Charmonium spectroscopy and Lattice QCD

Daniel Mohler

Geneva,
October 11, 2014

Fermilab

Outline

1. Introduction & Cautionary remarks
2. Low lying charmonium states at the physical point
    - New results on the 1S hyperfine splitting
    - Comparison of 1S hyperfine results
    - New results on charmonium 1P states and 2S states
3. Exploratory study of the $\Psi(2S)$ and $\Psi(3770)$
4. Conclusions & Outlook

Low-lying charmonium: A precision benchmark

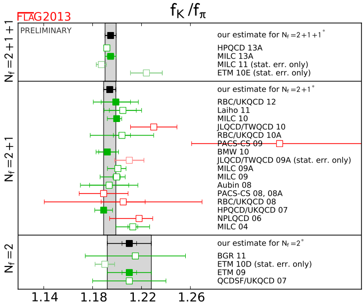
- Well understood from potential models
- Well determined in experiment

meson	mass	width
η_c	2983.7(7)	32.0(9) MeV
J/Ψ	3096.916(11)	92.9(2.8) keV
χ_{c0}	3414.75(31)	10.3(6) MeV
χ_{c1}	3510.66(3)	0.86(5) MeV
χ_{c2}	3556.20(9)	1.97(11) MeV
h_c	3525.38(11)	0.7(4) MeV
$\eta_c(2S)$	3639.4(1.3)	$11.3\left({}^{+3.2}_{-2.9}\right)$ MeV
$\Psi(2S)$	$3686.109\left({}^{+12}_{-14}\right)$	299(8) keV

- Spin-dependent mass splittings extremely sensitive to the charm-quark mass and heavy-quark discretization

Two kinds of progress...

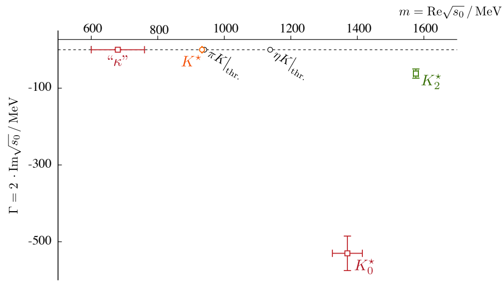
Precision results:



Example: FLAG review

See http://itpwiki.unibe.ch/flag/

Exploratory studies:



Example: πK-ηK-scattering

`Dudek et al.  PRL 113 182001 (2014)`

- I will report on both kind of progress with regard to charmonium
- There will be preliminary data - use with caution

Preliminary results from Fermilab-MILC

`Fermilab Lattice and MILC collaborations - to be published`

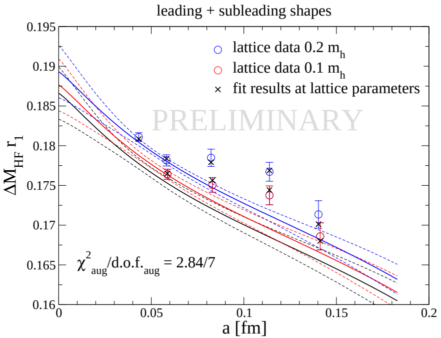
We use the *Fermilab method* for the charm quark

`El-Khadra et al., PRD 55, 3933 (1997)`

- m_c tuned by demanding the D_s meson kinetic mass to be physical
- We quote splittings among charmonium states
- 5 lattice spacings with two different light sea-quark masses
  → Controlled extrapolation to the chiral-continuum limit
- 2+1 flavor simulation with high statistics
- Follows previous efforts

  `T. Burch et al.  PRD 81 034508, 2010`
- Charm annihilation contributions are omitted

Fermilab-MILC results for the 1S hyperfine splitting

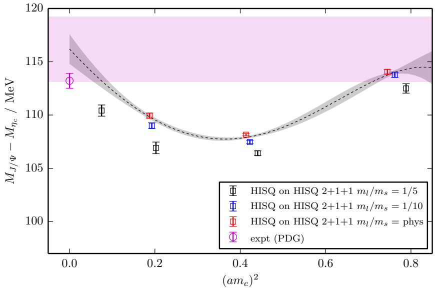


- Curves for physical (black), $0.1m_s$, and $0.2m_s$ light-quark masses
- This corresponds to $M_{J/\psi} - M_{\eta_c} = 118.1(2.1)\begin{pmatrix}-1.5\\-4.0\end{pmatrix}$
- Errors include statistics, chiral and continuum extrapolations
- Contribution from disconnected diagrams expected $\begin{pmatrix}-1.5\\-4.0\end{pmatrix}$

