Quarkonium spectral functions

and

Finite Temperature Charmonium Potentials

Chris Allton

Swansea University, U.K.

Quarkonium 2014

FASTSUM Collaboration

Gert Aarts[1], CRA[1], Alessandro Amato[1,2], Wynne Evans[1,3], Pietro Giudice[4], Simon Hands[1], Aoife Kelly[5], Seyong Kim[6], Maria-Paola Lombardo[7], Dhagash Mehta[8], Bugra Oktay[9], Sinead Ryan[10], Jon-Ivar Skullerud[5], Don Sinclair[11], Tim Harris[10]

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[8] North Carolina State University

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Outline

Setting the scene

- anisotropic lattices $a_\tau < a_s$
  - allowing better resolution, particularly at finite temperatures since $T = 1/(N_\tau a_\tau)$
- "2nd" generation lattice ensembles
  - moving towards continuum, infinite volume, realistic light quark masses
- bottom quark treated using NRQCD
- charm quark treated relativistically

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Physics/lattice parameters

1st Generation

2 flavours

smaller volume: $(2\text{fm})^3$
coarser lattices: $a_s = 0.167$ fm
quark mass: $M_\pi / M_\rho \sim 0.55$
temporal cut-off: $a_\tau \sim 7.4$ GeV

N_s	N_τ	T(MeV)	T/T_c
12	16	460	2.09
12	18	409	1.86
12	20	368	1.68
12	24	306	1.40
12	28	263	1.20
12	32	230	1.05
12	80	90	0.42

2nd Generation

2+1 flavours

larger volume: $(3\text{fm})^3 - (4\text{fm})^3$
finer lattices: $a_s = 0.123$ fm
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24	20	281	1.52
24, 32	24	235	1.27
24, 32	28	201	1.09
24, 32	32	176	0.95
24	36	156	0.84
24	40	141	0.76
32	48	117	0.63
16	128	44	0.24

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Physics/lattice parameters

$)^3$
$= 0.167$ fm
~ 0.55
~ 7.4 GeV

T/T_c
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Physics/lattice parameters

fm
eV

2nd Generation

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16	128	44	0.24

Physics/lattice parameters

2nd Generation

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24, 32	32	176	0.95
24	36	156	0.84
24	40	141	0.76
32	48	117	0.63
16	128	44	0.24

3rd Generation

2+1 flavours

larger volume: $(3\text{fm})^3 - (4\text{fm})^3$
finer lattices: $a_s = 0.123$ fm
quark mass: $M_\pi / M_\rho \sim 0.45$
temporal cut-off: $a_\tau \sim 11.2$ GeV

N_s	N_τ	T(MeV)	T/T_c
24, 32	32	352	1.90
24	40	281	1.52
24, 32	48	235	1.27
24, 32	56	201	1.09
24, 32	64	176	0.95
24	72	156	0.84
24	80	141	0.76
32	96	117	0.63
16	256	44	0.24

Lattice NRQCD

$$S(x + a_\tau e_\tau) = \left(1 - \frac{a_\tau H_0|_{\tau+a_\tau}}{2k}\right)^k U_\tau^\dagger(x) \left(1 - \frac{a_\tau H_0|_\tau}{2k}\right)^k (1 - a_\tau \delta H)\, S(x)$$

$$H_0 = -\frac{\Delta^{(2)}}{2m_b}, \qquad \text{with} \qquad \Delta^{(2n)} = \sum_{i=1}^{3} \left(\nabla_i^+ \nabla_i^-\right)^n \qquad (k = 1)$$

$$\begin{aligned}\delta H = & -\frac{\left(\Delta^{(2)}\right)^2}{8m_b^3} + \frac{ig_0}{8m_b^2}\left(\nabla^\pm \cdot E - E \cdot \nabla^\pm\right) \\ & - \frac{g_0}{8m_b^2}\sigma \cdot \left(\nabla^\pm \times E - E \times \nabla^\pm\right) - \frac{g_0}{2m_b}\sigma \cdot B \\ & + \frac{a_s^2 \Delta^{(4)}}{24m_b} - \frac{a_\tau \left(\Delta^{(2)}\right)^2}{16km_b^2}\end{aligned}$$

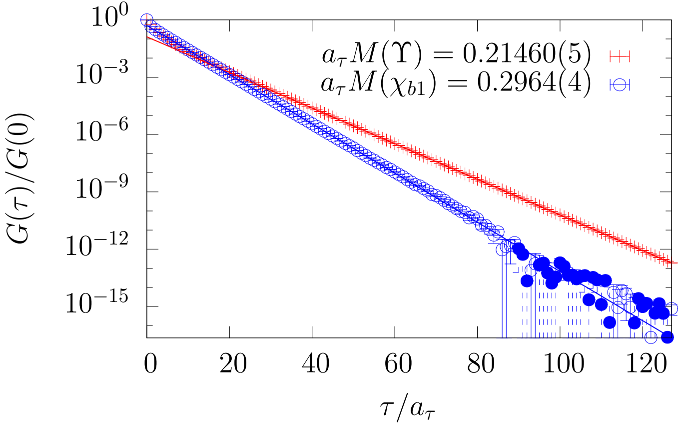
δH gives corrections to $\mathcal{O}(v^4), \mathcal{O}(a_s^2), \mathcal{O}(a_\tau)$

m_b set from spin averaged S-wave masses

Zero Temperature Bottomonium

Zero Temperature Correlators

$$G(\tau) \equiv \sum_x \langle 0|J(x,\tau)J^\dagger(0,0)|0\rangle \quad \xrightarrow{\tau\to\infty} \quad \frac{|\langle 0|J|\text{gnd}\rangle|^2}{2M}\, e^{-M\tau}$$



