Dark matter distributions around massive black holes: A general relativistic analysis

Francesc Ferrer

Washington University in St. Louis

TeVPA/IDM. Amsterdam, June 2014

Laleh Sadeghian, FF & Clifford M. Will, PRD88, 063522 (2013) [arXiv:1305.2619]

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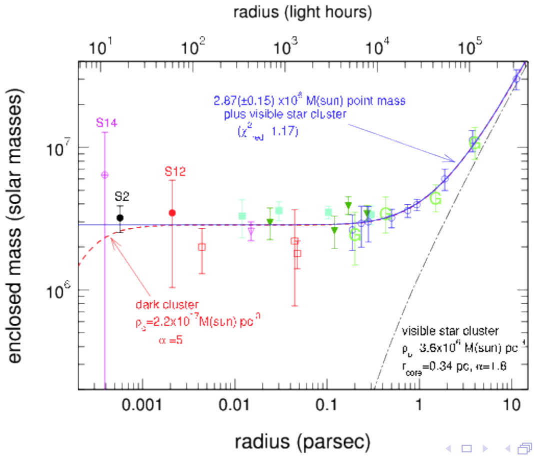
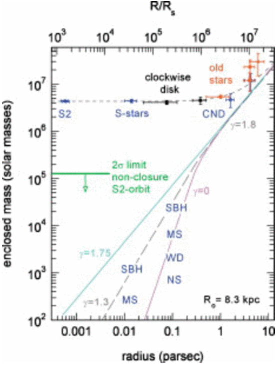
Super-massive black holes

- Will focus on the super-massive BH at the center of the Galaxy.
- Similar effects will occur in the cores of AGNs, or in IMBHs.



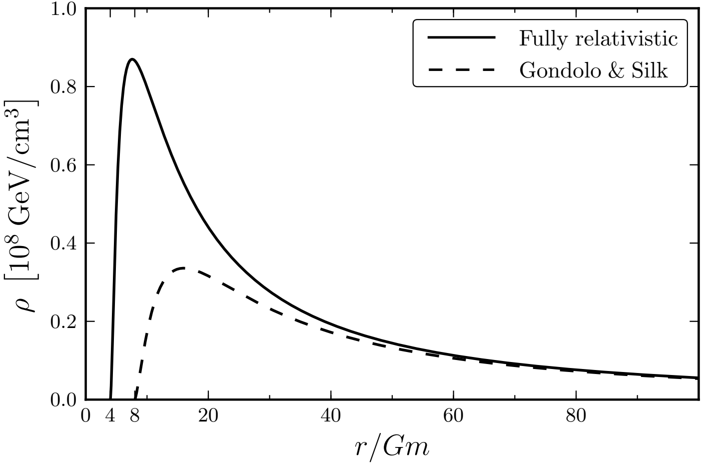
Super-massive black holes



Genzel, Eisenhauer & Gillessen, RMP 82, 3221 (2013)

We will assume that a black hole of mass $4 \times 10^6 M_\odot$ grows adiabatically over $\sim 10^{10}$yr.

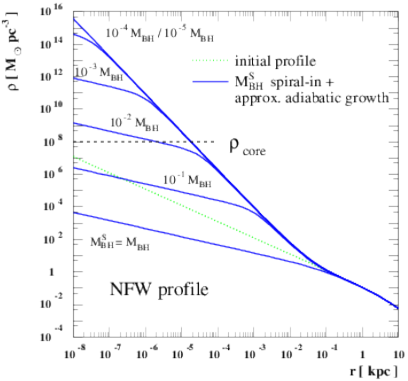
The DM spike



Fully relativistic
Gondolo & Silk
ρ [10^8 GeV/cm^3]
1.0
0.8
0.6
0.4
0.2
0.0
0
4
8
20
40
60
80
r/Gm

The central supermassive black hole

Caveats: Hierarchical mergers, initial BH seed off-center, kinetic heating of DM by stars, ...



Bertone, Hooper & Silk, Phys. Rep. 2004

Is the growth adiabatic?

