Global Fits of Supersymmetry

Roberto Ruiz de Austri (University of Valencia/IFIC)

Talk at the APP14 conference, June 23-28, Amsterdam

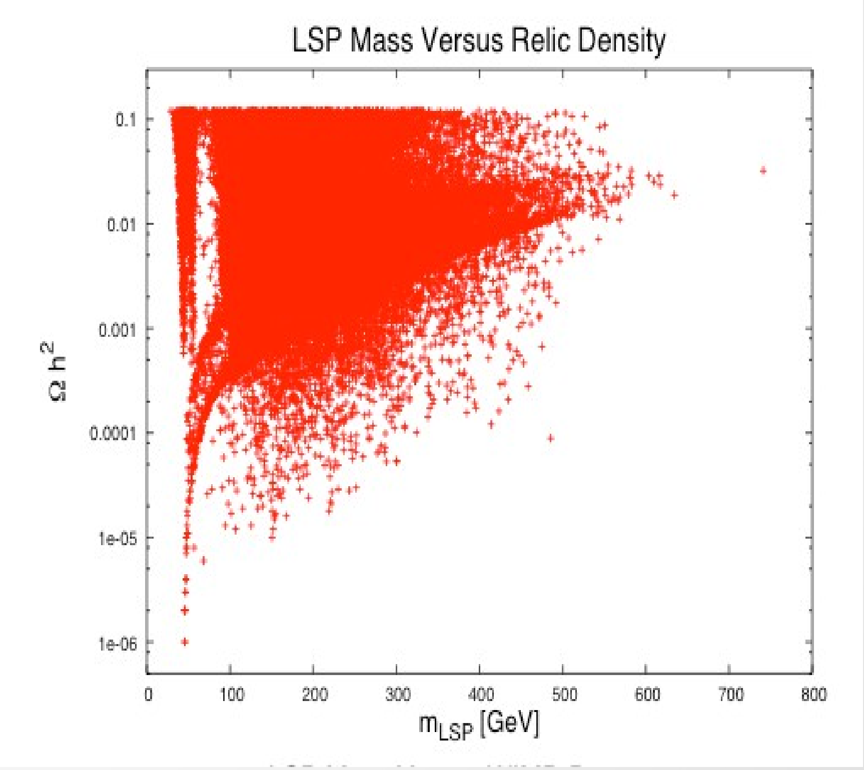
Outline

- Motivation for performing statistical inference in SUSY
- What does it consist of ?
- How it is done
- CMSSM and MSSM-15 case studies
- Conclusions

Random Scans

- Points accepted/rejected in a in/out fashion (e.g, 2σ cuts)
- No statistical measure attached to density of points: no probabilistic interpretation of results possible
- Inefficient in high dimensional parameter spaces (D > 5)
- HIDDEN PROBLEM: random scan explore only a very limited portion of the parameter spaces!

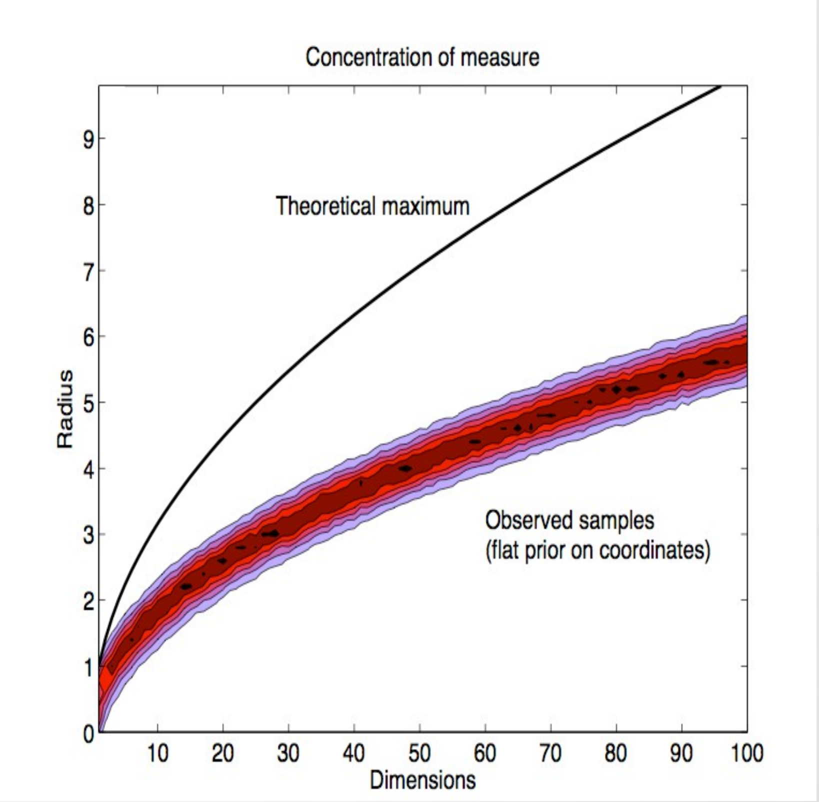
pMSSM scans (20 D)



Maths principle

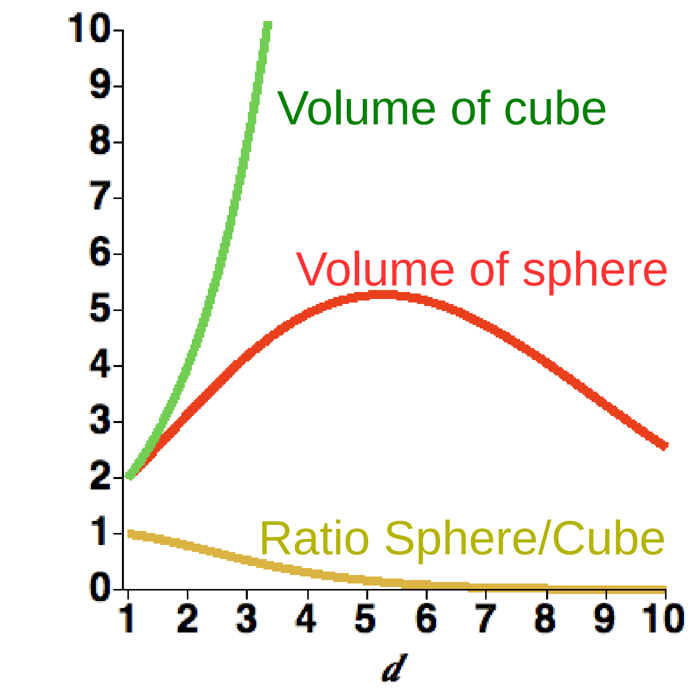
- Random scans of a high dimensional parameter space only probe a very limited sub-volume: this is the concentration of the measurement phenomenon

- Statistical fact: the norm of D draws from U[0,1] concentrates around $(D/3)^{1/2}$ with constant variance



Geometry

- Geometry fact: In D dimensions, most of the volume is near the boundary. The volume inside the spherical core of D-dimensional cube is negligible



Together, these two facts mean that random scans only explore a very small fraction of the available parameter space in high-dimensional models

The way out is to do statistical inference of the parameters of interest

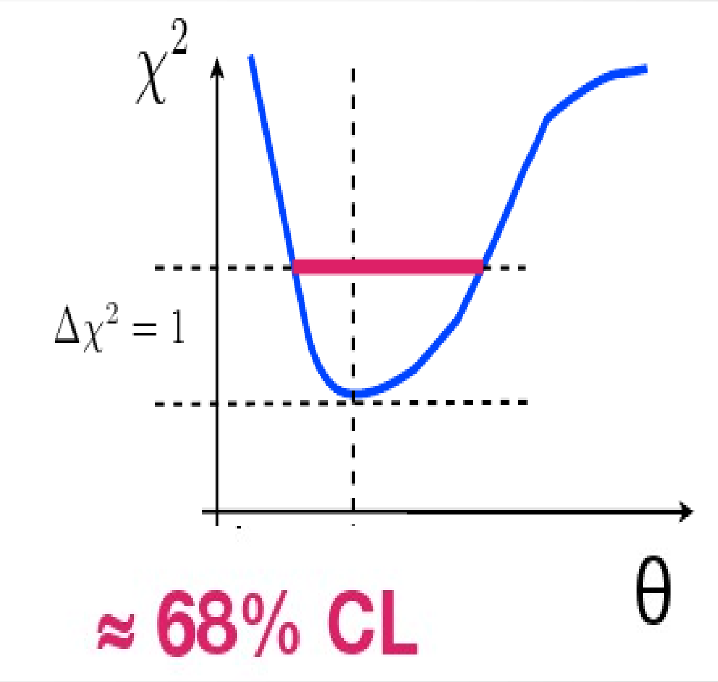
Likelihood based inference

Due to the weak nature of constraints, different scanning techniques and statistical methods will generally give different answers

- Likelihood-based methods: determine the best fit parameters by finding the minimum of -2 Log(Likelihood) = chi-squared

1. Markov Chain Monte Carlo and Minuit as “afterburner”
2. Simulated annealing
3. Genetic algorithms

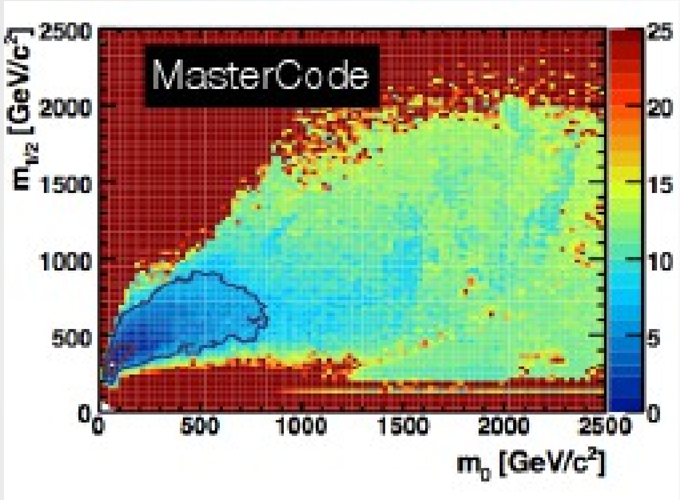
- Determine approximate confidence intervals: Local Δ(chi-squared) method
- Profile likelihood: way to treat nuisance



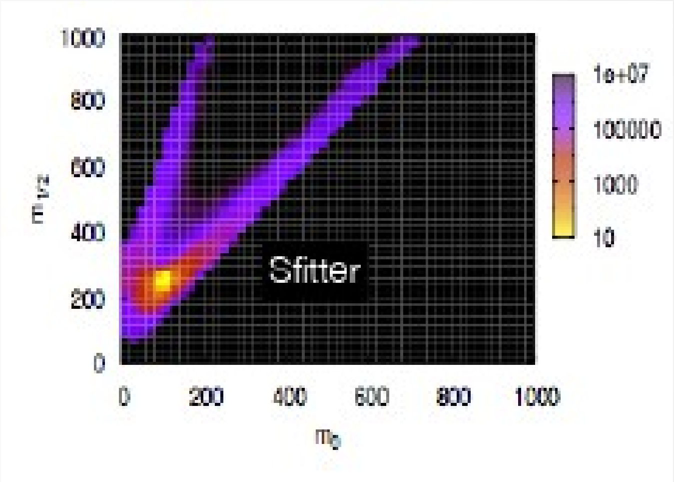
L(x,y) => PL(x) = max. L(x,y) for fixed x in y

Groups

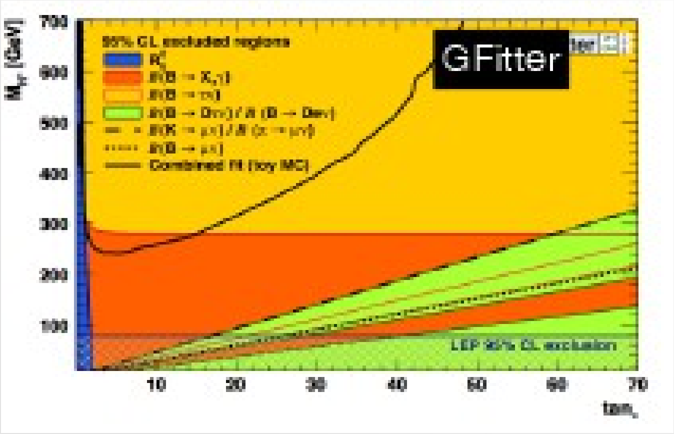
O. Buchmueller, R. Cavanaugh, A. De Roeck, Ellis, H.Flacher, S. Heinemeyer, G. Isidori, K.A. Olive, F.J. Ronga, G. Weiglein



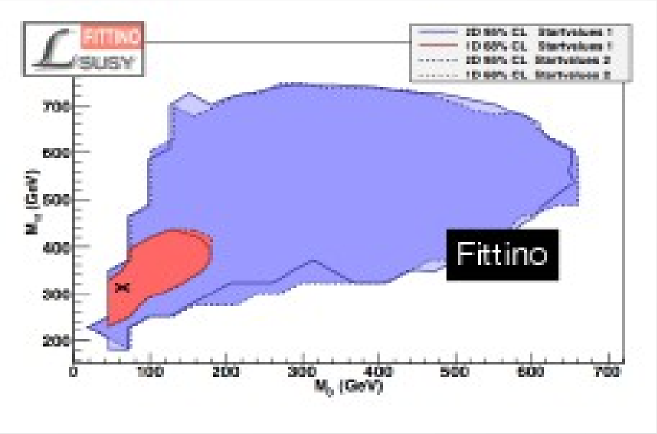
R. Lafaye , M. Rauch, T. Plehn, D. Zerwas



