Determinations of m_b, m_c, and α_s from quarkonium correlators

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For the HPQCD Collaboration

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Introduction

- Since 2003 the HPQCD collaboration has been computing the properties of mesons containing charm and “bottom” quarks using improved staggered fermions.
- I will report o the update of the charm quark mass, bottom quark mass, and α_s from $n_f = 2 + 1 + 1$ unquenched calculations.

The talk will largely be based on results from the recent paper below:

- High-precision quark masses and QCD coupling from nf=4 lattice QCD Bipasha Chakraborty, C.T.H. Davies, G.C. Donald, R.J. Dowdall, B. Galloway, P. Knecht, J. Koponen, G.P. Lepage, C. McNeile. arXiv:1408.4169

Basic idea of lattice QCD

What needs to be done

The equations of lattice QCD are numerically solved in Euclidean space via a Monte Carlo process.

Supercomputer Generate gauge configurations ("snapshots of the QCD vacuum")

Supercomputer Compute quark propagators.

Supercomputer Combine quark propagators into meson correlators and average over spatial volume $c(t)$.

PC Fit the meson correlators $c(t) \sim Ae^{-mt}$ to get masses m and amplitude A (from which decay constants).

PC Repeat calculations for different quark masses and lattice spacings and take the continuum limit.

The HISQ action

- There are many different choices for the lattice version of the Dirac operator.
- The HISQ (Highly Improved Staggered Action) action was designed by HPQCD to have reduced lattice spacing dependence over previous improved staggered actions (hep-lat/0610092).
- The leading lattice spacing corrections to continuum for the HISQ action are $O(\alpha_s a^2)$.
- The next "best" fermion action has leading lattice spacing corrections to continuum of $O(a^2)$.
- For example at a $= 0.09$ fm, $am_c = 0.4$, the discretization errors in the decay constant of the η_c are $O(2\%)$ (1203.3862).

Parameters of lattice QCD calculation

- In 2001 MILC, collaboration started generating gauge configs with ASQTAD staggered sea quarks (0903.3598). Used as basis for first HPQCD calculations.
- In 2008, the MILC collaboration started generating gauge configurations with HISQ staggered (light, strange, charm) sea quarks (1212.4768)

This calculation used the newer gauge configurations.

- HISQ sea and valence quarks.
- Lattice spacings used = 0.15, 0.12, 0.09, 0.06 fm. (Lattice spacing fixed from w_0, Wilson flow).
- Physical pion masses included.
- Valence heavy-heavy data included in the analysis.
- The finest lattice spacing was 0.06 fm, compared to 0.045 fm in the old $n_f = 2 + 1$ analysis (runs planned with sea HISQ quarks).

Extracting the quark masses and coupling

See short review on extracting quark masses and couplings from lattice QCD. (McNeile, 1306.3326).)

- Adjust the inputed quark masses to reproduce subset of hadron spectrum.
- Quark masses need to be converted to $\overline{MS}$ and evolved to a constant scale.
- Renormalisation factors cancel in ratio of quark masses.
- Lattice perturbation is more complicated than continuum perturbative theory, so normally use numerical/non-perturbative method.
- If the continuum limit of suitable physical quantity is taken, then continuum perturbation calculation can be used.
    - Moments method:
        - arXiv:0907.2110, K.G. Chetyrkin et al. (for example)
        - arXiv:0805.2999, HPQCD + K.G. Chetyrkin, J.H. Kuhn, M. Steinhauser, C. Sturm
        - arXiv:1004.4285 HPQCD, update to include bottom

Moments method

$j_5 = \overline{\psi}_h \gamma_5 \psi_h$:

$$G(t) = a^6 \sum_{\mathbf{x}} (am_{0h})^2 \langle 0 | j_5(\mathbf{x}, t) j_5(0,0) | 0 \rangle$$

where m_{0h} is the heavy quark's bare mass.
The moments of $G(t)$ are simple to analyze:

$$G_n \equiv \sum_t (t/a)^n G(t)$$

Replacing the moments with reduced moments:

$$R_n \equiv \begin{cases} G_4 / G_4^{(0)} & \text{for } n = 4, \\ \dfrac{am_{\eta_h}}{2am_{0h}} \left(G_n / G_n^{(0)} \right)^{1/(n-4)} & \text{for } n \geq 6, \end{cases}$$

where $G_n^{(0)}$ is the moment in lowest-order weak-coupling lattice perturbation theory.

