Review of CSR Instabilities in Electron Storage Rings

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Introduction

- Many ring based light sources operate at least part of the time in short-bunch mode, to let the beam radiate coherently producing THz radiation, *e.g.* MLS, BESSY-II, ANKA, ...
- The amount of current that can stably be stored in short-bunch mode is normally severely limited by the longitudinal microwave—or here the CSR—instability (though some, *e.g.* ANKA, are happy running above threshold)
- In recent years there has been lots of progress in understanding the CSR instability, as well as the wakefields and impedances that drive it, both in terms of theory and measurement
- I will give a brief review of the subject, focusing on the impedance and threshold calculations. The CSR instability is currently a very active field of research.
- Note: CSR is also an important topic in linac-based X-ray FELs, where *e.g.* it limits the amount a bunch can be compressed. Such effects of CSR will not be discussed here

Outline

- CSR short-range wakes
- Impedances
- Threshold calculations
- A short sampling of recent developments
- Conclusions/Discussion

I thank R. Bartolini, Y. Cai, A-.S. Mueller, R. Nagaoka, R. Warnock, ..., for helpful input

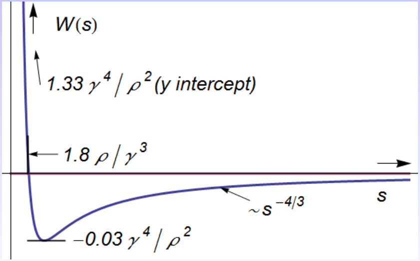
A good review of CSR instabilities can be found in Y. Cai's IPAC2011 report

Free Space CSR Model (Derbenev et al, 1995)

- Assume that two ultra-relativistic particles in free space follow each other on a circle of radius ρ. The longitudinal wake, with test particle at position s ahead of driving charge, can be approximated

$$W(s) = -\frac{2H(s)}{3^{4/3}\rho^{2/3}s^{4/3}} ,$$

with $H(s) = 1$ (0) for $s > 0$ ($s < 0$).



- Only particles in front of driving particle feel a wake (opposite to traditional wakes). Wake is negative (energy gain) and well approximated by above equation unless test particle is extremely close to driving particle (see sketch to left).

Bunch Wake

• For bunch wake, in case of smooth bunches with $\sigma_{z0} \gg \rho/\gamma^3$, don't need short range details of wake. Can integrate by parts:

$$W_\lambda(s) = -\int_0^\infty W(s')\lambda(s-s')\,ds' \approx \int_0^\infty S(s')\frac{d\lambda(s-s')}{ds'}\,ds' \ ,$$

with

$$S(s) = \int_0^s W(s')\,ds' = \frac{2H(s)}{3^{1/3}\rho^{2/3}s^{1/3}} \ .$$

[This is because $\int_0^\infty W(s)ds = 2\pi Z(0) = 0$.]

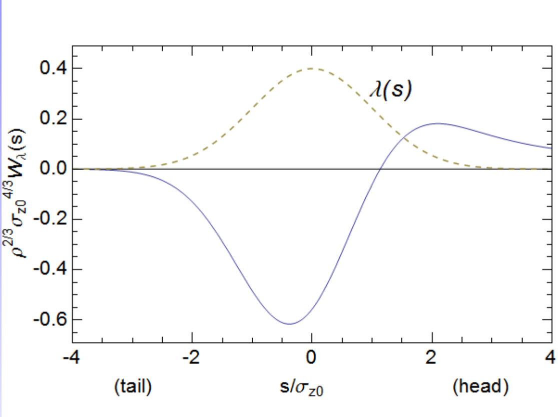
This type of calculation has long been done for similar wakes, *e.g.* the longitudinal resistive wall wake that varies as $(-s)^{-3/2}$.

