Depolarization effects at the IP of a linear collider

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- MOTIVATION: There is a physics requirement to produce polarized e+e- beams at a linear collider AND to know the actual state of polarization – need to know the extent of depolarization
- Review sources of depolarization and the equations which describe depolarization at the IP (T-BMT and the Sokolov-Ternov equations)
- Introduce full quantum treatment of beamstrahlung process by solving the Dirac equation in the field of the e+e- bunches
- Derive Sokolov-Ternov equation using Dirac equation solutions in external field (Volkov solutions) to determine approximations
- Discuss extension of beam field effects to 4-fermion processes
- Describe analytic calculation of 2nd order coherent processes

Polarization at a linear collider – physics requirements

- Precision physics eg. $e^+e^- \rightarrow Z \rightarrow f\,\underline{f}$ at the Z pole requires left right assymetry $\Delta A_{LR} < 10^{-4}$ Can only achieved by the Blondel scheme and polarized positron beam up to 60% required
- Polarised beams can increase effective luminosity by up to 50% by suppressing backgrounds
- Precision physics also requires uncertainty on luminosity-weighted polarisation to be $\leq 0.1\%$ We know depolarization from initial simulations to be
  - 0.1% due to storage ring, linac and bds
  - ~0.1% due to beam-beam processes at the IP
- Still need to reduce uncertainty – examine IP depolarization at the IP due to the effect of the strong bunch fields

Fermion spin

- Electrons of momentum p and spin s are described by spinors u(p,s)
- In the electron rest frame s is spatial (0,s') and is lorentz boosted to a frame with electron momentum p by

$$s^\mu = \left(\frac{\tilde{p}.\tilde{s}'}{m_0}, \tilde{s}' + \frac{\tilde{s}'.\tilde{p}}{m_0(E+m_0)} \tilde{p} \right)$$

Spin is normalised and orthogonal p^μ

- Spin projection operators Σ(s) suppress unwanted spin states and allow the transition rates of quantum processes of specific spin to be calculated using the usual trace sum.

$$\Sigma(s) = \tfrac{1}{2}(1 + \gamma_5 \not{s})$$

$$\Sigma(s) u(p,+s) = u(p,+s) \quad , \quad \Sigma(s) u(p,-s) = 0$$

Depolarization at the IP

- There is depolarization (spin flip) due to the QED process of beamsstrahlung, given by the Sokolov-Ternov equation

$$dW = -i\,\frac{\alpha m}{\sqrt{3}\pi\gamma}\left[\int_z^\infty K_{5/3}(z)\,dz + \frac{x^2}{1-x}K_{2/3}(z)\right]dx$$

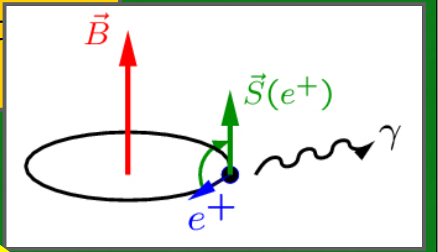
$$\text{where} \quad z = \frac{2}{3\upsilon\omega\epsilon_i}\,\frac{\omega_f}{\epsilon_i-\omega_f}$$

- The fermion spin can also precess in the bunch fields. Equation of motion of the spin given by the T-BMT equation

$$\frac{dS}{dt} = -\frac{e}{m\gamma}\left[(\gamma a+1)B - a(\gamma-1)(B.\hat{p})\hat{p}\right]\times\tilde{S}$$

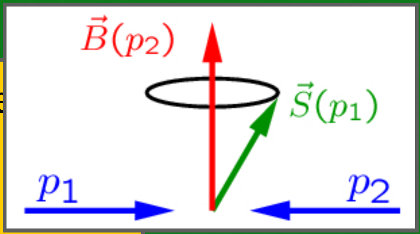
  - At the IP, the anomalous magnetic moment subject to radiative corrections in the presence of the bunch field

Stochastic spin diffusion from photon emission: Sokolov-Ternov effect



Parameter set	Depolarization ΔP_{lw}		
	T-BMT	S-T	total
Nominal	0.08%	0.02%	0.10%
low Q	0.04%	0.02%	0.06%
large Y	0.17%	0.02%	0.19%
low P	0.15%	0.09%	0.24%
TESLA	0.11%	0.03%	0.14%

Classical spin precession in inhomogeneous external fields: T-BMT equation.