`Levkova and DeTar, PRD 83 074504, 2011`

Preliminary results from the HPQCD Collaboration

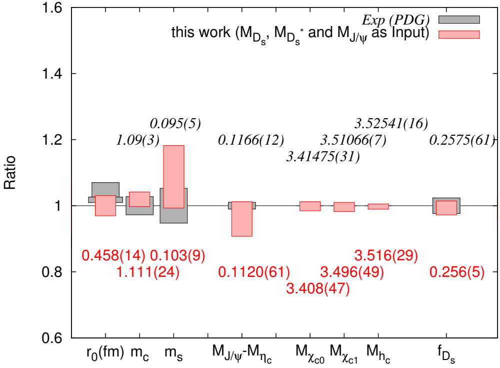
`HPQCD, Galloway et al. PoS LATTICE2014 (2014) 092`



- Uses MILC 2+1+1 flavor HISQ (Highly Improved Staggered Quarks) ensembles
- Shaded region shows the systematic uncertainty
- Systematics on 1S hyperfine dominated by estimate of annihilation effects

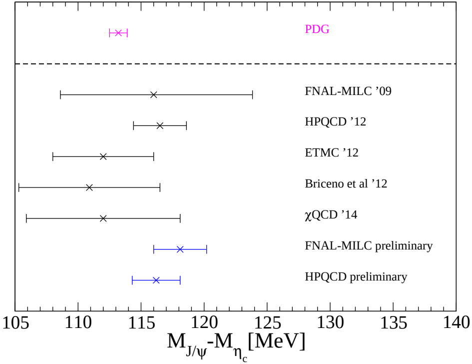
Results from the χQCD Collaboration

`χQCD Collaboration, Yang` *`et al.`* `arXiv:1410.3343`



- Results from Overlap fermions on 2+1 flavor domain wall gauge configurations
- Their 1S hyperfine uncertainty does not include annihilation effects

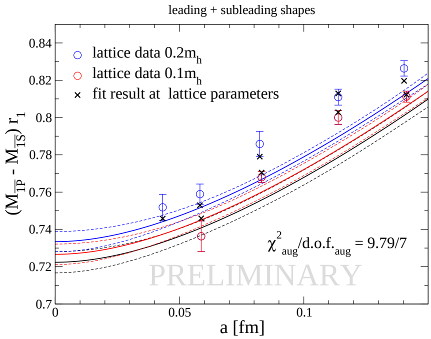
A comparison of hyperfine splittings

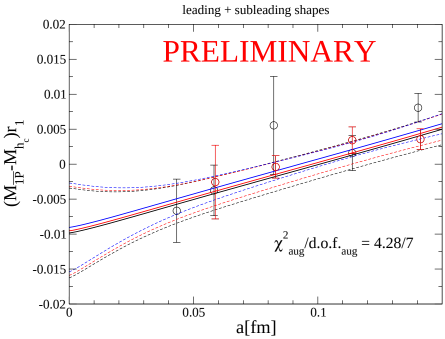


- All simulations report results at physical quark masses in the continuum limit
- Lattice numbers exclude (now significant) annihilation effects
- Estimate from data expects a shift of $\begin{pmatrix}-1.5\\-4.5\end{pmatrix}$ MeV

`Levkova and DeTar, PRD 83 074504, 2011`

Fermilab-MILC 1P-1S and 1P hyperfine splittings



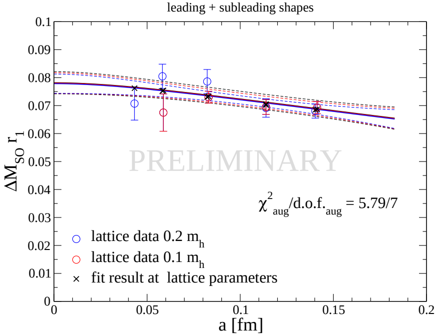


- Not yet included: Scale-setting uncertainty

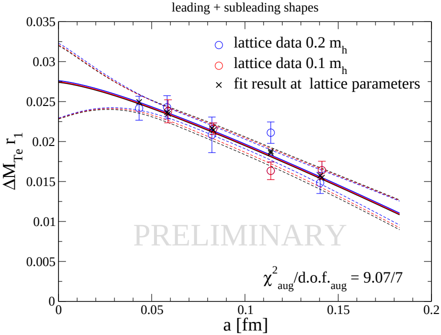
Mass difference	This analysis [MeV]	Experiment [MeV]
1P1S	457.3 ± 3.6	457.5 ± 0.3
1P hyperfine	-6.2 ± 4.1	-0.10 ± 0.22