- Growth time
- Dynamical time

$$\frac{r_h}{\sigma} \leq \frac{m}{\dot{m}_{\rm Edd}}$$

$$r_h \approx \frac{Gm}{\sigma^2} \to t_{\rm dyn} \approx 10^4 {\rm yr} \leq t_{\rm Salpeter} \approx 5 \times 10^7 {\rm yr}$$

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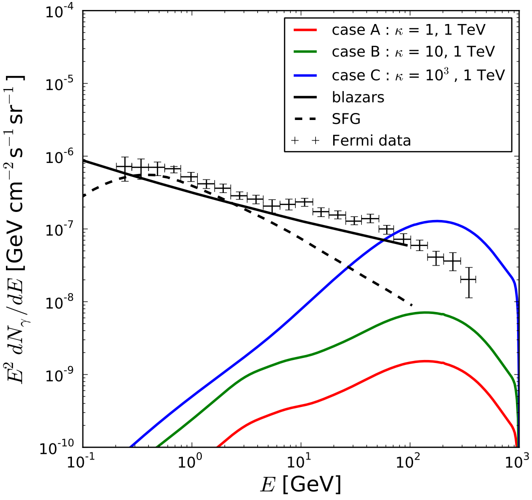
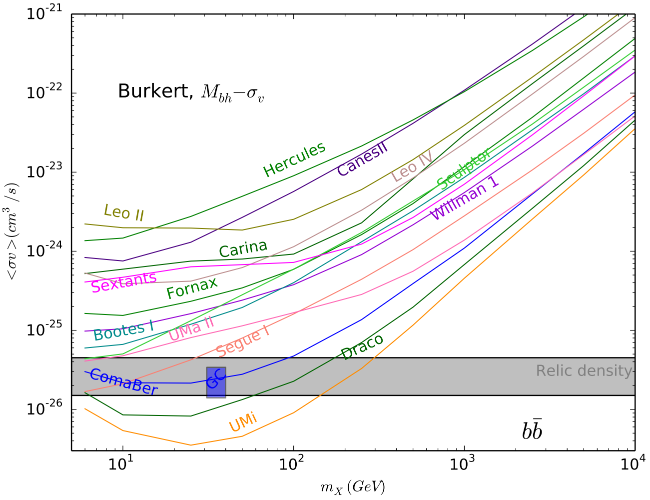
The diffuse γ-ray background



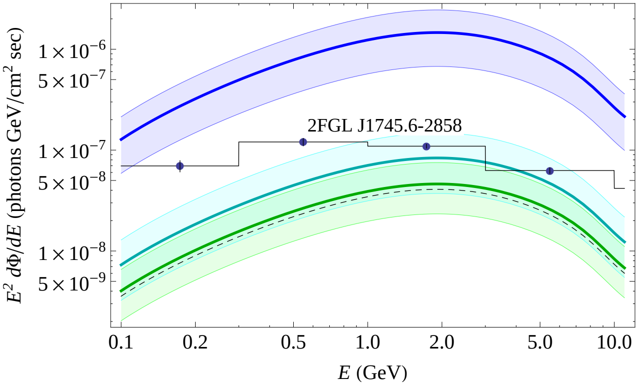
Belikov & Silk, 1312.0007

Limits from dwarf spheroidals



Gonzalez-Morales, Profumo & Queiroz, 1406.2424

The GC excess



Fields, Shapiro & Shelton, 1406.4856

Growing a BH: Newtonian analysis

We are interested in the DM density:

$$\begin{aligned}\rho &= \int f(E,L)d^3\boldsymbol{v} \\ &= 4\int dE \int LdL \int dL_z \frac{f(E,L)}{r^4|v_r||v^\theta|\sin\theta} \\ &= 4\pi \int dE \int LdL \frac{f(E,L)}{r^2|v_r|}\end{aligned}$$

The limits of integration are set by the requirements:

- $|v_r| = (2E - 2\Phi - L^2/r^2)^{1/2}$ real $\Rightarrow 0 \leq L \leq [2r^2(E-\Phi)]^{1/2}$.
- DM particle is bound to the halo $\Rightarrow \Phi(r) \leq E \leq 0$.

Take into account particles trapped inside the event horizon by modifying boundary conditions in an *ad hoc* manner: $L \geq 2cR_S$.

