The dark matter density profile in spherical systems: A simple way to get more information from the Jeans equation talk based on arxiv:1401.6195

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Precise measurement of Dark matter density profile ρ_{DM} in dwarf spheroidal galaxies /clusters is interesting because

- Indirect detection: J-factor for self-annihalation depends on ρ_{DM}^2

$$\frac{d\phi}{dE} = \frac{1}{4\pi} \frac{< \sigma v >}{2m_{DM}^2} \frac{dN}{dE} \times \int_{\Delta\Omega} \int \rho_{DM}^2(l, \Omega) dl d\Omega$$

- Test predictions from dark matter simulations (and corrections from baryons). Cusp/core debate.

- Most common method to infer ρ in a spherical system from its internal dynamics is with the spherical Jeans equation

$$\frac{d(\nu\sigma_r^2)}{dr} + \frac{2\beta}{r}\nu\sigma_r^2 + \nu\frac{GM}{r^2} = 0.$$

- Assumes collisionless tracers in a steady state. Smooth and spherically symmetric DF $f(r, \mathbf{v})$ and gravitational potential $\Phi(r)$.
- For a choice of tracer number density $\nu(r) = \int f d^3v$ and velocity anisotropy $\beta(r) = 1 - \sigma_t^2/2\sigma_r^2$ can calculate

$$\sigma_{los}^2(R) = \frac{2}{\Sigma(R)}\int_R^\infty (1 - \beta\frac{R^2}{r^2})\sigma_r^2 \frac{\nu(r)r}{\sqrt{r^2 - R^2}}dr$$

for a system with a total (stars+dark matter) mass profile $M(r)$ and compare it to observations.

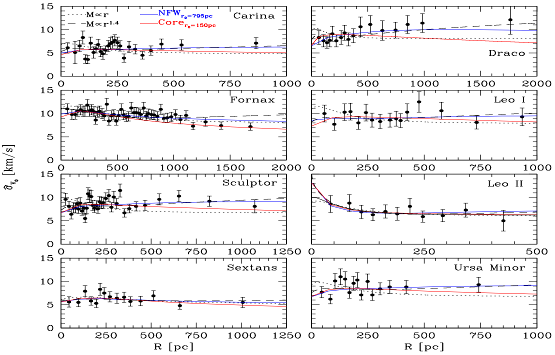


Figure: LOS dispersion profiles for MW dwarfs from Walker et al (2009) arxiv:0906.0341

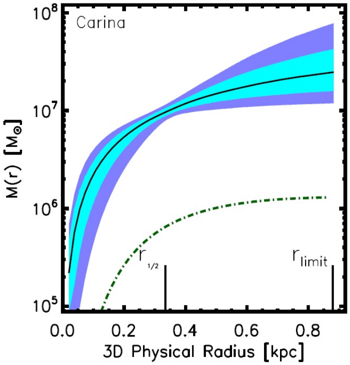


Figure: Mass estimates of Carina with Jeans equation from Wolf et al (2010) arxiv:0908.2995

Can fit many choices for ρ to the same LOS velocity dispersion data by tuning $\beta(r)$. Accurate mass estimates only possible close to the half-light radius.

Can we break the mass-anisotropy degeneracy with information from higher order moments of the LOSVD?

- First idea (Merritt 1987, Merrifield Kent 1990, Lokas 2002): Add constraints from fourth order Jeans equations on $\langle v_{\rm los}^4 \rangle(R)$.
- To marginalize over all possible choices of $f(r, \mathbf{v})$ need to add a new fourth order velocity anisotropy parameter β' (Richardson & Fairbairn 2013) [arxiv: 1207.1709].
- Global averages of $\langle v_{\rm los}^4 \rangle$ however only depend on quantities in the Jeans equation (Merrifield & Kent 1990)

$$\int_0^\infty \Sigma \langle v_{\rm los}^4 \rangle R \mathrm{dR} = \frac{2}{5} \int_0^\infty \nu (5 - 2\beta) \sigma_{\rm r}^2 \frac{\mathrm{d}\Phi}{\mathrm{dr}} \mathrm{r}^3 \mathrm{dr}$$

$$\int_0^\infty \Sigma \langle v_{\rm los}^4 \rangle R^3 \mathrm{dR} = \frac{4}{35} \int_0^\infty \nu (7 - 6\beta) \sigma_{\rm r}^2 \frac{\mathrm{d}\Phi}{\mathrm{dr}} \mathrm{r}^5 \mathrm{dr}$$

Can define very simple estimators (broadly analagous to the kurtosis),

$$\widehat{\zeta}_A = N_{\rm s} \frac{\sum_i^{N_{\rm s}} v_{\rm los,i}^4}{\left(\sum_i^{N_{\rm s}} v_{\rm los,i}^2\right)^2}$$

$$\widehat{\zeta}_B = N_{\rm s}^2 \frac{\sum_i^{N_{\rm s}} v_{\rm los,i}^4 R_i^2}{\left(\sum_i^{N_{\rm s}} v_{\rm los,i}^2\right)^2 \sum_i^{N_{\rm s}} R_i^2}.$$

that add extra shape information on the global LOS velocity distribution from the data. Expectation values for ζ parameters depend only on quantities in the Jeans equation.