Fermilab-MILC P-wave spin-orbit and tensor splittings

$$\Delta M_{\mathrm{Spin-Orbit}} = (5M_{\chi_{c2}} - 3M_{\chi_{c1}} - 2M_{\chi_{c0}})/9$$



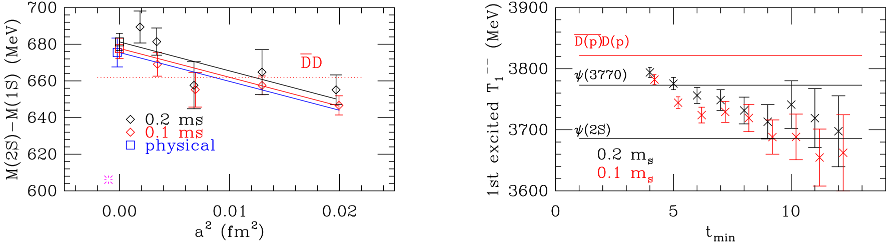
$$\Delta M_{\mathrm{Tensor}} = (3M_{\chi_{c1}} - M_{\chi_{c2}} - 2M_{\chi_{c0}})/9$$



Mass difference	This analysis [MeV]	Experiment [MeV]
1P spin-orbit	49.5 ± 2.5	46.6 ± 0.1
1P tensor	17.3 ± 2.9	16.25 ± 0.07

2S states

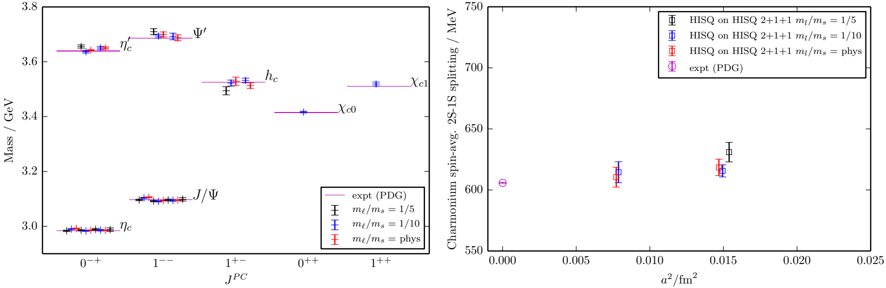
Fermilab/MILC, DeTar *et al.* arXiv:1410.3343



- Simple analysis of 2S states lead to splittings much larger than in experiment
- Strong dependence on fit range and very noisy data

Preliminary results from the HPQCD Collaboration

`HPQCD, Galloway et al. PoS LATTICE2014 (2014) 092`

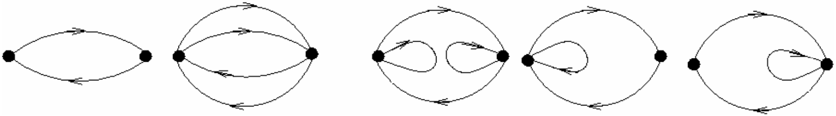


- Uses MILC 2+1+1 flavor HISQ (Highly Improved Staggered Quarks) ensembles
- No issue determining the 2S states

Exploratory study of the $\Psi(2S)$ and $\Psi(3770)$

`Lang, Leskovec, DM, Prelovsek, to be published.`

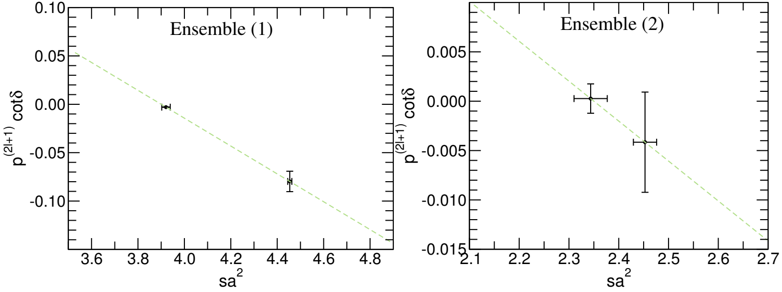
- The $\Psi(2S)$ is below DD, the $\Psi(3770)$ is a QCD resonance
- We use both $\bar{q}q$ and DD interpolators (in P-wave)
- We have to deal with backtracking quark lines



- Methods computationally expensive and currently not affordable on large (lattice) volumes

ID	$N_L^3 \times N_T$	N_f	a[fm]	L[fm]	#configs	m_π[MeV]	m_K[MeV]
(1)	$16^3 \times 32$	2	0.1239(13)	1.98	280/279	266(3)(3)	552(2)(6)
(2)	$32^3 \times 64$	2+1	0.0907(13)	2.90	196	156(7)(2)	504(1)(7)