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Each particle in an initial DM distribution $f(E,L)$, will react to the change in Φ caused by the growth of the BH by altering its E, L and L_z. However, *the adiabatic invariants remain fixed*:

$$
\begin{aligned}
I_r(E,L) &\equiv \oint v_r dr = \oint dr \sqrt{2E - 2\Phi - L^2/r^2}\,, \\
I_\theta(L,L_z) &\equiv \oint v_\theta d\theta = \oint d\theta \sqrt{L^2 - L_z^2 \sin^{-2}\theta} = 2\pi(L - L_z)\,, \\
I_\phi(L_z) &\equiv \oint v_\phi d\phi = \oint L_z d\phi = 2\pi L_z\,. \qquad (1)
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Newtonian BH

For a Newtonian point mass,

$$I_r(E,\, L) = 2\pi \left(-L + \frac{Gm}{\sqrt{-2E}} \right)$$

And we can find the final DM density in the form:

$$\rho(r) = \frac{4\pi}{r^2} \int_{-Gm/r}^{0} dE \int_{0}^{L_{\max}} LdL \frac{f'(E'(E,L),L)}{\sqrt{2E + 2Gm/r - L^2/r^2}}$$

Young 80, Quinlan *et al.* 95, Gondolo & Silk 99

Growing a BH: Relativistic analysis

1. Generalize the definition of density:

$$J^\mu(x) \equiv \int f^{(4)}(p) \frac{p^\mu}{\mu} \sqrt{-g}\, d^4p \,,$$

2. Use relativistic expressions to write it in terms of the invariants of motion (energy, angular momentum, ...).
3. Use relativistic expressions for the actions.

For Kerr:

$$\begin{aligned} \mathcal{E} &\equiv -u_0 = -g_{00}u^0 - g_{0\phi}u^\phi \,, && (2)\\ L_z &\equiv u_\phi = g_{0\phi}u^0 + g_{\phi\phi}u^\phi \,, && (3)\\ C &\equiv \Sigma^4 (u^\theta)^2 + \sin^{-2}\theta L_z^2 + a^2 \cos^2\theta (1-\mathcal{E}^2) \,, && (4)\\ g_{\mu\nu}p^\mu p^\nu &= -\mu^2 \,. && (5) \end{aligned}$$

And need to calculate the jacobian

$$d^4p = |J|^{-1} d\mathcal{E}\, dC\, dL_z\, d\mu$$

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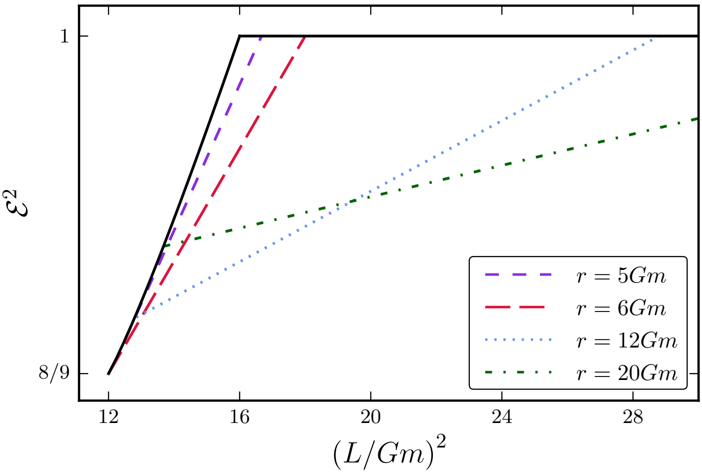
Example: Schwarzschild BH

The positivity of the radial action determines the boundary conditions, *including the effects of the horizon.*



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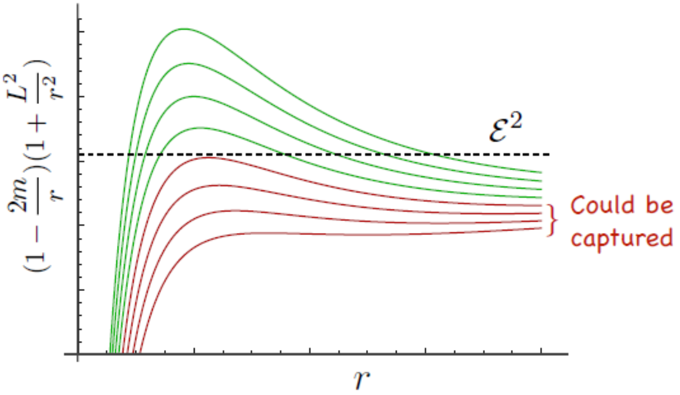