Bottomonium Spectrum Results

$n^{S+1}L_J$	State	$a_\tau M$	$E_0 + M$ (MeV) *(Lattice Postdiction)*	$M_{\rm expt}$ (MeV)
$1^1\mathrm{S}_0$	η_b	0.20549(4)	9409(12)	9398.0(3.2)
$2^1\mathrm{S}_0$	η_b'	0.311(3)	10004(21)	9999(4)
$1^3\mathrm{S}_1$	Υ	0.21460(5)	9460*	9460.30(26)
$2^3\mathrm{S}_1$	Υ'	0.318(3)	10043(22)	10023.26(31)
$1^1\mathrm{P}_1$	h_b	0.2963(4)	9920(15)	9899.3(1.0)
$1^3\mathrm{P}_0$	χ_{b0}	0.2921(4)	9896(15)	9859.44(52)
$1^3\mathrm{P}_1$	χ_{b1}	0.2964(4)	9921(15)	9892.78(40)
$1^3\mathrm{P}_2$	χ_{b2}	0.2978(4)	9928(15)	9912.21(40)

Maximum Entropy Method

Cont: $G(\tau) = \int K(\tau, \omega)\rho(\omega)d\omega$ Lat: $G(\tau_i) = \sum_j K(\tau_i, \omega_j)\, \rho(\omega_j)$

Input data: $\tau_i,\ i = \{1, \ldots, \mathcal{O}(10)\}$ Output data : $\omega_j,\ j = \{1, \ldots, \mathcal{O}(10^3)\}$

$\longrightarrow$ ill-posed

$$P[\rho|DH] = \frac{P[D|\rho H]P[\rho|H]}{P[D|H]} \propto \exp(-\chi^2 + \alpha S)$$

$H =$ prior knowledge $D =$ data

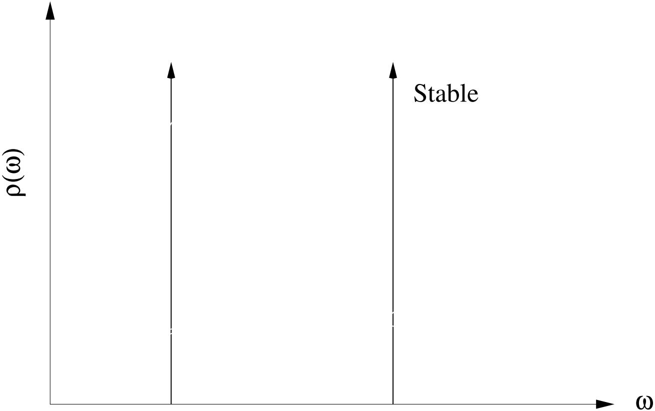
Shannon-Jaynes entropy: $S = \int_0^\infty \frac{d\omega}{2\pi} \left[\rho(\omega) - m(\omega) - \rho(\omega) \ln \frac{\rho(\omega)}{m(\omega)}\right]$

Competition between minimising χ^2 and maximising S

Asakawa, Hatsuda, Nakahara, Prog.Part.Nucl.Phys. 46 (2001) 459

Example Spectral Functions

$$G_2(t) \sim \int \rho(\omega)\; e^{-\omega t}\; d\omega$$



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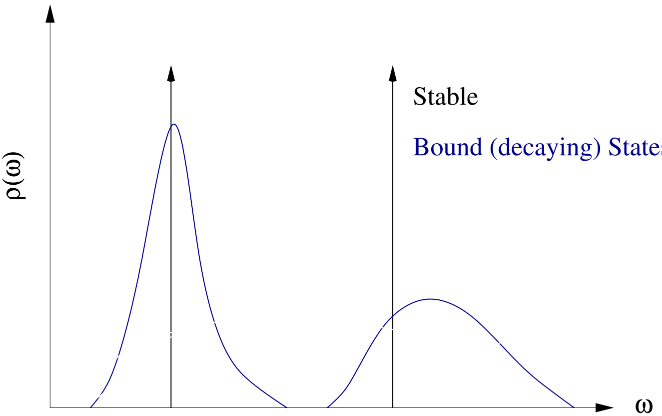

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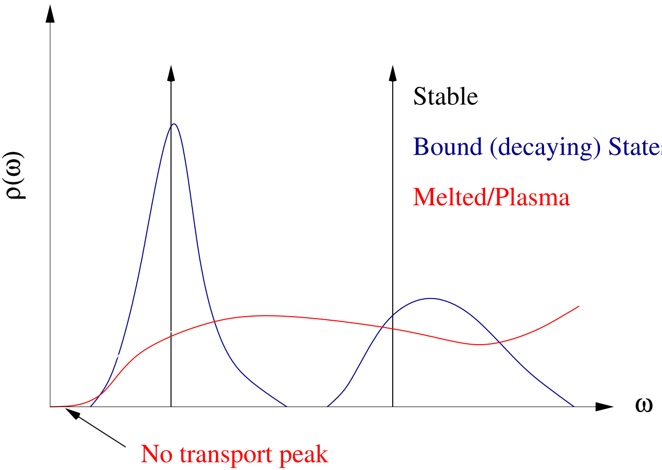