Preliminary Ψ(3770) resonance mass and coupling

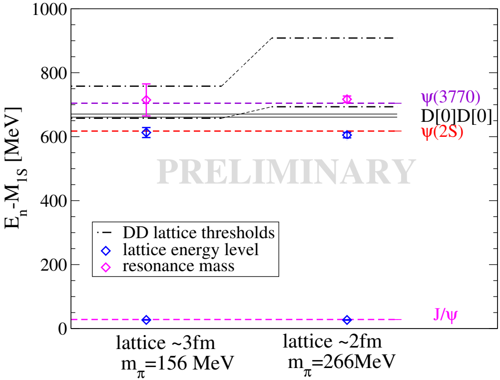


- Using Lüscher's method and assuming a Breit-Wigner shape gives

	Mass [MeV]	$g_{\Psi(3770)DD}$
Ensemble(1)	3785(7)(8)	11.4 (0.8)
Ensemble(2)	3783(49)(10)	21.6(14.9)
Experiment	3773.15(33)	$\approx$ 18.7

- Proof of principle - errors statistics and scale setting only
- Future studies will have to use physical quark masses, multiple lattice spacings and multiple volumes.

Low lying Ψ states



- J/Ψ and $\Psi(2S)$ are the lattice energy levels
- $\Psi(3770)$ shown at the resonance mass

Conclusions & Outlook

- Low-lying charmonium states are under good control
  - Agreement with experiment and among lattice collaborations is mostly excellent
  - The uncertainty on the charmonium hyperfine splitting is getting dominated by charm-annihilation effects
- 2S states are more challenging but various new results are encouraging
- Some non-exotic charmonium resonances can now be investigated
  - Coupled channel studies are in their infancy
  - Cases where 3-particle thresholds are important might remain an issue
- For lattice studies of X,Y,Z states see talks by Doi, Liu, Prelovsek

Thank you!

... to my collaborators Carleton DeTar, Andreas Kronfeld, Christian Lang, Song-Haeng Lee, Luka Leskovec, Ludmila Levkova, Sasa Prelovsek and Jim Simone
... to the Fermilab lattice and MILC collaborations, to Christine Davies, Ben Galloway and to Raul Briceno for providing me material

Backup slides

• • •

Interpolator basis and fit methodology

- Large correlator matrix from smeared stochastic wall sources
- Variational method

$$C(t)\vec{\psi}^{(k)} = \lambda^{(k)}(t)C(t_0)\vec{\psi}^{(k)}$$
$$\lambda^{(k)}(t) \propto \mathrm{e}^{-tE_k}\left(1 + \mathcal{O}\left(\mathrm{e}^{-t\Delta E_k}\right)\right)$$

Michael Nucl. Phys. B259, 58 (1985)
Lüscher and Wolff Nucl. Phys. B339, 222 (1990)
Blossier et al. JHEP 04, 094 (2009)

- (multi)exponential fits to the eigenvalues in $[t_{min}, t_{max}]$
- Keep t_0 and t_{min} approximately constant in fm across ensembles
- Pick t_{max} such that eigenvectors remain stable
- Correct data for unphysical κ_c where needed

Chiral and continuum fits

- Clear sea-quark mass dependence for some observables
- We need to take into account unphysical sea-quark masses
- Fit model

$$M = M_0 + c_1(2x_l + x_h) + c_2 f_1(a) + c_3 f_2(a) + \ldots$$

$$x_l = \frac{m_{ud,sea} - m_{ud,phys}}{m_{s,phys}}$$

$$x_h = \frac{m_{s,sea} - m_{s,phys}}{m_{s,phys}}$$

For the lattice spacing dependence

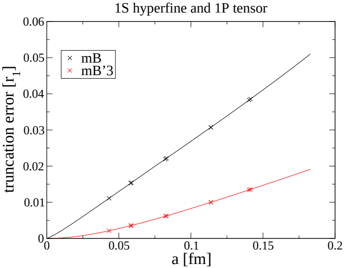
- 2-3 functions with shape estimated using NRQCD power counting

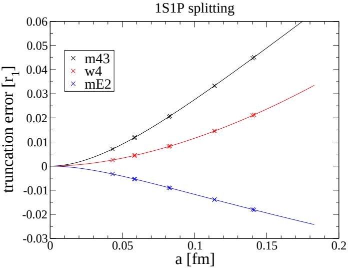
`Oktay and Kronfeld PRD 78 014504, 2008`

