Bayesian Reconstruction of the WIMP Velocity Distribution Function from Direct Dark Matter Detection Data

Chung-Lin Shan

Astroparticle Physics 2014, Amsterdam
June 27, 2014

Based on arXiv:1403.5610

Motivation

- Differential event rate for elastic WIMP-nucleus scattering

$$\frac{dR}{dQ} = \mathcal{A} F^2(Q) \int_{v_{\rm min}(Q)}^{v_{\rm max}} \left[\frac{f_1(v)}{v} \right] dv$$

Here

$$v_{\rm min}(Q) = \alpha \sqrt{Q}$$

is the minimal incoming velocity of incident WIMPs that can deposit the recoil energy Q in the detector,

$$\mathcal{A} \equiv \frac{\rho_0 \sigma_0}{2 m_\chi m_{\rm r,N}^2} \qquad \alpha \equiv \sqrt{\frac{m_{\rm N}}{2 m_{\rm r,N}^2}} \qquad m_{\rm r,N} = \frac{m_\chi m_{\rm N}}{m_\chi + m_{\rm N}}$$

ρ_0: WIMP density near the Earth

σ_0: total cross section ignoring the form factor suppression

$F(Q)$: elastic nuclear form factor

$f_1(v)$: one-dimensional velocity distribution of halo WIMPs

Reconstruction of the WIMP velocity distribution

❑ Normalized one-dimensional WIMP velocity distribution function

$$f_1(v) = \mathcal{N} \left\{ -2Q \cdot \frac{d}{dQ} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] \right\}_{Q = v^2/\alpha^2}$$

$$\mathcal{N} = \frac{2}{\alpha} \left\{ \int_0^\infty \frac{1}{\sqrt{Q}} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] dQ \right\}^{-1}$$

❑ Moments of the velocity distribution function

$$\langle v^n \rangle = \mathcal{N}(Q_{\rm thre}) \left(\frac{\alpha^{n+1}}{2} \right) \left[\frac{2Q_{\rm thre}^{(n+1)/2}}{F^2(Q_{\rm thre})} \left(\frac{dR}{dQ} \right)_{Q=Q_{\rm thre}} + (n+1) I_n(Q_{\rm thre}) \right]$$

$$\mathcal{N}(Q_{\rm thre}) = \frac{2}{\alpha} \left[\frac{2Q_{\rm thre}^{1/2}}{F^2(Q_{\rm thre})} \left(\frac{dR}{dQ} \right)_{Q=Q_{\rm thre}} + I_0(Q_{\rm thre}) \right]^{-1}$$

$$I_n(Q_{\rm thre}) = \int_{Q_{\rm thre}}^\infty Q^{(n-1)/2} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] dQ$$

[M. Drees and CLS, JCAP 0706, 011 (2007)]

Reconstruction of the WIMP velocity distribution

- Ansatz: the measured recoil spectrum in the nth Q-bin

$$\left(\frac{dR}{dQ}\right)_{\text{expt},\ Q\simeq Q_n} \equiv r_n\, e^{k_n(Q-Q_{s,n})} \qquad\qquad r_n \equiv \frac{N_n}{b_n}$$

Reconstruction of the WIMP velocity distribution

- Ansatz: the measured recoil spectrum in the nth Q-bin

$$\left(\frac{dR}{dQ}\right)_{\text{expt},\ Q \simeq Q_n} \equiv r_n\, e^{k_n (Q - Q_{s,n})} \qquad\qquad r_n \equiv \frac{N_n}{b_n}$$

- Logarithmic slope and shifted point in the nth Q-bin

$$\overline{Q - Q_n}|_n \equiv \frac{1}{N_n} \sum_{i=1}^{N_n} (Q_{n,i} - Q_n) = \left(\frac{b_n}{2}\right) \coth\left(\frac{k_n b_n}{2}\right) - \frac{1}{k_n}$$

$$Q_{s,n} = Q_n + \frac{1}{k_n} \ln\left[\frac{\sinh(k_n b_n/2)}{k_n b_n/2}\right]$$

Reconstruction of the WIMP velocity distribution

- Ansatz: the measured recoil spectrum in the nth Q-bin

$$\left(\frac{dR}{dQ}\right)_{\text{expt},\ Q \simeq Q_n} \equiv r_n\, e^{k_n (Q - Q_{s,n})} \qquad r_n \equiv \frac{N_n}{b_n}$$

- Logarithmic slope and shifted point in the nth Q-bin

$$\overline{Q - Q_n}|_n \equiv \frac{1}{N_n} \sum_{i=1}^{N_n} (Q_{n,i} - Q_n) = \left(\frac{b_n}{2}\right) \coth\left(\frac{k_n b_n}{2}\right) - \frac{1}{k_n}$$

$$Q_{s,n} = Q_n + \frac{1}{k_n} \ln\left[\frac{\sinh(k_n b_n/2)}{k_n b_n/2}\right]$$

