Lepton number violating mSUGRA and neutrino masses

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This talk represents edited highlights of

arXiv:0712.0852 (hep-ph) B. Allanach, CHK

Outline

- Neutrino masses in Lepton number violating (LNV) SUSY.
- Additional issues in high scale models (mSUGRA).
- Numerical procedure.
- Comments and Summary.

Neutrino oscillations

- Goal: obtain simple models with best-fit neutrino oscillations data Gonzalez-Garcia, Maltoni 0704.1800

$$\begin{aligned}
\Delta m^2_{21} &= 7.9^{+0.27}_{-0.28} \times 10^{-5}\text{eV}^2, \\
|\Delta m^2_{31}| &= 2.6 \pm 0.2 \times 10^{-3}\text{eV}^2, \\
\sin^2\theta_{12} \equiv s^2_{12} &= 0.31 \pm 0.02, \\
\sin^2\theta_{23} \equiv s^2_{23} &= 0.47^{+0.08}_{-0.07}, \\
\sin^2\theta_{13} \equiv s^2_{13} &= 0^{+0.008}_{-0.0}.
\end{aligned}$$

- $Z_{PMNS} = Z_l^\dagger Z_\nu =$

$$\begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}s_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13}
\end{pmatrix}.$$

LNV in SUSY

- Assume Standard Model (SM) (super-) field content:

$$\begin{array}{lllllllll} Q & : & (3,2,\frac{1}{6}), & \bar{U} & : & (\bar{3},1,-\frac{2}{3}), & \bar{D} & : & (\bar{3},1,\frac{1}{3}), \\ L & : & (1,2,-\frac{1}{2}), & H_d & : & (1,2,-\frac{1}{2}), & & & \\ \bar{E} & : & (1,1,1), & H_u & : & (1,2,\frac{1}{2}). & & & \end{array}$$

- Most general superpotential leads to fast proton decay:

$$\begin{aligned} \mathcal{W}_{RPC} &= (Y_E)_{ij} L_i H_d \bar{E}_j + (Y_D)_{ij} Q_i H_d \bar{D}_j + (Y_U)_{ij} Q_i H_u \bar{U}_j - \mu H_d H_u, \\ \mathcal{W}_{LNV} &= \frac{1}{2}\lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k - \mu_i L_i H_u + \frac{1}{2}\lambda''_{ijk} \bar{U}_i \bar{U}_j \bar{D}_k, \end{aligned}$$

  - Baryon parity Ibanez, Ross PLB 260 good alternative to R-parity.
  - Lepton number violated, with Majorana neutrino masses.

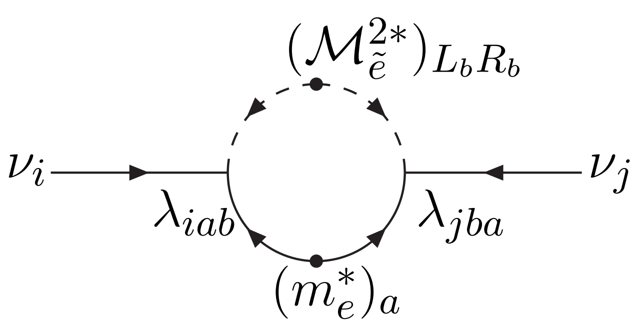
Seesaw mechanism: effective m_ν

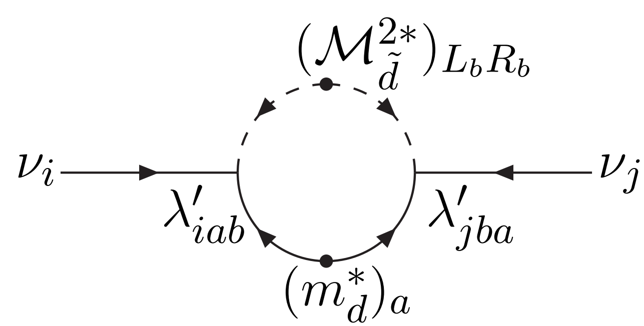
- ‘Electroweak seesaw’: neutralinos as the heavy fields
- At tree level ONE massive neutrino only Joshipura, Nowakowski PRD 51 : radiative corrections important