Sokolov-Ternov equation in more detail

- Derivation of S-T in the literature usually done with the semi-classical theory of Baier Katkov et al

$$\frac{\hbar\omega_0}{\epsilon_p} = \frac{B}{B_c}\left(\frac{m}{\epsilon_p}\right)^2$$

The energy levels of an ultrarelativistic electron in a magnetic field are very close together, so assume motion is classical i.e. Operators of the dynamical variables of the electron commute

- Would like to repeat using a more general technique – a full QED calulation using (Vokov) solutions of the Driac equation in the bunch field

- So, will derive Sokolov-Ternov using Volkov solutions to obtain full expression for beamstrahlung

- As a check we will make sure that the full expression reduces to the Sokolov-Ternov **and** discover the approximations required in order to do so

Solution of Dirac equation in beam field A^e

- External field described by its 4-momentum (ω,k), 4-potential A^e and field intensity $\nu^2=e|A^e|/mc^2$.
- Gauge condition allows $A^e(0,\underset{\sim}{a})$ and (a.k),(k.k)=0

$$\left[(p-eA^e)^2-m^2-\frac{ie}{2F^e_{\mu\nu}}\sigma^{\mu\nu}\right]\psi_V(x,p)=0$$

- Look for a solution of the form: $\psi_V(x,p)=u_s(p)F(\phi)$
- Substitution of the general solution for ψ_V yields a first order d.e. whose solution can be expanded in powers of k,A^e – terms > O(1) are zero

$$\psi_V(x,p)=\left[1+\frac{e}{2(kp)}\not{k}\not{A}^e\right]\exp[F(k,A^e)]e^{-ipx}u_s(p)$$

The Volkov solution in more detail

$$\psi_V(x,p)=\left[1+\frac{e}{2(kp)}\not{k}\not{A}^e\right]\exp\left[F(x,p,k,A^e)\right]e^{-ipx}u_s(p)$$

make Fourier transform to get linear term in x

$$\int dr\exp[-i(r+\nu^2/kp)kx]F_2(p,k,A^e)$$

- r term interpreted as a contribution from r external field photons (r can be -ve!)
- ν^2 term is a shift in electron momentum

non-external field Dirac solution

for ILC parameters

$$\frac{\omega}{m}\approx 0.06\ ,\ \nu^2=\frac{e|A^e|}{m}\approx 1$$

so for large E_p second term can be neglected

Volkov soln represented in Feynman diagrams by double straight lines

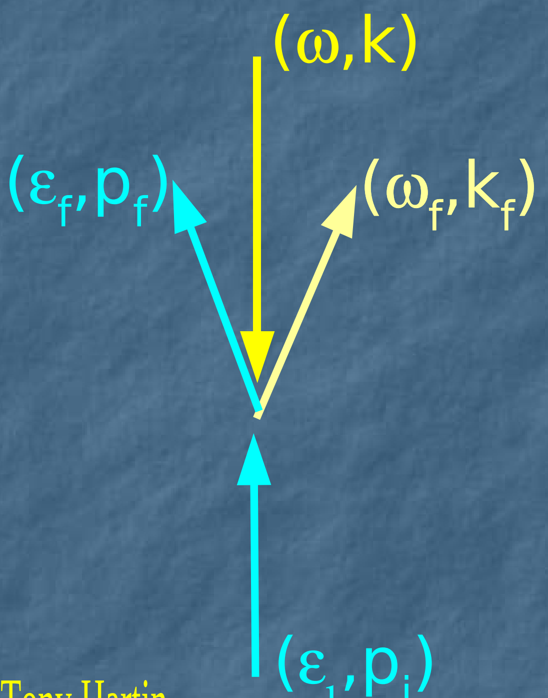
Deriving the Sokolov-Ternov equation

Sokolov-Ternov equation can be written down using the 'operator method' of Baier et al