H. Flächer, M. Goebel, J. Haller, A. Höcker, K. Mönig, J. Stelzer

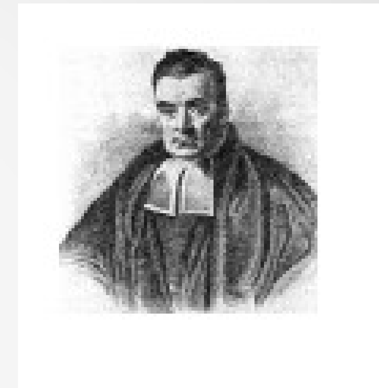


P. Bechtle, K. Desch M. Uhle, P. Wienemann



Bayesian based inference

$$P(H|d,I) = \frac{P(d|H,I)P(H|I)}{P(d|I)}$$



- H: hypothesis
- D: data
- I: external information
- Prior: what we know about H (given information I) before seeing the data
- Likelihood: the probability of obtaining data d if hypothesis H is true
- Posterior: the probability of obtaining data d if hypothesis H is true
- Evidence: normalization constant (independent of H), crucial for model comparison

Priors

- Ignoring the prior and identifying

$$p(\theta_i|\text{data}) \equiv p(\text{data}|\theta_i)$$

- implicitly assumes

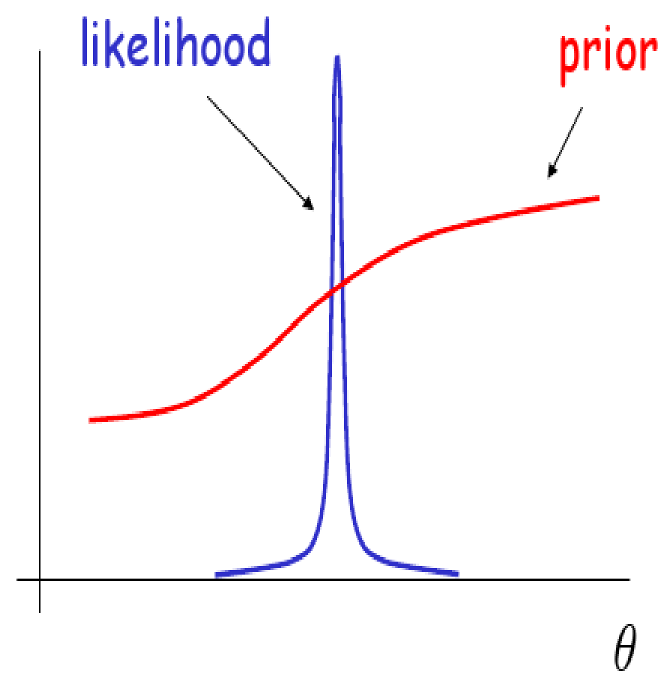
$$p(\theta_i) = \text{const.} \equiv \text{“flat”}$$

- But e.g.

$$\theta_i \longrightarrow \theta_i^2$$

$$\text{“flat”} \longrightarrow \text{“non-flat”}$$

- There is a vast literature on priors: Jeffreys’, conjugate, non-informative, ignorance, etc

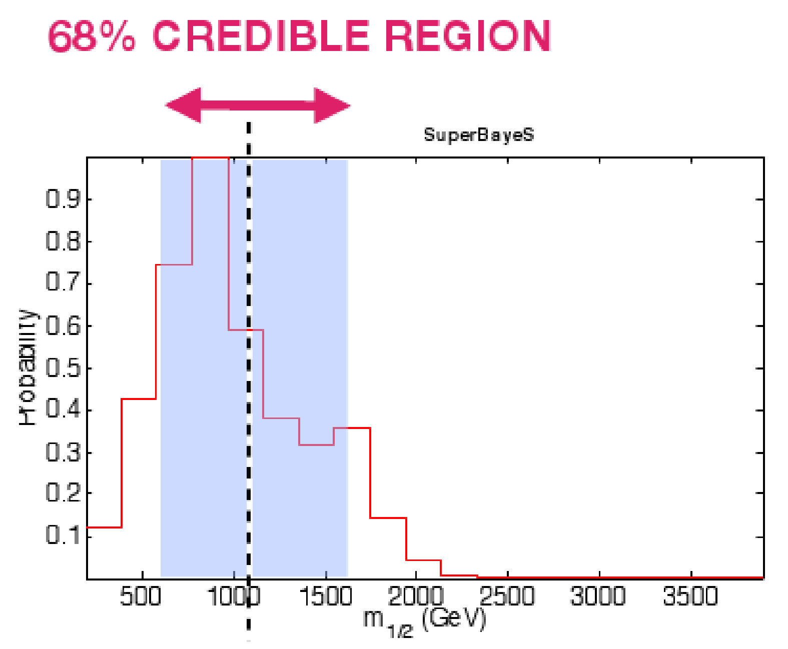


$$p(\theta_i|\text{data}) \equiv p(\text{data}|\theta_i)$$

If data are good enough to select a small region of {θ} then the prior p(θ) becomes irrelevant

Favoured regions: Bayesian Approach

- Bayesian methods: the best-fit has no special status. Focus on regions of large posterior probability mass instead
- Determine posterior credible regions: e.g. symmetric interval around the mean containing 68% of samples

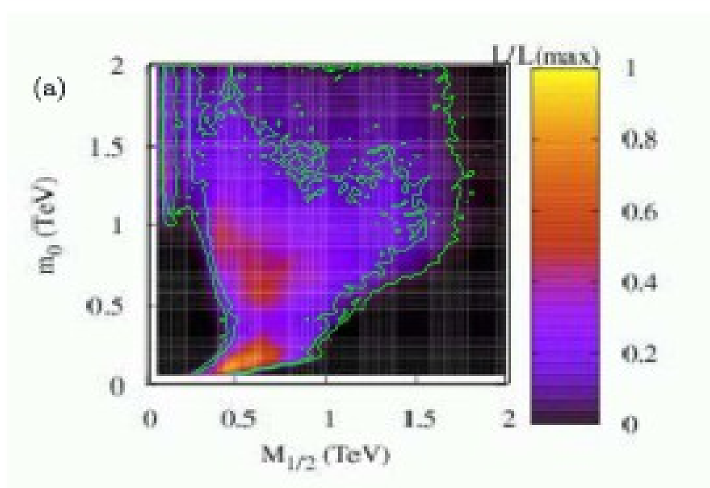


- Marginalisation: integration over hidden dimensions comes for free

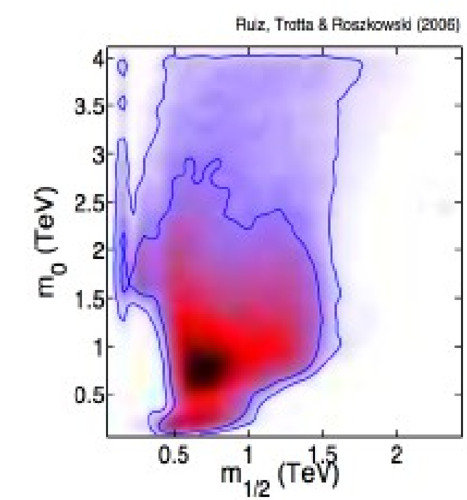
$$p(\theta_1|\text{data}) = \int d\theta_2 \cdots d\theta_N \; p(\theta_i|\text{data})$$

Groups

- Bayesian approach led by two groups (early work by Baltz & Gondolo)
- Ben Allanach (DAMPT) et al. (Allanach & Lester, 2006 onwards, Cranmerand others)
- RdA, Roszkowski & Roberto Trotta (2006 onwards) SuperBayeS public code (available from: superbayes.org) + Feroz & Hobson (MultiNest), + Silk (indirect detection) + de los Heros (IceCube) + Casas et al. (Naturalness) + Bertone et al. (pMSSM)
- BayesFITS: Roszkowski et al.



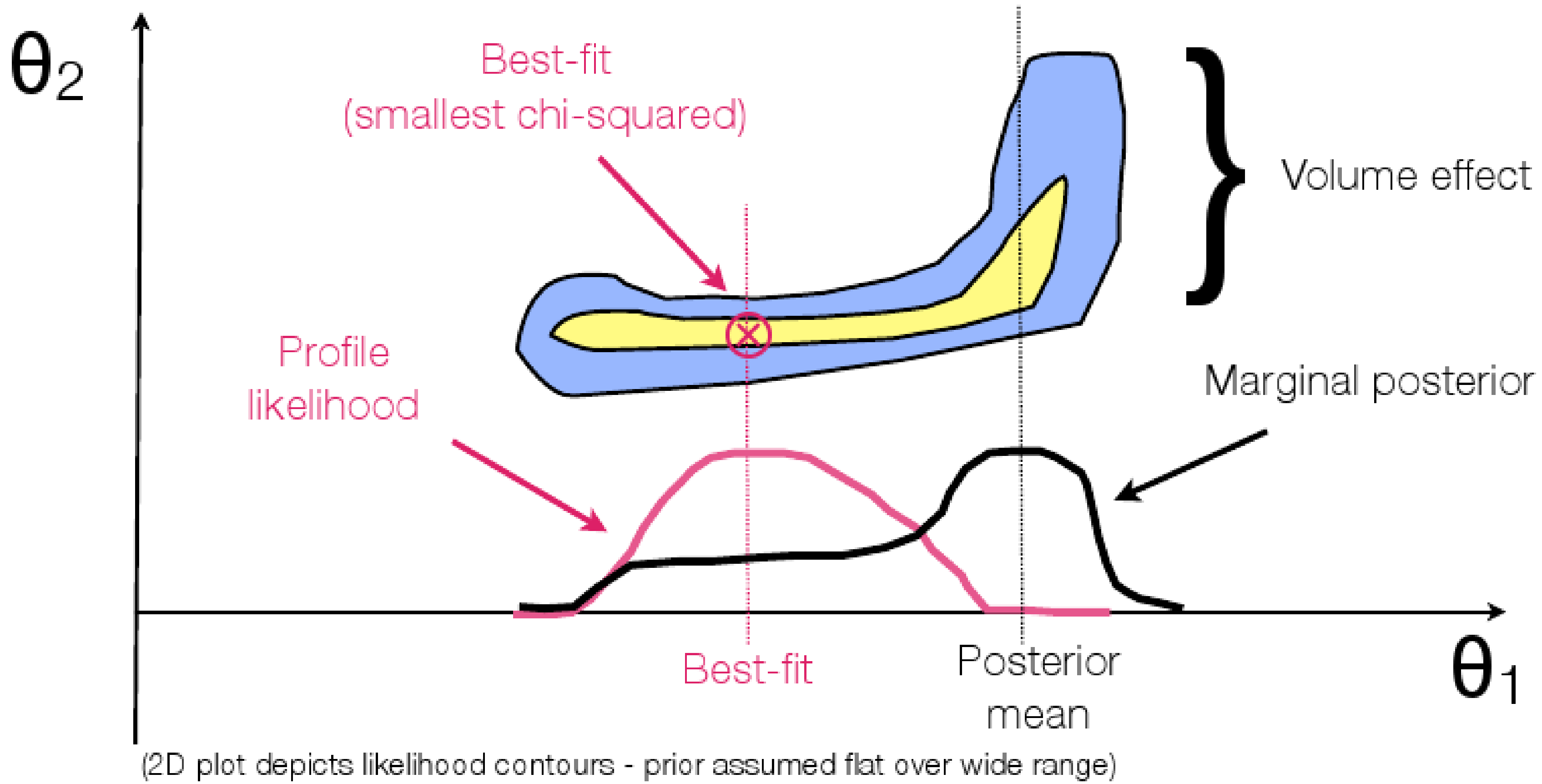
Allanach & Lester (2006)



Ruiz de Austri, Roszkowski & RT (2006)