- Reconstructing the one-dimensional WIMP velocity distribution

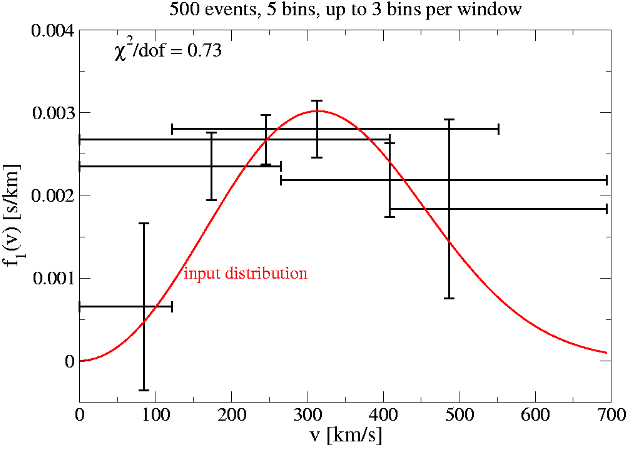
$$f_1(v_{s,n}) = \mathcal{N}\left[\frac{2 Q_{s,n} r_n}{F^2(Q_{s,n})}\right] \left[\frac{d}{dQ} \ln F^2(Q)\bigg|_{Q = Q_{s,n}} - k_n\right]$$

$$\mathcal{N} = \frac{2}{\alpha}\left[\sum_a \frac{1}{\sqrt{Q_a}\, F^2(Q_a)}\right]^{-1} \qquad v_{s,n} = \alpha\sqrt{Q_{s,n}}$$

[M. Drees and CLS, JCAP 0706, 011 (2007)]

Reconstruction of the WIMP velocity distribution

- Reconstructed $f_{1,\mathrm{rec}}(v_{s,n})$
  (^{76}Ge, 500 events, 5 bins, up to 3 bins per window)



[M. Drees and CLS, JCAP 0706, 011 (2007)]

Bayesian reconstruction of the WIMP velocity distribution function

Formalism

Formalism

- Bayesian analysis

$$p(\Theta|\text{data}) = \frac{p(\text{data}|\Theta)}{p(\text{data})} \cdot p(\Theta)$$

Formalism

- Bayesian analysis

$$p(\Theta|\text{data}) = \frac{p(\text{data}|\Theta)}{p(\text{data})} \cdot p(\Theta)$$

  - Θ: $\left\{a_1, a_2, \cdots, a_{N_{\rm Bayesian}}\right\}$, a specified (combination of the) value(s) of the fitting parameter(s)

Formalism

- Bayesian analysis

$$p(\Theta|\text{data}) = \frac{p(\text{data}|\Theta)}{p(\text{data})} \cdot p(\Theta)$$

  - Θ: $\{a_1, a_2, \cdots, a_{N_{\text{Bayesian}}}\}$, a specified (combination of the) value(s) of the fitting parameter(s)
  - $p(\Theta)$: prior probability, our degree of belief about Θ being the true value(s) of fitting parameter(s), often given in form of the (multiplication of the) probability distribution(s) of the fitting parameter(s)

Formalism

- Probability distribution functions for $p(\Theta)$

Formalism

- Probability distribution functions for $\mathrm{p}(\Theta)$
  - Without prior knowledge about the fitting parameter
    - Flat-distributed

$$\mathrm{p}_i(a_i) = 1 \qquad \text{for } a_{i,\mathrm{min}} \le a_i \le a_{i,\mathrm{max}},$$

[CLS, arXiv:1403.5610]

Formalism

- Probability distribution functions for p(Θ)
  - Without prior knowledge about the fitting parameter
    - Flat-distributed

$$\mathrm{p}_i(a_i) = 1 \qquad \text{for } a_{i,\min} \le a_i \le a_{i,\max},$$

  - With prior knowledge about the fitting parameter
    - Around a theoretical predicted/estimated or experimental measured value $\mu_{a,i}$
    - With (statistical) uncertainties $\sigma_{a,i}$
    - Gaussian-distributed

$$\mathrm{p}_i(a_i; \mu_{a,i}, \sigma_{a,i}) = \frac{1}{\sqrt{2\pi}\,\sigma_{a,i}}\, e^{-(a_i-\mu_{a,i})^2/2\sigma_{a,i}^2}$$

[CLS, arXiv:1403.5610]

Formalism

- Bayesian analysis

$$p(\Theta|\text{data}) = \frac{p(\text{data}|\Theta)}{p(\text{data})} \cdot p(\Theta)$$

  - Θ: $\{a_1, a_2, \cdots, a_{N_{\rm Bayesian}}\}$, a specified (combination of the) value(s) of the fitting parameter(s)
  - $p(\Theta)$: prior probability, our degree of belief about Θ being the true value(s) of fitting parameter(s), often given in form of the (multiplication of the) probability distribution(s) of the fitting parameter(s)
  - $p(\text{data})$: evidence, the total probability of obtaining the particular set of data

Formalism

- Bayesian analysis

$$p(\Theta|\text{data}) = \frac{p(\text{data}|\Theta)}{p(\text{data})} \cdot p(\Theta)$$

  - Θ: $\{a_1, a_2, \cdots, a_{N_{\text{Bayesian}}}\}$, a specified (combination of the) value(s) of the fitting parameter(s)
  - $p(\Theta)$: prior probability, our degree of belief about Θ being the true value(s) of fitting parameter(s), often given in form of the (multiplication of the) probability distribution(s) of the fitting parameter(s)
  - $p(\text{data})$: evidence, the total probability of obtaining the particular set of data
  - $p(\text{data}|\Theta)$: the probability of the observed result, once the specified (combination of the) value(s) of the fitting parameter(s) happens, usually be described by the "likelihood" function of Θ, $\mathcal{L}(\Theta)$.