Fit model for reduced moments

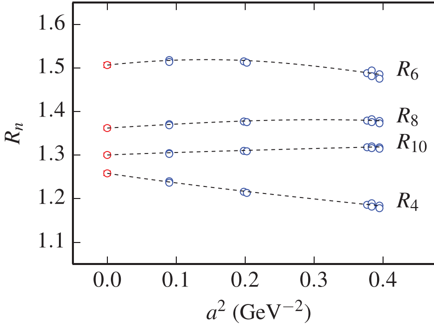
The reduced moments are analyzed using the fit model below with Bayesian fits.

$$R_n = \begin{Bmatrix} 1 & \text{for } n = 4 \\ m_{\eta_h}/2\xi_m m_h(\xi_\alpha \mu) & \text{for } n \geq 6 \end{Bmatrix}$$
$$\times\, r_n(\alpha_{\overline{\mathrm{MS}}}(\xi_\alpha \mu), \mu)$$
$$\times \left(1 + d_n^{\mathrm{cond}} \frac{\langle \alpha_s G^2/\pi \rangle}{(2m_h)^4}\right)$$
$$\times \left(1 + d_n^{h,c} \frac{m_{0h}^2 - m_{0c}^2}{m_{0h}^2}\right)$$
$$+ \left(\frac{am_{\eta_h}}{2.2}\right)^2 \sum_{i=0}^{N} c_i(m_{\eta_h}, n) \left(\frac{am_{\eta_h}}{2.2}\right)^{2i}$$

where r_n is the continuum perturbative factor. known through α_s^3 for moments with n = 4,6, 8, 10.

Continuum limit of R_n moments.

Charm-charm moments.



Summary of $n_f = 4$ analysis of the moments

The R_4 moment is sensitive to the coupling

$$\alpha_{\overline{\rm MS}}(M_Z, n_f = 5) = 0.11881(86).$$

This agrees well with 0.1183(7) from our $n_f = 3$ analysis(arXiv:1004.4285)
It also agrees well with the current world average 0.1185(6) from the PDG.

$$m_c(\mu, n_f = 4) = \begin{cases} 0.9915(57)\,\mathrm{GeV} & \mu = 3m_c \\ 0.9896(69)\,\mathrm{GeV} & \mu = 3\,\mathrm{GeV} \\ 1.281(11)\,\mathrm{GeV} & \mu = m_c. \end{cases}$$

These agree to within a standard deviation with our previous $n_f = 3$ analysis (arXiv:1004.4285) which gave 0.986(6) GeV for the mass at 3 GeV.

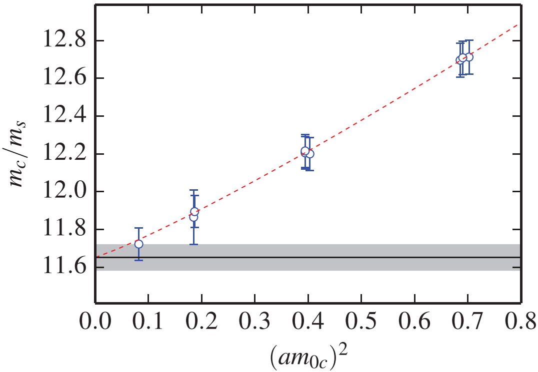
Joint $n_f = 3$, $n_f = 4$ Results

- Our earlier $n_f = 3$ analysis is more accurate than our new analysis because it includes results from a smaller lattice spacing (0.045 fm)
- The smaller lattice spacing also allowed us to include the b mass
- To merge our $n_f = 3$ and $n_f = 4$ results, we use the old code to generate predictions for $m_h(\mu = 3m_h, n_f = 4)$ at several values of m_{η_h}: 2.98, 4.6, 6.2, 7.8 and 9.4 GeV.