$$dW = -i\,\frac{\alpha m}{\sqrt{3}\pi\gamma}\left[\int_z^\infty K_{5/3}(z)\,dz + \frac{x^2}{1-x}K_{2/3}(z)\right]dx \quad \text{where} \quad z = \frac{2}{3\nu\omega\epsilon_i}\,\frac{\omega_f}{\epsilon_i-\omega_f}$$

but, more generally can be obtained within limits using full Volkov solutions

lets just try to get (discovering required approximations along the way)



$$d\sigma_{fi} = \frac{1}{v(2\pi)^2}\,\frac{m^2}{4\epsilon_i\omega_i}\,\delta(P_i - P_f)\,\overline{\sum_i}\sum_f |T_{fi}|^2\,\frac{dp_f\,dk_f}{\epsilon_f\,\omega_f}$$

- Matrix element contains one volkov solution per Feynman diagram order, so a product of solutions for S-T
- phase integral also contains an integration over the contribution from the bunch field

(Partial) S-T derivation (continued)

constant crossed field:

$$\begin{aligned} A^e_\mu(x) &= a_{1\mu}(kx) \\ (a_1 a_1) &= -a^2 \\ (a_1 k) &= 0 \end{aligned}$$

$$\begin{aligned} \Psi^V_p(x) &= E_p(x)\, u_p \\ E_p(x) &= \left[1 + \frac{e}{2(kp)} \not{k} \not{a}_1 (kx)\right] \\ &\times \exp\left(-iqx + i\frac{e^2a^2}{2(kp)}(kx) - i\frac{e(a_1p)}{2(kp)}(kx)^2 - i\frac{e^2a^2}{6(kp)}(kx)^3\right) \end{aligned}$$

where $q = p + \frac{e^2a^2}{2(kp)}k$

Simplification 1: k || p so that $(a_1p)=0$

$$F_{1,r} = \int_{-\infty}^{\infty} t\, \exp\left[i(r+Q)t - iQ\frac{t^3}{3}\right] = 2\pi i Q^{-\frac{2}{3}} \mathrm{Ai}'(z)$$

where $Q = \nu^2 \frac{(kk_f)}{(kp_i)((kp_i)-(kk_f))}$

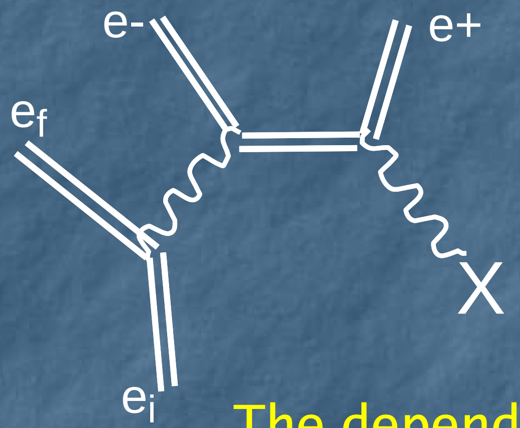
$$z = -(r+Q)Q^{-\frac{1}{3}}$$

Simplification 2: k || $-k_f$ and $\epsilon_i \gg m_e$ then $Q = \frac{\nu^2}{\omega\epsilon_i}\frac{\omega_f}{\epsilon_i - \omega_f}$