• Average wake loss is given by the loss factor

$$\varkappa = -\int_{-\infty}^\infty W_\lambda(s)\lambda(s)\,ds \ .$$

Bunch energy loss/turn: $\Delta E = Q^2\varkappa\mathcal{C}$, with Q the charge, $\mathcal{C}$ the ring circumference

Wake of Gaussian Bunch



Bunch wake for a Gaussian bunch using the free space CSR model. The dashed curve gives the bunch shape with the head to the right. The loss factor $\varkappa = 0.35\rho^{-2/3}\sigma_{z0}^{-4/3}$.

Shielded CSR (Murphy, Krinsky, Gluckstern, 1997)

- Bunch moving in a circle between two parallel plates, at $y = \pm h$, is given by: $W(q) = W_0(q) + W_1(q)$, with free/shielded terms

$$W_0(s) = -\frac{2}{3^{4/3}} H(s) \frac{1}{\rho^{2/3} s^{4/3}} ,$$

$$W_1(s) = -\frac{1}{2\pi\rho^{2/3}} \left(\frac{\Pi}{\sigma_{z0}}\right)^{4/3} G\left(\frac{\Pi s}{\sigma_{z0}}\right) ,$$

with shielding parameter $\Pi = \sigma_{z0}\rho^{1/2}/h^{3/2}$

- The function G is given by

$$G(\zeta) = 8\pi \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2} \frac{Y_k(\zeta)[3 - Y_k(\zeta)]}{[1 + Y_k(\zeta)]^3} ,$$

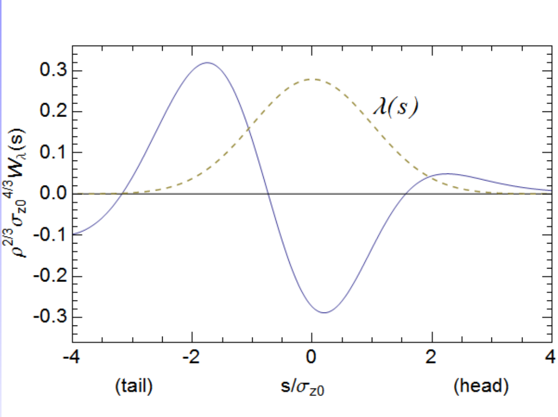
where Y_k is a root of the equation

$$Y_k - \frac{3\zeta}{k^{3/2}} Y_k^{1/4} - 3 = 0 .$$

- Typically k_{max} is taken to be ~ 25.

Shielding for Gaussian Bunch

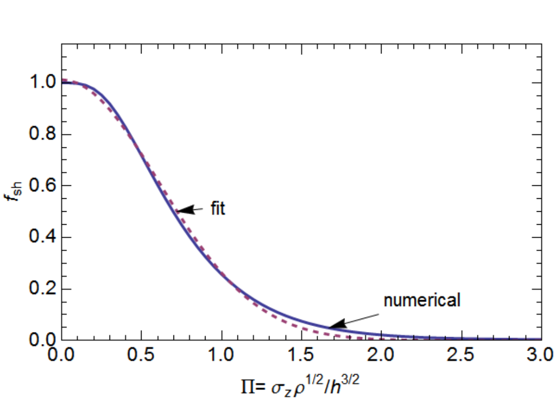
- We performed bunch wake calculations for a Gaussian bunch with shielding parameter $\Pi = 1$.



Bunch wake for a Gaussian bunch using the shielded CSR model with $\Pi = 1$. The dashed curve gives the bunch shape with the head to the right. The loss factor $\varkappa = 0.09\rho^{-2/3}\sigma_{z0}^{-4/3}$.