Profiling versus Marginalizing

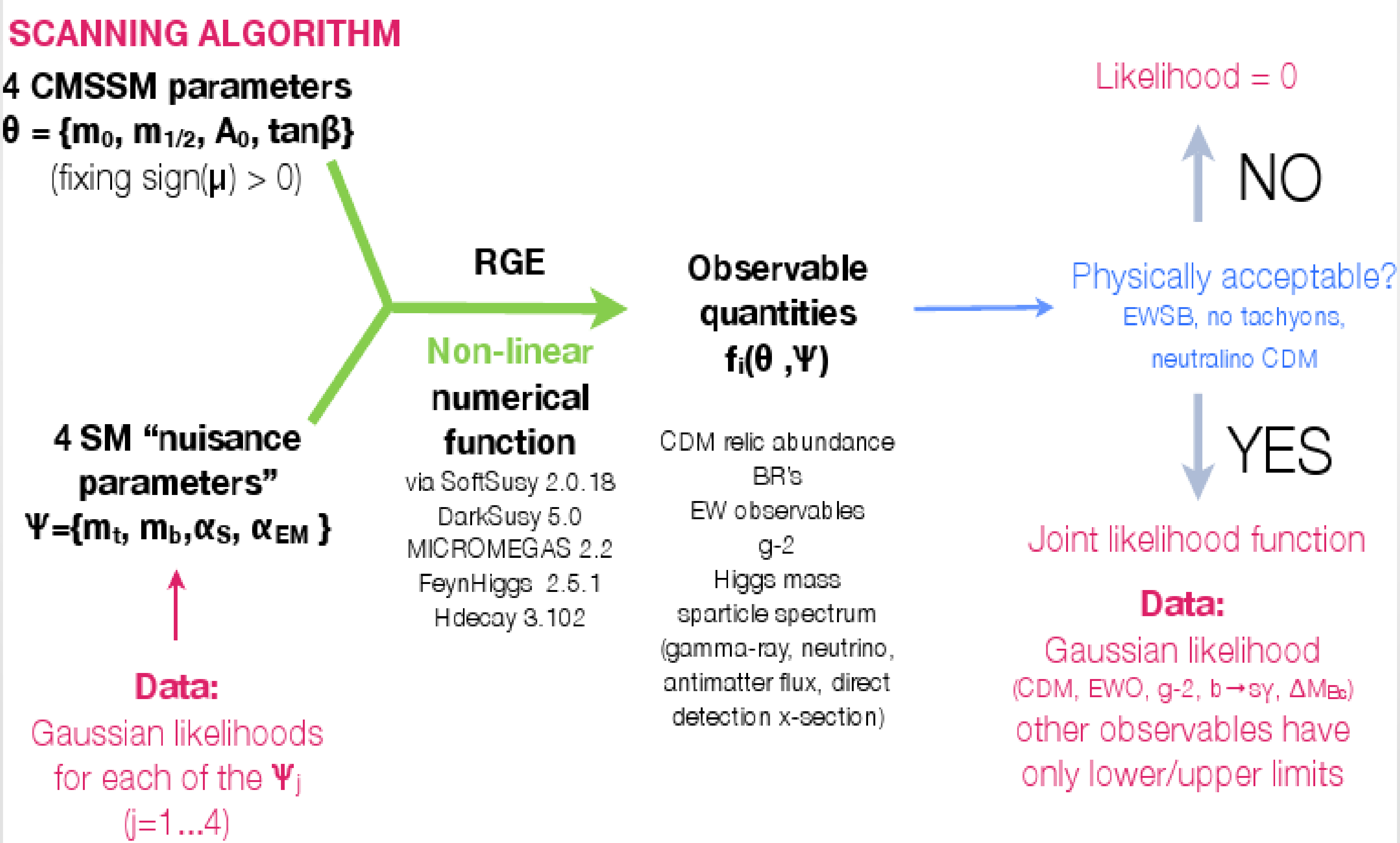
$$P(\theta_1|D) = \int L(\theta_1, \theta_2) p(\theta_1, \theta_2) d\theta_2 \quad L(\theta_1) = max_{\theta_2} L(\theta_1, \theta_2)$$



The CMSSM

Cabrera et al. (2010), (2013)
Strege et al. (2011), (2013)

Analysis pipeline



Analysis ingredients

Prior ranges

flat priors: CMSSM parameters
50 GeV $< m_0 <$ 4 TeV
50 GeV $< m_{1/2} <$ 4 TeV
$\lvert A_0 \rvert <$ 7 TeV
$2 < \tan\beta < 62$

Likelihood function

$$\mathcal{L} = p(\sigma, c|\xi(m)) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\chi^2}{2}\right]$$

$$\chi^2 = \frac{[\xi(m) - c]^2}{\sigma^2} \qquad \sigma \rightarrow s = \sqrt{\sigma^2 + \tau^2}$$

Data: indirect observables

Observable	Mean value μ	Uncertainties σ (exper.)	Uncertainties τ (theor.)
M_W [GeV]	80.399	0.023	0.015
$\sin^2\theta_{eff}$	0.23153	0.00016	0.00015
$\delta a_\mu^{SUSY} \times 10^{10}$	28.7	8.0	2.0
$BR(B \to X_s\gamma) \times 10^4$	3.55	0.26	0.30
$R_{\Delta M_{B_s}}$	1.04	0.11	-
$\frac{BR(B_u \to \tau\nu)}{BR(B_u \to \tau\nu)_{SM}}$	1.63	0.54	-
$\Delta_{0-} \times 10^2$	3.1	2.3	-
$\frac{BR(B \to D\tau\nu)}{BR(B \to De\nu)} \times 10^2$	41.6	12.8	3.5
R_{l23}	0.999	0.007	-
$BR(D_s \to \tau\nu) \times 10^2$	5.38	0.32	0.2
$BR(D_s \to \mu\nu) \times 10^3$	5.81	0.43	0.2
$BR(D \to \mu\nu) \times 10^4$	3.82	0.33	0.2
$\Omega_\chi h^2$	0.1109	0.0056	0.012
m_h [GeV]	125.8	0.6	2.0
$BR(\overline{B}_s \to \mu^+\mu^-)$	3.2×10^{-9}	1.5×10^{-9}	10%

	Limit (95% CL)	τ (theor.)
Sparticle masses	As in table 4 of Ref. [42].	
$m_0, m_{1/2}$	ATLAS, $\sqrt{s} = 8$ TeV, 5.8 fb^{-1} 2012 limits	
$m_A, \tan\beta$	CMS, $\sqrt{s} = 7$ TeV, 4.7 fb^{-1} 2012 limits	
$m_\chi - \sigma^{SI}_{\tilde{\chi}_1^0 - p}$	XENON100 2012 limits (224.6×34 kg days)	

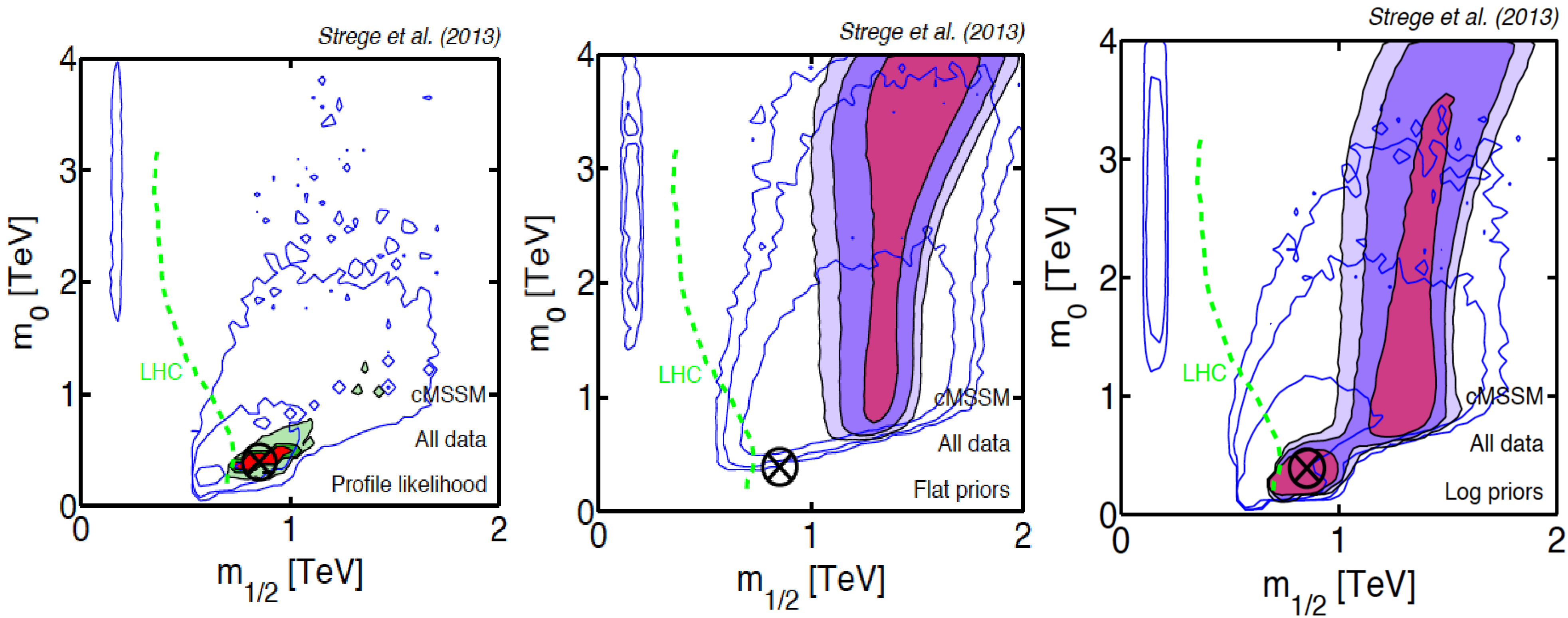
Nuisance parameters

	Gaussian prior	Range scanned
SM nuisance parameters		
M_t [GeV]	173.1 ± 1.3	(167.0, 178.2)
$m_b(m_b)^{\overline{MS}}$ [GeV]	4.20 ± 0.07	(3.92, 4.48)
$[\alpha_{em}(M_Z)^{\overline{MS}}]^{-1}$	127.955 ± 0.030	(127.835, 128.075)
$\alpha_s(M_Z)^{\overline{MS}}$	0.1176 ± 0.0020	(0.1096, 0.1256)
Astrophysical nuisance parameters		
ρ_{loc} [GeV/cm^3]	0.4 ± 0.1	(0.001, 0.900)
v_{lsr} [km/s]	230.0 ± 30.0	(80.0, 380.0)
v_{esc} [km/s]	544.0 ± 33.0	(379.0, 709.0)
v_d [km/s]	282.0 ± 37.0	(98.0, 465.0)
Hadronic nuisance parameters		
f_{Tu}	0.02698 ± 0.002	(0.010, 0.045)
f_{Td}	0.03906 ± 0.00395	(0.015, 0.060)
f_{Ts}	0.363 ± 0.119	(0.000, 0.85)

Use Multinest as sampling algorithm: Posterior and PL reconstruction

The CMSSM

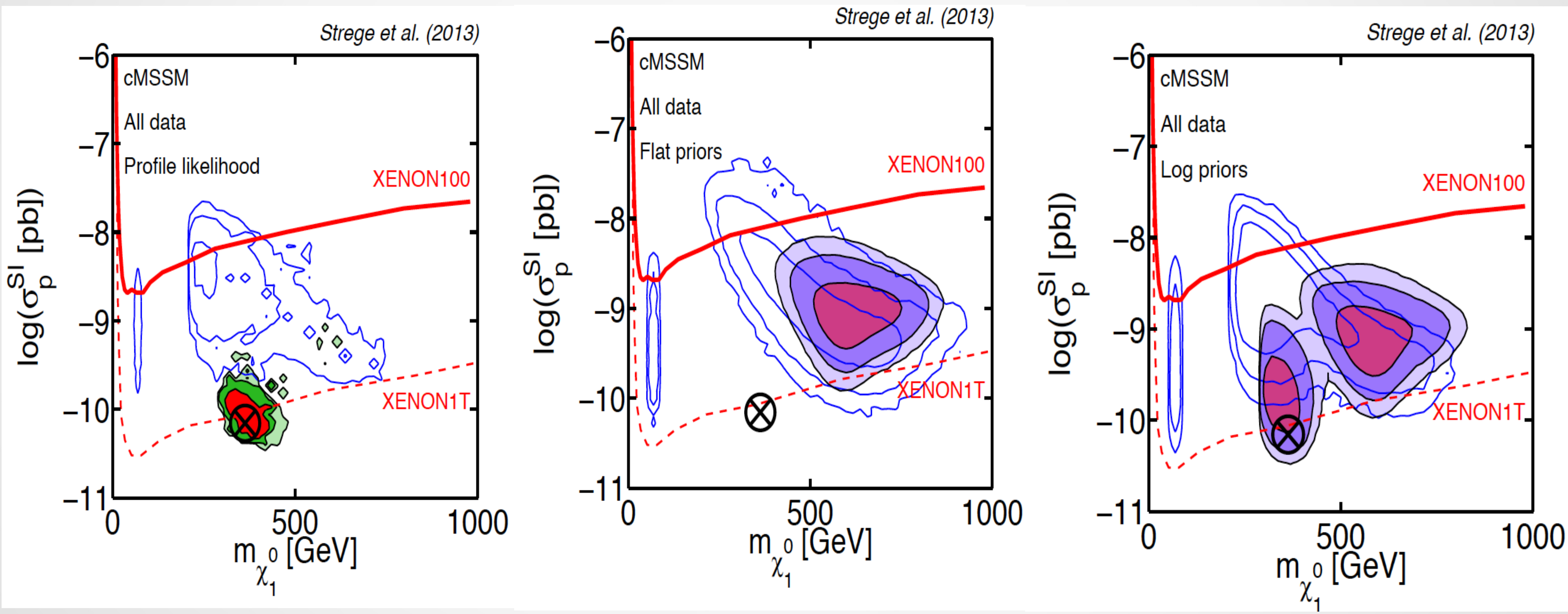
$$m_h^2 \lesssim m_Z^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + x_t^2\left(1 - \frac{x_t^2}{12}\right)\right] \quad M_S^2 \equiv \frac{1}{2}(M_{\tilde{t}_1}^2 + M_{\tilde{t}_2}^2),\ x_t \equiv X_t/M_S$$