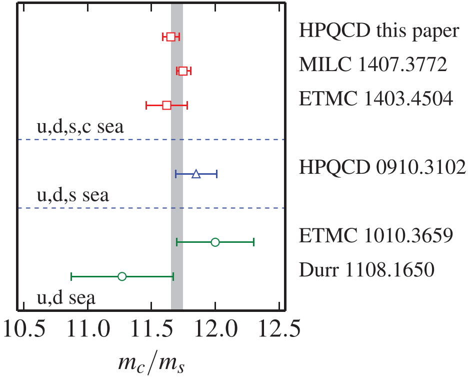
The results of a joint analysis of n_f =4 and n_f =3 data sets.

$$
\begin{aligned}
m_c(3\,\mathrm{GeV}, n_f = 4) &= 0.9864(41)\,\mathrm{GeV}, \\
m_b(10\,\mathrm{GeV}, n_f = 5) &= 3.625(25)\,\mathrm{GeV}, \\
m_b/m_c &= 4.54(3), \\
\alpha_{\overline{\mathrm{MS}}}(M_Z, n_f = 5) &= 0.11856(53).
\end{aligned}
$$

Continuum extrapolation of m_c/m_s ratio



Summary of m_c/m_s ratio



$$m_s(\mu, n_f = 3) = \begin{cases} 94.0(6)\,\text{MeV} & \mu = 2\,\text{GeV} \\ 84.9(6)\,\text{MeV} & \mu = 3\,\text{GeV}. \end{cases}$$

Testing the Georgi-Jarlskog relation

- The quarks and leptons are included in the same multiplets in GUTs.
- Sometimes this gives predictions for relations between quark and lepton masses.

From Georgi-Jarlskog relation, Phys.Lett. B86 (1979) 297-300, 698 citations.

$$\frac{m_b}{m_s} = \frac{3m_\tau}{m_\mu} = 50.45$$

This is over 5 σ from the HPQCD value of 52.90(44).

- There are threshold corrections from SUSY which modify the Georgi-Jarlskog relation.
- Ross and Serna, arXiv:0704.1248, Phys.Lett. B664 (2008) 97-102

Error budget in arXiv:1408.4169

The errors in m_c, m_b and α_s are broken down into component errors.

	$m_c(3)$	$m_b(10)$	m_b/m_c	$\alpha_{\overline{\mathrm{MS}}}(M_Z)$
Perturbation theory	0.1	0.1	0.1	0.2
Statistical, $n_f = 3$ errors	0.3	0.6	0.5	0.4
$a^2 \to 0$ extrapolation	0.1	0.2	0.3	0.2
$\delta m_{uds}^{\mathrm{sea}} \to 0$ extrapolation	0.1	0.1	0.1	0.0
$\delta m_c^{\mathrm{sea}} \to 0$ extrapolation	0.1	0.1	0.0	0.0
Uncertainty in w_0, w_0/a	0.1	0.1	0.1	0.1
δm_{η_c}: electromag., annih.	0.1	0.0	0.1	0.0
Total:	0.42%	0.70%	0.65%	0.48%

- Reduced errors from smaller lattice spacing and higher statistics.

Are more accurate results required?

- Can the errors on m_c, m_b, α_s be reduced?
- Do we need more accurate values of m_c, m_b, α_s?

Expected Precision of Higgs Boson Partial Widths within the Standard Model G. Peter Lepage, Paul B. Mackenzie, Michael E. Peskin. e-Print: arXiv:1404.0319

- BSM physics may contribute to the Higgs couplings.
- Precision standard model calculations of the Higgs couplings required with experimental measurements to expose possible BSM contribution.

Parameter dependence of Higgs couplings

- As part of Snowmass process(1401.6075) there have been some studies at looking for BSM contributions to Higgs decay.
- See analysis by Peskin 1312.4974

Decay of the Higgs to two particles $A\overline{A}$ in the Standard Model is

$$\Gamma(h \to A\overline{A}) = \frac{G_F}{\sqrt{2}} \frac{m_h m_A^2}{4\pi} \mathcal{F}$$

The $\mathcal{F}$ factor contains the perturbative factors.

