$\Gamma(\Upsilon(1S) \to \ell^+\ell^-)$ to NNLO

Matthias Steinhauser | (M. Beneke, Y. Kiyo, P. Marquard, A. Penin, J. Piclum, D. Seidel)

TTP KARLSRUHE

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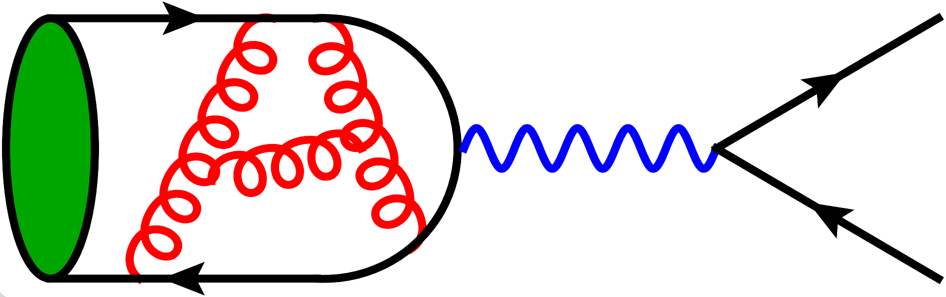


Outline

- Introduction
- c_v to 3 loops
- $\Gamma(\Upsilon(1S) \to \ell^+\ell^-)$ to NNNLO
- Summary

$\Gamma(\Upsilon(1S) \to \ell^+\ell^-)$

- $\Upsilon(1S)$ meson: simplest heavy quark bound state
- $\Gamma(\Upsilon(1S) \to \ell^+\ell^-)_{\rm exp} = 1.340(18)$ keV
- needed:
  - description of bound states
  - starting point: QCD
  - energy levels, wave function

- NLL: [Pineda'01]; NNLL-approx: [Pineda,Signer'07]; ...
- lattice [HPQCD'14]

Framework: potential NRQCD

scales: mass, m: hard $\gg$ momentum, mv: soft $\gg$ energy, mv^2: ultrasoft $\gg$ $\Lambda_{\rm QCD}$

bottom: 5 GeV 2.5 GeV 0.5 GeV

QCD $\Rightarrow$ NRQCD $\Rightarrow$ pNRQCD (potential non-relativistic QCD)

$\uparrow$ $m \gg mv, mv^2$

[Caswell,Lepage'86;
Bodwin,Braaten,Lepage'95]

$\uparrow$ integrate out all non-physical degrees of freedom

[Beneke,Smirnov'97; Pineda,Soto'98; Brambilla,Pineda,Soto,Vairo'00]

[alternative formulation: velocity NRQCD [Luke,Manohar,Rothstein'00; Hoang,Stewart'03]]

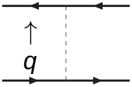
Effective Hamiltonian to N^{3}LO

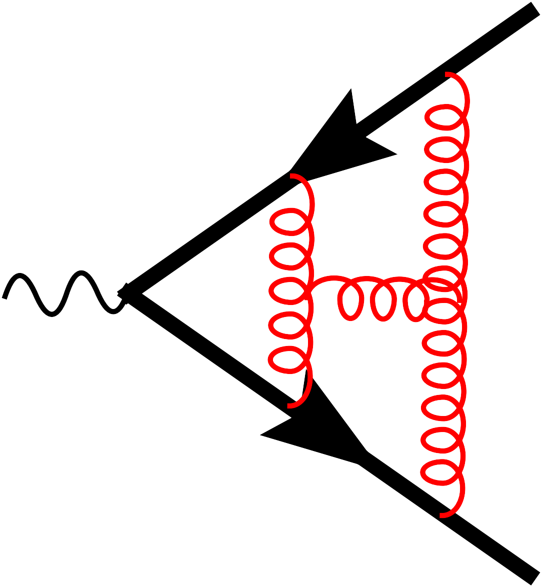
[Gupta,Radford'81,...,Manohar'97,...,Kniehl,Penin,Smirnov,Steinhauser'02,...,Beneke,Kiyo,Schuller'13]

$$\mathcal{H} = (2\pi)^3\delta(\vec{q})\left(\frac{\vec{p}^{\,2}}{m} - \frac{\vec{p}^{\,4}}{4m^3}\right) + C_c(\alpha_s)V_C(|\vec{q}|) + C_{1/m}(\alpha_s)V_{1/m}(|\vec{q}|)$$
$$+\frac{\pi C_F\alpha_s(\mu)}{m^2}\left[C_\delta(\alpha_s) + C_p(\alpha_s)\frac{\vec{p}^{\,2}+\vec{p}^{\,\prime 2}}{2\vec{q}^{\,2}} + C_s(\alpha_s)\vec{S}^{\,2}\right]$$