Effect of Shielding on Energy Loss

- We compared the loss factor $\varkappa$ for Gaussian bunches with and without shielding



Factor $f_{sh} = \varkappa_{sh}/\varkappa_{free}$ *vs.* shielding parameter Π for Gaussian beam moving between conducting plains. Shown are numerical results (blue) and the Gaussian fit $e^{-(\Pi/\sigma)^2/2}$, with $\sigma = .609$ (dashes)

Impedance

• At times it is more convenient to work in the frequency domain. The impedance is the Fourier transform of the point charge wake:

$$Z(k) = \frac{1}{c}\int_{-\infty}^{\infty} W(s)e^{iks}ds \ .$$

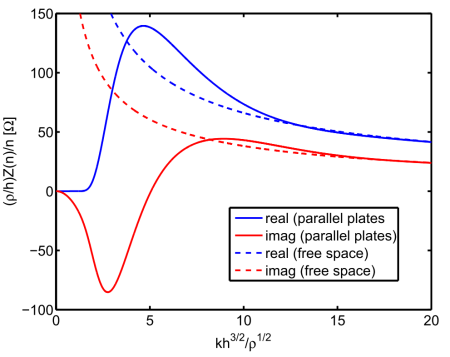
• For the case of an ultra-relativistic beam moving in a circle of radius ρ in free space (Faltens and Laslett, 1973):

$$Z(k) = \left(\frac{2\pi}{c}\right)\frac{\Gamma(2/3)(\sqrt{3}+i)}{3^{1/3}}(\rho k)^{1/3}$$

• The longitudinal instability threshold depends (as we will see) on the size of (Z/n), where $n \equiv \omega/\omega_0$, which here equals (ρk)

Shielded Impedance

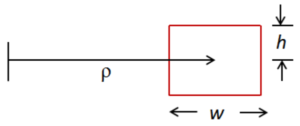
- Many have worked on this, *e.g.* Faltens, Laslett, 1973. Y. Cai has given the shielded impedance in scaled form, in terms of the Airy functions (Y. Cai, IPAC2011). Scaled frequency is $kh^{3/2}/\rho^{1/2}$



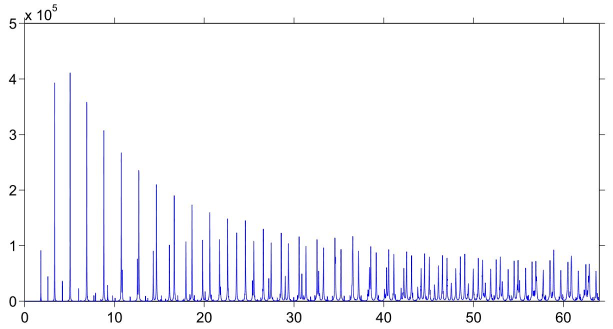
Scaled $Z(n)/n$ for CSR impedance with shielding and in free space (from Y. Cai, IPAC2011). Here h is the separation between plates

- Note the cut-off in $Re(Z)$ near the abscissa value of 2

Impedance in a Closed Vacuum Chamber



- Warnock and Morton, 1990, found the impedance in a torus with rectangular cross-section. They find whispering gallery-like modes giving spikes in $Re(Z)$



$ReZ(k)$ in ohms for ANKA parameters, *vs.* $k = 1/\lambda$ in cm^{-1}. Here $\rho = 5.6$ m, $h = 1.6$ cm, $w = 6.8$ cm (from Warnock et al, NaPAC13).

Impedance in a Closed Vacuum Chamber Cont'd

Some notes:

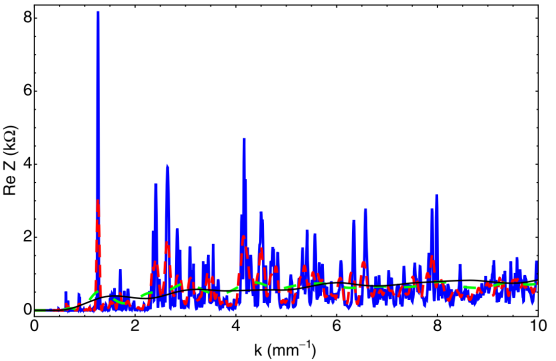
• Stupakov and Kotelnikov, 2003, assuming (h/ρ) is a small parameter, found that the paraxial approximation to the wave equation can simplify the short-range wake calculation.