$$\mathcal{M}^{\nu}_{\text{eff}} = \frac{(M_1 g_2^2 + M_2 g^2)}{2\mu[v_u v_d (M_1 g_2^2 + M_2 g^2) - \mu M_1 M_2]} \begin{pmatrix} \Lambda_e\Lambda_e & \Lambda_e\Lambda_\mu & \Lambda_e\Lambda_\tau \\ \Lambda_\mu\Lambda_e & \Lambda_\mu\Lambda_\mu & \Lambda_\mu\Lambda_\tau \\ \Lambda_\tau\Lambda_e & \Lambda_\tau\Lambda_\mu & \Lambda_\tau\Lambda_\tau \end{pmatrix},$$

$$\Lambda_i \equiv \mu v_i - v_d \mu_i, \qquad i = \{e, \mu, \tau\}.$$

- Example: mass corrections





Literature

Loop calculations

- Grossman, Rakshit PRD 69, Grossman, Haber PRL 78, Dedes, Rimmer, Rosiek JHEP 0608

Phenomenological models (weak scale)

- bilinear Kaplan, Nelson JHEP 0001, Nilles, Polonsky NPB 484
- trilinear Joshipura, Vempati PRD 60
- mixed Borzumati, Grossman, Nardi, Nir PLB 384, Cheung, Kong PRD 61, Dedes, Rimmer, Rosiek JHEP 0608

Neutrino masses in LNV mSUGRA

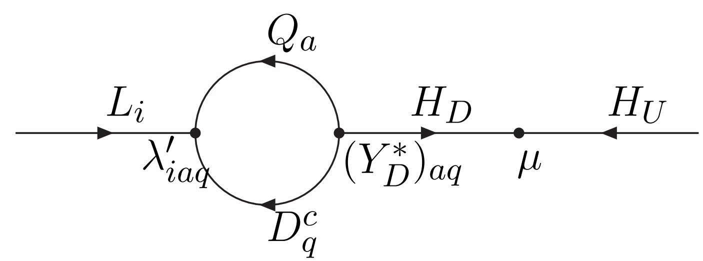
- At $M_X \sim 10^{16}$GeV Allanach, Dedes, Dreiner PRD 69 :
  - R-parity conserving (RPC) parameters: $m_0, M_{1/2}, A_0,$ sgnμ.
  - 0 bilinear LNV pars $\mu_i, \tilde{D}_i, m^2_{L_i H_d}$: rotated away.
  - 2 trilinear LNV pars: $\lambda_{ijk}, \lambda'_{ijk}$.
  - 3 charged lepton mixing angles.

  At M_Z: tan$\beta = v_u/v_d$.

Neutrino masses in LNV mSUGRA

- Renormalization effect important, e.g.

$$16\pi^2 \frac{d}{dt}\mu_i \simeq -\mu(\lambda_{ijk}(Y_E^*)_{jk} + 3\lambda'_{ijk}(Y_D^*)_{jk})$$



- Tree level m_ν dominates in general: suppression by interplay of the 2 LNV parameters.

Neutrinos in LNV mSUGRA: literature

Cosmological neutrino mass bound

- Allanach, Dedes, Dreiner PRD 69

Phenomenological models (GUT scale)

- bilinear Hempfling NPB 478, Hirsch, Diaz, Porod et al PRD 65
- trilinear Chun, Kang, Kim, Lee NPB 544

Numerical Procedure

MINUIT F. James finds best fit parameters of

$$\theta^l_{12}, \theta^l_{13}, \theta^l_{23}, \Lambda_1, \Lambda_2, \in \{\lambda_{ijk}, \lambda'_{ijk}\} @ M_X.$$

SOFTSUSY Allanach CPC 143 :

- Specify RPC parameters (SPS1a benchmark)

$$m_0 = 100\text{GeV}, M_{1/2} = 250\text{GeV}, A_0 = -100\text{GeV}, \text{sgn}\mu = + \quad @ \quad M_X,$$
$$\tan\beta = 10 \quad @ \quad M_Z,$$

- Rotate to diagonal lepton basis using θ^l @ M_X.
- Fit boundary conditions @ M_Z.
- EWSB conditions @ M_{SUSY}.
- Calculate m_ν @ M_{SUSY}.
- χ^2 of the neutrino oscillations data.