Static potential:	$V_C(\lvert\vec{q}\rvert) = -\frac{4\pi C_F\alpha_s(\lvert\vec{q}\rvert)}{\vec{q}^2}$	C_c	3 loops
$1/m$ potential:	$V_{1/m}(\lvert\vec{q}\rvert) = \frac{\pi^2 C_F\alpha_s^2(\lvert\vec{q}\rvert)}{m\lvert\vec{q}\rvert}$	$C_{1/m}$	2 loops
"Breit" potential:	$\propto 1/m^2$	$C_{\delta,p,s}$	1 loop

$$\vec{q} = \vec{p}^{\,\prime} - \vec{p}$$





c_V to 3 loops

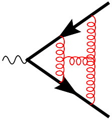
QCD $\longrightarrow$ NRQCD

$$j_v^\mu = \bar{Q}\gamma^\mu Q \quad \longrightarrow \quad \tilde{j}^i = \phi^\dagger \sigma^i \chi$$

$$j_v^i \quad = \quad c_v(\mu)\, \tilde{j}^i + \frac{d_v(\mu)}{6m_Q^2} \phi^\dagger \sigma^i \vec{D}^2 \chi + \ldots$$

$$\boxed{Z_2 \Gamma_v = c_v \tilde{Z}_2 \tilde{Z}_v^{-1} \tilde{\Gamma}_v + \ldots}$$

Γ_v:



$\tilde{\Gamma}_v \equiv 1 \quad \tilde{Z}_2 \equiv 1$

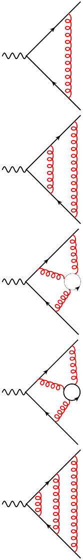
Z_2: [Melnikov,v.Ritbergen'00]
[Marquard,Mihaila,Piclum,Steinhauser'07]

$\tilde{Z}_v = 1 + \mathcal{O}(\alpha_s^2)$
[Beneke,Signer,Smirnov'98;
Kniehl,Penin,Steinhauser,Smirnov'03;
Marquard,Piclum,Seidel,Steinhauser'06;
Beneke,Kiyo,Penin'07]

($\overline{\mathrm{MS}}$ scheme)

c_V

$$c_V = 1 + \frac{\alpha_s}{\pi} c_V^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 c_V^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 c_V^{(3)} + \mathcal{O}(\alpha_s^4)$$



$c_V^{(1)}$	[Källen, Sarby'55]
$c_V^{(2)}$	[Czarnecki,Melnikov'97; Beneke,Signer,Smirnov'97]
$c_V^{(3),n_l}$	[Marquard,Piclum,Seidel,Steinhauser'06]
$c_V^{(3),n_h}$	[Marquard,Piclum,Seidel,Steinhauser'08]
$c_V^{(3)}$	[Marquard,Piclum,Seidel,Steinhauser'14]

massive vertices
on-shell quarks: $q_1^2 = q_2^2 = M_Q^2$
$(q_1 + q_2)^2 = 4M_Q^2$

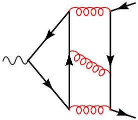
Results for c_v

[Marquard,Piclum,Seidel,Steinhauser'14]

$$c_v(\mu = m_Q) = 1 - 2.67\frac{\alpha_s}{\pi} + [-44.55 + 0.41 n_l]\left(\frac{\alpha_s}{\pi}\right)^2 + \left[-2091(2) + 120.66 n_l - 0.82 n_l^2\right]\left(\frac{\alpha_s}{\pi}\right)^3 + \text{singlet terms}$$

- $\mu = m_Q$
- large corrections
- singlet terms: small ($\leq 3\%$ of $c^{(2-loop)}$) at 2 loops (for axial-vector, scalar, pseudo-scalar current)

  [Kniehl,Onishchenko,Piclum,Steinhauser'06]



$\Gamma(\Upsilon(1S) \to \ell^+\ell^-)$

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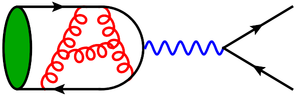