- Profile likelihood: At 99% C.L. contours squeezed around the stau-coannihilation
- Bayesian: Still there is a prior dependence though reduced

DM Direct Detection

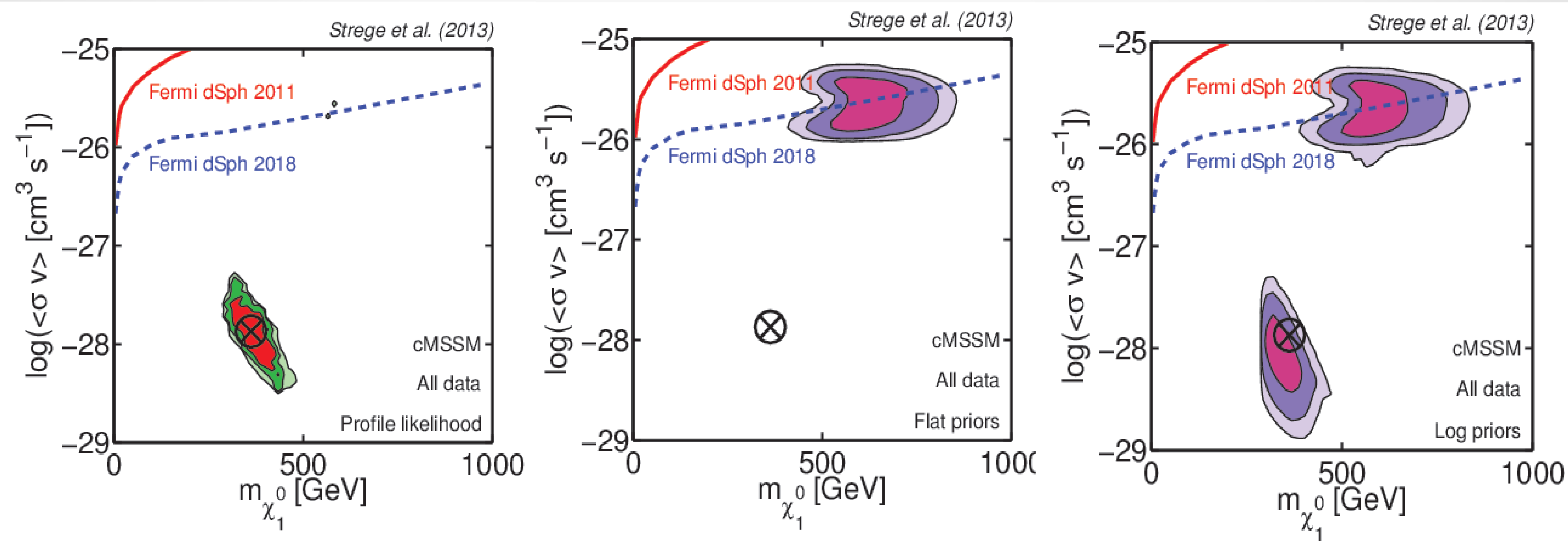
- Xenon100 data with 224.6 days of exposure



- Profile likelihood: 1 Ton scale can prove regions favoured at the 95% C.L.
- Bayesian: Bulk of the posterior covered by 1 Ton scale experiments

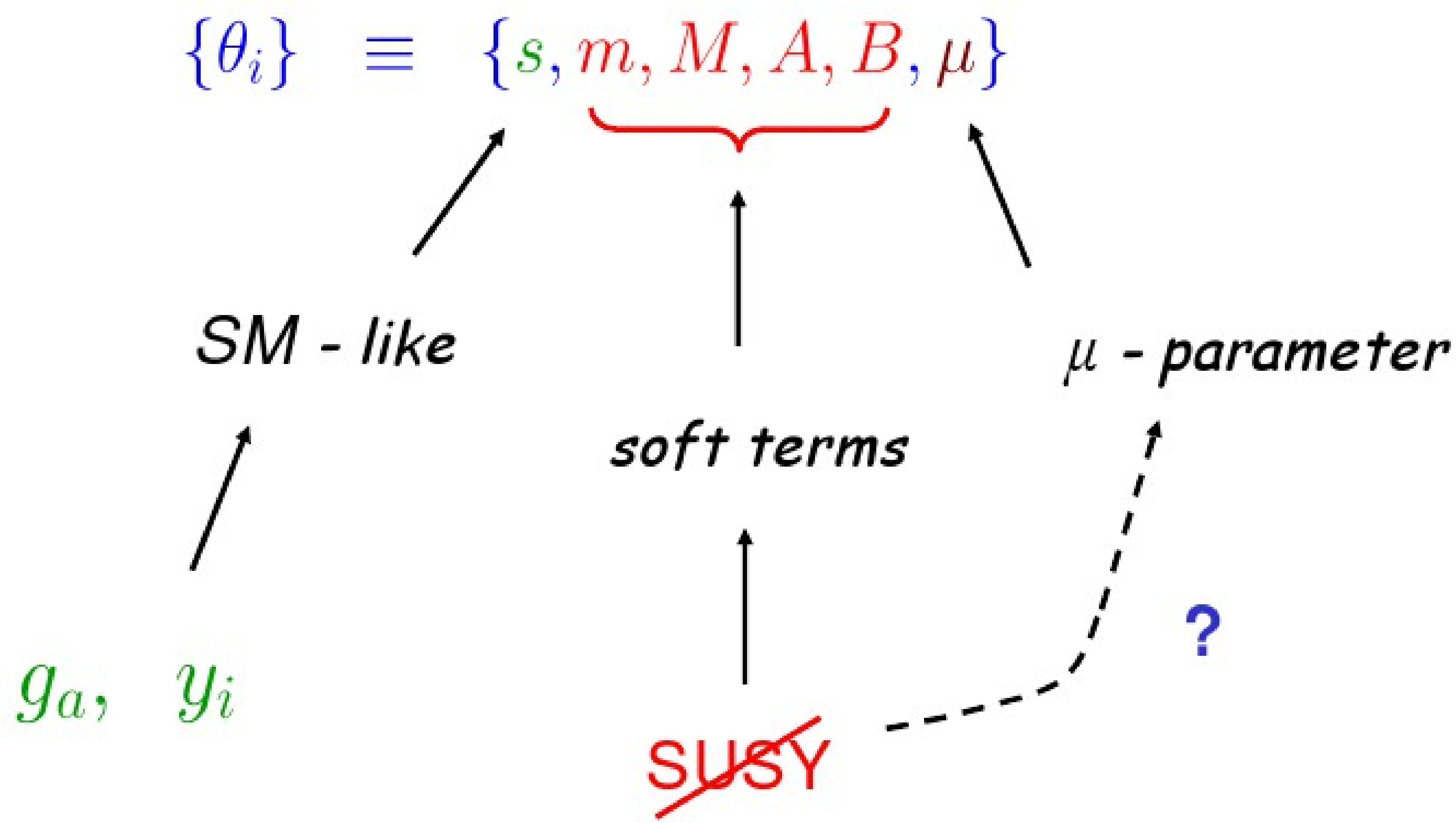
DM Indirect Detection

- Fermi constraints from dwarf spheroidal galaxies



- Profile likelihood: bf point out of the reach for Fermi
- Bayesian: it is going to probe a large fraction of the A-funnel region

Naturalness and the Bayesian approach



...

- Recall an usual assumption

$$\underbrace{m, M, A, B, \mu}_{(\equiv M_{\rm soft})}$$ should be < O(TeV)

In order to get a Natural Electroweak Symmetry Breaking (with no fine-tunnigs)

$$V(H_1, H_2) = m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 - 2B\mu H_1 H_2 + \frac{1}{8}(g^2 + g'^2)\left(|H_1|^2 - |H_2|^2\right)^2$$

$$M_Z^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - 2\mu^2$$

$$\sin 2\beta = \frac{2\mu}{B}\left(m_{H_1}^2 + m_{H_2}^2 + 2\mu_{\rm low}^2\right)$$

Unnatural fine-tuning unless $M_{\rm soft} \lesssim \mathcal{O}(\mathrm{TeV})$

...

- Instead solving μ^2 in terms of M_Z and the other soft-terms, treat as another exp. data
- Approximate the likelihood as

$$\mathcal{L} = N_Z\, e^{-\frac{1}{2}\left(\frac{M_Z - M_Z^{\exp}}{\sigma_Z}\right)^2} \mathcal{L}_{\rm rest} \simeq \delta(M_Z - M_Z^{\exp})\, \mathcal{L}_{\rm rest}$$

- Use M_Z to marginalize μ

$$p(s,m,M,A,B|\,\mathrm{data}) = \int d\mu\, p(s,m,M,A,B,\mu|\mathrm{data}) \simeq \mathcal{L}_{\rm rest}\left[\frac{d\mu}{dM_Z}\right]_{\mu_Z} p(s,m,M,A,B,\mu_Z)$$

$$p(s,m,M,A,B|\,\mathrm{data}) = 2\,\mathcal{L}_{\rm rest}\,\frac{\mu_Z}{M_Z}\,\frac{1}{c_\mu}\,p(s,m,M,A,B,\mu_Z) \qquad c_\mu = \frac{\partial\ \ln M_Z^2}{\partial\ \ln \theta_i}$$

$c^{-1} \sim$ Probability of cancellation between the varius contributions to get M_Z

...

$$\{\mu, y_t, B\} \xrightarrow{J} \{M_Z, m_t, \tan\beta\}$$

$$p(m_t, m, M, A, \tan\beta|\text{data}) \;=\; J|_{\mu=\mu_Z}\, p(y_t, m, M, A, B, \mu_Z)\, \mathcal{L}_{\text{rest}}$$

$$p_{\text{eff}}(m_t, m, M, A, \tan\beta)$$

$$p_{\text{eff}}(m_t, m, M, A, \tan\beta) \;\propto\; \left[\frac{E}{R_\mu^2}\right] \frac{y}{y_{\text{low}}} \frac{t^2-1}{t(1+t^2)} \frac{B_{\text{low}}}{\mu_Z}\, p(m, M, A, B, \mu=\mu_Z)$$

J

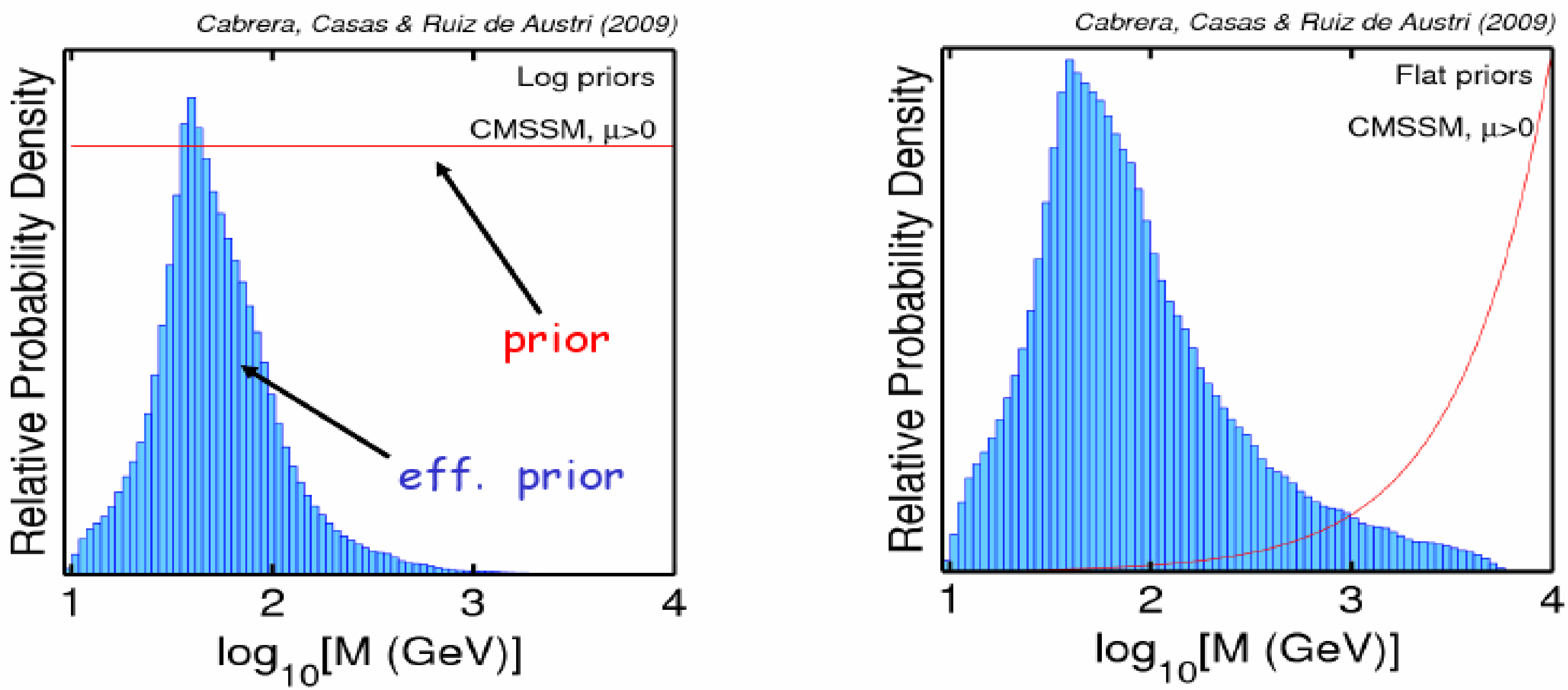
model-independent part !