Formalism

- Likelihood function for $p(\text{data}|\Theta)$

Formalism

- Likelihood function for p(data|Θ)
  - Theoretical one-dimensional WIMP velocity distribution function: $f_{1,\rm th}(v; a_1, a_2, \cdots, a_{N_{\rm Bayesian}})$

Formalism

- Likelihood function for p(data|Θ)
  - Theoretical one-dimensional WIMP velocity distribution function: $f_{1,\rm th}(v; a_1, a_2, \cdots, a_{N_{\rm Bayesian}})$
  - Assuming that the reconstructed data points are Gaussian-distributed around the theoretical predictions

Formalism

- Likelihood function for p(data|Θ)
  - Theoretical one-dimensional WIMP velocity distribution function: $f_{1,\mathrm{th}}(v; a_1, a_2, \cdots, a_{N_{\mathrm{Bayesian}}})$
  - Assuming that the reconstructed data points are Gaussian-distributed around the theoretical predictions

$$\mathcal{L}\left(f_{1,\mathrm{rec}}(v_{s,\mu}),\ \mu = 1,\ 2,\ \cdots,\ W;\ a_i,\ i = 1,\ 2,\ \cdots,\ N_{\mathrm{Bayesian}}\right) \equiv \prod_{\mu=1}^{W} \mathrm{Gau}\left(v_{s,\mu}, f_{1,\mathrm{rec}}(v_{s,\mu}), \sigma_{f_1,s,\mu}; a_1, a_2, \cdots, a_{N_{\mathrm{Bayesian}}}\right)$$

with

$$\mathrm{Gau}\left(v_{s,\mu}, f_{1,\mathrm{rec}}(v_{s,\mu}), \sigma_{f_1,s,\mu}; a_1, a_2, \cdots, a_{N_{\mathrm{Bayesian}}}\right) \equiv \frac{1}{\sqrt{2\pi}\,\sigma_{f_1,s,\mu}}\, e^{-\left[f_{1,\mathrm{rec}}(v_{s,\mu}) - f_{1,\mathrm{th}}(v_{s,\mu}; a_1, a_2, \cdots, a_{N_{\mathrm{Bayesian}}})\right]^2 / 2\sigma_{f_1,s,\mu}^2}$$

[CLS, arXiv:1403.5610]

Formalism

- Bayesian analysis

$$p(\Theta|\text{data}) = \frac{p(\text{data}|\Theta)}{p(\text{data})} \cdot p(\Theta)$$

  - Θ: $\left\{a_1, a_2, \cdots, a_{N_{\text{Bayesian}}}\right\}$, a specified (combination of the) value(s) of the fitting parameter(s)
  - $p(\Theta)$: prior probability, our degree of belief about Θ being the true value(s) of fitting parameter(s), often given in form of the (multiplication of the) probability distribution(s) of the fitting parameter(s)
  - $p(\text{data})$: evidence, the total probability of obtaining the particular set of data
  - $p(\text{data}|\Theta)$: the probability of the observed result, once the specified (combination of the) value(s) of the fitting parameter(s) happens, usually be described by the "likelihood" function of Θ, $\mathcal{L}(\Theta)$.
  - $p(\Theta|\text{data})$: posterior probability density function for Θ, the probability of that the specified (combination of the) value(s) of the fitting parameter(s) happens, given the observed result

Numerical results

Numerical results

- Input and fitting one-dimensional WIMP velocity distribution functions
  - "One-parameter" shifted Maxwellian velocity distribution

$$f_{1,\mathrm{sh},v_0}(v) = \frac{1}{\sqrt{\pi}} \left(\frac{v}{v_0 \, v_\mathrm{e}} \right) \left[e^{-(v - v_\mathrm{e})^2 / v_0^2} - e^{-(v + v_\mathrm{e})^2 / v_0^2} \right] \qquad v_\mathrm{e} = 1.05 \, v_0$$

Numerical results

- Input and fitting one-dimensional WIMP velocity distribution functions
  - "One-parameter" shifted Maxwellian velocity distribution

$$f_{1,\mathrm{sh},v_0}(v) = \frac{1}{\sqrt{\pi}} \left(\frac{v}{v_0 v_\mathrm{e}} \right) \left[e^{-(v-v_\mathrm{e})^2/v_0^2} - e^{-(v+v_\mathrm{e})^2/v_0^2} \right] \qquad v_\mathrm{e} = 1.05\, v_0$$

  - Shifted Maxwellian velocity distribution

$$f_{1,\mathrm{sh}}(v) = \frac{1}{\sqrt{\pi}} \left(\frac{v}{v_0 v_\mathrm{e}} \right) \left[e^{-(v-v_\mathrm{e})^2/v_0^2} - e^{-(v+v_\mathrm{e})^2/v_0^2} \right]$$

  - "Variated" shifted Maxwellian velocity distribution

$$f_{1,\mathrm{sh},\Delta v}(v) = \frac{1}{\sqrt{\pi}} \left[\frac{v}{v_0 \, (v_0 + \Delta v)} \right] \left\{ e^{-[v-(v_0+\Delta v)]^2/v_0^2} - e^{-[v+(v_0+\Delta v)]^2/v_0^2} \right\}$$