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- $$\Gamma(\Upsilon(1S) \to \ell^+\ell^-) = \frac{4\pi\alpha^2}{9m_b^2} \left|\psi_1(0)\right|^2 c_v \left[c_v - \frac{E_1}{m_b}\left(c_v + \frac{d_v}{3}\right) + \ldots\right]$$
- d_v: matching constant of sub-leading $b\bar{b}$ current 1 loop: [Luke,Savage'98]
- $\psi_1(0)$ wave function of the $(b\bar{b})$ system $\quad \left|\psi_1^{\rm LO}(0)\right|^2 = \frac{8m_b^3\alpha_s^3}{27\pi}$
- $M_{\Upsilon(1S)} = 2m_b + E_1 \qquad E_1^{p,{\rm LO}} = -(4m_b\alpha_s^2)/9$
- many building blocks necessary; most recent ones:
  - a_3 [Smirnov,Smirnov,Steinhauser'08; Smirnov,Smirnov,Steinhauser'09; Anzai,Kiyo,Sumino'09]
  - c_v [Marquard,Piclum,Seidel,Steinhauser'14]
  - $\psi_1(0)$: ultrasoft contribution [Beneke,Kiyo,Penin'07]
  - $\psi_1(0)$: single- and double-potential insertions [Beneke,Kiyo,Schuller'08'13]
  - $\mathcal{O}(\epsilon)$ term of 2-loop $1/(m_b r^2)$ pNRQCD potential [Penin,Smirnov,Steinhauser'13]
- NNLL-approx: [Pineda,Signer'07]

Result

$$\Gamma(\Upsilon(1S) \to \ell^+\ell^-)|_{\rm pole} =$$

$$\frac{2^5\alpha^2\alpha_s^3 m_b}{3^5}\Big[1 + \alpha_s\left(-2.003 + 3.979\,L\right) + \alpha_s^2\left(9.05 - 7.44\,\ln\alpha_s - 13.95\,L + 10.55\,L^2\right)$$

$$+\alpha_s^3\big(-0.91 + 4.78_{a_3} + 22.07_{b_2\epsilon} + 30.22_{c_f} - 134.8(1)_{c_g} - 14.33\,\ln\alpha_s - 17.36\,\ln^2\alpha_s$$

$$+(62.08 - 49.32\,\ln\alpha_s)\,L - 55.08\,L^2 + 23.33\,L^3\big) + \mathcal{O}(\alpha_s^4)\Big]$$

$$\stackrel{\mu=3.5\text{ GeV}}{=} \frac{2^5\alpha^2\alpha_s^3 m_b}{3^5}\big[1 + 1.166\alpha_s + 15.2\alpha_s^2 + (66.5 + 4.8_{a_3}$$

$$+22.1_{b_2\epsilon} + 30.2_{c_f} - 134.8(1)_{c_g})\,\alpha_s^3 + \mathcal{O}(\alpha_s^4)\big]$$

$$= \frac{2^5\alpha^2\alpha_s^3 m_b^{\rm pole}}{3^5}[1 + 0.28 + 0.88 - 0.16]$$

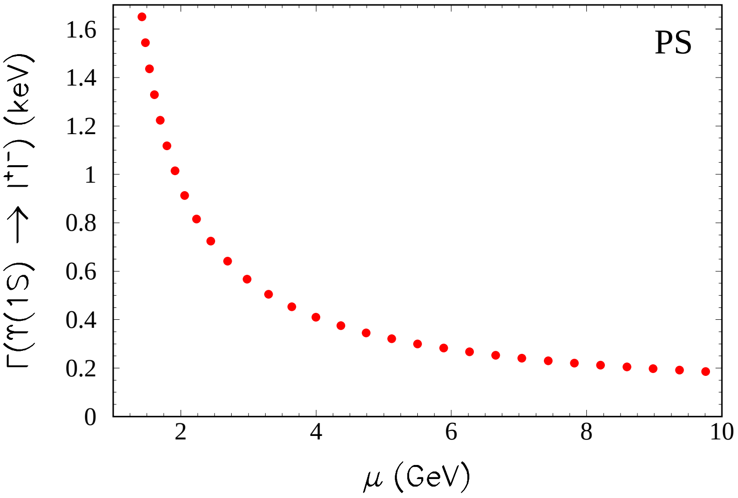
$$= [1.04 \pm 0.04(\alpha_s)^{+0.02}_{-0.15}(\mu)]\text{ keV}$$