Numerical results @ SPS1a

Normal hierarchy						
Λ_1	Λ_2	θ^l_{12}	θ^l_{13}	θ^l_{23}	Δ_{FT}	χ^2
$^a\lambda'_{233} = -2.49978 \times 10^{-6}$	$\lambda_{233} = 4.06508 \times 10^{-5}$	0.459520	0.388989	0.304863	8.09	-
$^a\lambda'_{233} = -2.50019 \times 10^{-6}$	$\lambda_{211} = 4.06533 \times 10^{-5}$	1.98935	1.08162	0.632130	8.10	-
$^a\lambda'_{233} = -3.41336 \times 10^{-6}$	$\lambda_{321} = 9.86746 \times 10^{-5}$	0.448321	0.400030	2.89062	12.4	-
$^b\lambda'_{122} = -1.14066 \times 10^{-4}$	$\lambda_{122} = 4.06346 \times 10^{-5}$	1.19298	0.190538	1.17391	11.8	-
$^b\lambda'_{122} = -8.97777 \times 10^{-5}$	$\lambda_{123} = 1.02771 \times 10^{-4}$	2.10672	0.174800	1.18124	9.44	-
$^b\lambda'_{122} = -8.59626 \times 10^{-5}$	$\lambda_{133} = 4.09647 \times 10^{-5}$	0.997963	0.281922	0.417935	8.00	-
Inverted hierarchy						
Λ_1	Λ_2	θ^l_{12}	θ^l_{13}	θ^l_{23}	Δ_{FT}	χ^2
$^a\lambda'_{233} = -5.69116 \times 10^{-6}$	$\lambda_{233} = 1.36023 \times 10^{-4}$	1.55779	0.815115	0.146126	755	0.01
$^a\lambda'_{233} = -5.69126 \times 10^{-6}$	$\lambda_{211} = 1.32365 \times 10^{-4}$	1.38843	0.760045	0.140903	758	0.06
$^a\lambda'_{233} = -5.67940 \times 10^{-6}$	$\lambda_{123} = 1.42938 \times 10^{-4}$	1.81386	-0.757538	0.141975	726	0.05
$^b\lambda'_{122} = -1.96283 \times 10^{-4}$	$\lambda_{122} = 1.24824 \times 10^{-4}$	0.134765	0.101938	0.798282	988	0.43
$^b\lambda'_{122} = -1.93673 \times 10^{-4}$	$\lambda_{132} = 1.47364 \times 10^{-4}$	3.03234	0.0866645	0.931616	743	2.85
$^b\lambda'_{122} = -1.96175 \times 10^{-4}$	$\lambda_{123} = 1.45708 \times 10^{-4}$	0.144386	0.0943266	0.689708	736	0.52

Comment

- Mild tuning in normal hierarchy. Stronger fine tuning in inverted hierarchy.
- Near tri-bi maximal mixing accidental.
- 2 LNV couplings: phenomenological studies manageable.

Summary

- Lepton number violating SUSY models provides alternative See-Saw mechanism to neutrino masses.
- Possible to promote to unified models, e.g. minimal supergravity (mSUGRA), with additional issues.
- Performed quantitative study of mSUGRA with 2 GUT-scale trilinear LNV couplings and three charged lepton mixing angles.

Thank you.

Decay widths

λ_{ijk}	Channel	BR	Channel	BR
λ_{122}	$e^- \ \nu_\mu \ \mu^+$	0.006	$\mu^- \ \nu_e \ \mu^+$	0.006
λ_{132}	$e^- \ \nu_\tau \ \mu^+$	0.007	$\tau^- \ \nu_e \ \mu^+$	0.007
λ_{123}	$e^- \ \nu_\mu \ \tau^+$	0.029	$\mu^- \ \nu_e \ \tau^+$	0.029
λ_{133}	$e^- \ \nu_\tau \ \tau^+$	0.034	$\tau^- \ \nu_e \ \tau^+$	0.034
λ_{231}	$\mu^- \ \nu_\tau \ e^+$	0.005	$\tau^- \ \nu_\mu \ e^+$	0.005
λ_{232}	$\mu^- \ \nu_\tau \ \mu^+$	0.027	$\tau^- \ \nu_\mu \ \mu^+$	0.028
λ_{233}	$\mu^- \ \nu_\tau \ \tau^+$	0.138	$\tau^- \ \nu_\mu \ \tau^+$	0.140

(a) Normal hierarchy. Best fit: $\lambda_{233} = 4.07e^{-5}, \lambda'_{233} = -2.50e^{-6}$, $\theta^l_{12} = 0.460, \theta^l_{13} = 0.389, \theta^l_{23} = 0.305$ @ SPS1a.

