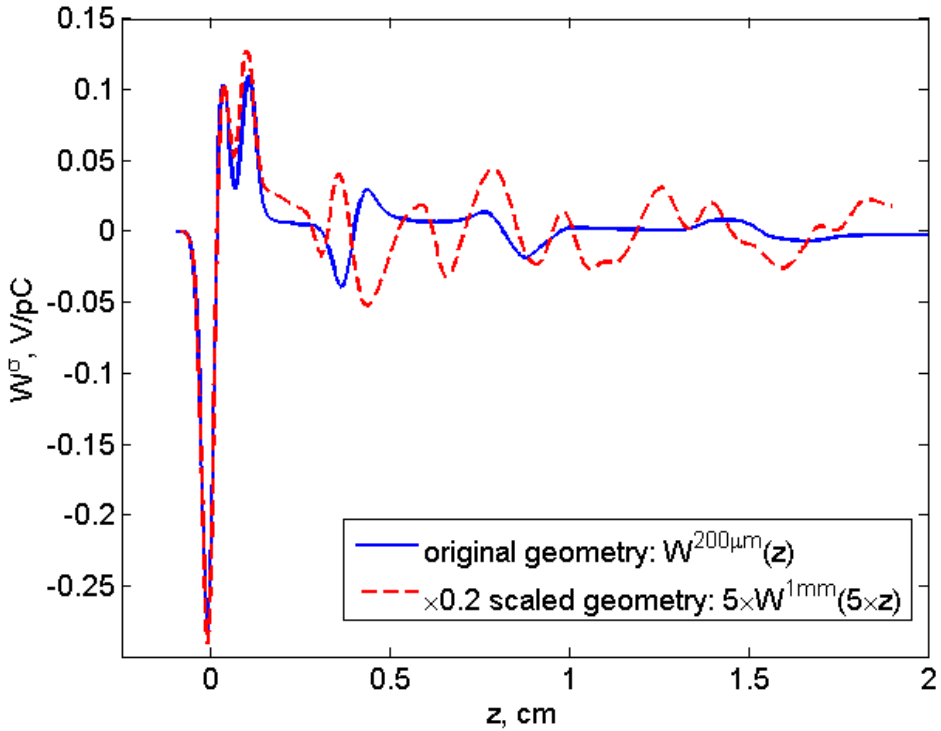


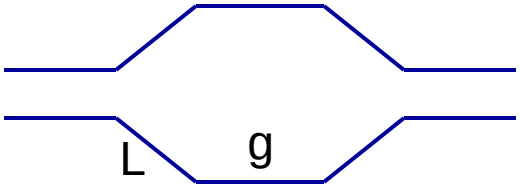
FIG. 7. The 3D test example is a symmetric, small angle transition. Here we show, in cut view, the geometry of one of the tapers of this transition.

G. Stupakov, K. L. F. Bane, I. Zagorodnov, PRST-AB 14, 014402 (2011)

Scaling Breakdown Example: Small Angle Tapered Cavity



Geometry before Scaling
r_{min}=1 cm,
r_{max}=1.05 cm,
g=20 cm
L=2.5 cm



- ECHO results for the original and λ=1/5 scaled geometries are shown
- Clearly there is a disagreement in the long range wake
- In ω-domain this disagreement extends way above the cutoff
- In general, scaling in a very large range λ<<1, doesn't seem accurate
- More on breakdown is in A. Blednykh, S. Krinsky, PRSTAB 15, 054405 (2012)

Recent Method to Find Point-Charge Wakes

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS
16, 024401 (2013)

Point-charge wakefield calculations from finite length bunch wake potentials

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(Received 2 November 2012; published 15 February 2013)

- **In WEODB1 NAPAC'13 paper we extended this method to transverse wakes**

Introducing the Method: Wake of a Step-Out

- Wake-potentials are singular at $\sigma \longrightarrow 0$
- Subtracting singular part (optical model) we obtain a well-defined limit (black line) at $\sigma \longrightarrow 0$

$$D^{\sigma}(z) = W^{\sigma}(z) - W_s^{\sigma}(z)$$

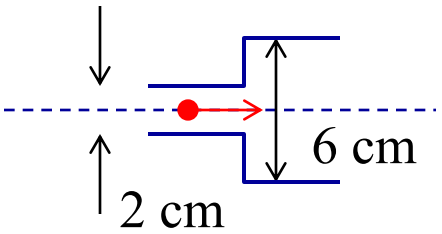
$$D^{\delta}(z) = \lim_{\sigma \to 0} D^{\sigma}(z)$$

- This function is approximated by

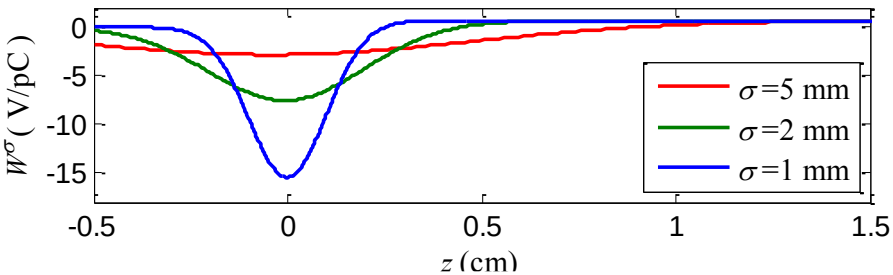
$$D^{\delta}(z) \approx (\alpha + \beta z) H(z)$$

step-function from causality

- Coefficients α and β can be found by fitting (next VG).
- Thus we reconstruct point-charge wakefield (at short z-range)



$$W_s^{\sigma}(z) = W_{opt}^{\sigma}(z) = \frac{Z_0 c\, Ln(3)}{2^{1/2} \pi^{3/2} \sigma} e^{-\frac{z^2}{2\sigma^2}}$$

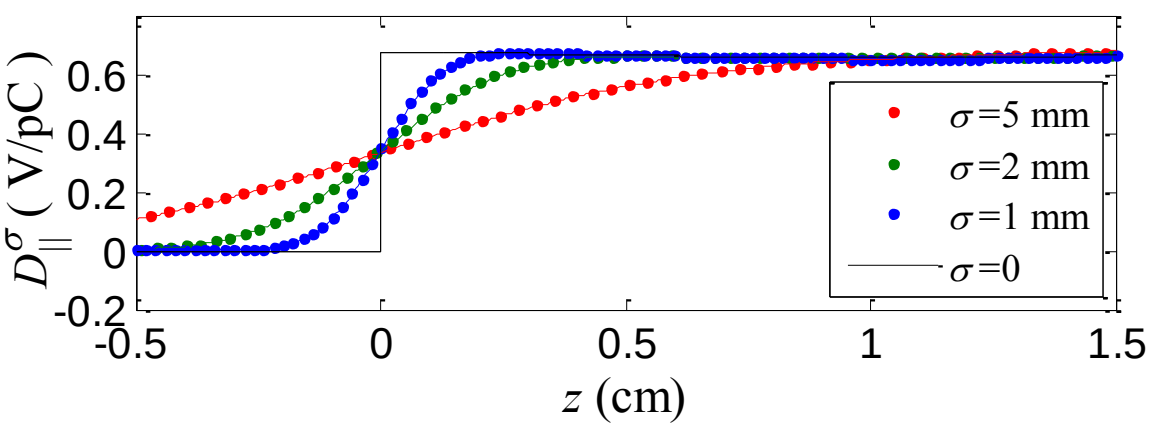


Wake of a Step-Out Con't: fitting for α and β

- Point-charge and Gaussian bunch functions are related:

$$D^{\delta}(z) = (\alpha + \beta z) H(z) \qquad D^{\sigma}(z) = \frac{\alpha + \beta z}{2}\left(1 + \mathrm{erf}\left(\frac{z}{\sqrt{2}\sigma}\right)\right) + \frac{\beta\sigma}{\sqrt{2\pi}} e^{-\frac{z^2}{2\sigma^2}}.$$