Their method works for arbitrary cross-section and has been developed into algorithms (Agoh and Yokoya, 2004; D. Zhou et al, 2010).

• A bunch can excite the chamber modes in just an arc of a ring, if the arc is longer than a catch-up distance (for a wave reflecting off the outer wall)

• The CSR wake in a chamber will ring behind a driving bunch $\Rightarrow$ multibunch effects (see *e.g.* Warnock et al, NaPAC13)

Closed Chamber Cont'd



Calculated $ReZ(k)$ in one of two arc sections of the SuperKEKB e^+ damping ring (blue curve). The arc sections comprise 16 cells each, with each cell containing two opposing bend magnets and drifts. The chamber cross-section was taken to be square (D. Zhou et al, JJAP 2012).

Microwave Instability: Two Types

• Coasting beam analysis gives a (conservative) estimate of the threshold to the microwave instability (Boussard, 1975):

$$\frac{e\widehat{I}_{th}|Z/n|}{2\pi\alpha E\sigma_{p0}^2} = 1$$

with $n = ck/\omega_0$ and $|Z/n|$ taken at a representative frequency, *e.g.* at $1/\sigma_z$

• In 1994 the SLC damping ring vacuum chamber were replaced with smoother chambers to raise the threshold. Surprisingly the threshold dropped, from $(N_b)_{th} = 3 \times 10^{10}$ to 1.5–2.0 $\times 10^{10}$.

Analysis showed that we found a new kind of instability, which we call a *weak* instability. Unlike the normal (strong) microwave instability it is sensitive to the damping time $[(N_b)_{th} \sim \tau_d^{-1/2}]$ and can be suppressed by Landau damping (see *e.g.* Oide, 1994)

[Fortunately for the SLC the instability was weak, and the current could be increased to $\sim 4.5 \times 10^{10}$ in normal operations]

Microbunching Instability

- Coasting beam analysis gives the dispersion relation, $\omega(k)$:

$$1 = i\frac{r_0 c n_b Z(k)}{\gamma}\int \frac{\lambda'_\delta(\delta)d\delta}{\omega + ck\eta\delta}$$

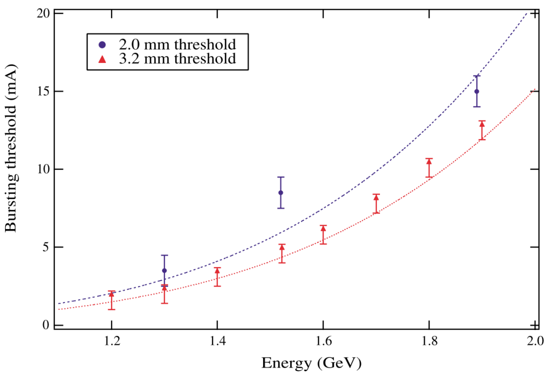
Stupakov and Heifets, 2002, solved this equation for the free space CSR model, yielding the stability criterion (if $\eta > 0$):
$k\rho > 2.0\Lambda^{3/2}$, with $\Lambda = n_b r_0/(\eta\gamma\sigma_{\delta 0}^2)$

- For electron bunch, let $n_b = N_b/(\sqrt{2\pi}\sigma_{z0})$, and require that $k > 1/\sigma_{z0}$ and the shielding parameter $\sigma_{z0}\rho^{1/2}/h^{3/2} \lesssim 0.5$

- Y. Cai (IPAC2011) repeated the calculation for the parallel plate impedance model and finds the result $(N_b)_{th} \sim \Pi^{2/3}$

Microbunching Instability Cont'd

- At the ALS bursting was detected in the far infrared



Bursting threshold as function of electron beam energy at 3.2 and 2 mm wavelength at the ALS. Measurements are given as symbols; the lines are calculated thresholds using the Stupakov/Heifets result with nominal ALS parameters (Byrd et al, PRL 2002).