$$= \frac{2^5\alpha^2\alpha_s^3 m_b^{\rm PS}}{3^5}[1 + 0.37 + 0.95 - 0.04]$$

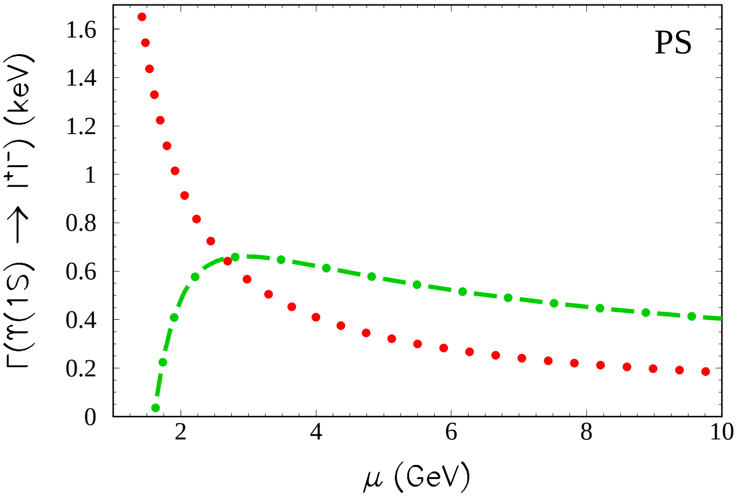
$$= [1.08 \pm 0.05(\alpha_s)^{+0.01}_{-0.20}(\mu)]\text{ keV}$$

$$\alpha_s = 0.1184 \pm 0.001$$
$$3 \le \mu \le 10\text{ GeV}$$
$$L = \ln[\mu/(4m_b\alpha_s/3)]$$

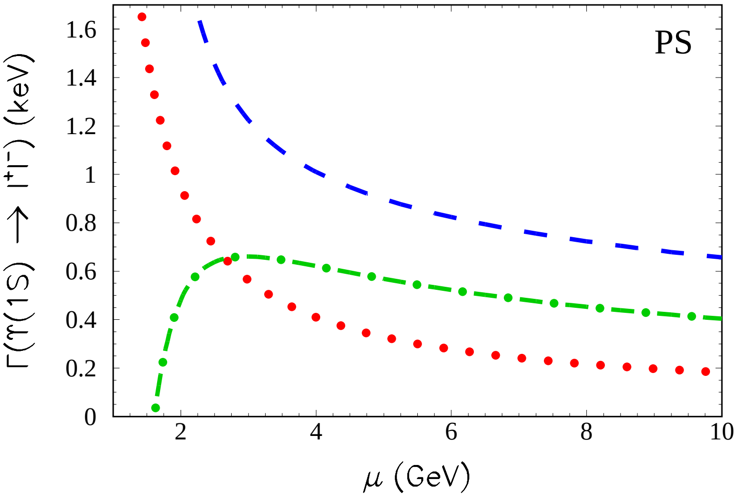
$\Gamma(\Upsilon(1S) \to \ell^+\ell^-)$: μ dependence



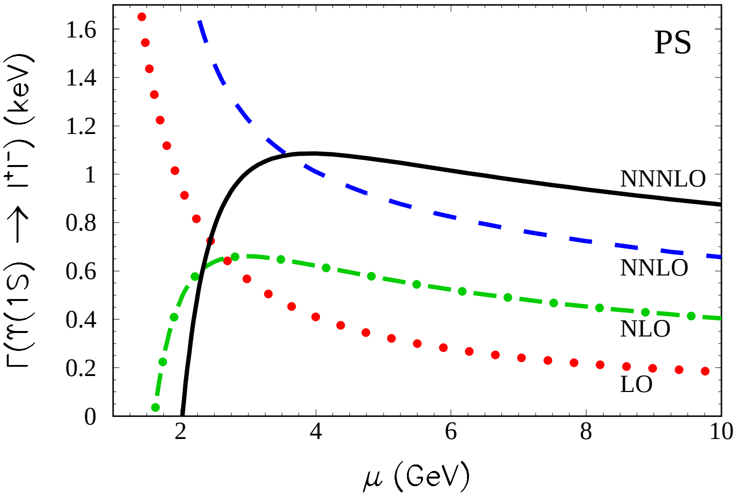
$\Gamma(\Upsilon(1S) \to \ell^+\ell^-)$: μ dependence