- **$\int$(P.I.)dr yields r→Q**
- **$Q^{-2/3}\,\mathrm{Ai}'(Q^{-2/3}) = K_{2/3}(2Q/3)$**

$$(S.T.)\, z = \frac{2}{3\omega\epsilon_i}\frac{\omega_f}{\epsilon_i - \omega_f} \qquad (volkov)\, z = \frac{2}{3(kp_i)}\frac{(kk_f)}{(kp_i)-(kk_f)}$$

4-fermion IFQED process

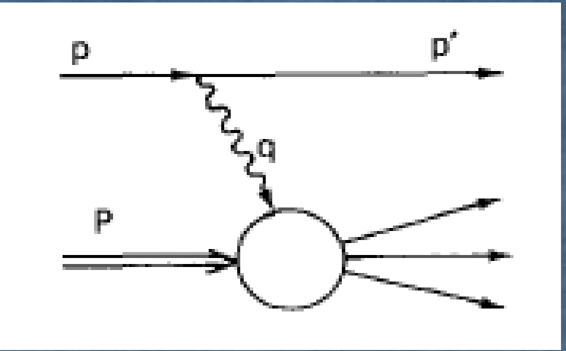


- To do bunch field effect properly replace all fermion lines by Volkov solutions
- 4^{th} order external field process is intimidating so look for an 'external field' EPA

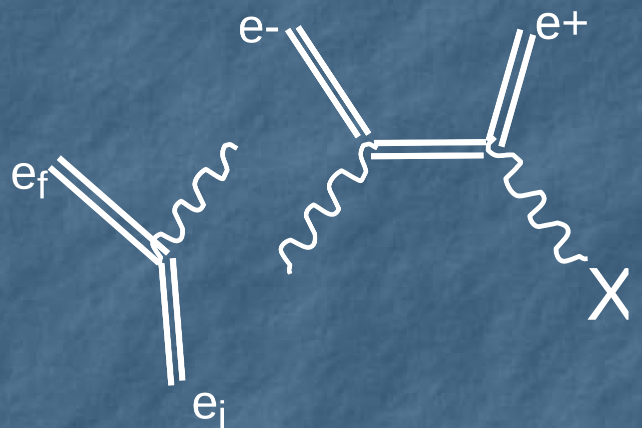
k_f

The dependency on fermion momenta have to be modified $P_\mu \to P_\mu + k_\mu \, \nu^2/2(kp)$ and $P^2 \to P^2 + \nu^2$

$$d\sigma = \frac{\alpha}{4\pi^2|q^2|}\left[\frac{(qP)^2 - q^2P^2}{(pP)^2 - p^2P^2}\right]^{1/2}(2\rho^{++}\sigma_T + \rho^{00}\sigma_S)\,\frac{d^3p'}{E'}$$