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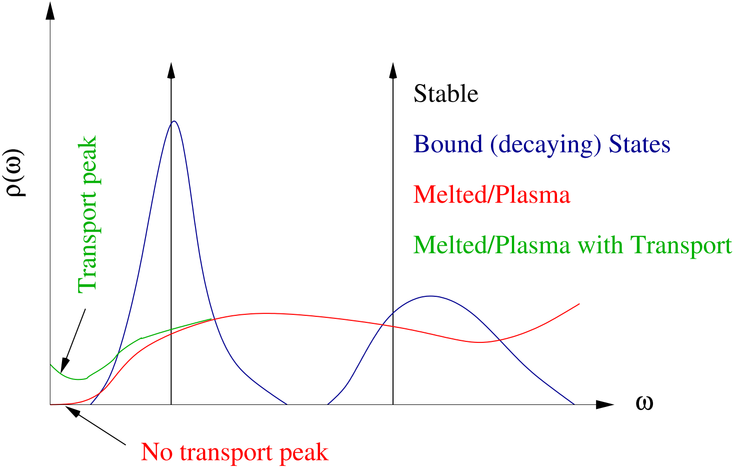
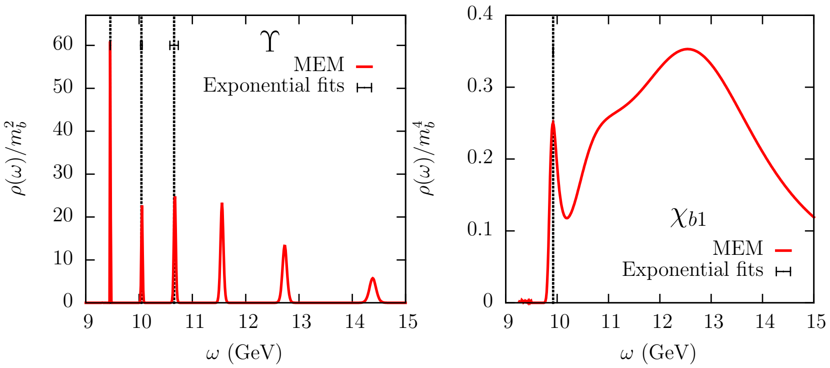


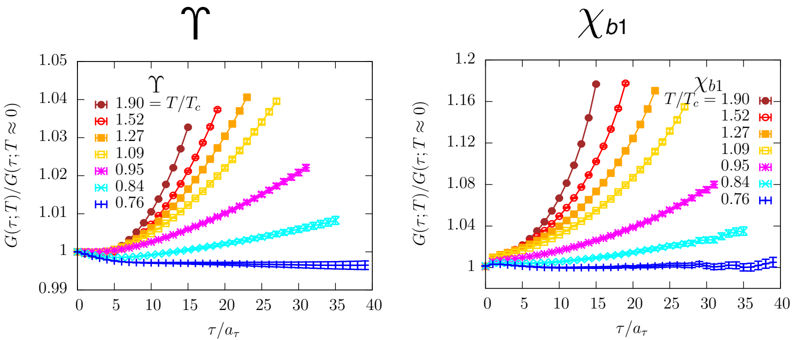
Zero Temperature Spectral Functions

$$G(\tau) = \int_{\omega_{\min}}^{\omega_{\max}} \frac{d\omega}{2\pi} K(\tau,\omega)\,\rho(\omega), \qquad K(\tau,\omega) = e^{-\omega\tau}.$$



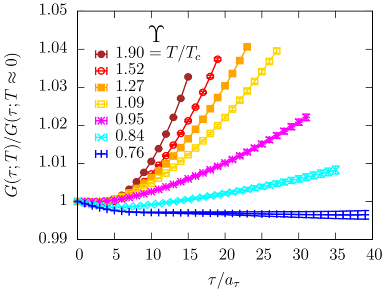
Non-zero Temperature Bottomonium

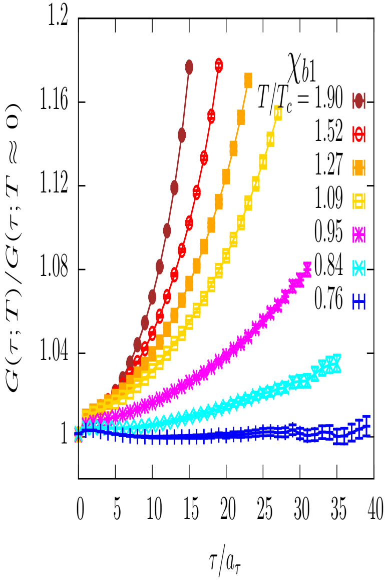
Thermal modification of correlation functions



Clear difference between S- and P-waves

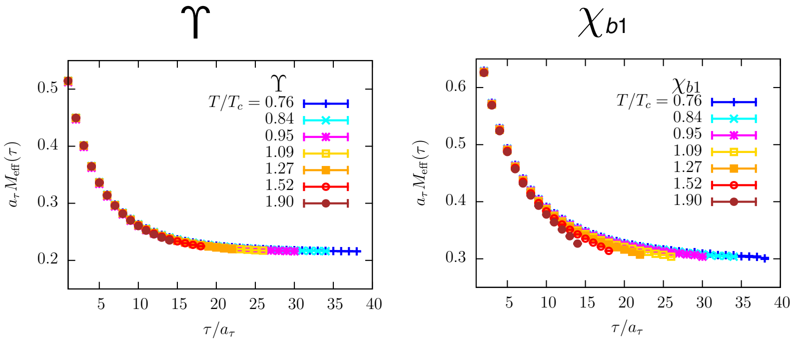
Thermal modification of correlation functions





Thermal modification of effective mass

$$M_{\text{eff}}(\tau) \equiv -\frac{1}{G(\tau)}\frac{dG(\tau)}{d\tau} \quad \xrightarrow{G(\tau)\sim e^{-M\tau}} \quad M$$