Simulations: Haïssinski Solution

- For an electron ring below the threshold to the microwave instability, the beam energy distribution is Gaussian with rms $\sigma_{\delta 0}$, and the longitudinal bunch distribution is given by the solution to the Haïssinski equation (Haïssinski 1973):

$$\lambda(s) = \frac{e^{-\frac{1}{2}\left(\frac{s}{\sigma_{z0}}\right)^2} - \frac{I\mathcal{C}}{\sigma_{z0}} \int_0^\infty S(s')\lambda(s-s')\,ds'}{\int_{-\infty}^{\infty} \lambda(s')\,ds'}$$

with $S(s) = \int_0^s W(s')\,ds'$, $\mathcal{C}$ is ring circumference, and normalized current

$$I = \frac{e^2 N_b}{2\pi\nu_{s0} E_0 \sigma_{\delta 0}} \ .$$

- R. Warnock has developed an efficient algorithm for solving this equation

Macroparticle Tracking (A. Renieri, 1976)

- The development of longitudinal phase space can be followed using macroparticle tracking
- Track $i = M$ particles, with energy p_i (normalized to $\sigma_{\delta 0}$) and longitudinal position q_i (normalized to σ_{z0}), over $n = N_T$ time steps:

$$\left(\begin{array}{c} p_i \\ q_i \end{array} \right)_{n+1} = \left[\begin{array}{cc} 1 - c_d & -\theta \\ (1 - c_d)\theta & 1 - \theta^2 \end{array} \right] \left(\begin{array}{c} p_i \\ q_i \end{array} \right)_n + \left[\begin{array}{c} 1 \\ \theta \end{array} \right] (\theta I \mathcal{C} W_\lambda(q_i \sigma_{z0}) + r_e)$$

with time step $\theta = 2\pi \Delta t / T_s$, and c_d and r_e represent effects of radiation damping and quantum excitation

- After each time step the bunch distribution $\lambda(s)$ and then $W_\lambda(s)$ are recomputed
- Algorithm is easy to implement (see *e.g. Elegant*), though results tend to be noisier than the Vlasov-Fokker-Planck solutions

Vlasov-Fokker-Planck (VFP) Equation Solver

- Beam longitudinal density distribution $\psi(\theta, q, p)$ follows

$$\frac{\partial\psi}{\partial\theta} - \{H, \psi\} = 2\beta \frac{\partial}{\partial p}(p\psi + \frac{\partial\psi}{\partial p}) \ .$$

The independent variable $\theta = \omega_{s0} t$, with ω_{s0} synchrotron frequency and t time. Here phase space variables $q = z/\sigma_{z0}$ and $p = -\delta/\sigma_{\delta 0}$; $\{f, g\}$ is a Poisson bracket; $\beta = 1/\omega_s \tau_p$

- The Hamiltonian

$$H(\theta, q, p) = \frac{1}{2}(q^2 + p^2) - I \int_{-\infty}^{q} dq'' \int_{-\infty}^{\infty} dq' \lambda(\theta, q')\, w(q'' - q') \ ,$$

with longitudinal distribution $\lambda(\theta, q) = \int_{-\infty}^{\infty} \psi(\theta, q, p) dp$, and $w(q) \equiv \mathcal{C} W(q\sigma_{z0})$ is point charge wake (per turn).