λ_{ijk}	Channel	BR	Channel	BR
λ_{122}	$e^- \ \nu_\mu \ \mu^+$	0.063	$\mu^- \ \nu_e \ \mu^+$	0.063
λ_{132}	$e^- \ \nu_\tau \ \mu^+$	0.056	$\tau^- \ \nu_e \ \mu^+$	0.057
λ_{123}	$e^- \ \nu_\mu \ \tau^+$	0.067	$\mu^- \ \nu_e \ \tau^+$	0.067
λ_{133}	$e^- \ \nu_\tau \ \tau^+$	0.059	$\tau^- \ \nu_e \ \tau^+$	0.060

(b) Inverted hierarchy. Best fit: $\lambda_{233} = 1.36023e^{-4}, \lambda'_{233} = -5.69116e^{-6}$, $\theta^l_{12} = 1.55779, \theta^l_{13} = 0.815155, \theta^l_{23} = 0.146126$ @ SPS1a.

Alignment

Tree level effective mass matrix

$$(\mathcal{M}^{\nu}_{\text{eff}})_{in} \quad \propto \quad \Big[\lambda_{ijk}(Y^*_E)_{jk}\lambda_{nlm}(Y^*_E)_{lm}\Big].$$

One loop mass corrections

$$(m_\nu)_{in} \quad \simeq \quad \sum_{j,k,l,m}\Big\{\frac{1}{(4\pi)^2}\lambda_{ijk}\lambda_{nlm}\frac{(m^*_e)_{jm}(\mathcal{M}^{2*}_{\tilde{e}_{LR}})_{lk}}{\overline{\mathcal{M}}^2_{\tilde{e}_{LL}}-\overline{\mathcal{M}}^2_{\tilde{e}_{RR}}}\ln\Big(\frac{(\overline{\mathcal{M}}^2_{\tilde{e}_{LL}})}{(\overline{\mathcal{M}}^2_{\tilde{e}_{RR}})}\Big)\Big[1+\mathcal{O}\Big(\frac{\delta\mathcal{M}^2}{\overline{\mathcal{M}}^2}\Big)\Big]$$
$$+(i\leftrightarrow n)\Big\}.$$

One LNV coupling: i=n, j=l, k=m:

$$\mathcal{M}^{\nu}_{\text{eff}} \quad \propto \quad m_\nu + m_\nu\mathcal{O}\Big(\frac{\delta\mathcal{M}^2}{\overline{\mathcal{M}}^2}\Big).$$

Tree- loop mass ratio

Tree level mass scale

$$m_\nu^{tree} \quad \simeq \quad -\frac{8\pi\alpha_{GUT}}{5M_{1/2}}[\frac{v_d}{16\pi^2}]^2[\ln\frac{M_X}{M_Z}]^2[\lambda_{ijq}(Y_E^*)_{jq}]^2 f^2(\frac{\mu^2}{m_0^2};\frac{A_0^2}{m_0^2};\frac{\tilde{B}}{m_0^2};\tan\beta),$$

Loop level mass scale

$$m_\nu^{loop} \quad \equiv \quad \sum_{i=1}^{3}\left(\Sigma_{D\,ii}\right) \simeq 2\sum_{i,j,k}\lambda_{iL_jR_k}\lambda_{iL_kR_j}\frac{m^*_{f_j}}{(4\pi)^2}\frac{(\mathcal{M}^{2*}_{\tilde{f}})_{L_kR_k}}{(\mathcal{M}^2_{\tilde{f}})_{L_kL_k}-(\mathcal{M}^2_{\tilde{f}})_{R_kR_k}}\ln\frac{(\mathcal{M}^2_{\tilde{f}})_{L_kL_k}}{(\mathcal{M}^2_{\tilde{f}})_{R_kR_k}}$$

Tree-loop mass ratio $m_\nu^{tree}/m_\nu^{loop}$

$$\simeq \quad -n_c\frac{\alpha_{GUT}\ln^2(M_X/M_Z)}{10\pi M_{1/2}(A_0-\mu\tan\beta)}\frac{(\mathcal{M}^2_{\tilde{f}})_{L_kL_k}-(\mathcal{M}^2_{\tilde{f}})_{R_kR_k}}{\ln((\mathcal{M}^2_{\tilde{f}})_{L_kL_k}/(\mathcal{M}^2_{\tilde{f}})_{R_kR_k})}f^2(\frac{\mu^2}{m_0^2};\frac{A_0^2}{m_0^2};\frac{\tilde{B}}{m_0^2};\tan\beta)$$

Experimental contraints

- Bounds on FCNC, rare decays, etc Allanach, Dedes, Dreiner PRD 60 .