still undefined

It contains the fine-tuning penalization

It penalizes large tan β

The ElectroWeak Scale

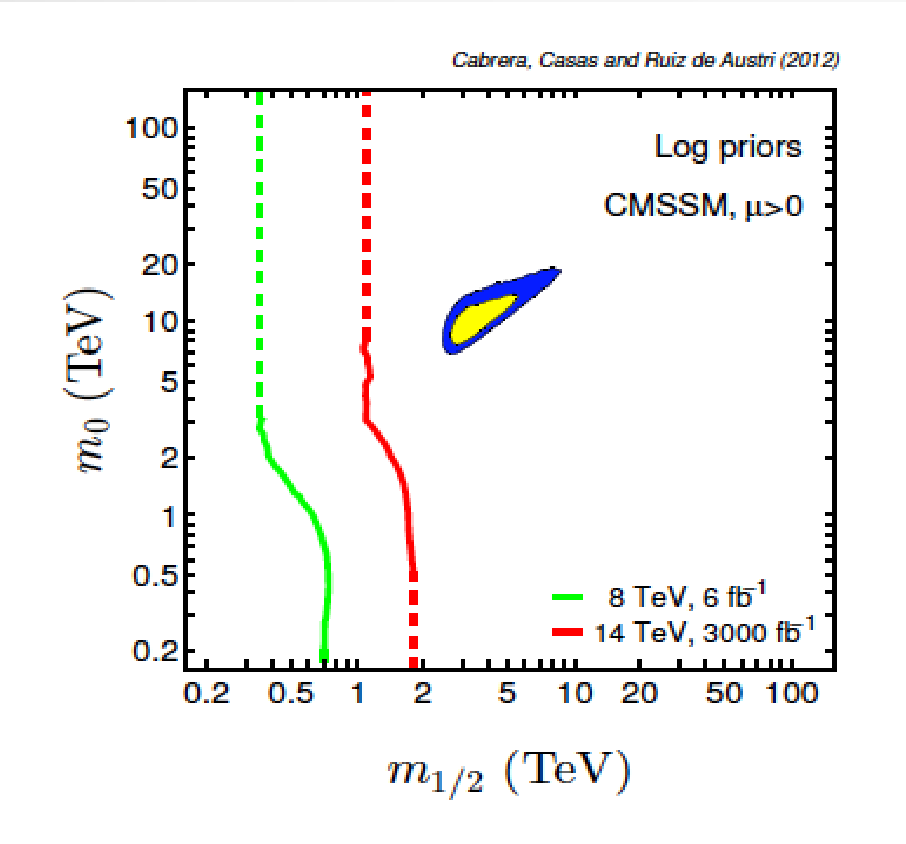


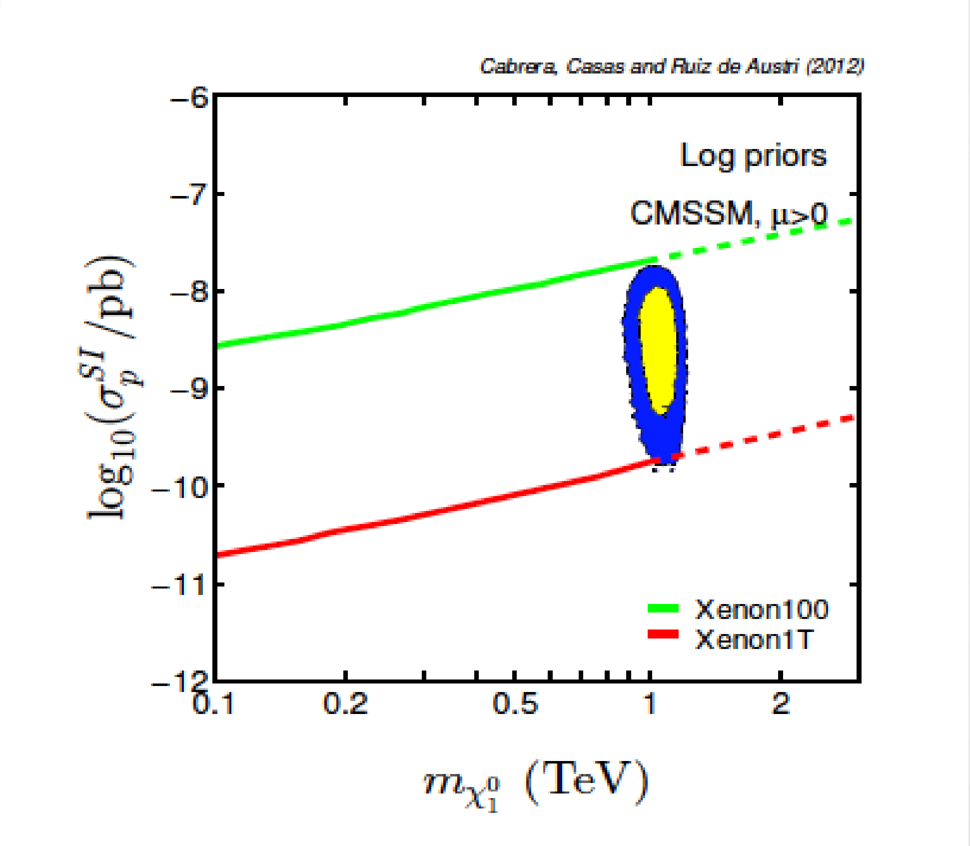
M_Z brings SUSY to the EW region

- We may vary M_{soft} up to M_X the results do not depend on the range chosen
- This suggests that large soft-masses are disfavoured

CMSSM and Naturalness

- Single-component DM scenario

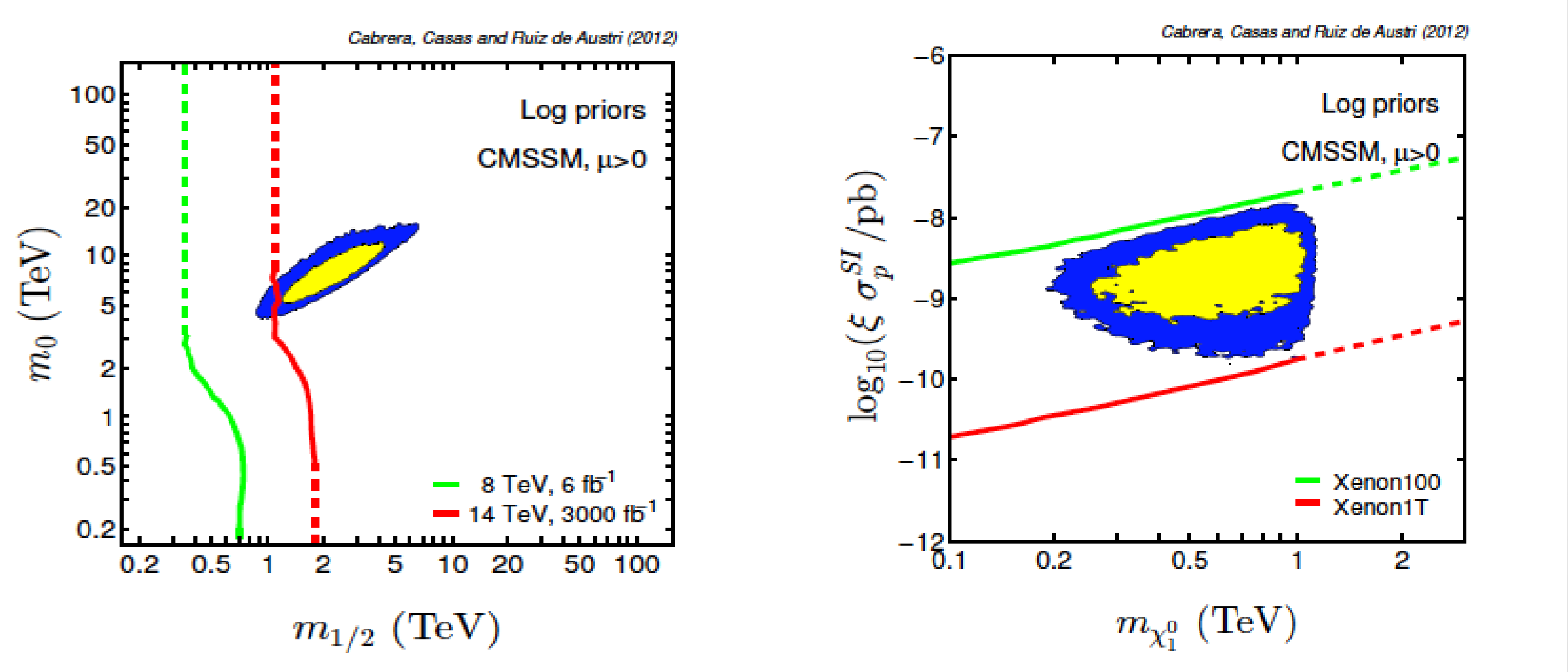




- The 95% credible region is not accesible to the LHC
- However it is fully accesible to 1 Tone scale DM DD experiments

...

- Multi-component DM scenario: $\xi \equiv \rho_\chi/\rho_{DM} = \Omega_\chi/\Omega_{DM}$



- The 95% the credible region is partially accesible to the LHC
- However it is fully accesible to 1 Tone scale DM DD experiments

MSSM-15

Strege et al. (2014)

MSSM-15

- M_1, M_2, M_3: the bino, wino and gluino masses
- m_L, m_Q: the first/second generation sfermion masses
- m_{L3}, m_{E3}, m_{Q3}, m_{U3}, m_{D3}: third generation sfermion masses
- A_0: universal trilinear bottom, tau coupling
- A_t: top trilinear coupling
- μ: Higgsino mass
- m_A: the CP-odd Higgs mass
- $\tan\beta$: the ratio of the vevs of the two-Higgs doublet fields

All except A_0 defined at SUSY scale

Analysis ingredients

Priors

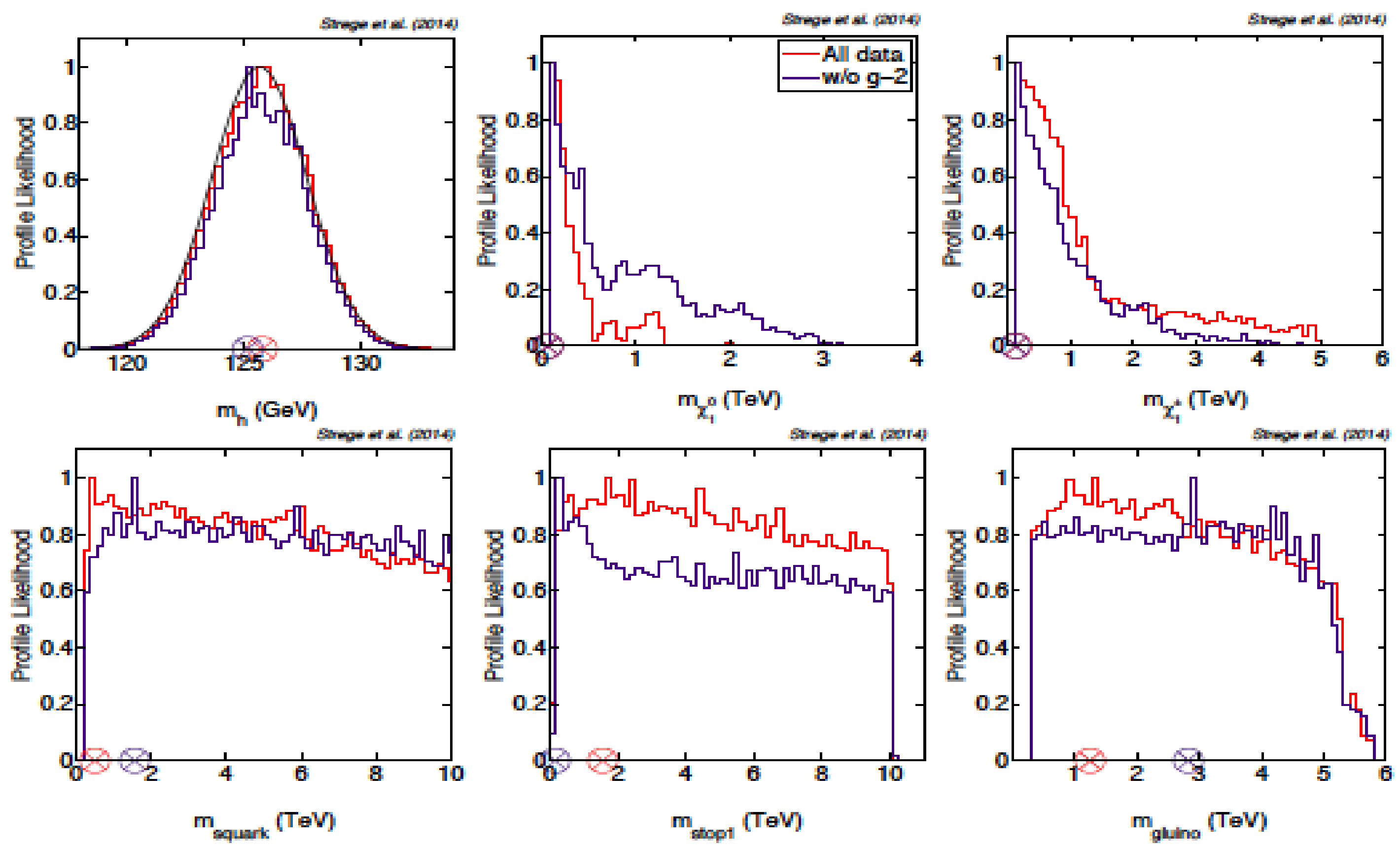
MSSM-15 parameters and priors			
Flat priors		Log priors	
M_1 [TeV]	(-5, 5)	$\text{sgn}(M_1)\log\lvert M_1\rvert/\text{GeV}$	$(-3.7, 3.7)$
M_2 [TeV]	(0.1, 5)	$\log M_2/\text{GeV}$	$(2, 3.7)$
M_3 [TeV]	(-5, 5)	$\text{sgn}(M_3)\log\lvert M_3\rvert/\text{GeV}$	$(-3.7, 3.7)$
m_L [TeV]	(0.1,10)	$\log m_L/\text{GeV}$	$(2, 4)$
m_{L_3} [TeV]	(0.1,10)	$\log m_{L_3}/\text{GeV}$	$(2, 4)$
m_{E_3} [TeV]	(0.1,10)	$\log m_{E_3}/\text{GeV}$	$(2, 4)$
m_Q [TeV]	(0.1,10)	$\log m_Q/\text{GeV}$	$(2, 4)$
m_{Q_3} [TeV]	(0.1,10)	$\log m_{Q_3}/\text{GeV}$	$(2, 4)$
m_{U_3} [TeV]	(0.1,10)	$\log m_{U_3}/\text{GeV}$	$(2, 4)$
m_{D_3} [TeV]	(0.1,10)	$\log m_{D_3}/\text{GeV}$	$(2, 4)$
A_t [TeV]	(-10, 10)	$\text{sgn}(A_t)\log\lvert A_t\rvert/\text{GeV}$	$(-4, 4)$
A_0 [TeV]	(-10,10)	$\text{sgn}(A_0)\log\lvert A_0\rvert/\text{GeV}$	$(-4, 4)$
μ [TeV]	(-5,5)	$\text{sgn}(\mu)\log\lvert\mu\rvert/\text{GeV}$	$(-3.7, 3.7)$
m_A [TeV]	(0.01, 5)	$\log m_A/\text{GeV}$	$(1, 3.7)$
$\tan\beta$	(2, 62)	$\tan\beta$	$(2, 62)$
M_t [GeV]	173.2 ± 0.87 [17] (Gaussian prior)		