- Warnock and Ellison in 2000 developed a robust algorithm for solving the VFP equation. It follows the development of the distribution function ψ on a rectangular mesh in phase space

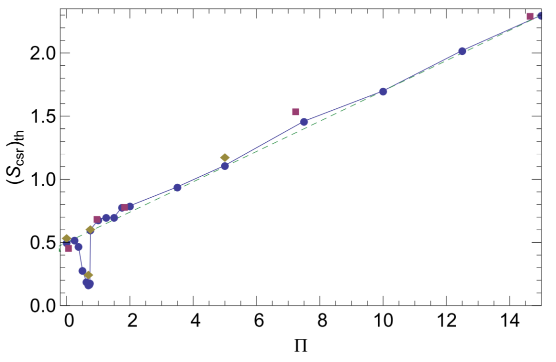
Threshold Study for Shielded CSR

- Bane, Cai, Stupakov, 2010, used Warnock and Ellison's algorithm, for the case of the shielded CSR wake, to find the normalized threshold current, $S_{\rm csr} = I\rho^{1/3}/\sigma_{z0}^{4/3}$, as function of shielding parameter, $\Pi = \rho^{1/2}\sigma_{z0}/h^{3/2}$.

- Every run was begun with the nominal Gaussian distribution in energy, and the Haïssinski distribution in position (for a quiet start), and phase space was followed for many synchrotron periods.

- The current at which the energy spread σ_p begins to grow was designated the threshold current, $(S_{csr})_{th}$

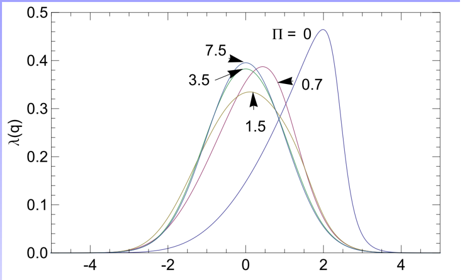
Threshold Current vs Shielding Parameter

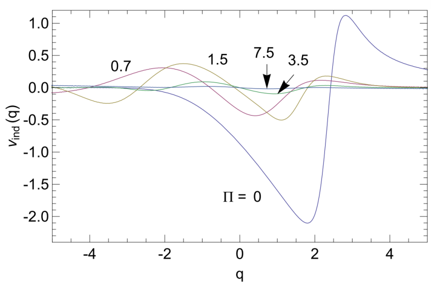


For the CSR wake, threshold value of $S_{\mathrm{csr}} = I\rho^{1/3}/\sigma_{z0}^{4/3}$ *vs.* shielding parameter, $\Pi = \rho^{1/2}\sigma_{z0}/h^{3/2}$. Symbols give results of the VFP solver (blue circles) and the VFP solver with twice stronger radiation damping (olive diamonds).

- Except in a "dip region" around $\Pi = 0.7$, curve fits well to $(S_{csr})_{th} = 0.5 + 0.12\Pi$ (the dashed line in the figure)

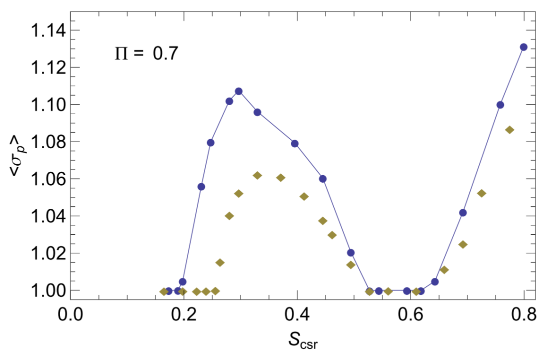
Bunch Shape and Induced Voltage at Threshold





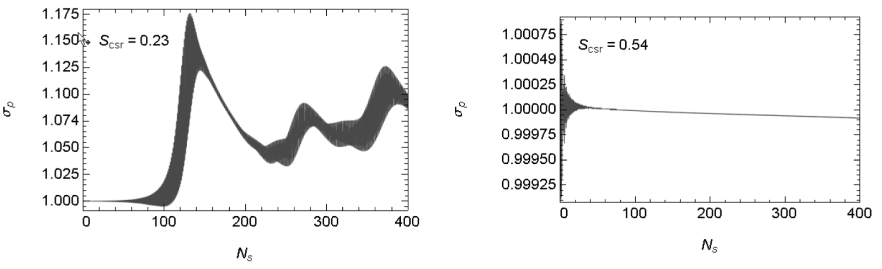
Haïssinski solution at threshold and wake induced voltage for shielded CSR, for the cases $\Pi = 0.0, 0.7, 1.5, 3.5, 7.5$. Note that bunch head is to the right.