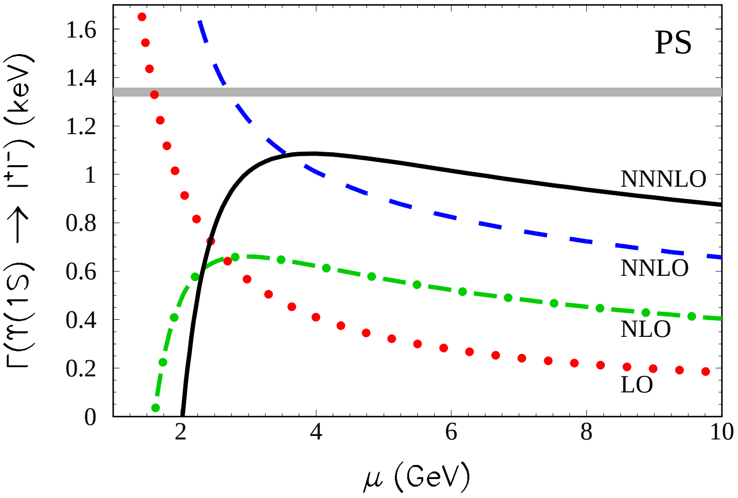
$\Gamma(\Upsilon(1S) \to \ell^+\ell^-)$: μ **dependence**



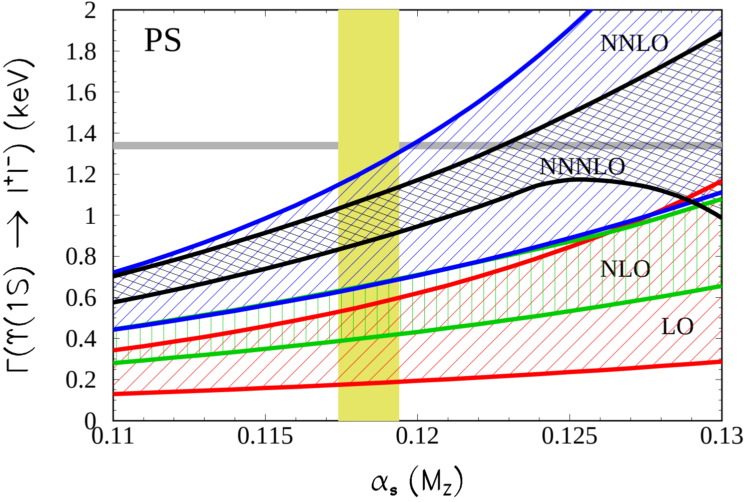
$\Gamma(\Upsilon(1S) \to \ell^+\ell^-)$: μ dependence



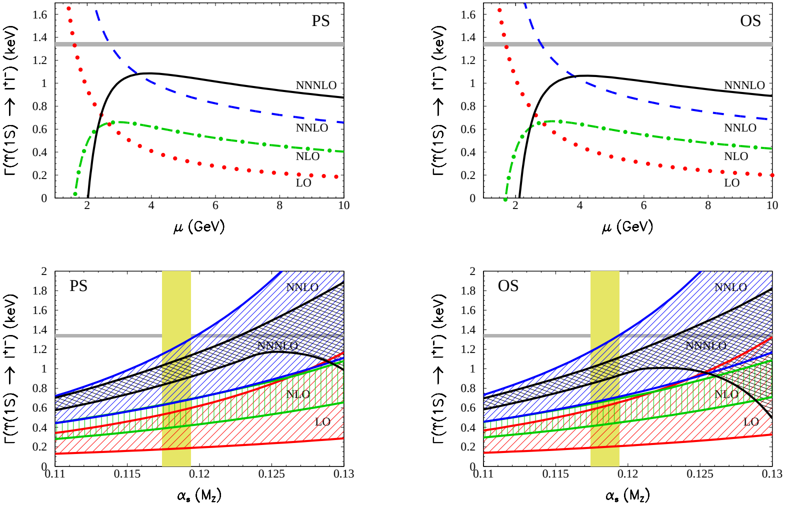
$\Gamma(\Upsilon(1S) \to \ell^+\ell^-)$: μ dependence



$\Gamma(\Upsilon(1S) \to \ell^+\ell^-)$: α_s dependence



$\Gamma(\Upsilon(1S) \to \ell^+\ell^-)$: PS vs. OS



Result

- $\Gamma(\Upsilon(1S) \to \ell^+\ell^-)|_{\rm pole} = [1.04 \pm 0.04(\alpha_s)^{+0.02}_{-0.15}(\mu)]$ keV
- $\Gamma(\Upsilon(1S) \to \ell^+\ell^-)|_{\rm PS} \;\; = [1.08 \pm 0.05(\alpha_s)^{+0.01}_{-0.20}(\mu)]$ keV