Clear difference between S- and P-waves

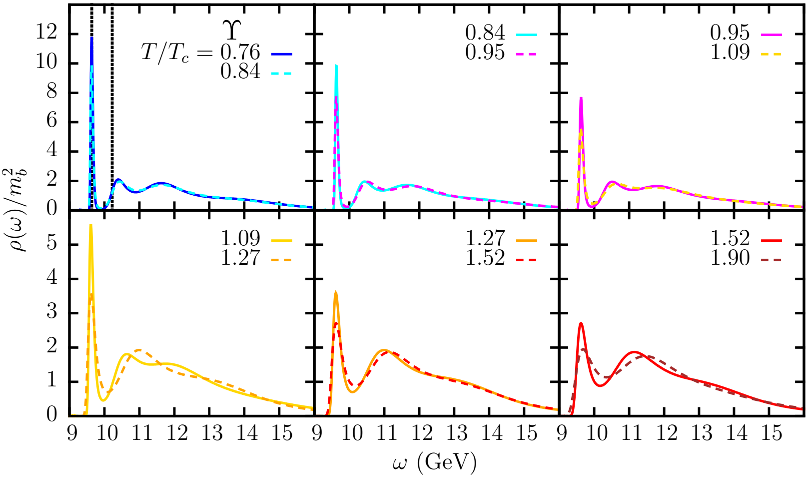
Free spectral functions and effective mass

$$\rho_{\text{free}}(\omega) \propto (\omega - \omega_0)^{\alpha}\, \Theta(\omega - \omega_0) \quad \text{where} \quad \alpha = \begin{cases} 1/2 & \text{S-wave} \\ 3/2 & \text{P-wave} \end{cases}$$

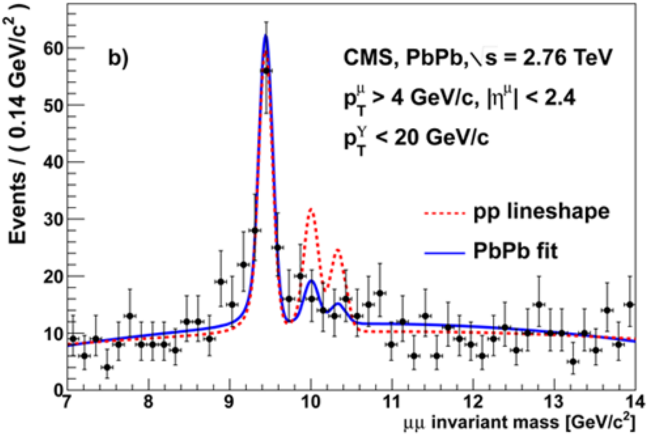
$$G_{\text{free}}(\tau) \propto \frac{e^{-\omega_0 \tau}}{\tau^{\alpha+1}}$$

$$M_{\text{eff}}(\tau) \equiv -\frac{1}{G(\tau)} \frac{dG(\tau)}{d\tau} \quad \xrightarrow{G=G_{\text{free}}} \quad \omega_0 + \frac{\alpha+1}{\tau}$$

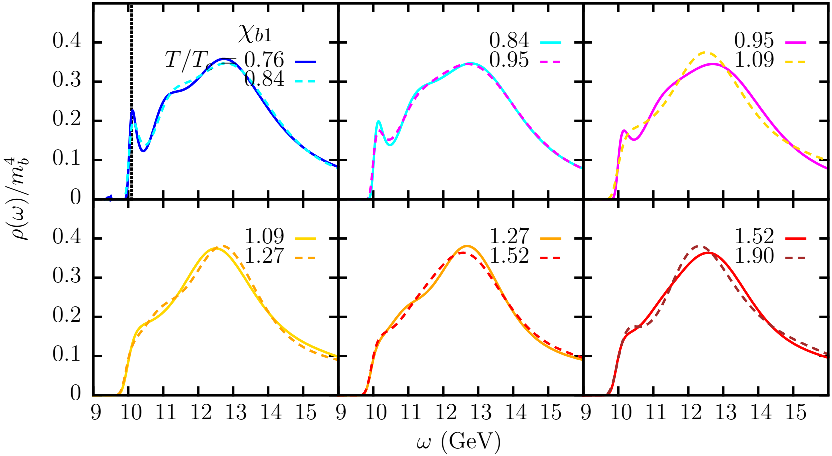
Thermal modification of ϒ spectral function



CMS pp versus PbPb



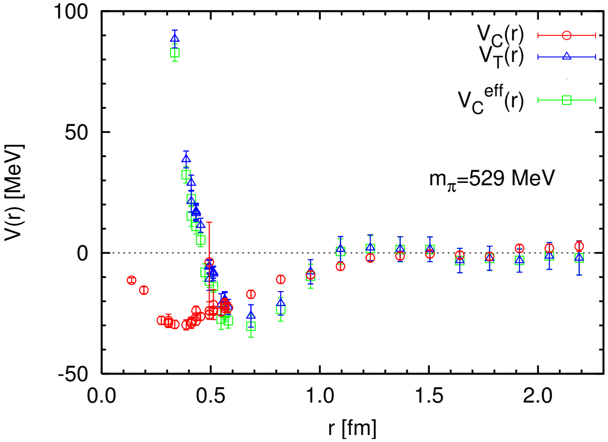
Thermal modification of χ_{b1} spectral function



Charmonium Potentials at Finite Temperature

Lattice goes Nuclear

N-N potential



HAL QCD Collaboration, Aoki, Doi, Hatsuda, Ikeda, Inoue, Ishii, Murano, Nemura, Sasaki

Schrödinger Equation Approach

Hatsuda, PoS CD09 (2009) 068

Schrödinger equation used to "reverse engineer" the potential, $V(r)$, given the Nambu- Bethe-Salpeter wavefunction, $\psi(r)$:

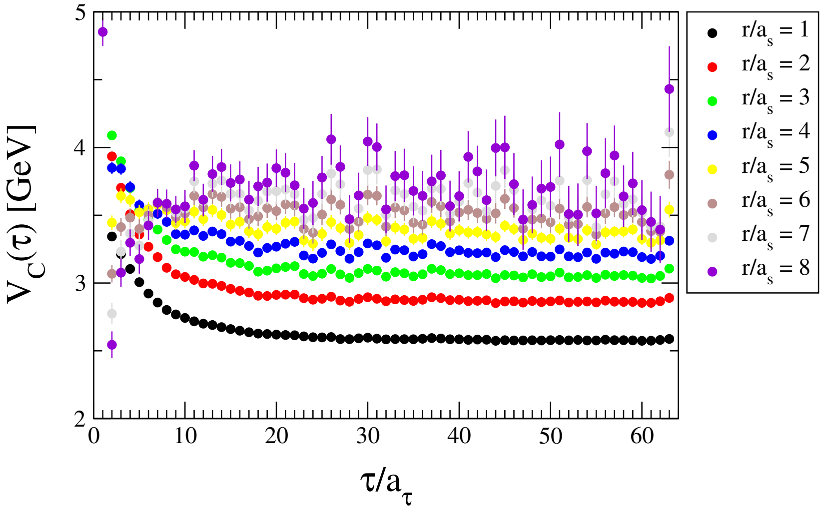
$$\left(\frac{p^2}{2M} + \underset{\substack{\downarrow \\ \text{output}}}{V(r)}\right) \overset{\substack{\text{input} \\ \downarrow}}{\psi(r)} = \overset{\substack{\text{input} \\ \downarrow}}{E} \ \overset{\downarrow}{\psi(r)}$$

$\psi(r)$ is determined from correlators of *non-local* (point-split) operators,

$$\begin{aligned} J(x;\vec{r}) &= q(x)\,\Gamma\,U(x, x+\vec{r})\overline{q}(x+\vec{r}) \\ C(\vec{r},t) &= \sum_{\vec{x}} < J(0;\vec{r})\,J(x;\vec{r}) > \\ &\longrightarrow |\psi(r)|^2\,e^{-Mt} \end{aligned}$$