Numerical results

- Input and fitting one-dimensional WIMP velocity distribution functions
  - "One-parameter" shifted Maxwellian velocity distribution

$$f_{1,\mathrm{sh},v_0}(v) = \frac{1}{\sqrt{\pi}} \left(\frac{v}{v_0 v_e} \right) \left[e^{-(v-v_e)^2/v_0^2} - e^{-(v+v_e)^2/v_0^2} \right] \qquad v_e = 1.05\, v_0$$

  - Shifted Maxwellian velocity distribution

$$f_{1,\mathrm{sh}}(v) = \frac{1}{\sqrt{\pi}} \left(\frac{v}{v_0 v_e} \right) \left[e^{-(v-v_e)^2/v_0^2} - e^{-(v+v_e)^2/v_0^2} \right]$$

  - "Variated" shifted Maxwellian velocity distribution

$$f_{1,\mathrm{sh},\Delta v}(v) = \frac{1}{\sqrt{\pi}} \left[\frac{v}{v_0 (v_0 + \Delta v)} \right] \left\{ e^{-[v-(v_0+\Delta v)]^2/v_0^2} - e^{-[v+(v_0+\Delta v)]^2/v_0^2} \right\}$$

  - Simple Maxwellian velocity distribution

$$f_{1,\mathrm{Gau}}(v) = \frac{4}{\sqrt{\pi}} \left(\frac{v^2}{v_0^3} \right) e^{-v^2/v_0^2}$$

Numerical results

- Input and fitting one-dimensional WIMP velocity distribution functions
  - "One-parameter" shifted Maxwellian velocity distribution

$$f_{1,\mathrm{sh},v_0}(v) = \frac{1}{\sqrt{\pi}} \left(\frac{v}{v_0 v_e} \right) \left[e^{-(v-v_e)^2/v_0^2} - e^{-(v+v_e)^2/v_0^2} \right] \qquad v_e = 1.05\, v_0$$

  - Shifted Maxwellian velocity distribution

$$f_{1,\mathrm{sh}}(v) = \frac{1}{\sqrt{\pi}} \left(\frac{v}{v_0 v_e} \right) \left[e^{-(v-v_e)^2/v_0^2} - e^{-(v+v_e)^2/v_0^2} \right]$$

  - "Variated" shifted Maxwellian velocity distribution

$$f_{1,\mathrm{sh},\Delta v}(v) = \frac{1}{\sqrt{\pi}} \left[\frac{v}{v_0 (v_0 + \Delta v)} \right] \left\{ e^{-[v-(v_0+\Delta v)]^2/v_0^2} - e^{-[v+(v_0+\Delta v)]^2/v_0^2} \right\}$$

  - Simple Maxwellian velocity distribution

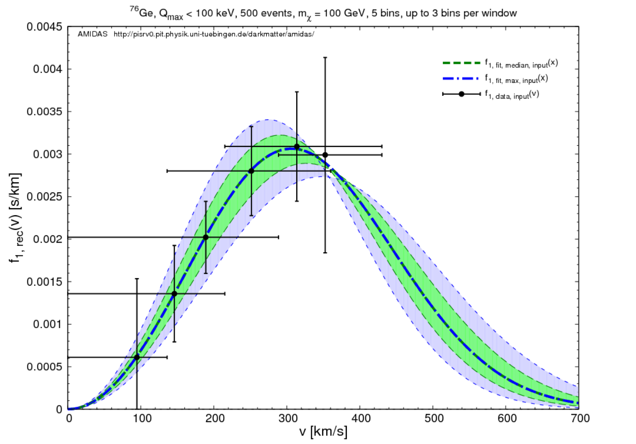
$$f_{1,\mathrm{Gau}}(v) = \frac{4}{\sqrt{\pi}} \left(\frac{v^2}{v_0^3} \right) e^{-v^2/v_0^2}$$

  - "Modified" simple Maxwellian velocity distribution

$$f_{1,\mathrm{Gau},k}(v) = \frac{v^2}{N_{f,k}} \left(e^{-v^2/k v_0^2} - e^{-v_{\max}^2/k v_0^2} \right)^k \qquad \text{for } v \le v_{\max}$$

Numerical results

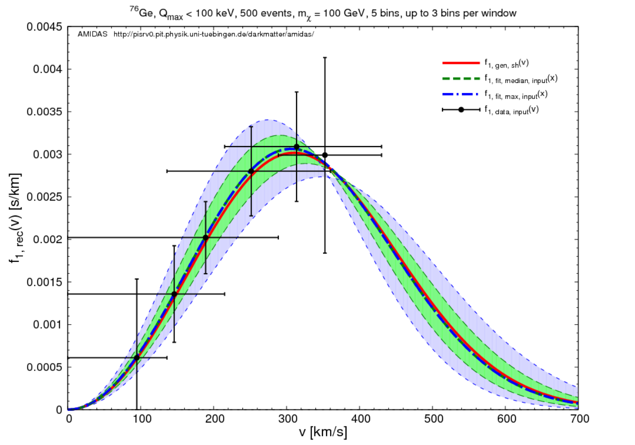
- Reconstructed $f_{1,\mathrm{Bayesian}}(v)$ with an input WIMP mass
  (^{76}Ge, 0 - 100 keV, 500 events, $m_\chi = 100$ GeV, $f_{1,\mathrm{sh},v_0}(v) \Rightarrow f_{1,\mathrm{sh},v_0}(v)$, flat-dist.)