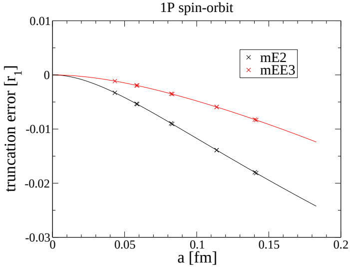
- We consider the leading two contributions arising at either v^4 or v^6
- Qualitatively, we expect errors of order a^2 and $\alpha_s a$ for quarkonium

Discretization uncertainties II

Expected shape of discretization uncertainties ($c_i = 1 \forall i$)







Testing our tuning: charm and light

	Ensemble (1)	Ensemble (2)	Experiment
m_π	266(3)(3)	156(7)(2)	139.5702(4)
m_K	552(1)(6)	504(1)(7)	493.677(16)
m_ϕ	1015.8(1.8)(10.7)	1018.4(2.8)(14.6)	1019.455(20)
m_{η_s}	732.3(0.9)(7.7)	692.9(0.5)(9.9)	688.5(2.2)*
$m_{J/\Psi} - m_{\eta_c}$	107.9(0.3)(1.1)	107.1(0.2)(1.5)	113.2(0.7)
$m_{D_s^*} - m_{D_s}$	120.4(0.6)(1.3)	142.1(0.7)(2.0)	143.8(0.4)
$m_{D^*} - m_D$	129.4(1.8)(1.4)	148.4(5.2)(2.1)	140.66(10)
$2m_{\overline{D}} - m_{\overline{c}c}$	890.9(3.3)(9.3)	882.0(6.5)(12.6)	882.4(0.3)
$2M_{\overline{D_s}} - m_{\overline{c}c}$	1065.5(1.4)(11.2)	1060.7(1.1)(15.2)	1084.8(0.6)
$m_{D_s} - m_D$	96.6(0.9)(1.0)	94.0(4.6)(1.3)	98.87(29)

- A single ensemble: Discrepancies due to discretization and unphysical light-quark masses expected

Testing our tuning: charm and beauty

	Ensemble (1)	Ensemble (2)	Experiment
$m_{J/\psi} - m_{\eta_c}$	107.9(0.3)(1.1)	107.1(0.2)(1.5)	113.2(0.7)
$m_{D_s^*} - m_{D_s}$	120.4(0.6)(1.3)	142.1(0.7)(2.0)	143.8(0.4)
$m_{D^*} - m_D$	129.4(1.8)(1.4)	148.4(5.2)(2.1)	140.66(10)
$2m_{\overline{D}} - m_{\overline{c}c}$	890.9(3.3)(9.3)	882.0(6.5)(12.6)	882.4(0.3)
$2M_{\overline{D_s}} - m_{\overline{c}c}$	1065.5(1.4)(11.2)	1060.7(1.1)(15.2)	1084.8(0.6)
$m_{D_s} - m_D$	96.6(0.9)(1.0)	94.0(4.6)(1.3)	98.87(29)
$m_{B^*} - m_B$	-	54.3(8.1)(0.8)	45.78(35)
$m_{B_{s^*}} - m_{B_s}$	-	47.6(1.0)(0.7)	$48.7^{+2.3}_{-2.1}$
$m_{B_s} - m_B$	-	83.9(7.9)(1.2)	87.35(23)
$m_\Upsilon - m_{\eta_b}$	-	44.1(0.5)(0.6)	62.3(3.2)
$2m_{\overline{B}} - m_{\overline{b}b}$	-	1196.0(6.3)(17.1)	1182.7(1.0)
$2m_{\overline{B_s}} - m_{\overline{b}b}$	-	1353.6(1.7)(19.4)	1361.7(3.4)
$2m_{B_c} - m_{\eta_b} - m_{\eta_c}$	-	168.1(0.6)(2.4)	167.3(4.9)

Errors statistical and scale setting only

Bottom quark still slightly too light

Preliminary comparison to experiment

- Uncertainties include statistics, chiral and continuum extrapolations
- Not yet included: Scale-setting uncertainty (significant for ΔM_{HF} and ΔM_{1P1S})
- Second uncertainty on the 1S hyperfine splitting is best-estimate for disconnected contributions
- Volume effects are expected to be negligible

Mass difference	This analysis [MeV]	Experiment [MeV]
1P1S	457.3 ± 3.6	457.5 ± 0.3
1S hyperfine	$118.1 \pm 2.1^{-1.5}_{-4.0}$	113.2 ± 0.7
1P spin-orbit	49.5 ± 2.5	46.6 ± 0.1
1P tensor	17.3 ± 2.9	16.25 ± 0.07
1P hyperfine	-6.2 ± 4.1	-0.10 ± 0.22

Low lying ψ states