- α and β can be found by fitting $D^{\sigma}(z)$ from EM solver for i.e. $|z/\sigma|<3$.
- Take σ_0=2 mm and apply the fitting. Then use α and β obtained to reconstruct wakes for other values of σ:

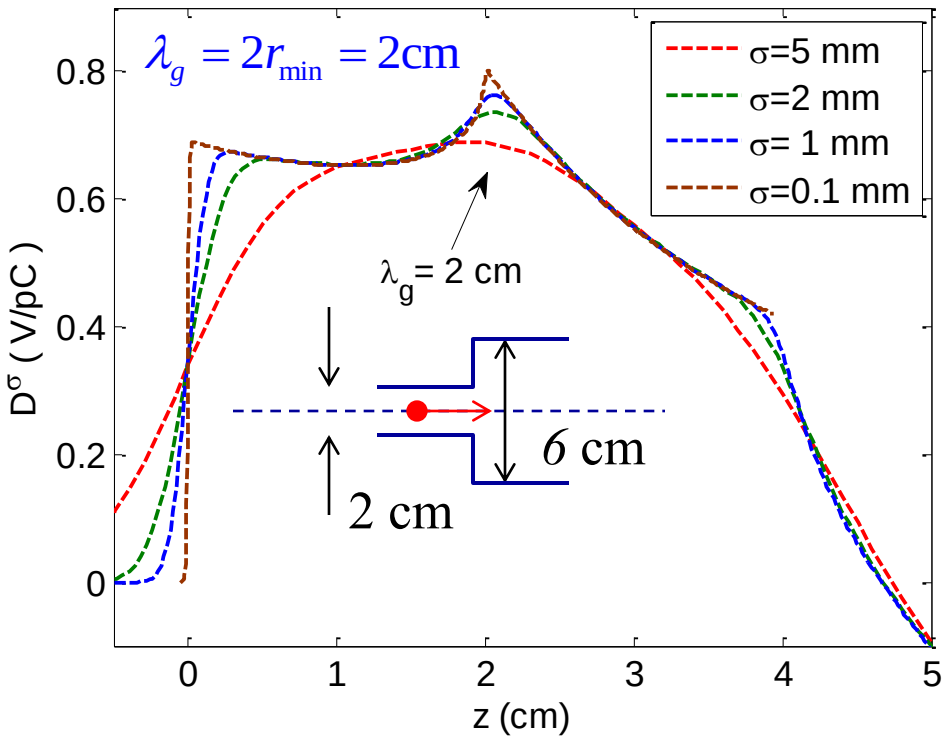


dots: ECHO
lines: rec'd from $W^{2\mathrm{mm}}_{\mathrm{ECHO}}$

- Reconstructed wakes agree well with direct ECHO calculation

How to Pick σ_0 in EM solver

- Why did the σ_0=2 mm fit work well? Because $\sigma_0 << \lambda_g$.
- Parameter λ_g>0 is the first location of the wake singularity (or singularity of its derivatives) closest to *z*=0.
- $D^{\delta}(z) = (\alpha + \beta z) H(z)$ cannot be extended beyond $z = \lambda_g$ since the wake derivative is singular ("kink").

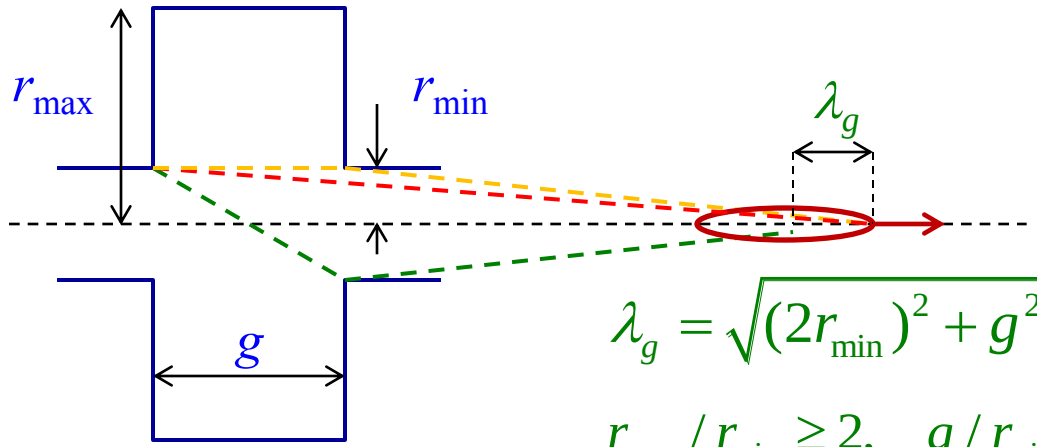


•Run EM solver with $\sigma_0 << \lambda_g$, typically σ_0/λ_g=0.1-0.15 is O.K.

•Running with shorter bunch gives no new information about the wake!

•λ_g can be found by simple geometry analysis.

λ_g Parameter for a Cavity



$$\lambda_g = \sqrt{(2r_{min})^2 + g^2} - g$$

$$r_{max} / r_{min} \geq 2, \quad g / r_{min} \geq 2^{-1/2}$$

- Red ray (spherical wave front) eventually catches up with ALL particles in the bunch, thus affecting the wakefield for all values of *z*.
- Green ray travels λ_g/c behind and it will never catch up with the front of the bunch, so λ_g emerges in the front portion of the wake.
- For other ratios between r_{min}, r_{max}, and g, other combinations may define λ_g , i.e. $\lambda_g = 2g$ for a short cavity or, for a shallow one,

$$\lambda_g = \sqrt{4(r_{max} - r_{min})^2 + g^2} - g$$

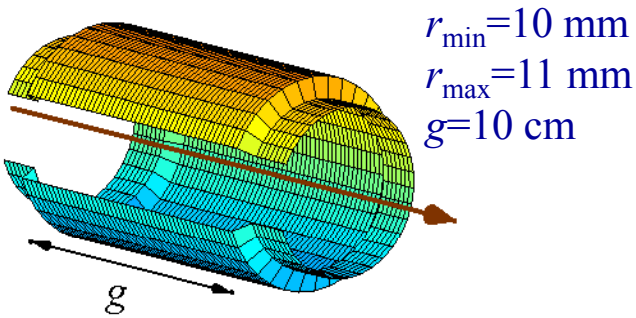
Similarly λ_g one can find for arb. geometry (see PRST-AB paper)