[CLS, arXiv:1403.5610]

Numerical results

- Reconstructed $f_{1,\text{Bayesian}}(v)$ with an input WIMP mass
(^{76}Ge, 0 - 100 keV, 500 events, $m_\chi = 100$ GeV, $f_{1,\text{sh},v_0}(v) \Rightarrow f_{1,\text{sh},v_0}(v)$, flat-dist.)

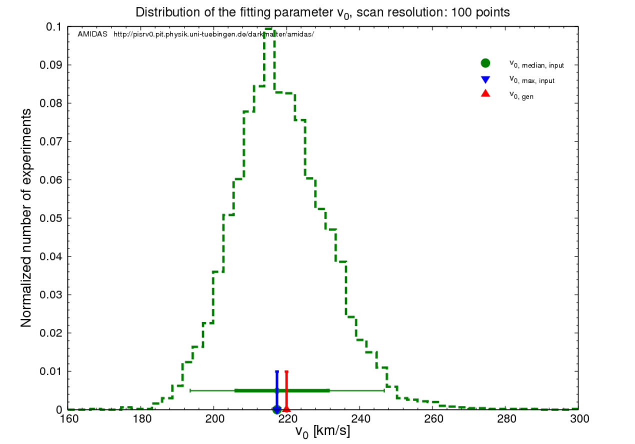


[CLS, arXiv:1403.5610]

Numerical results

- Distribution of the reconstructed v_0

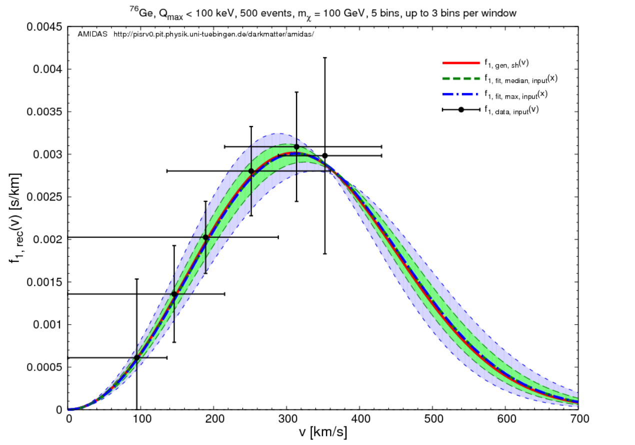
(^{76}Ge, 0 - 100 keV, 500 events, $m_\chi = 100$ GeV, $f_{1,\mathrm{sh},v_0}(v) \Rightarrow f_{1,\mathrm{sh},v_0}(v)$, flat-dist.)



[CLS, arXiv:1403.5610]

Numerical results

- Reconstructed $f_{1,\text{Bayesian}}(v)$ with an input WIMP mass
  (^{76}Ge, 0 - 100 keV, 500 events, $m_\chi = 100$ GeV, $f_{1,\text{sh},v_0}(v) \Rightarrow f_{1,\text{sh},v_0}(v)$, Gaussian-dist.)

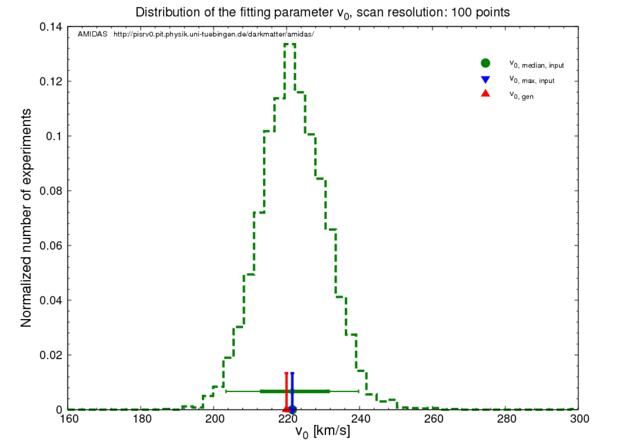


[CLS, arXiv:1403.5610]

Numerical results

- Distribution of the reconstructed v_0

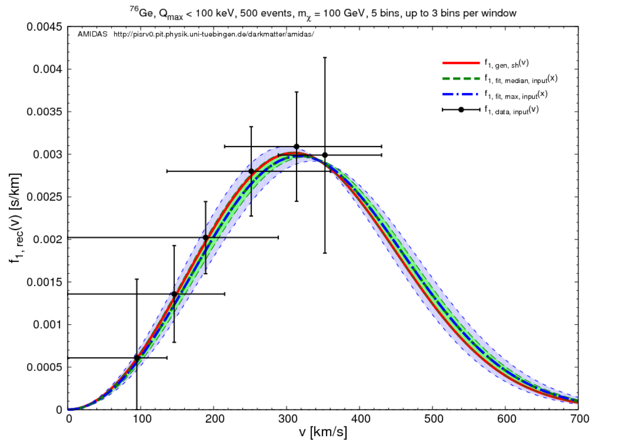
(^{76}Ge, 0 - 100 keV, 500 events, $m_\chi = 100$ GeV, $f_{1,\mathrm{sh},v_0}(v) \Rightarrow f_{1,\mathrm{sh},v_0}(v)$, Gaussian-dist.)