Data

Observable	Mean value μ	Standard deviation σ (exper.)	Standard deviation τ (theor.)	Ref.
M_W [GeV]	80.385	0.015	0.01	[48]
$\sin^2\theta_{\text{eff}}$	0.23153	0.00016	0.00010	[48]
Γ_Z [GeV]	2.4952	0.0023	0.001	[48]
σ^0_{had} [nb]	41.540	0.037	-	[48]
R^0_l	20.767	0.025	-	[48]
R^0_b	0.21629	0.00066	-	[48]
R^0_c	0.1721	0.003	-	[48]
$^{\#}A^{0,l}_{FB}$	0.0171	0.001	-	[48]
$^{\#}A^{0,b}_{FB}$	0.0992	0.0016	-	[48]
$^{\#}A^{0,c}_{FB}$	0.0707	0.0035	-	[48]
$^{\#}A_l(SLD)$	0.1513	0.0021	-	[48]
$^{\#}A_b$	0.923	0.02	-	[48]
$^{\#}A_c$	0.670	0.027	-	[48]
$\delta a^{\text{SUSY}}_\mu \times 10^{10}$	28.7	8.0	2.0	[62]
$BR(B \to X_s\gamma) \times 10^4$	3.55	0.26	0.30	[49]
$R_{\Delta M_{B_s}}$	1.04	0.11	-	[50]
$\frac{BR(B_u\to\tau\nu)}{BR(B_u\to\tau\nu)_{SM}}$	1.63	0.54	-	[49]
$\Delta_{0-} \times 10^2$	3.1	2.3	1.75	[54]
$^{\#}\frac{BR(B\to D\tau\nu)}{BR(B\to De\nu)} \times 10^2$	41.6	12.8	3.5	[63]
$^{\#}R_{l23}$	0.999	0.007	-	[64]
$A_{FB}(B \to K^*\mu^+\mu^-)$	-0.18	0.063	0.05	[51]
$BR(D_s \to \tau\nu) \times 10^2$	5.44	0.22	0.1	[49]
$^{\#}BR(D_s \to \mu\nu) \times 10^3$	5.54	0.24	0.2	[49]
$^{\#}BR(D \to \mu\nu) \times 10^4$	3.82	0.33	0.2	[49]
$BR(\overline{B}_s \to \mu^+\mu^-) \times 10^9$	3.2	1.5	0.38	[52]
$\Omega_\chi h^2$	0.1186	0.0031	0.012	[55]
m_h [GeV]	125.66	0.41	2.0	[65, 66]
$^{\dagger}\mu_{\gamma\gamma}$	0.78	0.27	15%	[68]
$^{\dagger}\mu_{W^+W^-}$	0.76	0.21	15%	[69]
$^{\dagger}\mu_{ZZ}$	0.91	0.27	15%	[70]
$^{\dagger}\mu_{b\bar{b}}$	1.3	0.65	15%	[72]
$^{\dagger}\mu_{\tau^+\tau^-}$	1.1	0.4	15%	[71]

	Limit (95% CL)	τ (theor.)	Ref.
Sparticle masses	LEP, Tevatron. As in Table 4 of Ref. [18].		[18]
†0-lepton SUSY search	ATLAS, $\sqrt{s} = 7$ TeV, 4.7 fb^{-1}		[73]
†3-lepton SUSY search	ATLAS, $\sqrt{s} = 7$ TeV, 4.7 fb^{-1}		[74]
$m_\chi - \sigma^{\text{SI}}_{\tilde{\chi}^0_1-p}$	XENON100 2012 limits (224.6×34 kg days)		[58]
$m_\chi - \sigma^{\text{SD}}_{\tilde{\chi}^0_1-p}$	XENON100 2012 limits (224.6×34 kg days)		[59]

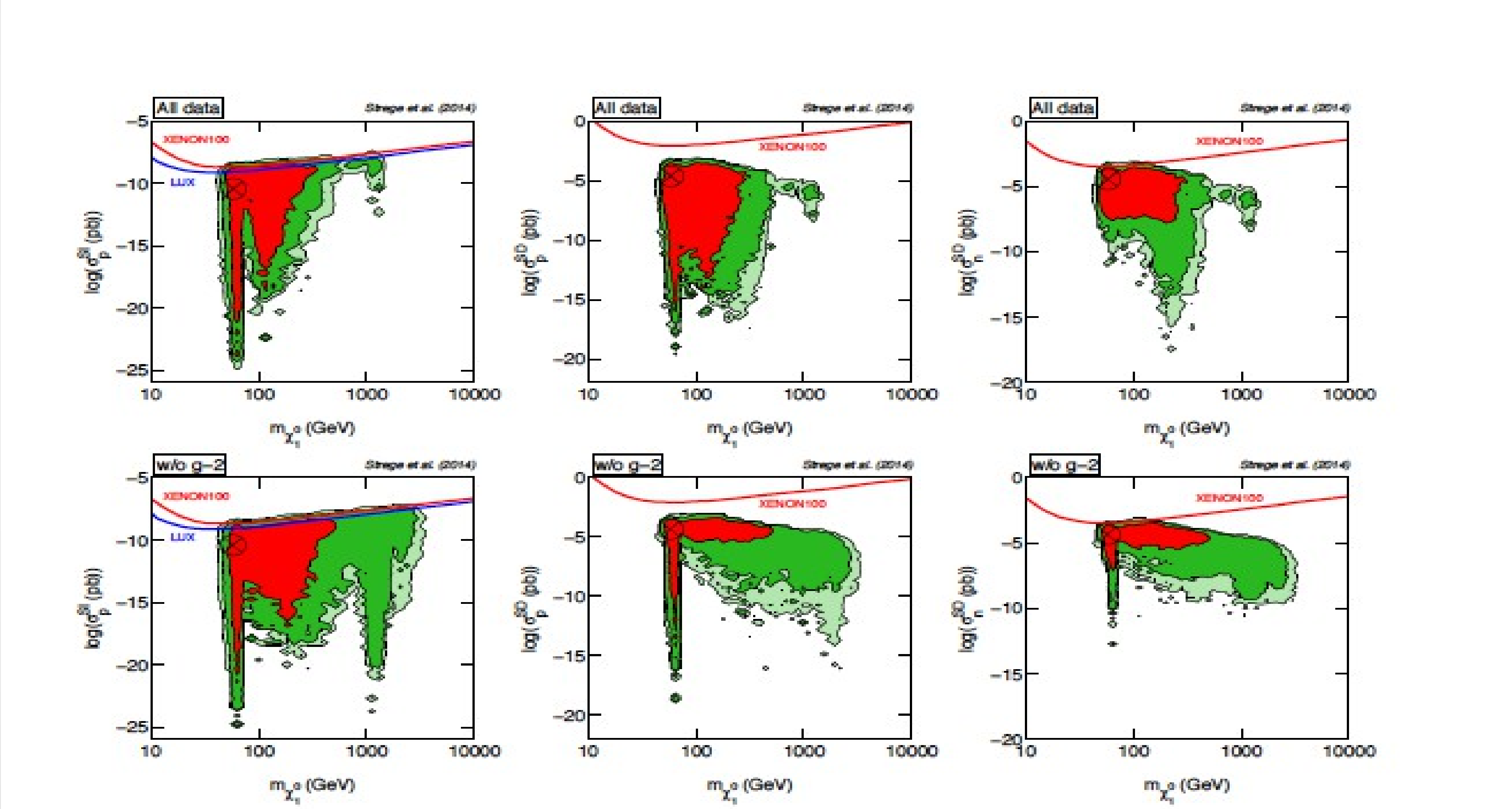
Pre-LHC data



- Only EWkinos are effectively constrained

DM DD

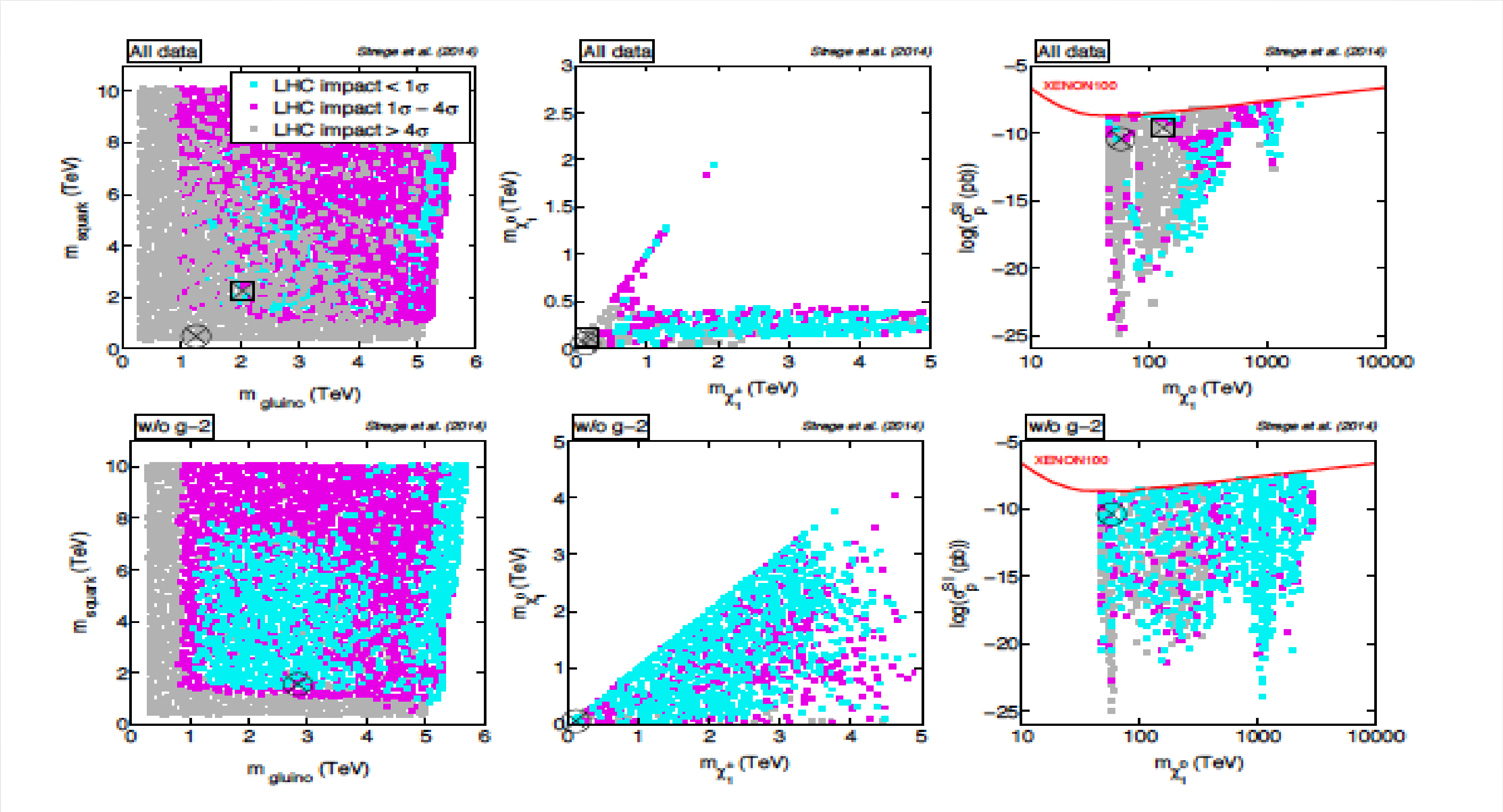
- SI + SD parts added to compute the signal