Shallow Cavity Example

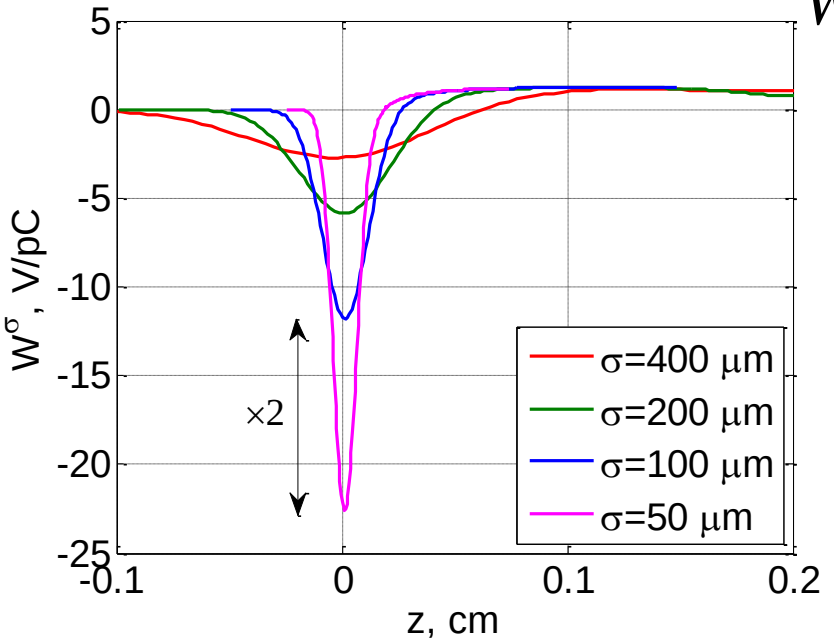
- Our method is NOT shortening the bunch until the EM solver wakes converge (this can fail!)
- For this cavity wakes are ~Gaussian; they double as σ shortens each factor of 2, suggesting
- This is totally incorrect!
- It is diffraction model instead

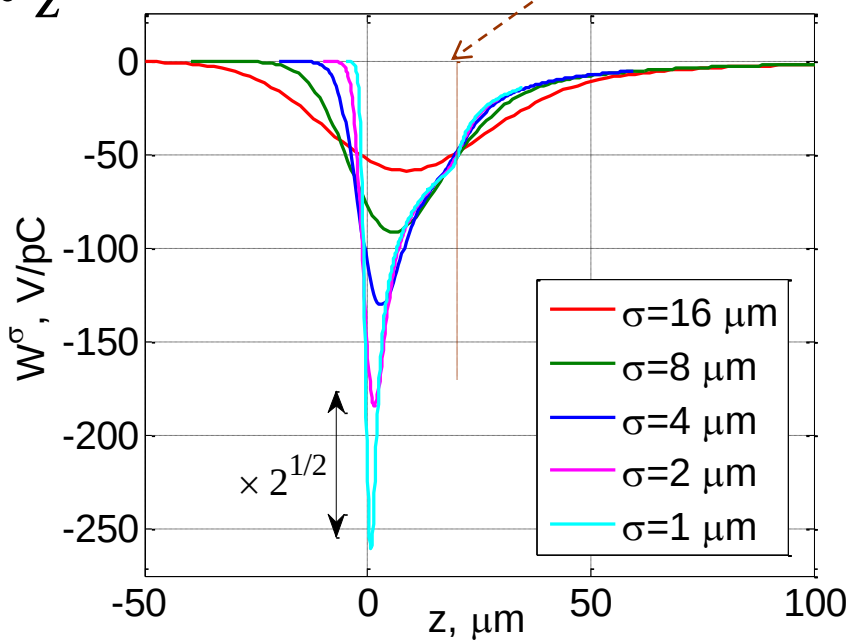
$$\cancel{W^{\delta}(z) \sim \delta(z)}$$

$$W^{\delta}(z) \sim z^{-1/2}$$



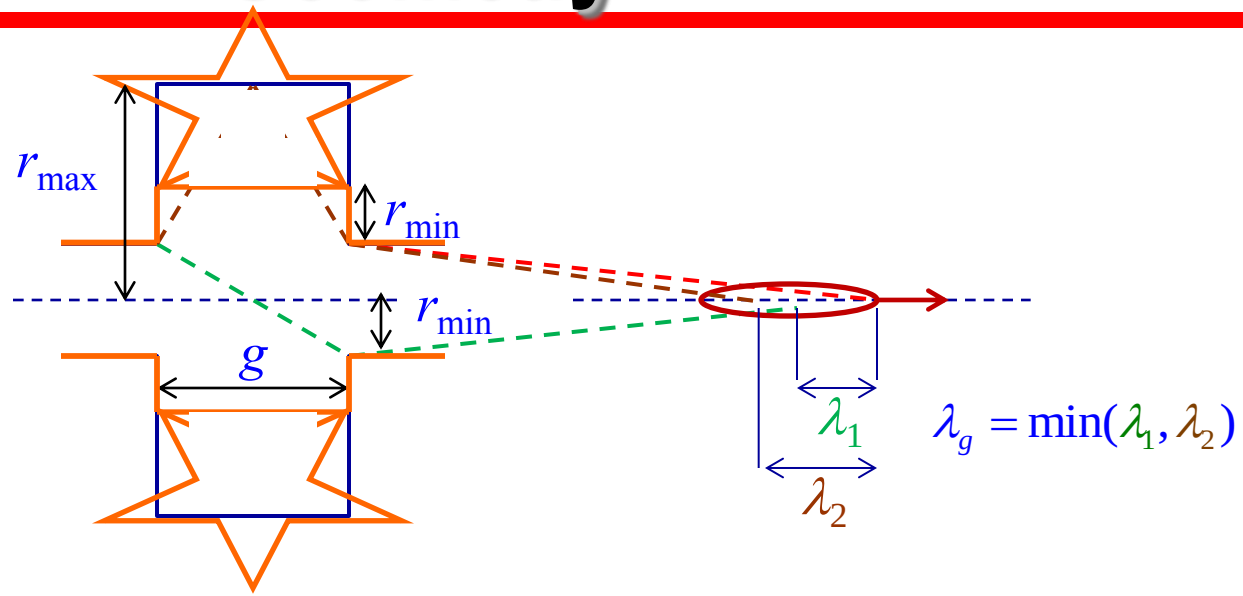
$$\lambda_g \approx 2(r_{max} - r_{min})^2 / g = 20\ \mu m$$





Important to Know λ_g <u>before</u> Running the EM Solver

Why λ_g is Easy to Find for Arbitrary Geometry



- Green ray does not affect short-range wake for $z < \sqrt{(2r_{min})^2 + g^2} - g$
- Brown ray does not affect short-range wake for $z < \sqrt{4(r_{max} - r_{min})^2 + g^2} - g$
  This is longer for deep cavities with $r_{max} > 2r_{min}$.
- By causality, any cavity with radial boundary, $r(s)$, that coincides with the figure for $r(s) < 2r_{min}$, but otherwise is arbitrarily complex, must have the same short-range wake for $z < \lambda_g$.
- $=>$ λ_g is defined by the geometry near r_{min}