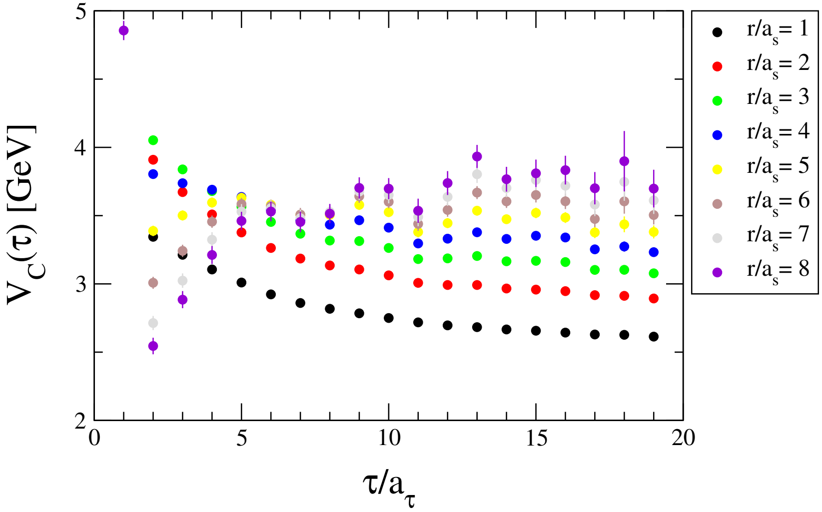
Spin-Independent Time-Slice Potential

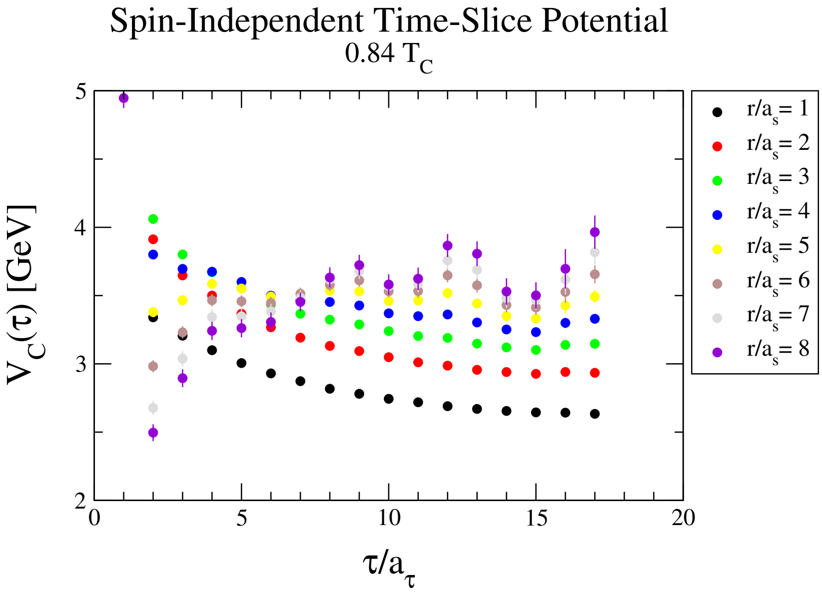
Zero Temperature



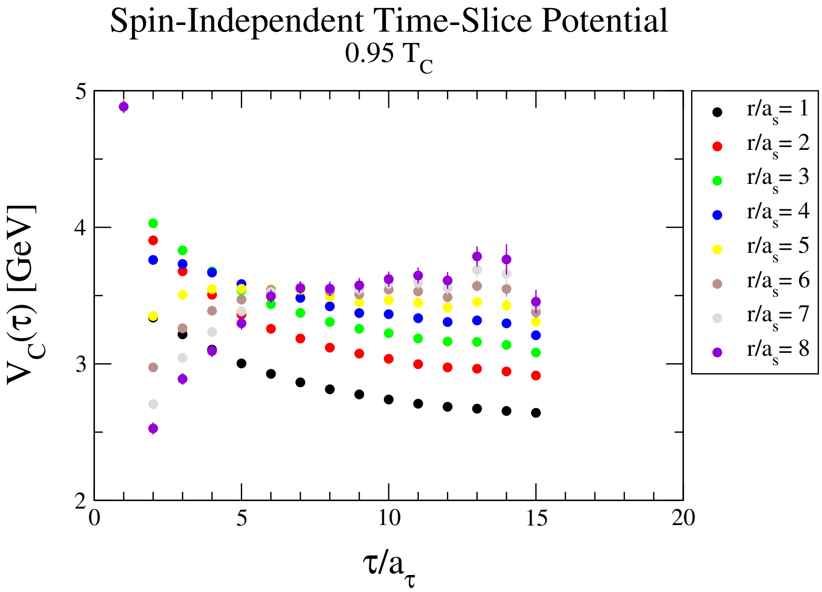
Spin-Independent Time-Slice Potential

0.76 T_C





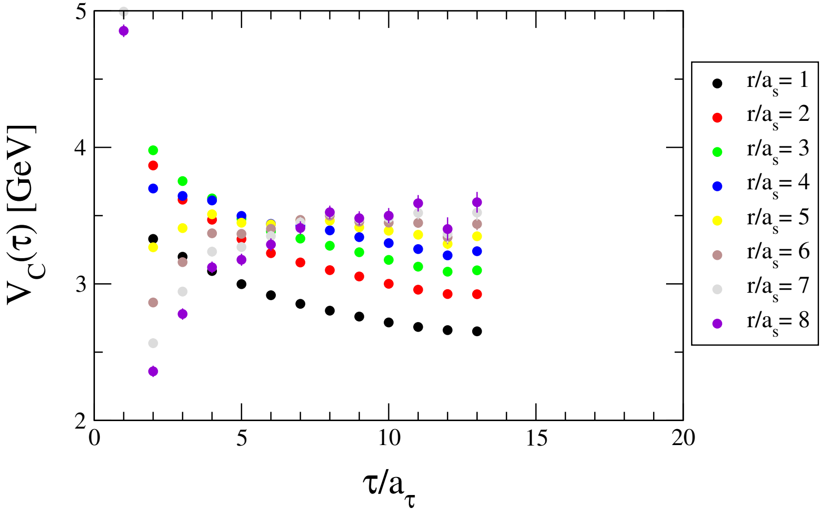
Spin-Independent Time-Slice Potential
0.84 T_C
V_C(τ) [GeV]
τ/a_τ
r/a_s = 1
r/a_s = 2
r/a_s = 3
r/a_s = 4
r/a_s = 5
r/a_s = 6
r/a_s = 7
r/a_s = 8
2
3
4
5
0
5
10
15
20



Spin-Independent Time-Slice Potential
0.95 T_C
$V_C(\tau)$ [GeV]
τ/a_τ
r/a_s = 1
r/a_s = 2
r/a_s = 3
r/a_s = 4
r/a_s = 5
r/a_s = 6
r/a_s = 7
r/a_s = 8