	$BR_{exp}(\mu \to e\gamma)$	$< 1.2 \times 10^{-11}$	90%CL	MEGA PRD 65 ,
	$BR_{exp}(\tau \to e\gamma)$	$< 1.1 \times 10^{-7}$	90%CL	BABAR PRL 96 ,
	$BR_{exp}(\tau \to \mu\gamma)$	$< 6.8 \times 10^{-8}$	90%CL	BABAR PRL 95 ,
	$BR_{exp}(B_s \to \mu^+\mu^-)$	$< 1.0 \times 10^{-7}$	95%CL	CDF 2006 ,
$2.76 \times 10^{-4} <$	$BR(b \to s\gamma)$	$< 4.34 \times 10^{-4}$	2σ	

  Allanach, Bernhardt, Dreiner, CHK, Richardson PRD 75 .

- Normal hierarchy $\frac{m_3}{m_2} = 5.74 \pm 0.32,$ @ 1σ.
- Inverted hierarchy $\frac{m_2}{m_1} = 1.0151 \pm 0.0769.$ @ 1σ.

Fine-tuning measure

- Ellis, Enqvist, Nanopoulos, Zwirner Mod PLA 1

$$\Delta_{FT} = \left|\frac{\partial \ln(\mathcal{O}(\lambda'))}{\partial \ln\lambda'}\right|,$$

- Our numerical results:

$$\Delta_{FT} \simeq \left|\frac{\ln(\mathcal{O}(\lambda'_a)) - \ln(\mathcal{O}(\lambda'_b))}{\ln\lambda'_a - \ln\lambda'_b}\right|,$$

$$\lambda'_a = \lambda'_b \times (1 - 2.0 \times 10^{-4}).$$

Neutrino mass generation

Some examples

- Dirac neutrinos
  - Neutrino singlet, lepton number conservation: $\mathcal{L} = (Y_N)_{ij} L_i H_u N_j$
- Majorana neutrinos
  - Neutrino singlet, lepton number violation: $\mathcal{L} = (Y_N)_{ij} L_i H_u N_j + \frac{1}{2} M_{ij} N_i N_j$.
  - Higgs triplet, lepton number violation: $\mathcal{L} = (Y_N)_{ij} L_i \phi L_j$.

Soft SUSY breaking lagragian with LNV

- Most general soft $\not{S}USY$ lagrangian consistent with Baryon parity.

$$
\begin{aligned}
-\mathcal{L}_{soft} &= \mathcal{L}^{mass}_{RPC} + \mathcal{L}^{int}_{RPC} + \mathcal{L}_{LNV}, \\
\mathcal{L}^{mass}_{RPC} &= \frac{1}{2}M_1\tilde{B}\tilde{B} + \frac{1}{2}M_2\tilde{W}\tilde{W} + \frac{1}{2}M_3\tilde{g}\tilde{g} + h.c. \\
&\quad + \tilde{Q}^\dagger(m^2_{\tilde{Q}})\tilde{Q} + \tilde{\bar{U}}^\dagger(m^2_{\tilde{\bar{U}}})\tilde{\bar{U}} + \tilde{\bar{D}}^\dagger(m^2_{\tilde{\bar{D}}})\tilde{\bar{D}} + \tilde{L}^\dagger(m^2_{\tilde{L}})\tilde{L} + \tilde{\bar{E}}^\dagger(m^2_{\tilde{\bar{E}}})\tilde{\bar{E}} \\
&\quad + m^2_{h_u}h_u^\dagger h_u + m^2_{h_d}h_d^\dagger h_d, \\
\mathcal{L}^{int}_{RPC} &= (h_E)_{ij}\tilde{L}_i h_d \tilde{\bar{E}}_j + (h_D)_{ij}\tilde{Q}_i h_d \tilde{\bar{D}}_j + (h_U)_{ij}\tilde{Q}_i h_u \tilde{\bar{U}}_j - \tilde{B}h_d h_u + h.c., \\
\mathcal{L}_{LNV} &= \frac{1}{2}h_{ijk}\tilde{L}_i\tilde{L}_j\tilde{\bar{E}}_k + h'_{ijk}\tilde{L}_i\tilde{Q}_j\tilde{\bar{D}}_k - \tilde{D}_i\tilde{L}_i h_u \\
&\quad + h_d^\dagger m^2_{H_d L_i}\tilde{L}_i + h.c.
\end{aligned}
$$