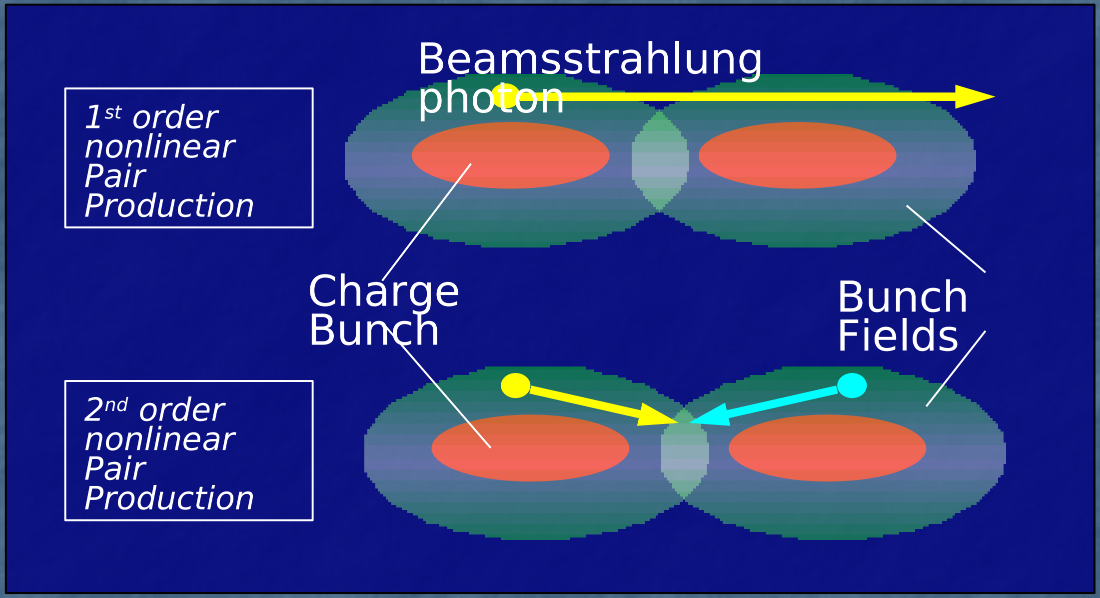


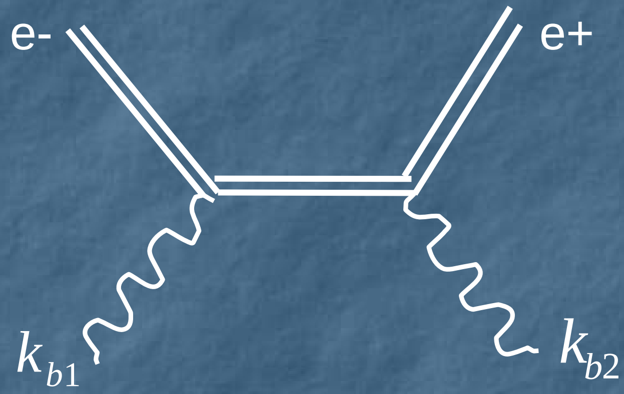
- So assuming EPA can still be used we are left with the 1^{st} order external field process (Sokolv-Ternov). Known to be determined by the intensity of the bunch field Υ



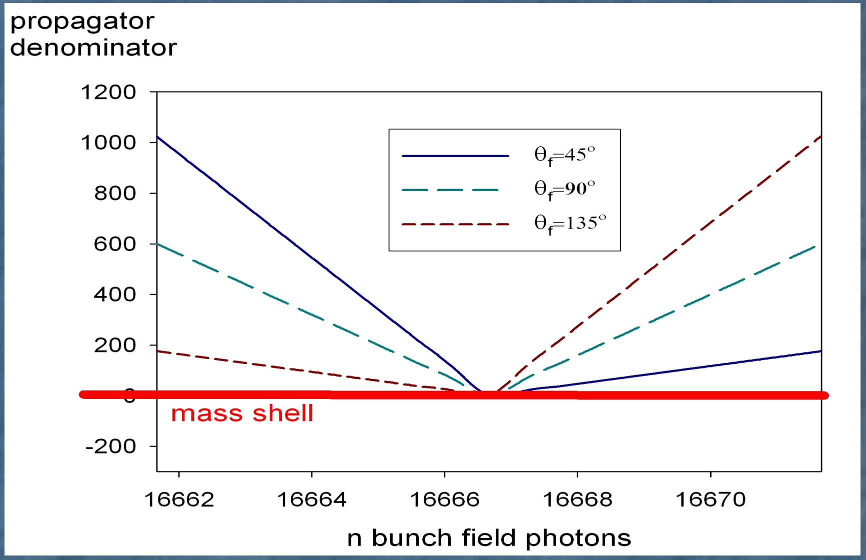
- The 2^{nd} order external field processes need special treatment. Propagators can reach the mass shell, the x-sections can exceed S-T and the effect does not necessarily have a simple relationship with Υ