[CLS, arXiv:1403.5610]

Numerical results

- Reconstructed $f_{1,\mathrm{Bayesian}}(v)$ with an input WIMP mass
  (^{76}Ge, 0 - 100 keV, 500 events, $m_\chi = 100$ GeV, $f_{1,\mathrm{sh},v_0}(v) \Rightarrow f_{1,\mathrm{sh}}(v)$, Gaussian-dist.)

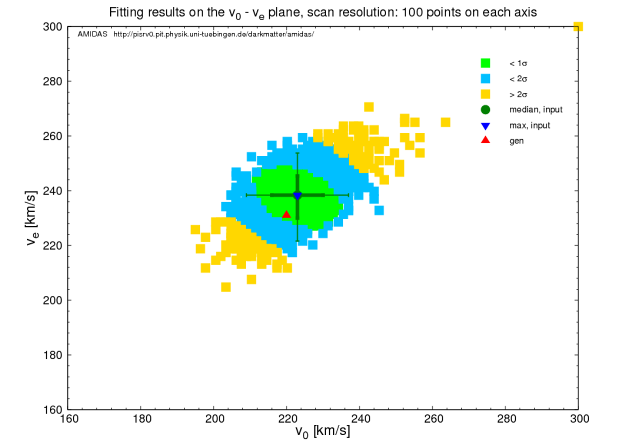


[CLS, arXiv:1403.5610]

Numerical results

- Distribution of the reconstructed $v_0 - v_e$

(^{76}Ge, 0 - 100 keV, 500 events, $m_\chi = 100$ GeV, $f_{1,\rm sh, v_0}(v) \Rightarrow f_{1,\rm sh}(v)$, Gaussian-dist.)

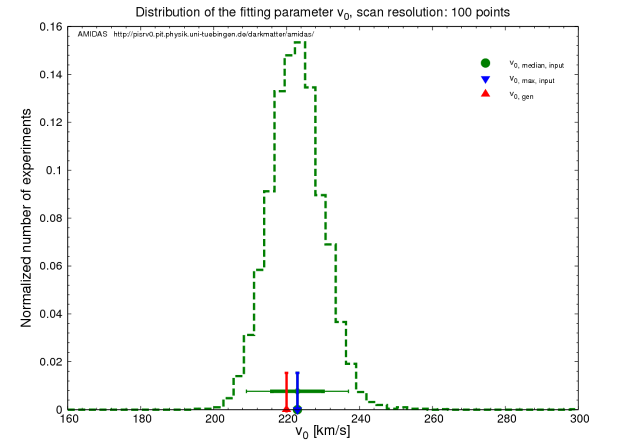


[CLS, arXiv:1403.5610]

Numerical results

- Distribution of the reconstructed v_0

(^{76}Ge, 0 - 100 keV, 500 events, $m_\chi = 100$ GeV, $f_{1,\mathrm{sh},v_0}(v) \Rightarrow f_{1,\mathrm{sh}}(v)$, Gaussian-dist.)

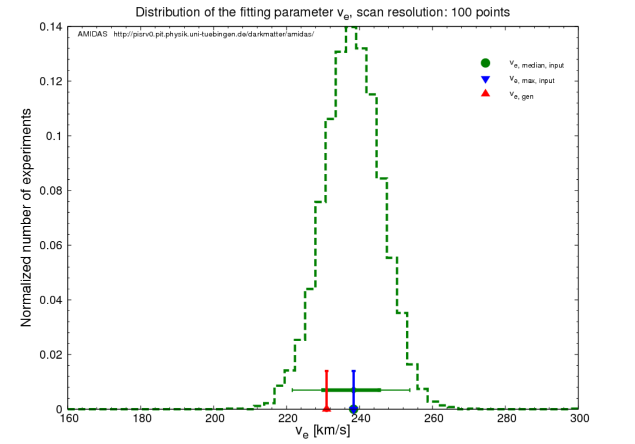


[CLS, arXiv:1403.5610]

Numerical results

- Distribution of the reconstructed v_e

  (^{76}Ge, 0 - 100 keV, 500 events, $m_\chi = 100$ GeV, $f_{1,\mathrm{sh},v_0}(v) \Rightarrow f_{1,\mathrm{sh}}(v)$, Gaussian-dist.)