LNV in MSSM

- Can suppress proton decay by R-parity Farrar, Fayet PLB 76
  - $$\begin{array}{rcl} (L,\bar{E},Q,\bar{U},\bar{D}) & \rightarrow & - \quad (L,\bar{E},Q,\bar{U},\bar{D}). \\ (H_u,H_d) & \rightarrow & + \quad (H_u,H_d). \end{array}$$
  - $\mathcal{W}_{LNV} = \mathcal{W}_{BNV} = 0$: R-parity conservation (RPC).
- Also possible to forbid $\mathcal{W}_{BNV}$ alone - Baryon parity Ibanez, Ross PLB 260 .
  - $$\begin{array}{rcl} (Q,\bar{U},\bar{D}) & \rightarrow & - \quad (Q,\bar{U},\bar{D}). \\ (L,\bar{E},H_u,H_d) & \rightarrow & + \quad (L,\bar{E},H_u,H_d). \end{array}$$
  - $\mathcal{W}_{LNV} \neq 0$: lepton number violation (LNV).
  - L_i and H_d have same transformation properties: $H_d \equiv L_0$.

Neutrino masses in LNV SUSY: tree level

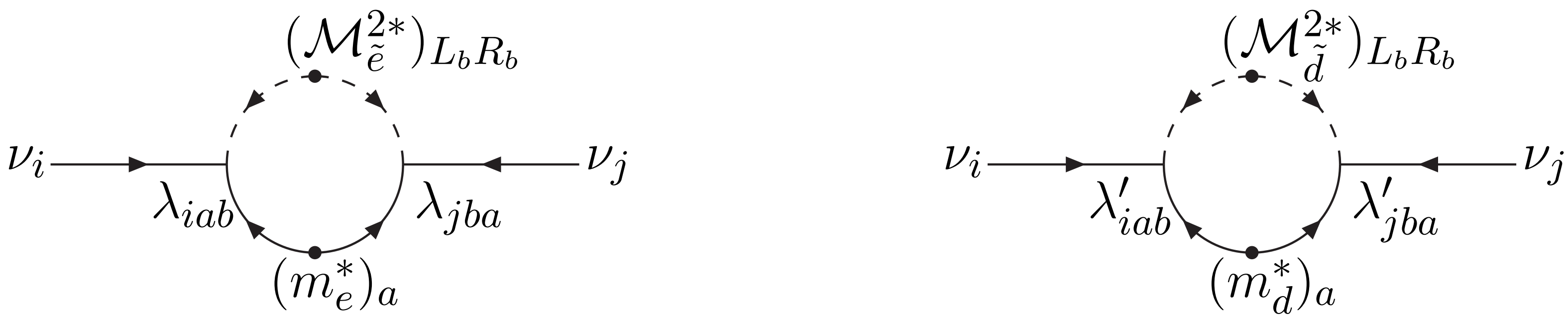
- Neutrinos mix with neutralinos.
- Involves bilinear μ_i and sneutrino VEVs v_i.