How It All Works Together

1. Determine analytical singular wake model: $W_s^{\delta}(z)$ & $W_s^{\sigma}(z)$
2. Determine λ_g
3. Calculate the wake-potential with your favourite EM solver for $\sigma_0 << \lambda_g$: $W_{ECHO}^{\sigma_0}(z)$
4. Subtract the singular wake: $D^{\sigma_0}(z) = W_{ECHO}^{\sigma_0}(z) - W_s^{\sigma_0}(z)$
5. Fit the remainder, $D^{\sigma_0}(z)$, with the function: $\frac{\alpha+\beta z}{2}\left(1+\mathrm{erf}(\frac{z}{\sqrt{2}\sigma_0})\right)+\frac{\beta\sigma_0}{\sqrt{2\pi}}\mathrm{e}^{-\frac{z^2}{2\sigma_0^2}}$

   (fit range $|z/\sigma_0|<3$ works well)
6. Short-bunch wake (for arb. $\sigma \le \sigma_0$) is then:

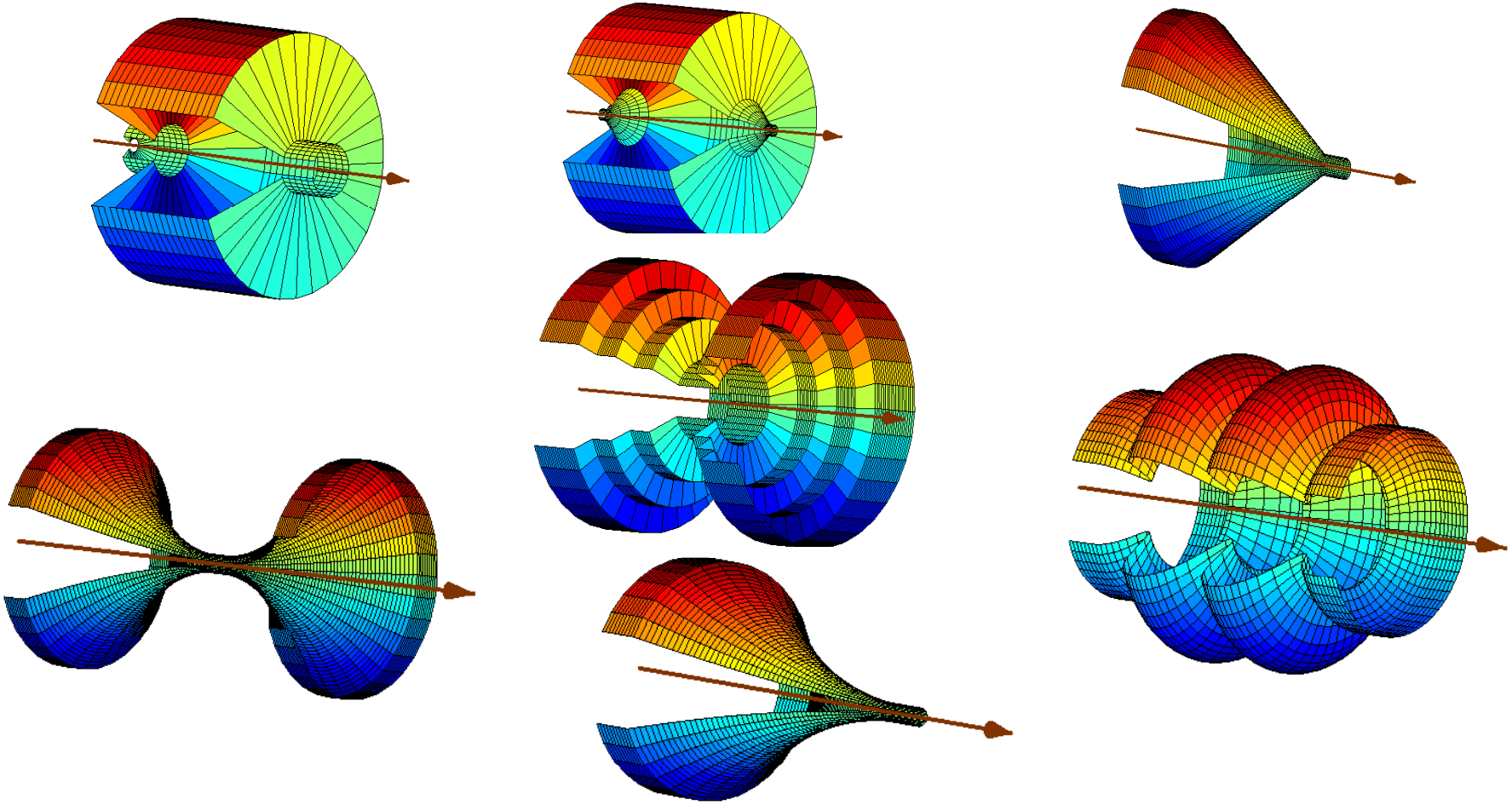
$$W^{\sigma}(z \le 3\sigma_0) = \frac{\alpha+\beta z}{2}\left(1+\mathrm{erf}(\frac{z}{\sqrt{2}\sigma})\right)+\frac{\beta\sigma}{\sqrt{2\pi}}\mathrm{e}^{-\frac{z^2}{2\sigma^2}}+W_s^{\sigma}(z) \qquad W^{\sigma}(z > 3\sigma_0) = W_{ECHO}^{\sigma_0}(z)$$

7. For point-charge:

$$W^{\delta}(z \le 3\sigma_0) = (\alpha+\beta z)H(z)+W_s^{\delta}(z) \qquad W^{\delta}(z > 3\sigma_0) = W_{ECHO}^{\sigma_0}(z)$$