- If A is the bottom or charm quark, then the errors on the quark masses are an important contribution to the decay rate.

Parameterize the uncertainty on the Higgs decay on A

$$\delta_A = \frac{1}{2} \frac{\Delta\Gamma(h \to A\bar{A})}{\Gamma(h \to A\bar{A})}.$$

Improvements in lattice QCD moment calculations

Repeat this moment analysis with the "conservative" improvements"

LS	work at lattice spacing 0.03 fm
LS^2	work at lattice spacing 0.023 fm
ST	100 times the statistics
PT	4th order in perturbation theory

Pessimism

- Fine lattice spacings will produce problems with autocorrelation of topological observables, but may not be important for more physical quantities.

Optimism

- There are many techniques in lattice QCD, which have not been used to their full potential.

Future lattice QCD calculations and Higgs couplings

From the paper arXiv:1404.0319. Current input from HPQCD 1004.4285.

	% errors					
	Lattice QCD input			Higgs coupling errors		
	$\delta m_b(10)$	$\delta\alpha_s(m_Z)$	$\delta m_c(3)$	δ_b	δ_c	δ_g
current errors	0.70	0.63	0.61	0.77	0.89	0.78
$+$ PT	0.69	0.40	0.34	0.74	0.57	0.49
$+$ LS	0.30	0.53	0.53	0.38	0.74	0.65
$+$ LS^2	0.14	0.35	0.53	0.20	0.65	0.43
$+$ PT $+$ LS	0.28	0.17	0.21	0.30	0.27	0.21
$+$ PT $+$ LS^2	0.12	0.14	0.20	0.13	0.24	0.17
$+$ PT $+$ LS^2 $+$ ST	0.09	0.08	0.20	0.10	0.22	0.09
ILC goal				0.30	0.70	0.60

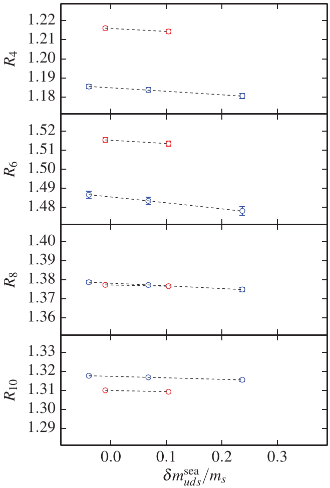
Conclusions

- The HPQCD collaboration have reported new values for m_c, α_s and m_b from lattice QCD calculation including light, strange, and charm quarks in the sea.
- These new results from 2+1+1 are consistent with older results with 2+1 sea quarks. Including charm in the sea didn't cause problems.
- There will be some improvements when results which include lattice spacings at 0.045 fm are included: More radical improvements from:
    - Ensembles at lattice spacings 0.03 fm and 0.023 fm
    - Statistics increase by 100.
- Improved lattice QCD calculations will reduce the error on the Higgs decay to two hadrons. The detailed comparison between theory and experiment at ILC may help to find BSM physics.

Backup

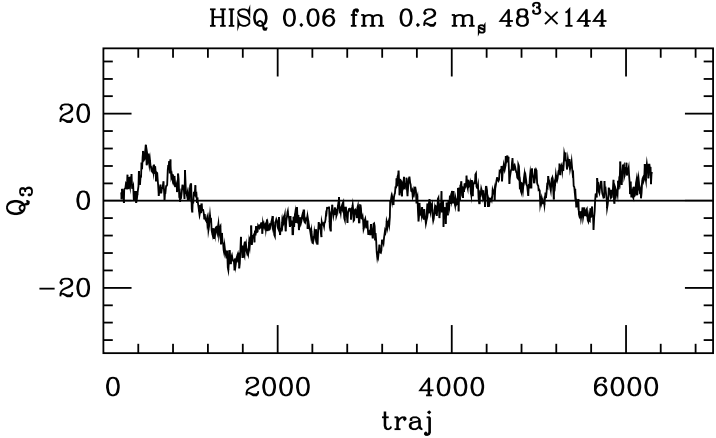
Sea quark mass dependence of R_n moments.

Charm-charm



Topological charge

MILC collaboration, 1212.4768



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