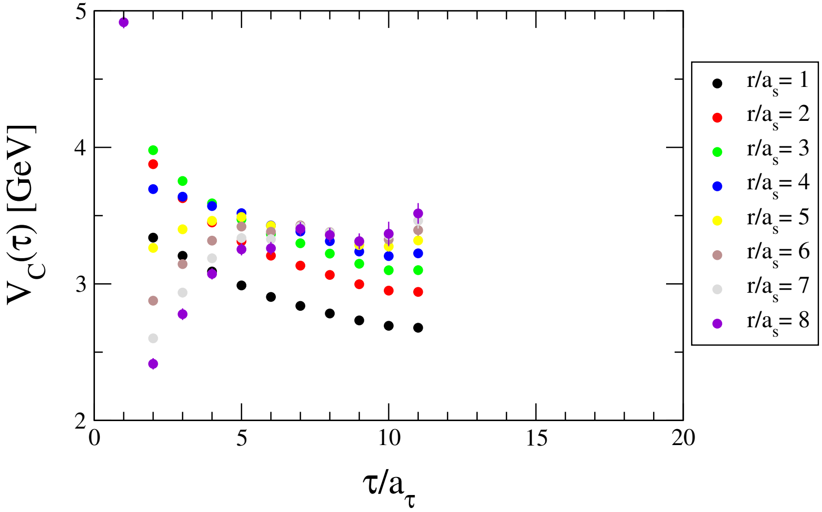
Spin-Independent Time-Slice Potential

1.09 T_C

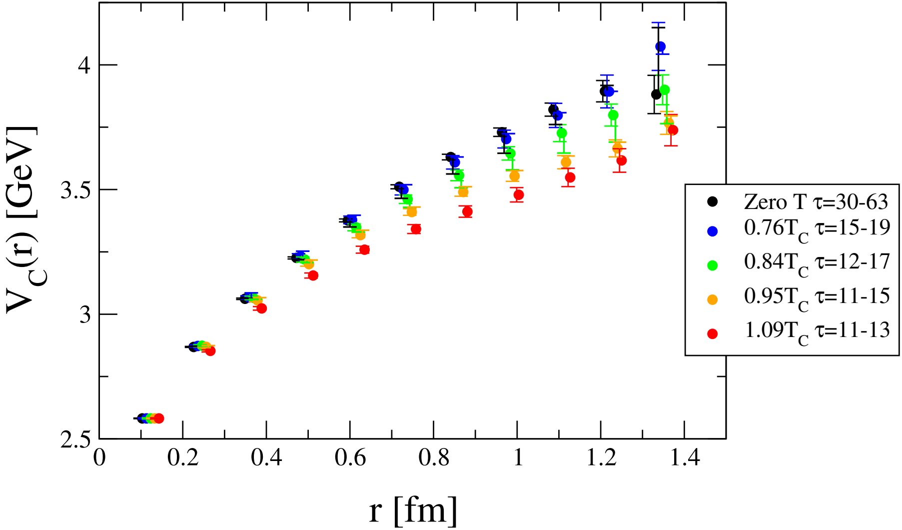


Spin-Independent Time-Slice Potential

$1.27\ T_C$



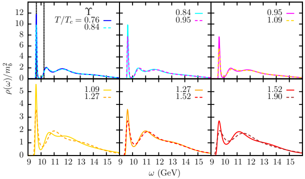
Spin-Independent Potential



Summary

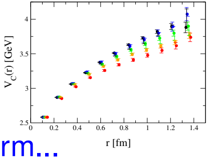
Bottomonium Spectral Functions

- S-waves don't melt until at least $\sim 2T_c$
- P-waves appear to melt at $\sim T_c$



Inter-quark potential in charmonium at finite temperature

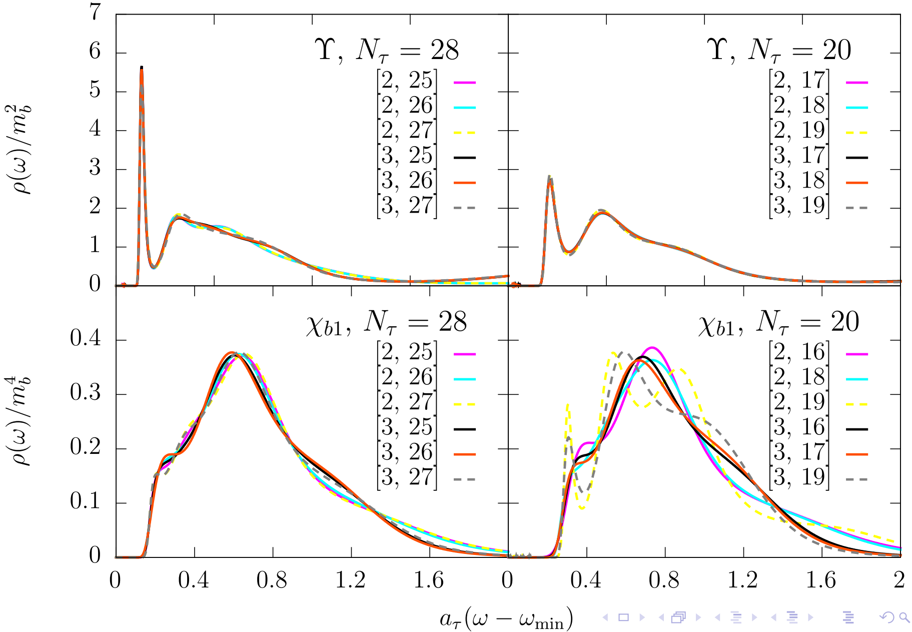
- Clear temperature dependent effect
- Relativistic quarks rather than static quarks
- No issue with Free Energy and the Entropy Term...
- Finite temperature rather than $T = 0$



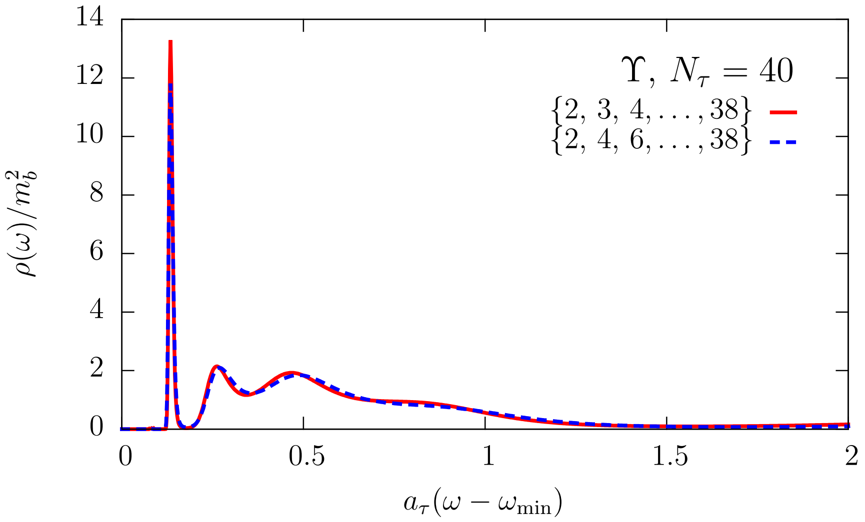
SLIDES TO HELP ME ANSWER TRICKY QUESTIONS