$$\mathcal{L} = -\frac{1}{2}\Big(-i\widetilde{\mathcal{B}} \quad -i\widetilde{\mathcal{W}}^3 \quad \tilde{h}_u^0 \quad \tilde{h}_d^0 \quad \nu_i \Big) \mathcal{M}_N \begin{pmatrix} -i\widetilde{\mathcal{B}} \\ -i\widetilde{\mathcal{W}}^3 \\ \tilde{h}_u^0 \\ \tilde{h}_d^0 \\ \nu_j \end{pmatrix},$$

$$\mathcal{M}_N = \begin{pmatrix} (\mathcal{M}_{\chi^0})_{4\times 4} & m^T \\ m & m_\nu \end{pmatrix} = \begin{pmatrix} M_1 & 0 & \frac{1}{2}gv_u & -\frac{1}{2}gv_d & -\frac{1}{2}gv_j \\ 0 & M_2 & -\frac{1}{2}g_2v_u & \frac{1}{2}g_2v_d & \frac{1}{2}g_2v_j \\ \frac{1}{2}gv_u & -\frac{1}{2}g_2v_u & 0 & -\mu & -\mu_j \\ -\frac{1}{2}gv_d & \frac{1}{2}g_2v_d & -\mu & 0 & 0_j \\ -\frac{1}{2}gv_i & \frac{1}{2}g_2v_i & -\mu_i & 0_i & 0_{ij} \end{pmatrix}.$$

Neutrino masses: radiative corrections

- Focus on trilinear LNV couplings.
- Mass corrections:



$$(m_\nu)_{ij} \simeq \sum_{a,b}\Big\{ \frac{1}{(4\pi)^2}\lambda_{iab}\lambda_{jba}\frac{(m^*_e)_{aa}(\mathcal{M}^{2*}_{\tilde{e}_{LR}})_{bb}}{\overline{\mathcal{M}}^2_{\tilde{e}_{LL}} - \overline{\mathcal{M}}^2_{\tilde{e}_{RR}}}\ln\Big(\frac{(\overline{\mathcal{M}}^2_{\tilde{e}_{LL}})}{(\overline{\mathcal{M}}^2_{\tilde{e}_{RR}})}\Big)\Big[1+\mathcal{O}\Big(\frac{\delta\mathcal{M}^2}{\overline{\mathcal{M}}^2}\Big)\Big] + (i \leftrightarrow j)\Big\}.$$

- Wavefunction corrections also important: included in our full 1-loop calculation.

Neutrino masses in LNV mSUGRA

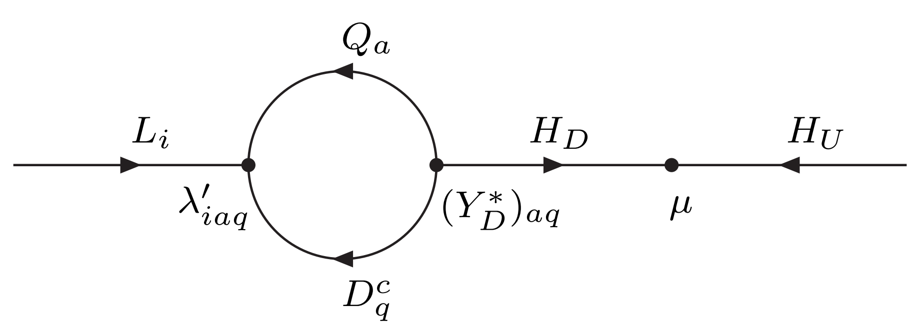
How many *dominant* trilinear LNV parameters @ M_X ?

- One λ or λ' in a weak interaction basis:
  - Renormalization effect: tree level mass matrix dominates.
  - Alignment: tree mass and loop corrections tend to be proportional.
- One λ *and* λ' in a weak interaction basis:
  - Partial cancellation may suppress tree level mass.
  - Alignment also weakens.