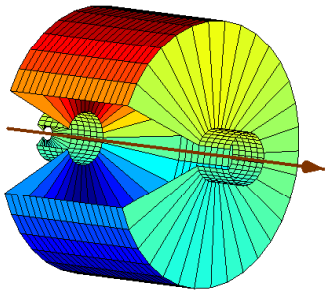
Transverse is very similar

The Method Was Applied to Many Geometries



It worked well for all of them

Simple Cavity Example



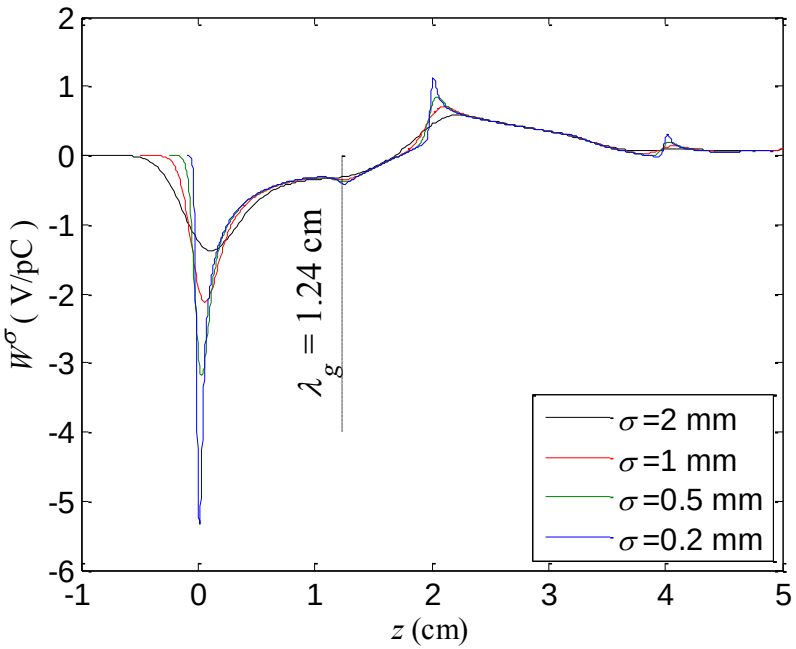
$r_{\min}$=1 cm
$r_{\max}$=5 cm
g=1 cm

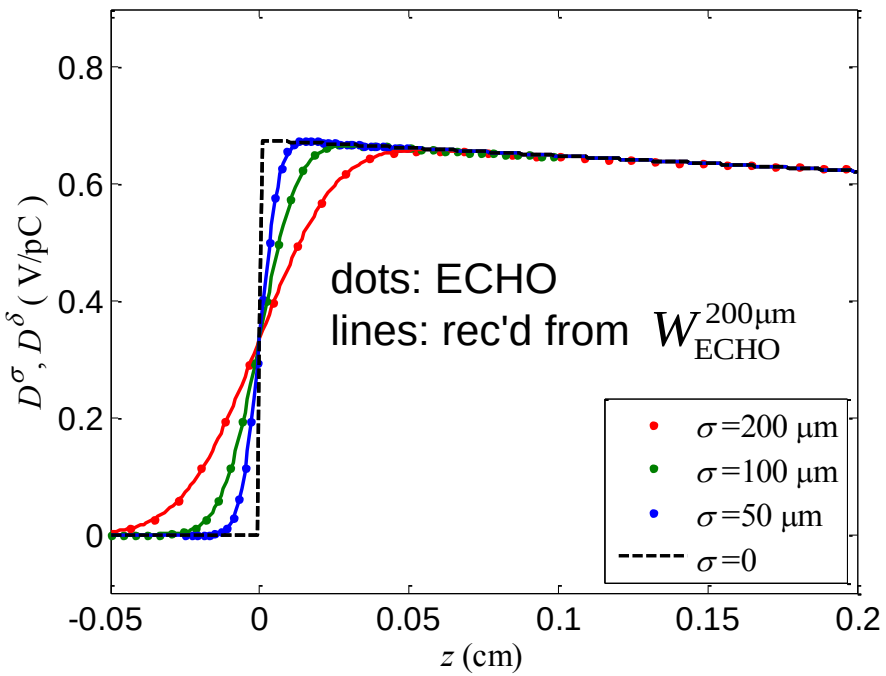
$$\lambda_g = \sqrt{(2r_{\min})^2 + g^2} - g = 1.24\ \text{cm}$$

- Diffraction-model behaviour near z=0
- Pick σ_0=200μm<< λ_g

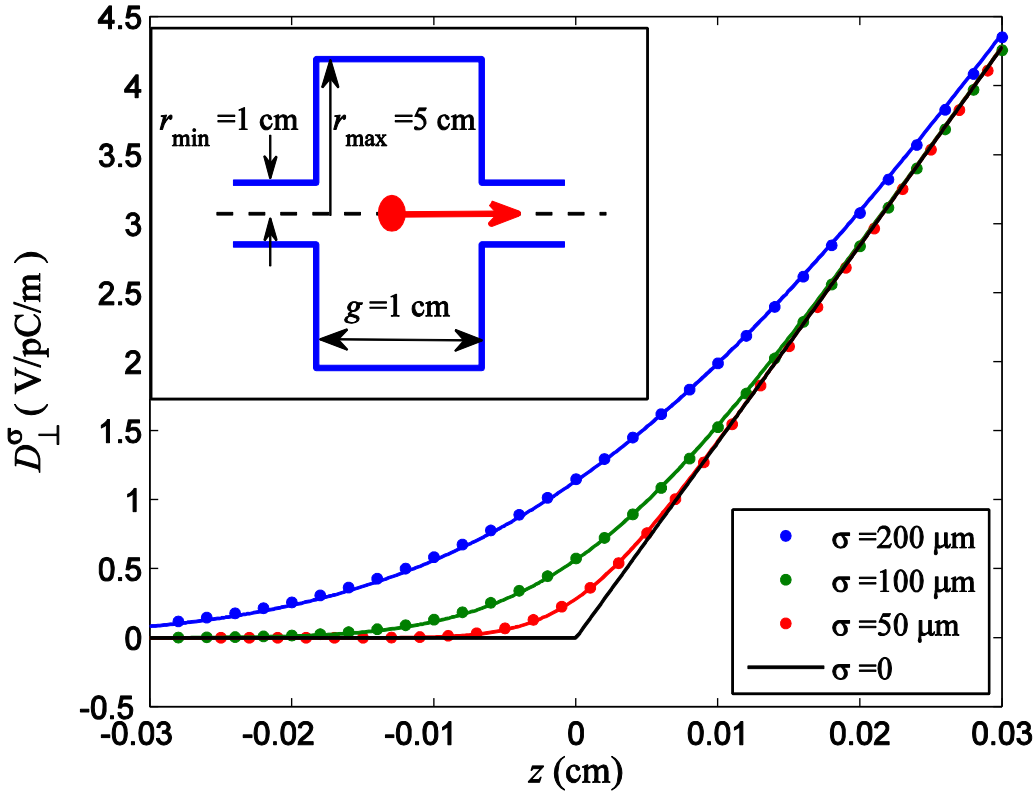
$$D^{\sigma_0}(z) = W_{ECHO}^{\sigma_0}(z) - W_d^{\sigma_0}(z)$$

- Short-bunch wake reconstructed well.





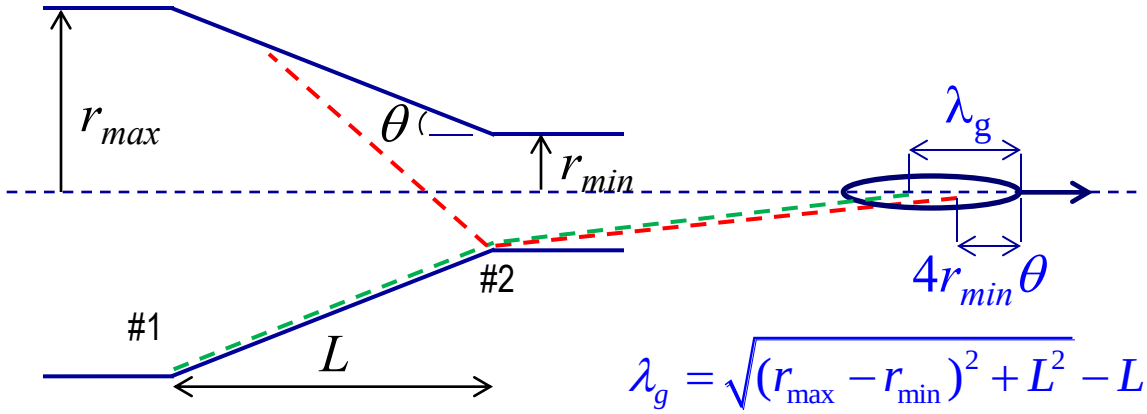
Transverse Wake for the Same Cavity



dots: direct ECHO
lines: rec'd from $W_{\mathrm{ECHO}}^{2\mathrm{mm}}$

- The same algorithm works well in the transverse (except the transverse diffraction model is non-singular).
- Reconstructed wakes from σ_0 =2 mm agree perfectly with direct ECHO calculations