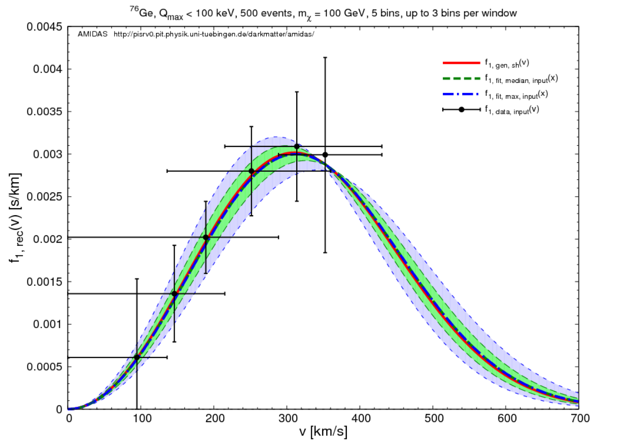


[CLS, arXiv:1403.5610]

Numerical results

- Reconstructed $f_{1,\text{Bayesian}}(v)$ with an input WIMP mass

  (^{76}Ge, 0 - 100 keV, 500 events, $m_\chi = 100$ GeV, $f_{1,\text{sh},v_0}(v) \Rightarrow f_{1,\text{sh},\Delta v}(v)$, Gaussian-dist.)

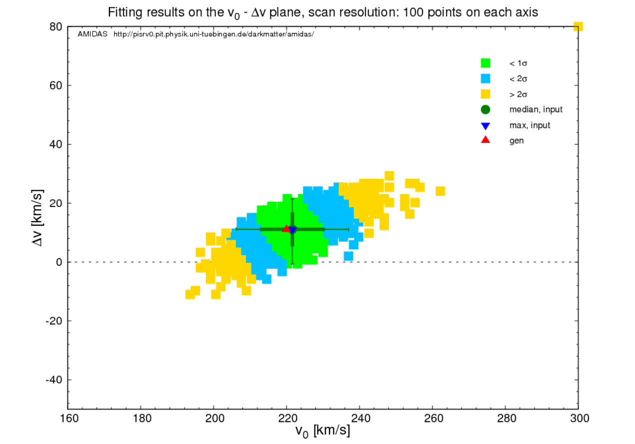


[CLS, arXiv:1403.5610]

Numerical results

- Distribution of the reconstructed $v_0 - \Delta v$

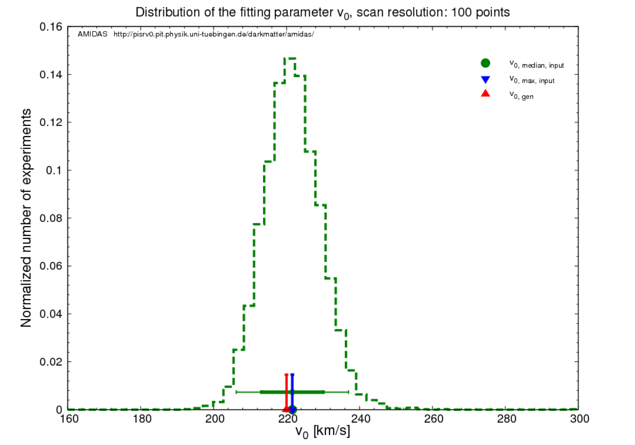
(^{76}Ge, 0 - 100 keV, 500 events, $m_\chi = 100$ GeV, $f_{1,\text{sh},v_0}(v) \Rightarrow f_{1,\text{sh},\Delta v}(v)$, Gaussian-dist.)



[CLS, arXiv:1403.5610]

Numerical results

- Distribution of the reconstructed v_0
  (^{76}Ge, 0 - 100 keV, 500 events, $m_\chi = 100$ GeV, $f_{1,\mathrm{sh},v_0}(v) \Rightarrow f_{1,\mathrm{sh},\Delta v}(v)$, Gaussian-dist.)

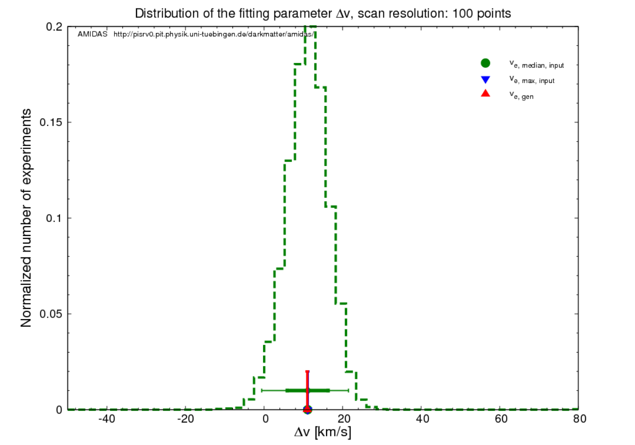


[CLS, arXiv:1403.5610]

Numerical results

- Distribution of the reconstructed Δv

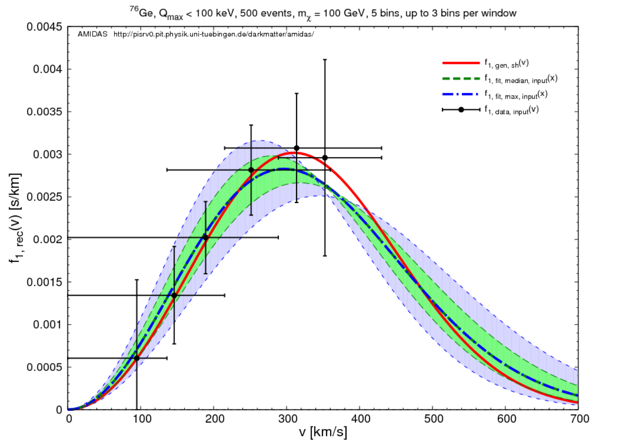
(^{76}Ge, 0 - 100 keV, 500 events, $m_\chi = 100$ GeV, $f_{1,\mathrm{sh},v_0}(v) \Rightarrow f_{1,\mathrm{sh},\Delta v}(v)$, Gaussian-dist.)