- $\Gamma(\Upsilon(1S) \to \ell^+\ell^-)|_{\rm exp} \;\; = [1.340 \pm 0.018]$ keV

- third-order perturbative result is $\approx 30\%$ too low
- possible explanation: sizeable non-perturbative contribution

Non-perturbative contribution

1. $\delta_{\rm np}|\psi_1(0)|^2 = |\psi_1^{\rm LO}(0)|^2 \times 17.54\pi^2 K$ [Leutwyler'81,Voloshin'82]

- $K = \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{m_b^4 (\alpha_s C_F)^6}$
- $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012~\text{GeV}^4$ ⇨ $\boxed{\delta_{\rm np}\Gamma_{\ell\ell}(\Upsilon(1S)) = 1.67_{\rm pole}/2.20_{\rm PS}~\text{keV}}$
  $[\alpha_s(3.5~\text{GeV}) \approx 0.24]$

? value of $\langle \frac{\alpha_s}{\pi} G^2 \rangle$
? scale of α_s
? dimension-6 condensate contribution [Pineda'97]

$$\Gamma(\Upsilon(1S) \to \ell^+\ell^-)\Big|_{\rm exp} - \Gamma(\Upsilon(1S) \to \ell^+\ell^-)\Big|_{\rm pert. N^3LO} \approx 0.3~\text{keV}$$

Non-perturbative contribution

1. $\delta_{\rm np}|\psi_1(0)|^2 = |\psi_1^{\rm LO}(0)|^2 \times 17.54\pi^2 K$ [Leutwyler'81,Voloshin'82]
   - $K = \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{m_b^4(\alpha_s C_F)^6}$
2. $M_{\Upsilon(1S)} = 2m_b + E_1^{\rm p} + \frac{624\pi^2}{425}\, m_b(\alpha_s C_F)^2\, K$
   - use PS scheme ➪ perturbation theory converges [Beneke,Kiyo,Schuller'05]

$$➪\ \delta_{\rm np}\Gamma_{\ell\ell}(\Upsilon(1S)) = \frac{4\alpha^2\alpha_s}{9}\,\frac{17.54\times 425}{3744}\,\delta M^{\rm np}_{\Upsilon(1S)}$$

$$\approx \left[1.28^{+0.17}_{-0.18}(\alpha_s) \pm 0.42(m_b)^{+0.20}_{-0.57}(\mu) \pm 0.12(m_c)\right] \text{keV}$$

(charm effects [Hoang'00])

? $m_b^{\overline{\rm MS}} = 4.163$ GeV $\to 4.203$ GeV ➪ $\delta_{\rm np}\Gamma_{\ell\ell}(\Upsilon(1S)) \approx 0.3$ keV

$$\alpha_s = 0.1184 \pm 0.001, \quad m_b = 4.163 \pm 0.016 \text{ GeV}, \quad 3 \le \mu \le 10 \text{ GeV}$$

[PDG] [Chetyrkin et al.'09]

Non-perturbative contribution

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   - $K = \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{m_b^4 (\alpha_s C_F)^6}$

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   - use PS scheme ➪ perturbation theory converges [Beneke,Kiyo,Schuller'05]

➪ $\delta_{\rm np}\Gamma_{\ell\ell}(\Upsilon(1S)) = \frac{4\alpha^2\alpha_s}{9} \frac{17.54\times 425}{3744} \delta M_{\Upsilon(1S)}^{\rm np}$

$\approx [1.28^{+0.17}_{-0.18}(\alpha_s) \pm 0.42(m_b)^{+0.20}_{-0.57}(\mu) \pm 0.12(m_c)]$ keV

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? $m_b^{\overline{\rm MS}} = 4.163$ GeV $\to 4.203$ GeV ➪ $\delta_{\rm np}\Gamma_{\ell\ell}(\Upsilon(1S)) \approx 0.3$ keV

Summary: perturbation theory: solid prediction
non-perturbative contribution: unclear

Conclusions

- c_v to 3 loops
- $\Gamma(\Upsilon(1S) \to \ell^+\ell^-)$ to NNNLO
- $\Gamma(\Upsilon(1S) \to \ell^+\ell^-) =$

$$= 1.340 \pm 0.018 \text{ keV (experiment)}$$
$$= 1.19 \pm 0.11 \text{ keV (HPQCD)}$$
$$= 1.08 \; {}^{+0.05}_{-0.21} \text{ keV } (\text{N}^3\text{LO})$$