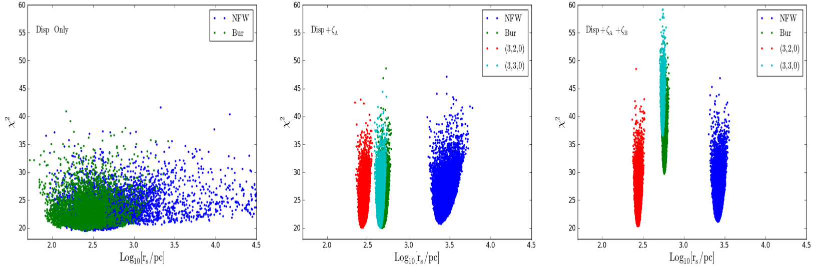
$$\zeta_A = \frac{\langle v_{\rm los}^4 \rangle_\star}{\langle v_{\rm los}^2 \rangle_\star^2} = \frac{9 N_{\rm tot}}{10} \frac{\int_0^\infty \nu (5-2\beta)\sigma_r^2 \frac{{\rm d}\Phi}{{\rm dr}} r^3 {\rm dr}}{\left(\int_0^\infty \nu \frac{{\rm d}\Phi}{{\rm dr}} r^3 {\rm dr}\right)^2}$$

$$\zeta_B = \frac{\left(\langle v_{\rm los}^4 \rangle R^2\right)_\star}{\langle v_{\rm los}^2 \rangle_\star^2 R_\star^2} = \frac{9 N_{\rm tot}^2}{35} \frac{\int_0^\infty \nu (7-6\beta)\sigma_r^2 \frac{{\rm d}\Phi}{{\rm dr}} r^5 {\rm dr}}{\left(\int_0^\infty \nu \frac{{\rm d}\Phi}{{\rm dr}} r^3 {\rm dr}\right)^2 \int_0^\infty \Sigma(R) R^3 {\rm dR}}.$$

New fitting procedure

- Assign suitable parametrisations for β and ρ (should also allow freedom in ν for uncertainty in star counts)
- For a choice of ρ and β use Jeans equation to find $\sigma^2_{\mathrm{los}}(R|\beta,\rho)$ and calculate both zeta parameters $\zeta(\beta,\rho)$.
- Use the LOS velocity data to calculate the binned dispersion profile and the estimators $\widehat{\zeta}$
- Evaluate the model (β,ρ) with a joint likelihood function by adding new factors $P(\widehat{\zeta}|\zeta)$ in product with the usual likelihood for the dispersion data.
- Use a technique such as MCMC to find posterior samples from the $\beta-\rho$ parameter space and the best fitting regions.

..........So how do the ζ parameters restrict the ρ parameter space in practice? Can they solve the core/cusp problem in dwarf spheroidal galaxies?!



$$\rho(r) = \frac{\rho_0 r_s^3}{r\,(r^2 + r_s^2)}, \qquad \rho(r) = \frac{\rho_0 r_s^3}{(r_s + r)\,(r_s^2 + r^2)}$$