[CLS, arXiv:1403.5610]

Numerical results

- Reconstructed $f_{1,\mathrm{Bayesian}}(v)$ with an input WIMP mass
  (^{76}Ge, 0 - 100 keV, 500 events, $m_\chi = 100$ GeV, $f_{1,\mathrm{sh},v_0}(v) \Rightarrow f_{1,\mathrm{Gau}}(v)$, flat-dist.)

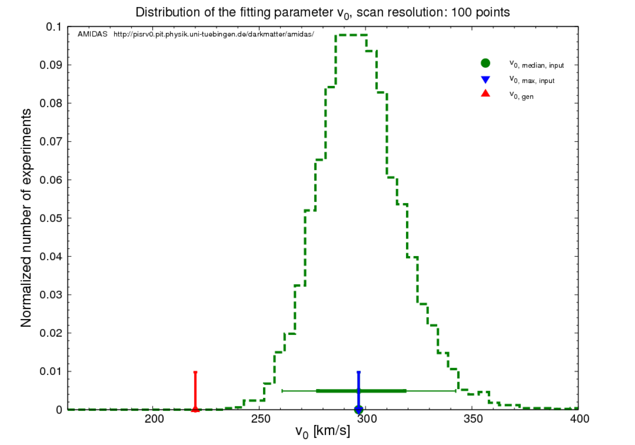


[CLS, arXiv:1403.5610]

Numerical results

- Distribution of the reconstructed v_0

(^{76}Ge, 0 - 100 keV, 500 events, $m_\chi = 100$ GeV, $f_{1,\mathrm{sh},v_0}(v) \Rightarrow f_{1,\mathrm{Gau}}(v)$, flat-dist.)



[CLS, arXiv:1403.5610]

Summary

Summary

- By using Bayesian analysis/fitting one could reconstruct $f_1(v)$ very precisely:
  - small or even negligible systematic deviations of v_0, v_e and the peak position of $f_1(v)$
  - 1σ statistical uncertainties of $\lesssim$ 20 km/s on v_0, v_e and the peak position of $f_1(v)$

Summary

- By using Bayesian analysis/fitting one could reconstruct $f_1(v)$ very precisely:
  - small or even negligible systematic deviations of v_0, v_e and the peak position of $f_1(v)$
  - 1σ statistical uncertainties of $\lesssim$ 20 km/s on v_0, v_e and the peak position of $f_1(v)$
- With prior knowledge about fitting parameters
  - systematic deviations and statistical uncertainties could be reduced (significantly)
  - even when the expected values are (slightly) incorrect

Summary

- By using Bayesian analysis/fitting one could reconstruct $f_1(v)$ very precisely:
  - small or even negligible systematic deviations of v_0, v_e and the peak position of $f_1(v)$
  - 1σ statistical uncertainties of $\lesssim$ 20 km/s on v_0, v_e and the peak position of $f_1(v)$
- With prior knowledge about fitting parameters
  - systematic deviations and statistical uncertainties could be reduced (significantly)
  - even when the expected values are (slightly) incorrect
- Using variated analytic form of the same fitting $f_1(v)$
  - systematic deviations could be reduced (significantly)
  - statistical uncertainties might however be (a bit) larger

Summary

- Using an improper fitting $f_1(v)$
  - information about the true velocity distribution function (e.g. the peak position of $f_1(v)$) could still be reconstructed approximately
  - with clearly 2σ - 6σ deviations of v_0 and v_e from the theoretical expections

Summary

- Using an improper fitting $f_1(v)$
  - information about the true velocity distribution function (e.g. the peak position of $f_1(v)$) could still be reconstructed approximately
  - with clearly 2σ - 6σ deviations of v_0 and v_e from the theoretical expections
- For light m_χ:
  - (very) shape recoil energy spectrum
  - only with very few "reconstructed-input" data points

Summary

- Using an improper fitting $f_1(v)$
  - information about the true velocity distribution function (e.g. the peak position of $f_1(v)$) could still be reconstructed approximately
  - with clearly 2σ - 6σ deviations of v_0 and v_e from the theoretical expections
- For light m_χ:
  - (very) shape recoil energy spectrum
  - only with very few "reconstructed-input" data points
- For heavy m_χ:
  - large statistical fluctuation of and uncertainty on $m_{\chi,\rm rec}$
  - only with "reconstructed-input" data points in the low-velocity range

Summary

- Using an improper fitting $f_1(v)$
  - information about the true velocity distribution function (e.g. the peak position of $f_1(v)$) could still be reconstructed approximately
  - with clearly 2σ - 6σ deviations of v_0 and v_e from the theoretical expections
- For light m_χ:
  - (very) shape recoil energy spectrum
  - only with very few "reconstructed-input" data points
- For heavy m_χ:
  - large statistical fluctuation of and uncertainty on $m_{\chi,\mathrm{rec}}$
  - only with "reconstructed-input" data points in the low-velocity range
- With a small fraction of unrejected background events

Thank you very much for your attention!