Neutrino masses in LNV mSUGRA

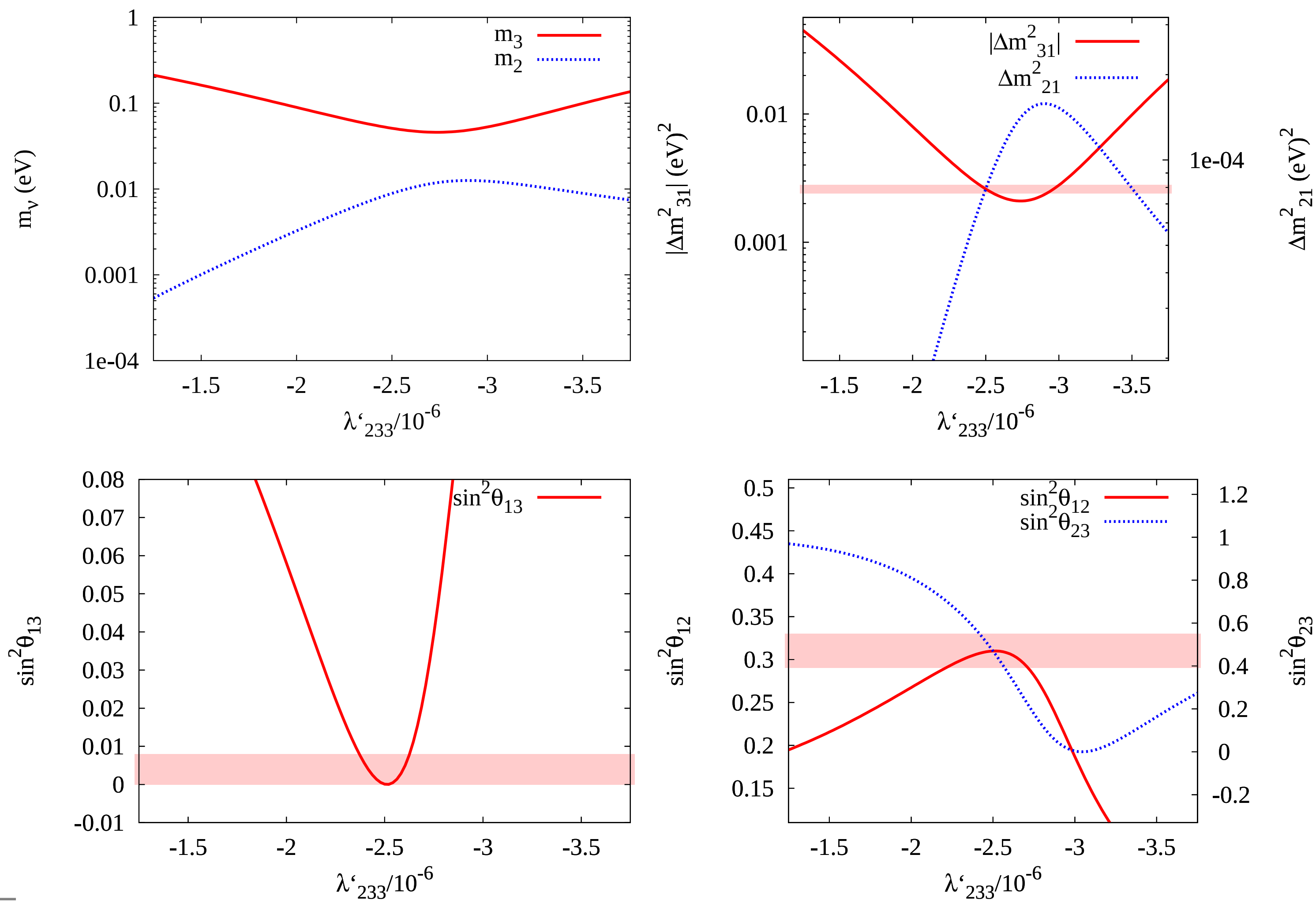
- Boundary conditions at unification scale $M_X \sim 10^{16}$GeV Allanach, Dedes, Dreiner PRD 69 :
  - RPC parameters: $m_0, M_{1/2}, A_0,$ tan$\beta,$ sgnμ.
  - Rotate away dimensionful bilinear LNV parameters $\mu_i, \tilde{D}_i$.
  - Trilinear LNV couplings: $\lambda_{ijk}, \lambda'_{ijk}$.
- Include renormalization effects: IMPORTANT ! e.g.

$$16\pi^2 \frac{d}{dt}\mu_i \quad \simeq \quad -\mu(\lambda_{ijk}(Y_E^*)_{jk} + 3\lambda'_{ijk}(Y_D^*)_{jk})$$



Results: normal hierarchy

Best fit: $\lambda_{233} = 4.07e^{-5}, \lambda'_{233} = -2.50e^{-6}$,
$\theta^l_{12} = 0.460, \theta^l_{13} = 0.389, \theta^l_{23} = 0.305$ @ SPS1a.



Results: inverted hierarchy

Best fit: $\lambda_{233} = 1.36023e^{-4}, \lambda'_{233} = -5.69116e^{-6}$, $\theta^l_{12} = 1.55779, \theta^l_{13} = 0.815155, \theta^l_{23} = 0.146126$ @ SPS1a.