λ_g Parameter for a Small-Angle Linear Taper

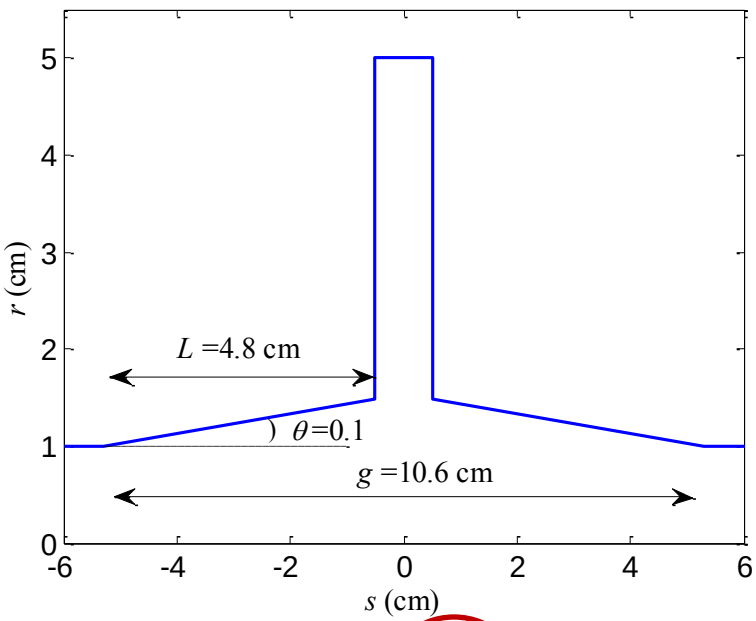


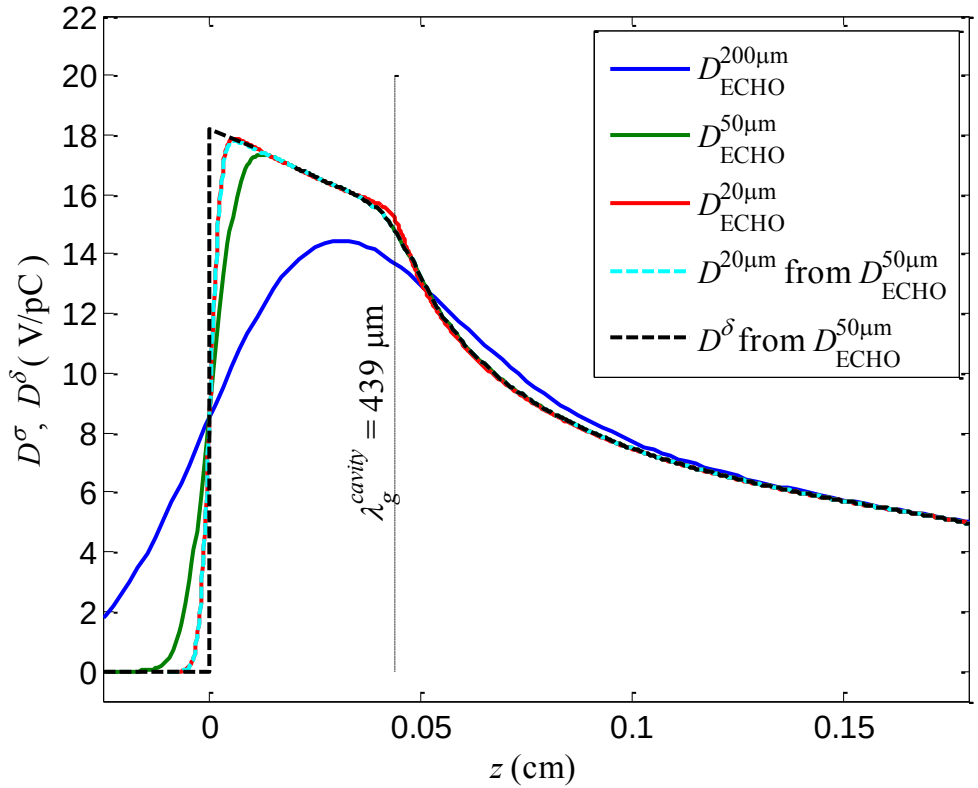
$$\lambda_g = \sqrt{(r_{\max} - r_{\min})^2 + L^2} - L \approx L\theta^2/2, \quad (\theta << 1)$$

- Wake length-scales can be understood from causality.
- Beam fields scattered at corner #1 (shown in green) will never catch up with the head, as they must clear corner #2 getting (at least) λ_g/c delay.
- Thus λ_g emerges in the front portion of the wake-field. $W^\delta(z<\lambda_g)$ depends on r_{min} and θ only, while at larger z it depends on L and r_{max} as well.
- If $r_{max}>9r_{min}$, a shorter length scale, $\lambda_g=4r_{min}\theta$, shows up, because fields that "cut across" (shown in red) acquire a delay of $4r_{min}\theta < L\theta^2/2$
- $W^\delta(z<\lambda_g)$ is very simple (see BP IPAC2012 for analytical results).

Example of a Structure with a Linear Taper

- Since λ_g is known, tapered structures can be easily handled with our method.





$$\lambda_g^{taper} = \min\left(\tfrac{1}{2} L\theta^2, 4r_{\min}\theta\right)$$

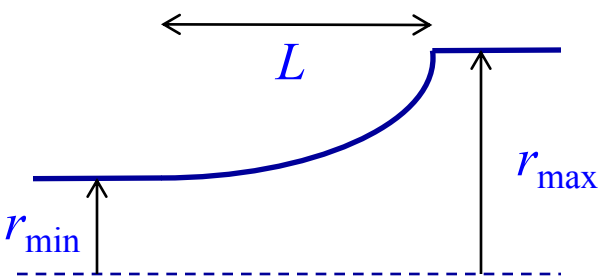
$$\lambda_g^{cavity} = \frac{\lambda_g^{taper}}{1 - L/g}$$

Smooth Non-Linear Transitions

- Smooth non-linear transitions are common (i.e. SC cavities)
- Esp. important is the case with the slope, *r'*(*s*), matched to zero at $r_{\min}$
- This case results in additional wake singularity.
- Simplest case is a quadratically varying boundary near $r_{\min}$.

$$W_{smooth}^{\delta}(0 < z << \lambda_g^{smooth}) = \kappa z^{-1/3}$$

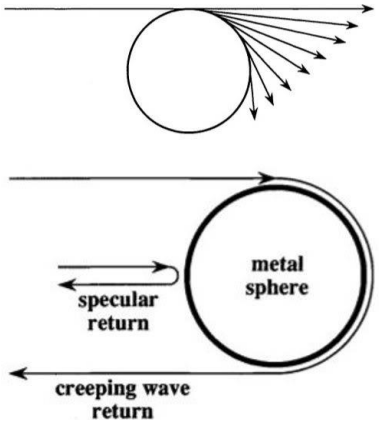
$$\lambda_g^{smooth} = \int_0^L \left(1 + r'(s)^2\right)^{\frac{1}{2}} ds - L$$



- Wake-potential can be expressed through hypergeometric functions.
- For simple cases κ can be found analytically, see BP IPAC2012.

Diversion to Creeping Waves

- Diffraction on a smoothed out edge is related to creeping waves
- A creeping wave is diffracted around the shadowed surface of a smooth body.