- Large cancellations occur for bino-like neutralinos

Post-LHC

- 0l + 3l (7 TeV and 4.7 fm^{-1}) + Higgs signal strength modifiers



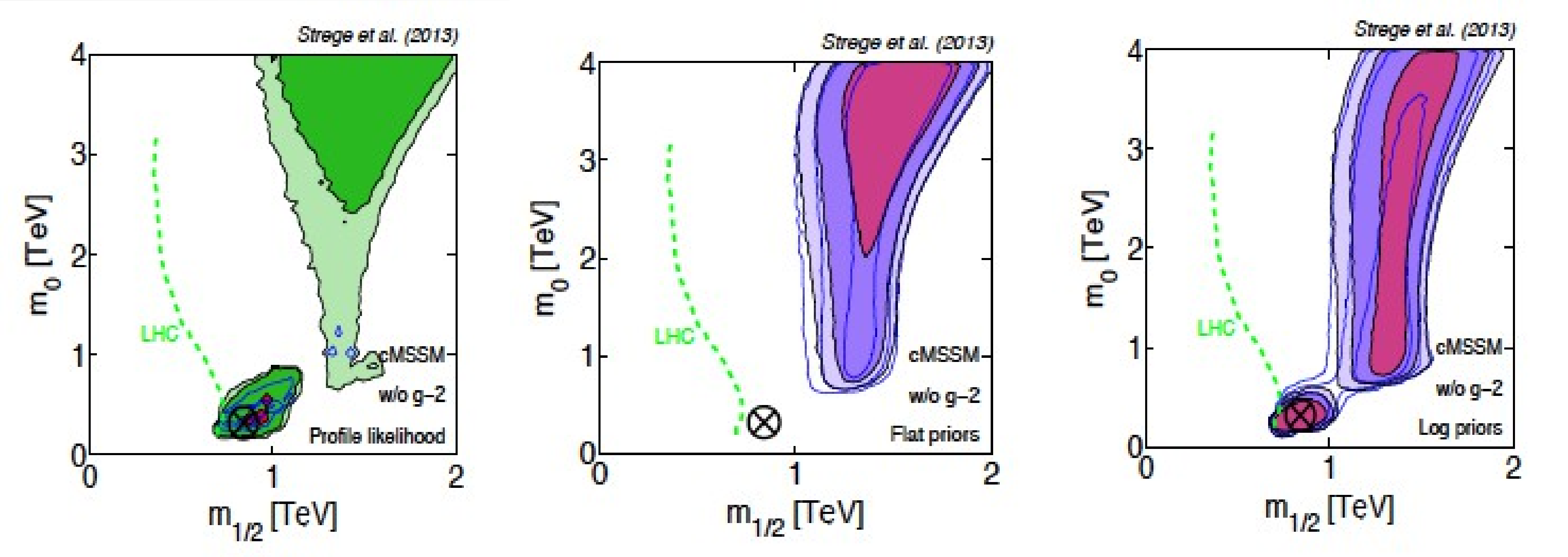
- LHC searches are able to rule-out regions unaccesible to DM Direct detection experiments

Conclusions

- SUSY phenomenology provides a timely and challenging problem for parameter inference
- DM Direct Detection experiments and LHC SUSY and Higgs searches already reject/disfavour large portions of SUSY models
- High complementarity of LHC searches with direct detection methods

Thanks !!!

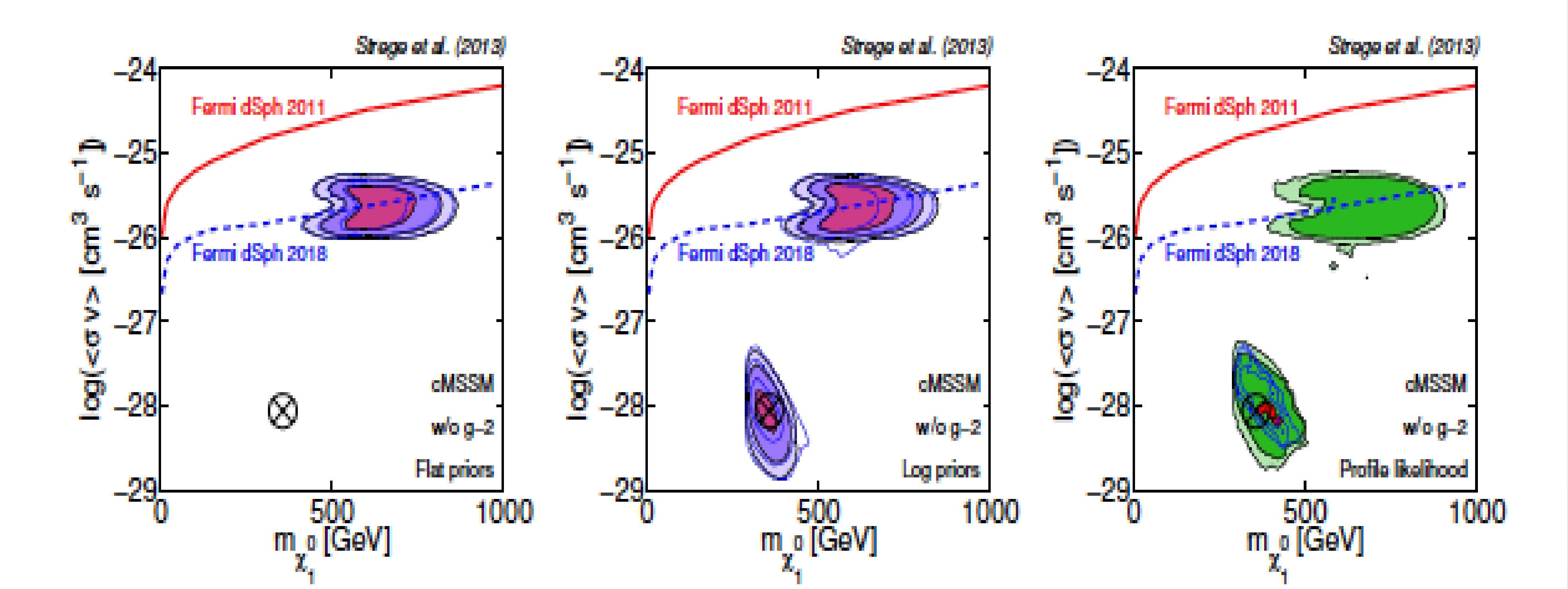
CMSSM w/o gm2



A-funnel viable at the 95% CL

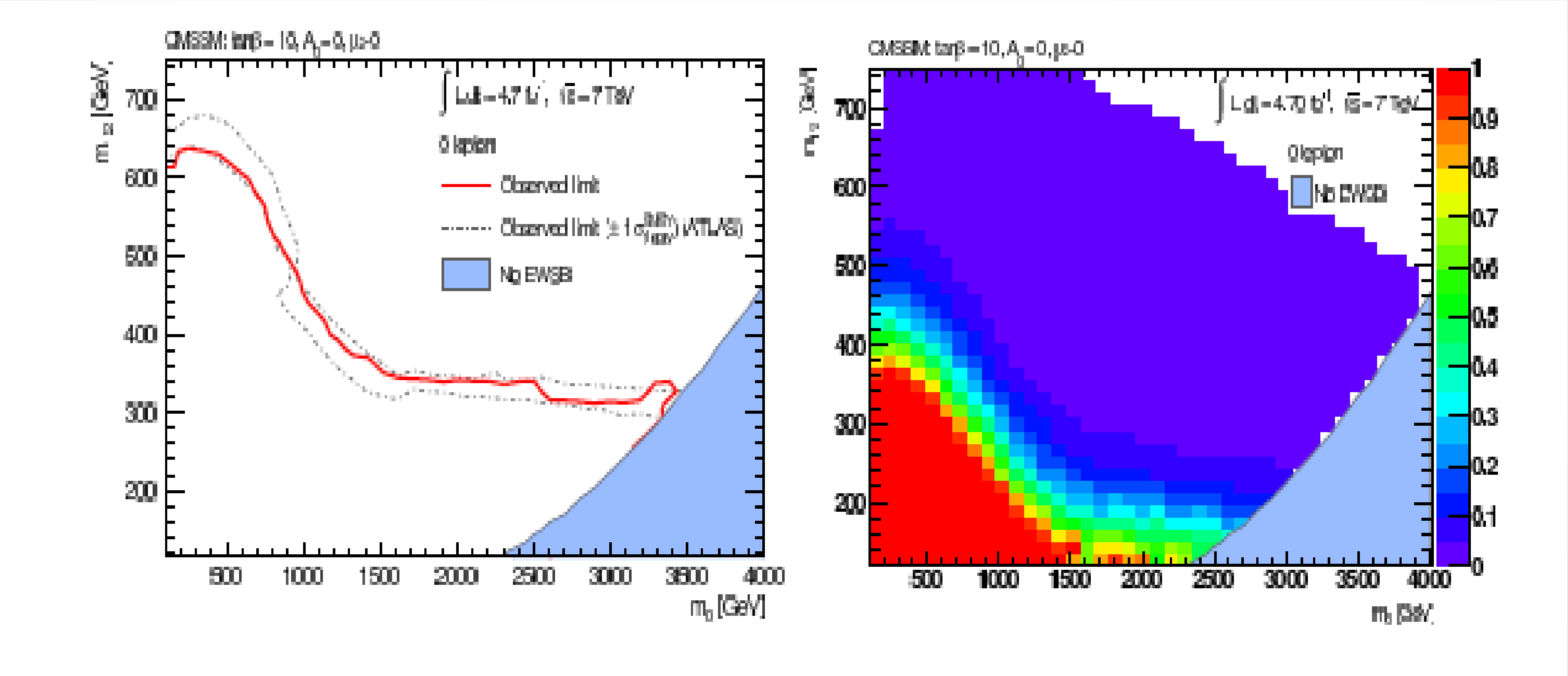
DM Indirect Detection

- Fermi and dwarfs

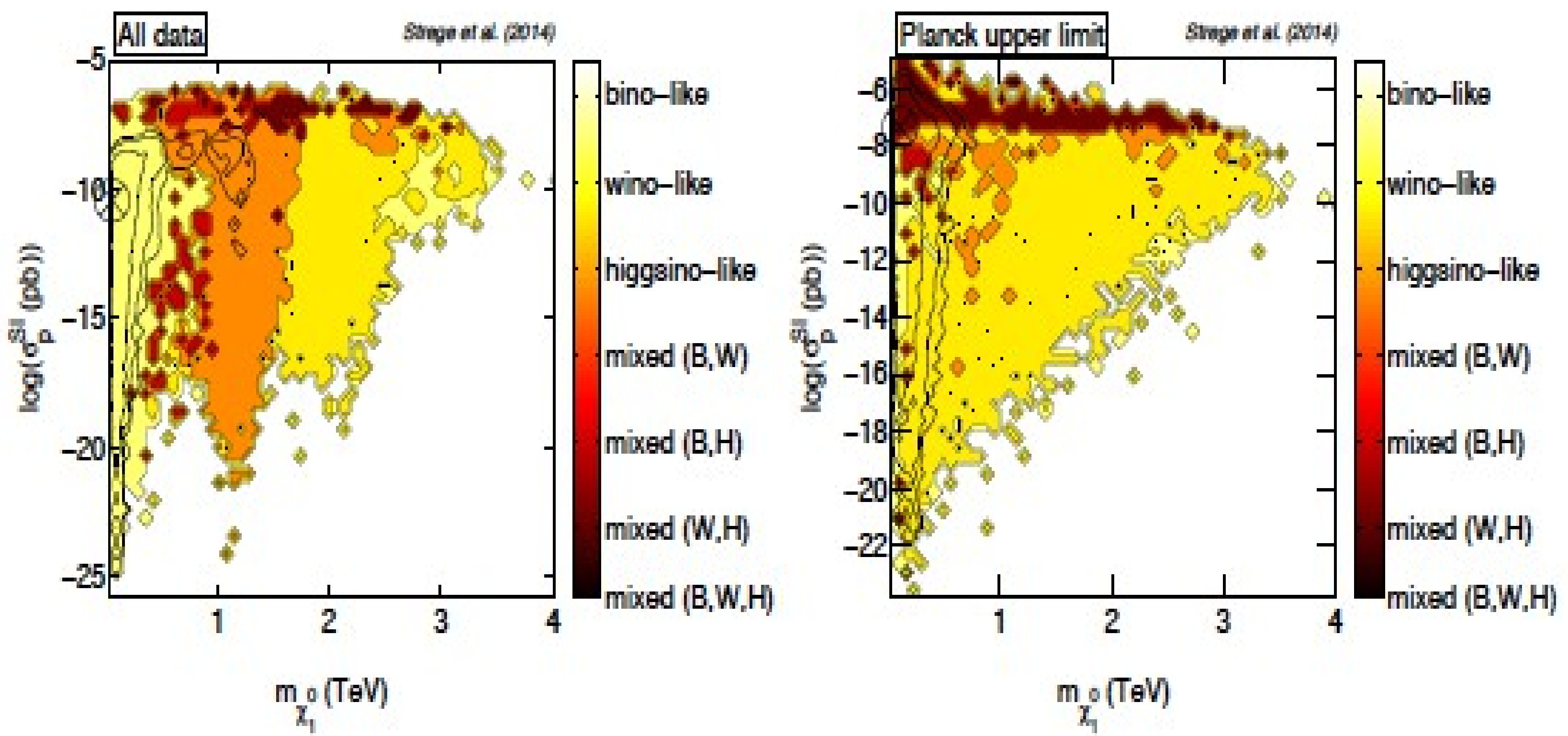


- Profile likelihood: bf point out of the reach for Fermi
- Bayes: conclusions rather depend on the prior