2^{nd} order external field process: Coherent Breit-Wheeler (CBW) process



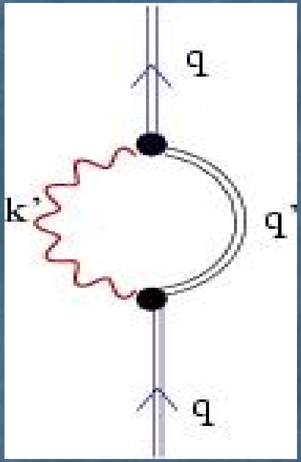


- 2^{nd} order process contains twice as many Volkov E_p
- Double integrals over products of 4 Airy functions – mathematical challenge!
- spin structure same as ordinary Breit-Wheeler

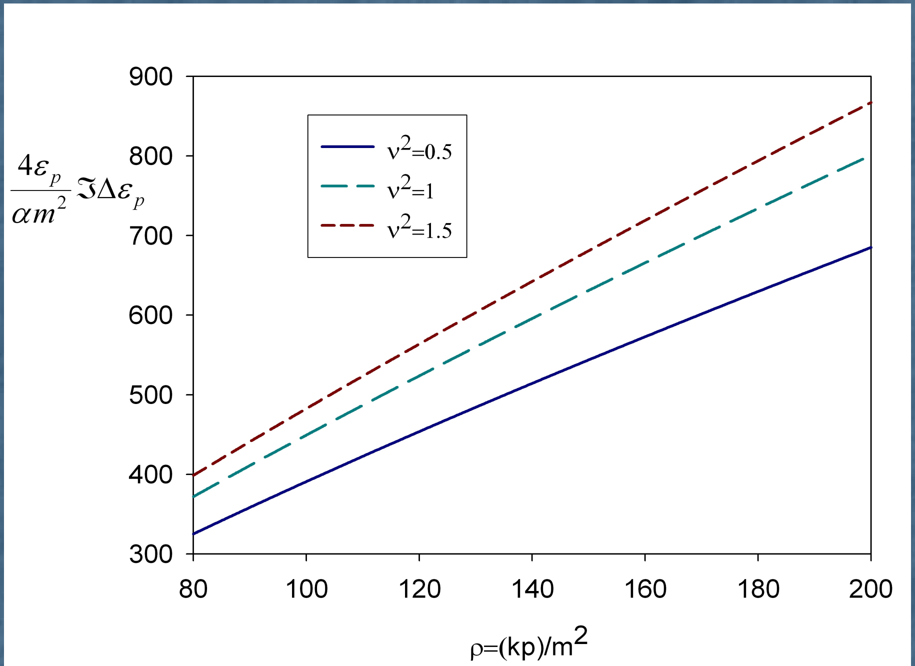


fermions recieve a mass shift due to bunch field and the propagator can reach mass shell whenever $r\omega \sim \omega_b$

Calculation of Resonance widths



- The Electron Self Energy must be included in the Coherent Breit-Wheeler process
- This is a 2nd order IFQED process in its own right.
- Renormalization/Regularization reduces to that of the non-external field case



- The Electron Self Energy in external CIRCULARLY POLARISED e-m field originally due to Becker & Mitter 1975 for low field intensity parameter $(ea/m)^2$. Has been recalculated for general field intensity parameter
- ESE in external CONSTANT CROSSED field is due to Ritus, 1972
- Optical theorem: the imaginary part of the ESE has the same form as the Sokolov-Ternov equations

Summary

- We want highly polarized beams at a linear e+e- collider.
- We want to know precisely what the state of polarization is and consequently the depolarization at the IP
- The Sokolov-Ternov equation is contains various approximations concerning particle energies and scattering angles
- Full quatum treatment of beamstrahlung involves Dirac equation solutions for fermions in the external field
- Calculating the beamstrahlung process using the full solutions reveals the approximations made in the Sokolov-Ternov equation
- To treat bunch field effects consistently we should modify 4-fermion processes by replacing **all** fermion lines by full Volkov solutions
- EPA has to be studied for validity when external fields are included
- 2^{nd} order IFQED Coherent Breit-Wheeler discussed. Calculation has some mathematical challenges