Study of Dip Region



For shielded CSR with $\Pi = 0.7$, the value of σ_p, averaged over 400 synchrotron periods, $\langle\sigma_p\rangle$, as given by the VFP simulations. The case of nominal damping $\beta = 1.25 \times 10^{-3}$ is given by blue circles (connected by straight lines), the case of $\beta = 2.5 \times 10^{-3}$ by olive diamonds.

Study of Dip Region Cont'd



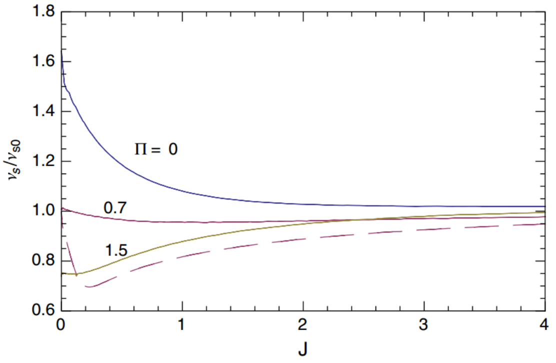
For shielded CSR with $\Pi = 0.7$ and nominal damping, rms of p coordinate, σ_p, *vs.* synchrotron period number, N_s, for cases $S_{\rm csr} = 0.23$ and 0.54.

• We believe the first instability (beginning at $\Pi \sim 0.2$) is a weak instability (sensitive to τ_d), and the second one (at $\Pi \sim 0.6$) is a normal, strong one (relatively insensitive to τ_d)

Study of Dip Region Cont'd

- Incoherent tune in potential well $u(q)$:

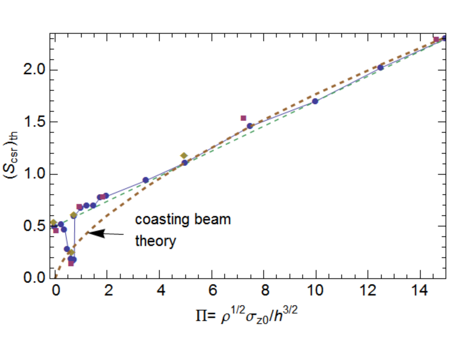
$$\nu_s(J) = \nu_{s0} \left[\frac{1}{\sqrt{2}\pi} \int_{q_b}^{q_e} \frac{dq}{\sqrt{u(q_b) - u(q)}} \right]^{-1}$$



Incoherent tune at threshold due to the distorted potential well as function of action for values of $\Pi = 0, 0.7, 1.5$ (the solid curves). Dashed curve is in second quiet region of $\Pi = 0.7$, with $S_{csr} = 0.54$

Compare with Threshold of Coasting Beam Theory

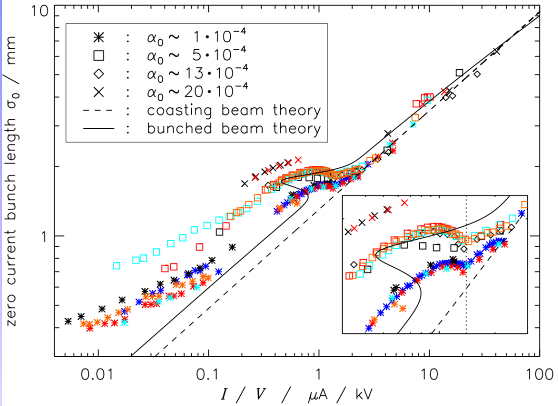
Coasting beam theory predicts, for parallel plate impedance, $(S_{csr})_{th} = \frac{3}{\sqrt{2}\pi^{3/2}}\Pi^{2/3}$ (Y. Cai IPAC2011)



Comparing threshold current $(S_{csr})_{th}$ as function of shielding parameter Π as obtained by coasting beam theory (brown, dashed curve) to that given by the fit to the VFP simulations.