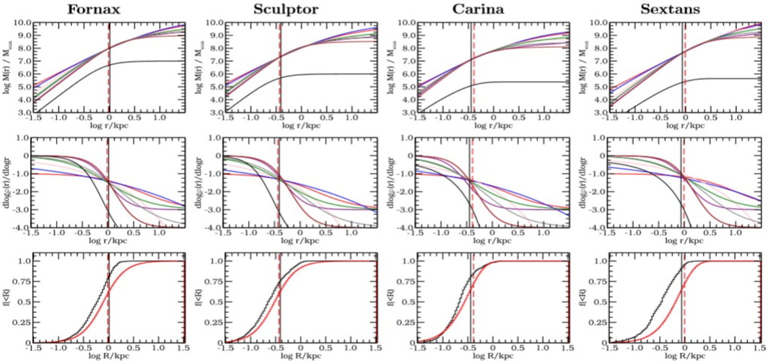
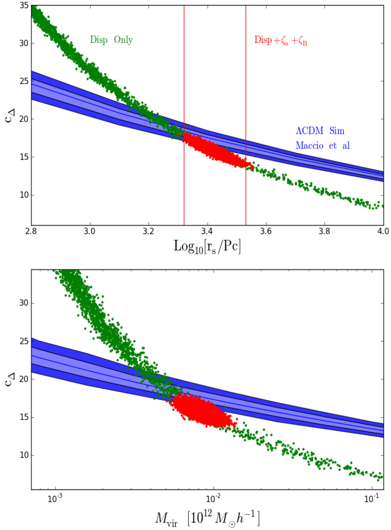
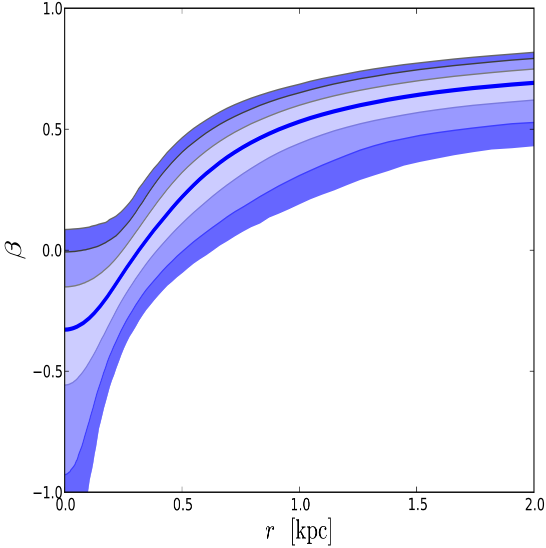


Figure: Mass constraints on MW dSphs from Breddels & Helmi (2013) arxiv:1304.2976


Disp Only
Disp + ζa + ζB
ΛCDM Sim
Maccio et al
cΔ
Log10[rs/Pc]
Mvir [10^12 M⊙ h^-1]

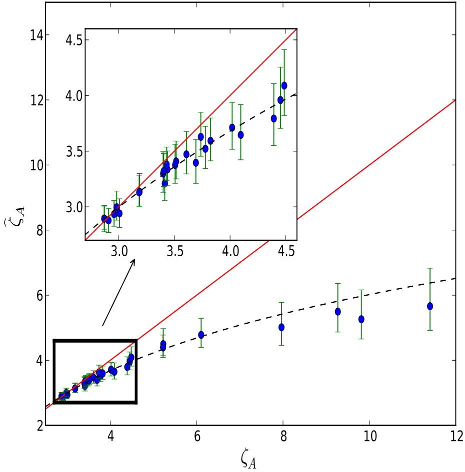

1.0
0.5
0.0
−0.5
−1.0
0.0
0.5
1.0
1.5
2.0
β
r [kpc]

Summary of results

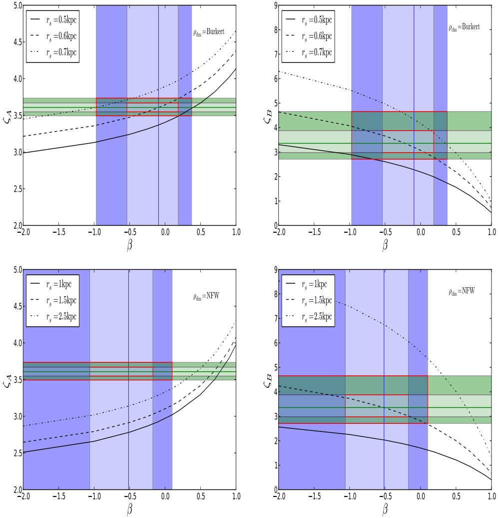
- Adding ζ parameters to the Jeans equation does not break the mass-anisotropy completely but leaves a smaller space of solutions where cusped models are degenerate with more concentrated cored models.
- More specifically, the dark matter density slope appears to be most tightly constrained near the half-light radius. If the density slope is less than one then both cored and cusped models are viable. Discrepancy with multiple population studies?
- For simple 2 parameter density profiles such as NFW and Burkert, ζ can isolate the concentration and the anisotropy parameter. Tests for ΛCDM and concept of 'universal' density profile.

Final comments

- The ζ parameters can be used to reduce the range of acceptable density profiles for **ALL** applications of the Jeans equation.
- No additional data or fitting parameters are required
- No additional assumptions are made on the form of the DF
- Gains are made with little extra complexity or computational cost (though more care is neeed for the statistical analysis)



14
12
10
8
6
4
2
4
6
8
10
12
ζ_A
$\hat{\zeta}_A$
4.5
4.0
3.5
3.0
3.0
3.5
4.0
4.5



r_s =0.5kpc
r_s =0.6kpc
r_s =0.7kpc
ρ_{dm} =Burkert
ζ_A
ζ_B
β
r_s =1kpc
r_s =1.5kpc
r_s =2.5kpc
ρ_{dm} =NFW