LHC analysis validation

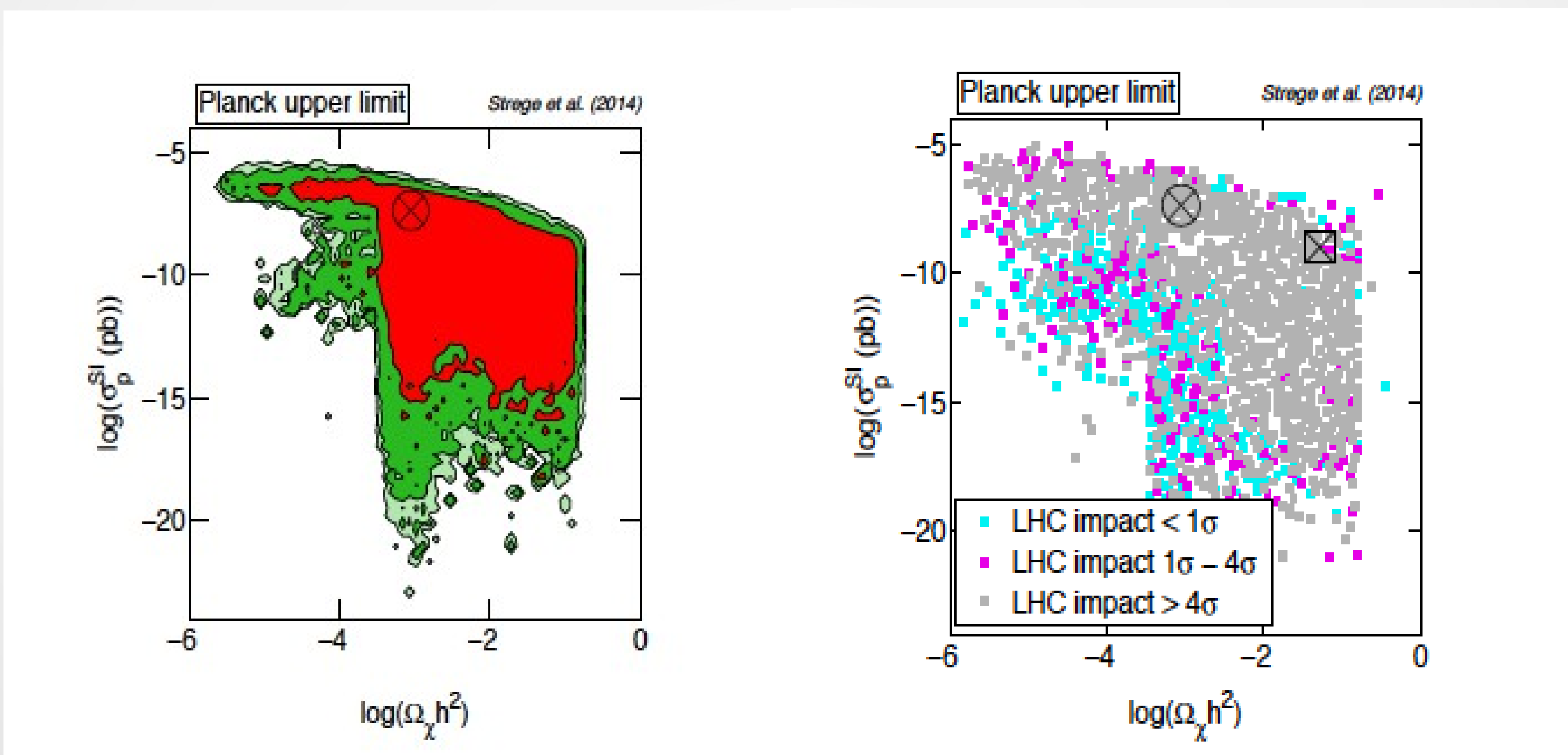


Neutralino composition

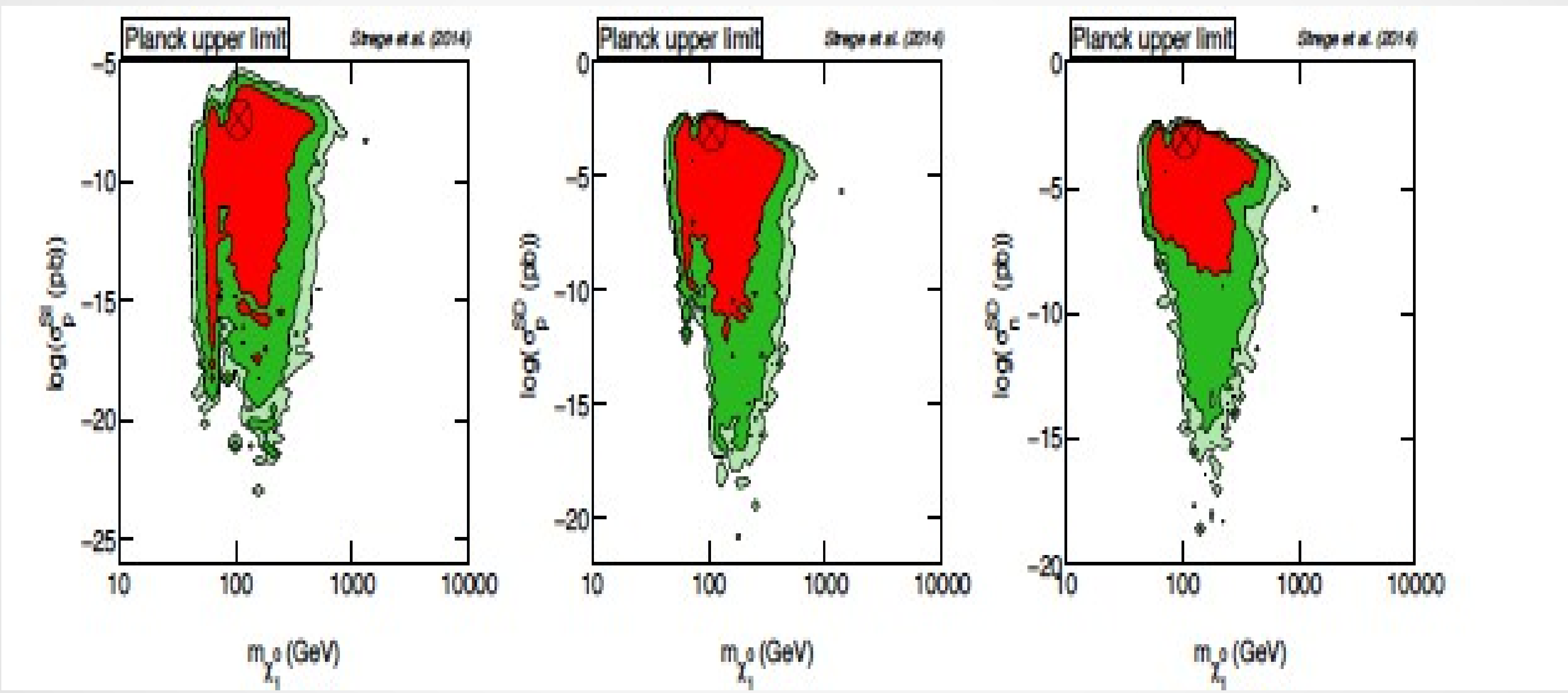


Multi-component DM

- th

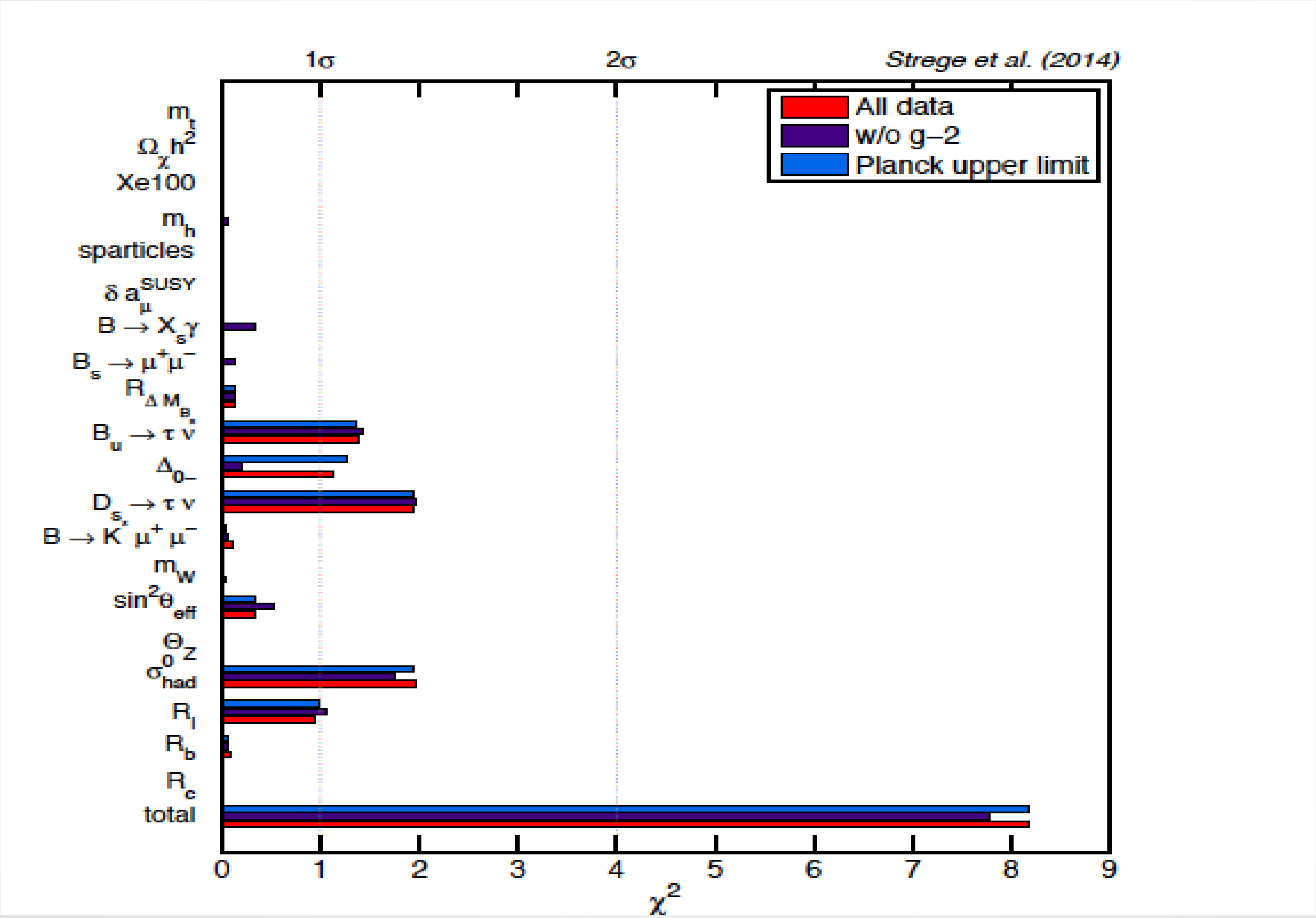


• ds



Planck upper limit
Strege et al. (2014)
-5
-10
-15
-20
-25
10
100
1000
10000
m (GeV)
Planck upper limit
Strege et al. (2014)
0
-5
-10
-15
-20
10
100
1000
10000
m (GeV)
Planck upper limit
Strege et al. (2014)
0
-5
-10
-15
-20
10
100
1000
10000
m (GeV)

Statistical pull

- 

Homogeneous exploration