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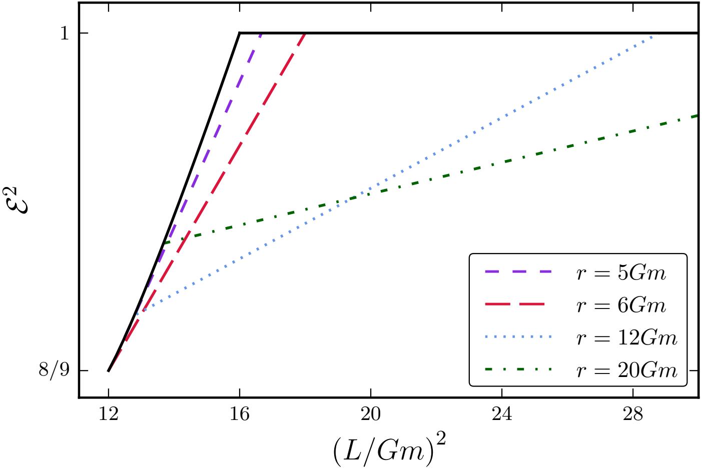
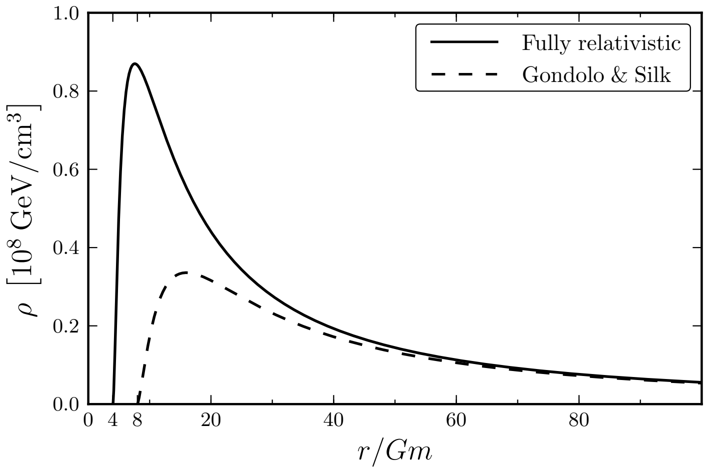


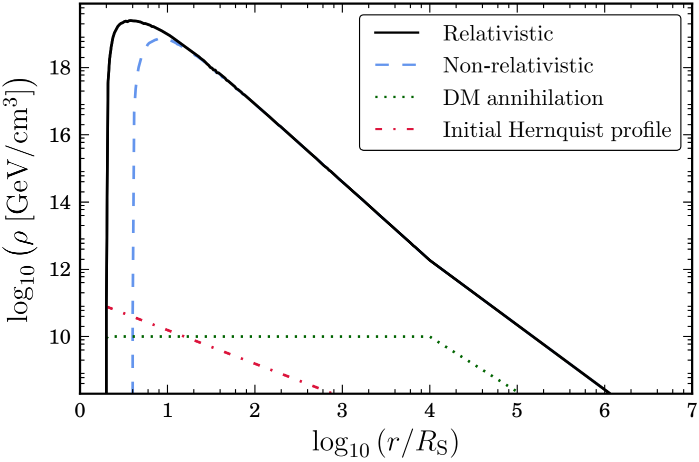
Example: Schwarzschild BH

For a constant phase-space distribution:

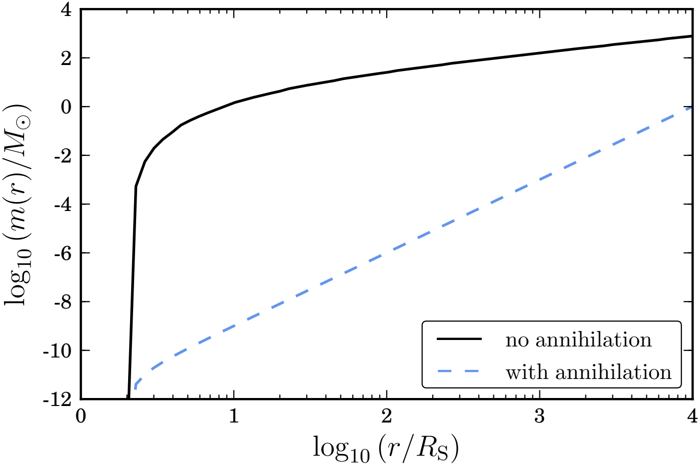


Example: Schwarzschild BH

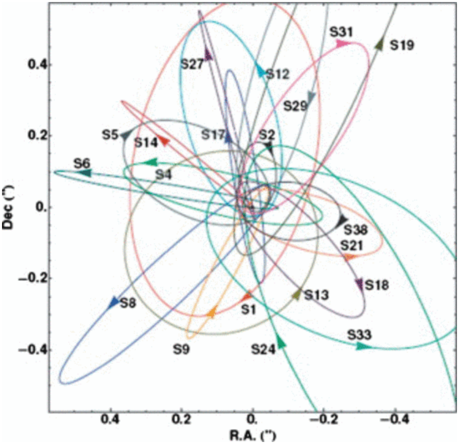
For a, more realistic, cuspy DM distribution:



The gravitational potential

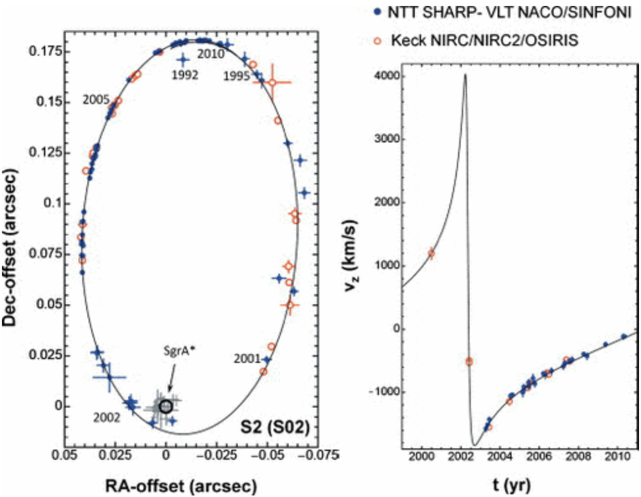


S stars



Genzel, Eisenhauer & Gillessen, RMP 82, 3221 (2013)

S2 star



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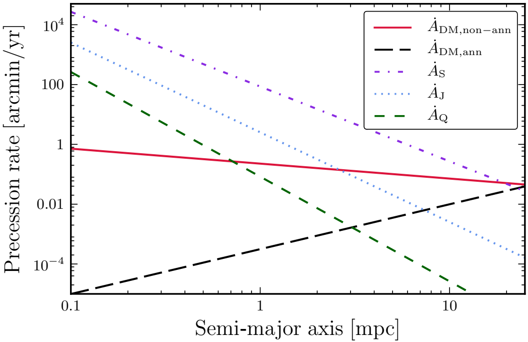
Tests of the *no-hair* theorem



$$\boxed{Q_2 = -J^2/m}$$

Conclusions

- A full general relativistic treatment confirms that the DM distribution can develop a dense spike in the vincinity of a super-massive black hole. The spike continues down to $r \approx 4M$.
- γ-ray limits from the diffuse cosmological background, dwarf spheroidal galaxies, the galactic center can be strengthened by a few orders of magnitude.
- Several astrophysical processes could dilute the central spike, although they are not universal. Efforts are underway to take into account the effects of the rotation of the black hole.
- If DM does not self-annihilate, it could affect future precission tests of no-hair theorems based on the precession of the orbital plane.

GR details

$$J^\mu = \rho u^\mu \Rightarrow \rho = \frac{J^0}{\sqrt{-g_{00} - 2g_{0j}v^j - g_{ij}v^i v^j}} \tag{6}$$

$$V(r) = \left(1 + \frac{a^2}{r^2} + \frac{2Gma^2}{r^3}\right)\mathcal{E}^2 \tag{7}$$

$$- \frac{\Delta}{r^2}\left(1 + \frac{C}{r^2}\right) + \frac{a^2 L_z^2}{r^4} - \frac{4Gma\mathcal{E}L_z}{r^3} \tag{8}$$

$$J_0 = -2\int \mathcal{E}d\mathcal{E}\int dC \int dL_z \frac{f(\mathcal{E},C)}{\Sigma^2\Delta|u_r||u^\theta|\sin\theta} \tag{9}$$

$$J_\phi = 2\int d\mathcal{E}\int dC \int L_z dL_z \frac{f(\mathcal{E},C)}{\Sigma^2\Delta|u_r||u^\theta|\sin\theta} \tag{10}$$