1948 г. Ноябрь *Т. XXXVI, вып. 3*

УСПЕХИ ФИЗИЧЕСКИХ НАУК

ЗАКОНЫ ОТРАЖЕНИЯ ФРЕНЕЛЯ И ЗАКОНЫ ДИФФРАКЦИИ

В. А. Фок

В 1821 г. французский учёный Френель установил формулы, определяющие интенсивности и направления колебаний в отражённом и преломлённом луче света, падающего на плоскую поверхность прозрачного тела.

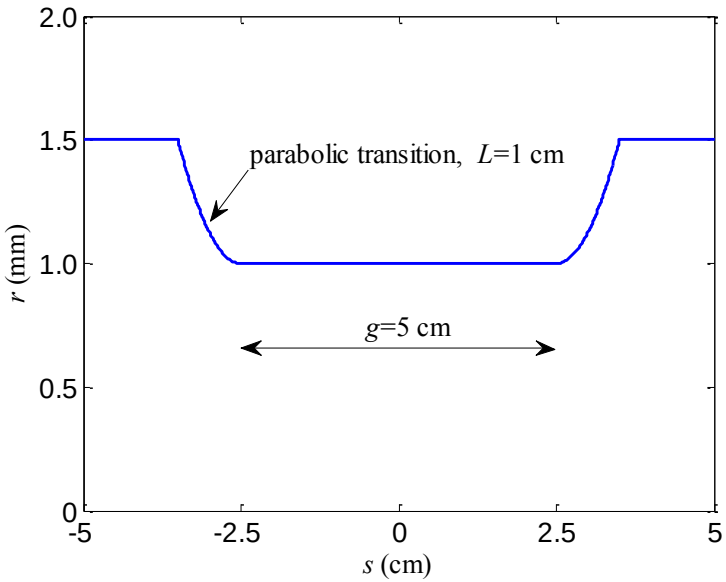
Vladimir Fock

Born	December 22, 1898 St. Petersburg, Russia
Died	December 27, 1974 (aged 76) St. Petersburg (Leningrad), Russia

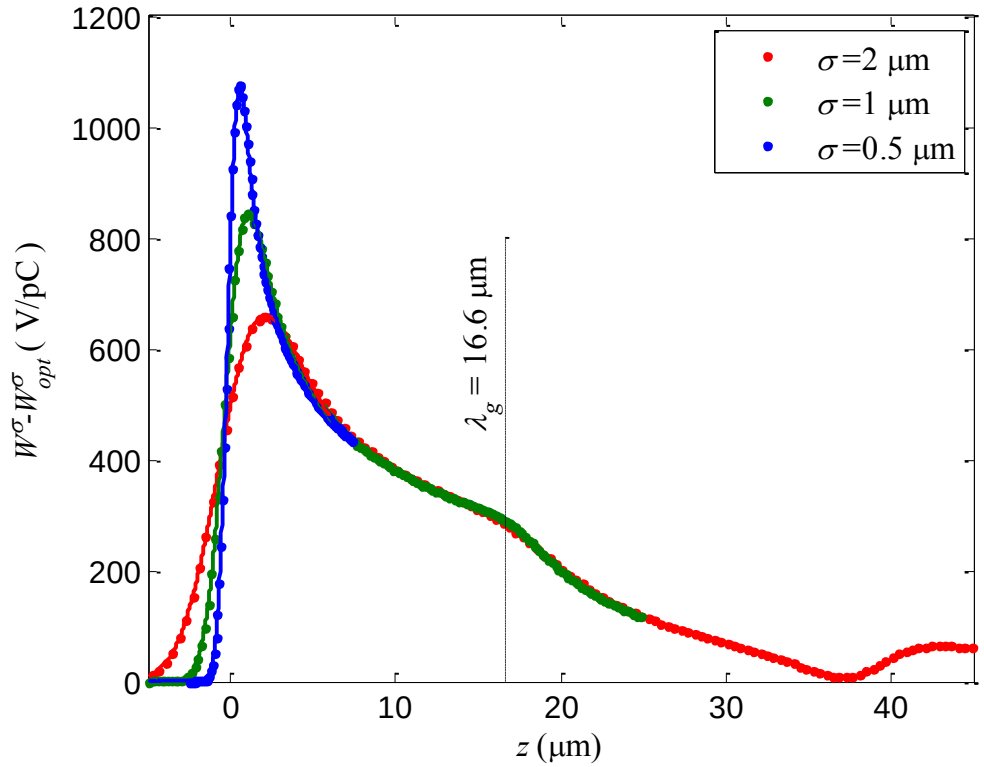
From Wikipedia

Example of a Smooth Parabolic Transition

- Since singular wakefields and λ_g are known, such structures can be easily handled with our method (for complex structures coefficient κ could be found by fitting).