- The two curves agree well for the longer bunches, for $\Pi \gtrsim 3.5$

Metrology Light Source Measurement

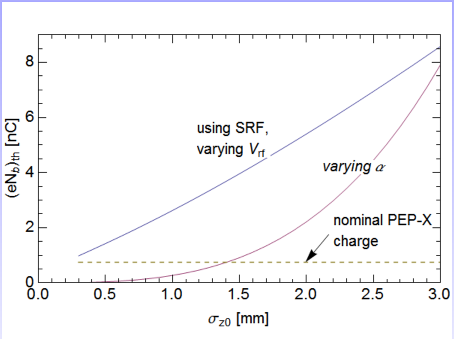


Scaled values of measured bursting thresholds. Different symbols correspond to different α_0. Colors indicate measurement series within one α_0 set. The small inset is a zoom in the region where the sets start to spread. The dotted line marks $I/V = 1.4\ \mu A/kV$ (from M. Ries et al, IPAC2012).

Adding RF for a Shorter Bunch Length

$(S_{csr})_{th} = 0.5 + 0.12\Pi$ can be expanded out (*cf.* Y. Cai talk, 2011)

$$\sigma_{z0}^{7/3} = \frac{Z_0 c^2}{8\pi^2} \frac{(eN_b)_{th}}{(S_{csr})_{th}} \frac{\rho^{1/3}}{f_{rf} V_{rf} \cos\phi_s}$$



Microwave threshold in PEP-X due to shielded CSR, when using SRF and varying V_{rf} (blue), or normal RF and adjusting α (red). Note: SRF has factor 3 higher frequency ($f_{rf} = 1.4$ GHz)

Some Recent Topics

- *More RF:* BESSY II is considering installing SRF to increase the RF gradient by factor 100 and thus increase the current that can be stably stored in short bunch mode by factor of ~ 100 (Wuestefeld et al, IPAC2011)
- *Multi-bunch wake:* Using a fast bolometer, THz radiation was measured in ANKA that can distinguish contributions from individual bunches. Such measurements, and others at the CLS, can be used to compare the impedance with theoretical models
- *Fast monitor:* A near-field, electro-optical bunch length monitor has been installed in ANKA that possibly can record the CSR wake as well as the bunch shape variations in bursting mode (Hiller et al, IPAC2013)

Some Recent Topics Cont'd

- *Numerical study of CSR induced microbunching instability:* There is an ambitious program to understand specific features in simulated phase space of an unstable beam—such as "fingers"—and their connection with spectra obtained by bolometer-type measurements (Evain et al, EPL 2012)
- *Burst control:* At the ALS bursting was excited during laser "slicing" of the beam; the bursting amplitude was random but the timing was synchronized to the laser (Byrd et al, 2006). At the UVSOR facility in Japan bursting seeding experiments were performed and compared to simulations (Roussel et al, 2013)
- There have been studies at Diamond, Soleil, ...

Conclusions/Discussion

- We have presented the models that are used to represent the CSR wake/impedance, and the resulting predictions for the threshold to the microwave instability
- We have presented a simple relation for the threshold to the fast (strong) instability, and have also indicated a parameter region where a weak instability is excited
- With the shielded CSR model, in the region of significant shielding ($\Pi \gtrsim 3.5$), numerical simulations are in good accord with the coasting beam threshold
- For machines like MLS, BESSY II, ANKA, thresholds are in reasonably good agreement with the calculated results, including indications of a second, weak instability region, indicating impedances that are dominated by CSR
- Many topics not covered: details of closed vacuum chamber impedances, beam behavior above threshold, connection of beam to bolometer measurements of radiation, ...