SLIDES TO HELP ME ANSWER DUMB QUESTIONS

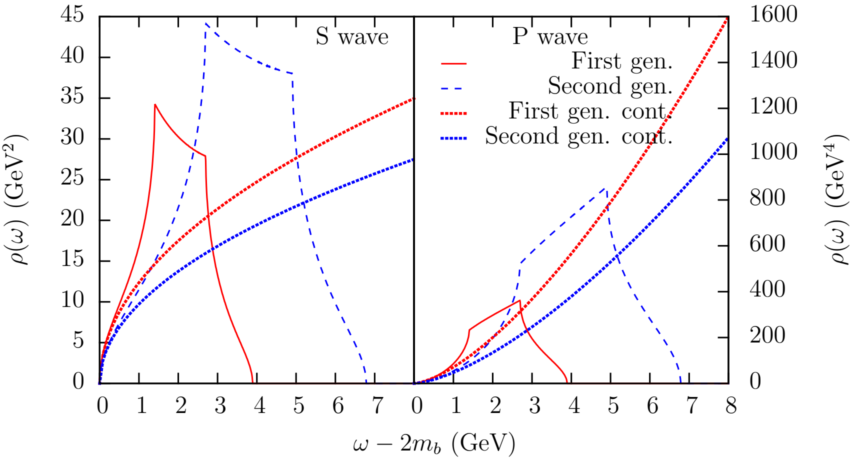
Stability of MEM w.r.t. τ window endpoints



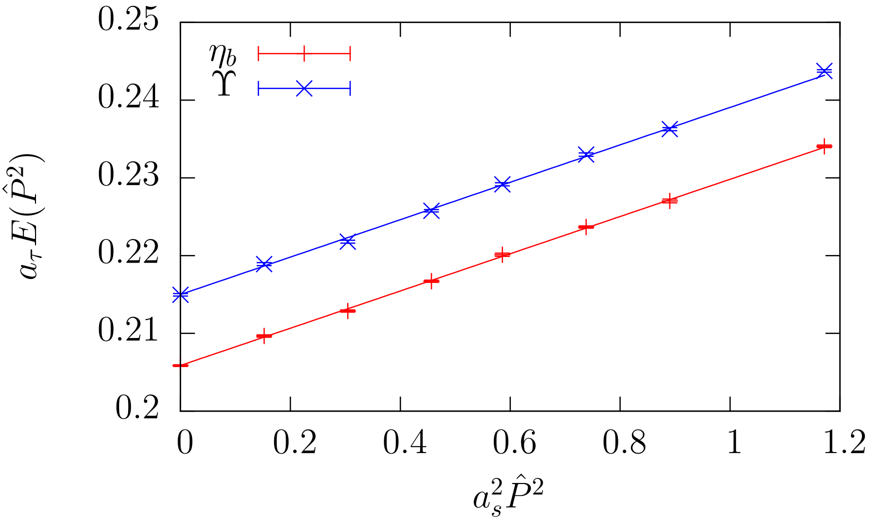
Stability of MEM w.r.t. coarse choice of τ window



Free lattice spectral functions



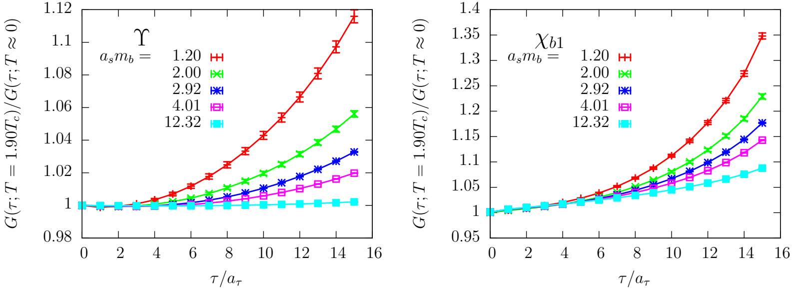
Zero temperature dispersion relations



Energy range used in MEM

	Υ	χ_{b1}
N_τ	$a_\tau\omega_{\rm min}$, $a_\tau\omega_{\rm max}$	$a_\tau\omega_{\rm min}$, $a_\tau\omega_{\rm max}$
128	0.12, 2.12	0.18, 2.18
40	0.08, 2.08	0.16, 2.16
36	0.08, 2.08	0.16, 2.16
32	0.08, 2.08	0.16, 2.16
28	0.08, 2.08	0.10, 2.10
24	0.08, 2.08	0.08, 2.08
20	0.00, 2.00	0.00, 2.00
16	-0.04, 1.96	-0.04, 1.96

Mass dependency of cor f'n



The HAL QCD time-dependent Method

Charm treated relativistically

Charmonium Operators: $J_\Gamma(x;\mathbf{r}) = q(x)\,\Gamma\,U(x,x+\mathbf{r})\,\overline{q}(x+\mathbf{r})$

Correlation F'ns: $C_\Gamma(\mathbf{r},\tau) = \sum_{\mathbf{x}} \langle J_\Gamma(\mathbf{x},\tau;\mathbf{r})\,J_\Gamma{}^\dagger(0;\mathbf{0})\rangle$

$$= \sum_j \frac{\psi_j(\mathbf{r})\psi_j^*(\mathbf{0})}{2E_j}\left(e^{-E_j\tau} + e^{-E_j(N_\tau-\tau)}\right)$$

Schrödinger Eq'n $\left[-\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + V_\Gamma(r)\right]\psi_j(r) = E_j\psi_j(r)$

Apply this to C_Γ :

$$\frac{\partial C_\Gamma(r,\tau)}{\partial\tau} = \sum_j \left(\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} - V_\Gamma(r)\right)\frac{\psi_j^*(0)\psi_j(r)}{2E_j}e^{-E_j\tau}$$

$$= \left(\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} - V_\Gamma(r)\right)C_\Gamma(r,\tau)$$

This gives an algebraic equation for $V_\Gamma(r,\text{“}\tau\text{”})$