$$\kappa = Z_0 c (6r'')^{-1/3} (2\pi r_{\min})^{-1}$$

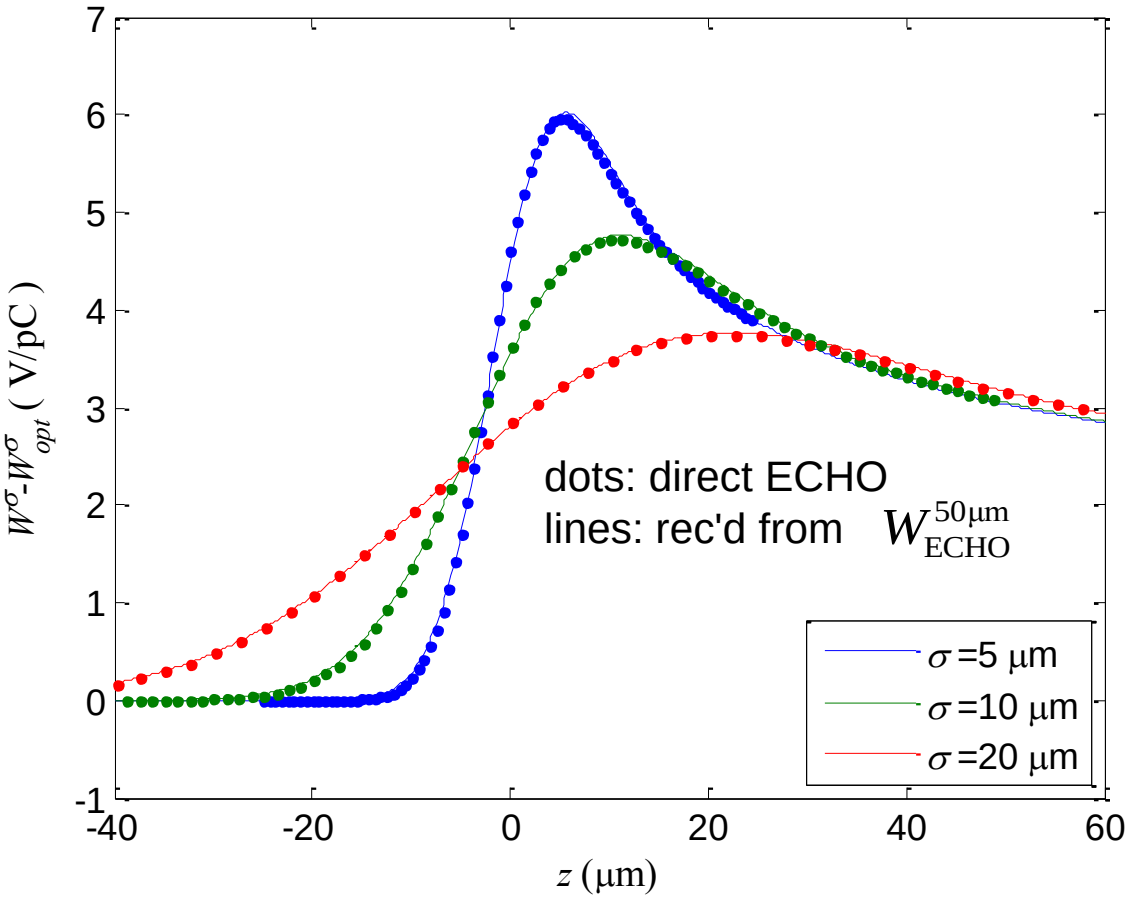


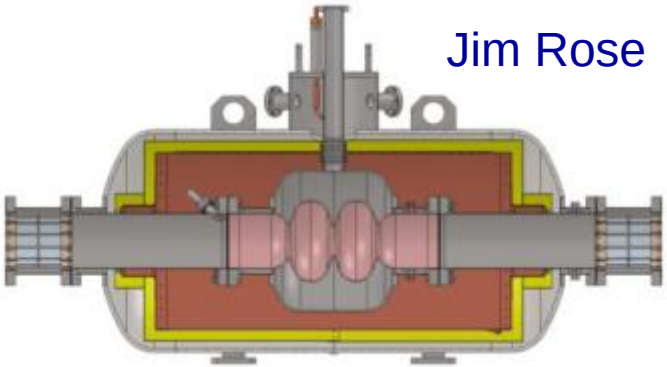
dots – direct calculations with ECHO

lines – reconstructed from ECHO calc's for 2μm bunch

NSLS-II Landau Cavity

- 1.5 GHz dual cell cavity, $r_{\text{side\_pipe}} = 6$ cm
- Final results for the short-range wakes:





Jim Rose

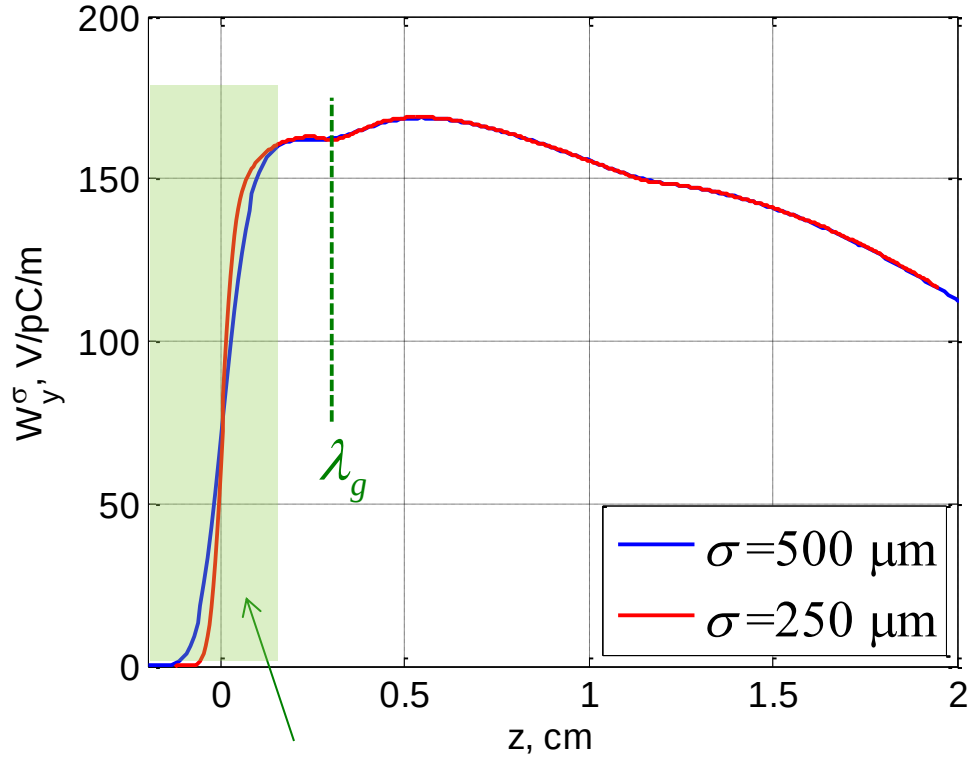
To find 10μm bunch wake

Brute force: ~480 hours of Intel(R) Xeon(R) 5570@2.93 GHz CPU to z_{max}=1 cm.

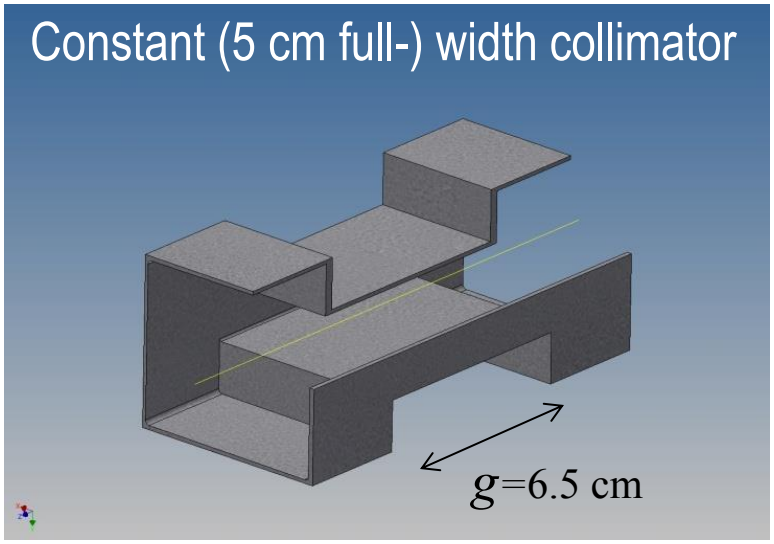
Our method: uses only σ=50μm calc's, saves a factor of 5^3 on CPU time and 5^2 on memory. Gives a model of the point-charge wake as a bonus.

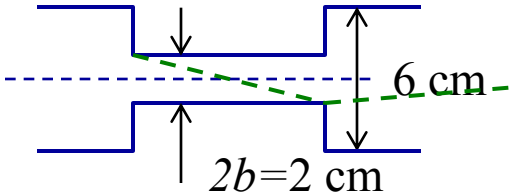
3D Example

Wakes by I. Zagorodnov,
3D ECHO + CST mesher



~(1+erf($z/\sigma/2^{1/2}$)) behavior





$$\lambda_g = \sqrt{(2b)^2 + g^2} - g = 3\,\text{mm}$$

- Observe λ_g, where expected
- Expected behavior near the origin; can easily fit point-charge wake α & β
- Same for longitudinal (+ optical model), and for quadrupolar wakes

Summary

- Wakefield calculation is important task for modern accelerators. For large and smooth accelerator structures and short bunches, direct EM solver calc's can be extremely time-consuming.
- We reviewed several recent developments in geometric wakefield calculations, including new (or more general) short-bunch asymptotic wakefield models, use of scaling laws for small angle transitions, and a new method to obtain point-charge wakefields from an EM solver.
- All the works reviewed provide valuable physics intuition, while some of them also directly result in great savings in computing time required to calculate wake-potentials due to very short bunches.
- The works described are applicable to 2D or 3D geometries.

Thank you