

# Probing Quarkonium Production Mechanisms with Jet Substructure

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Review of Quarkonium Production Theory

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Heavy Quarkonium Fragmenting Jet Functions

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New Tests of NRQCD Using Jet Observables

# Non-Relativistic QCD (NRQCD) Factorization Formalism

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(Bodwin, Braaten, Lepage)

$$\sigma(gg \rightarrow J/\psi + X) = \sum_n \sigma(gg \rightarrow c\bar{c}(n) + X) \langle \mathcal{O}^{J/\psi}(n) \rangle$$

$n = {}^{2S+1}L_J^{(1,8)}$

double expansion in  $\alpha_s, v$

## NRQCD long-distance matrix element (LDME)

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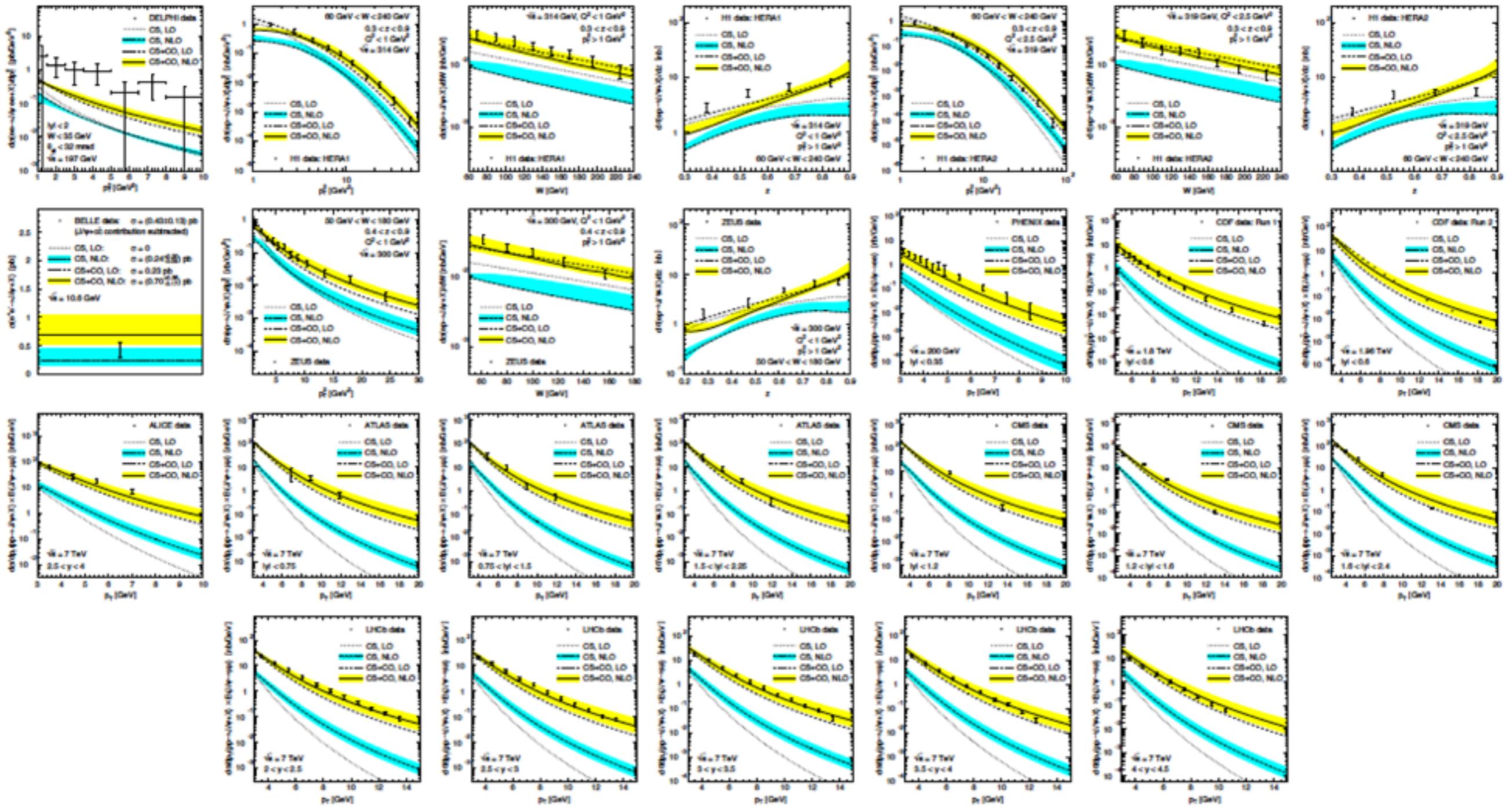
$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]}) \rangle \sim v^3$$

CSM - lowest order in  $v$

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]}) \rangle, \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]}) \rangle, \langle \mathcal{O}^{J/\psi}({}^3P_J^{[8]}) \rangle \sim v^7$$

color-octet mechanisms

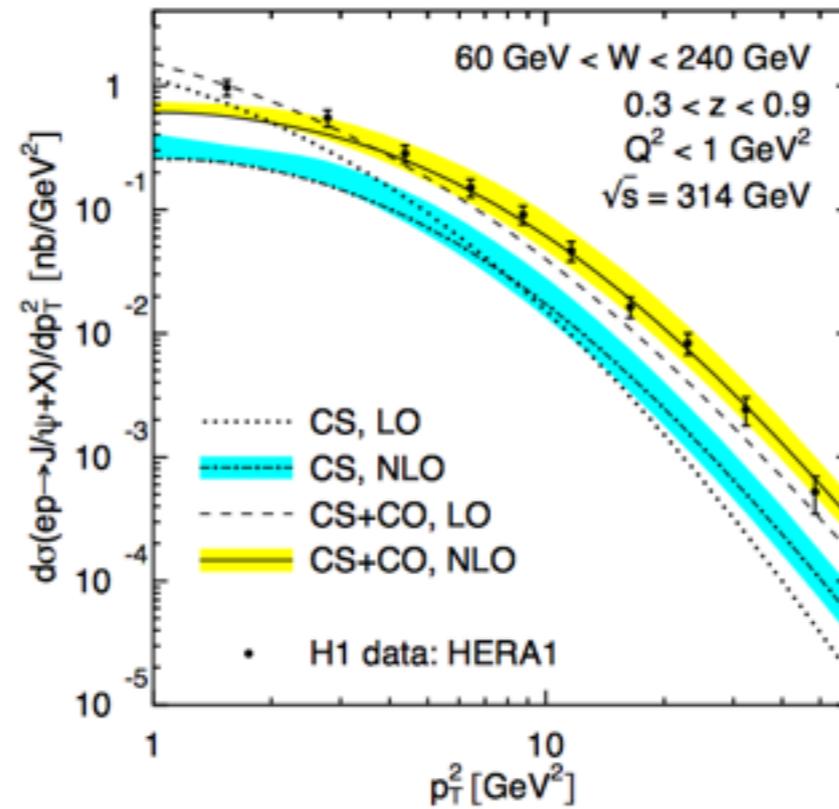
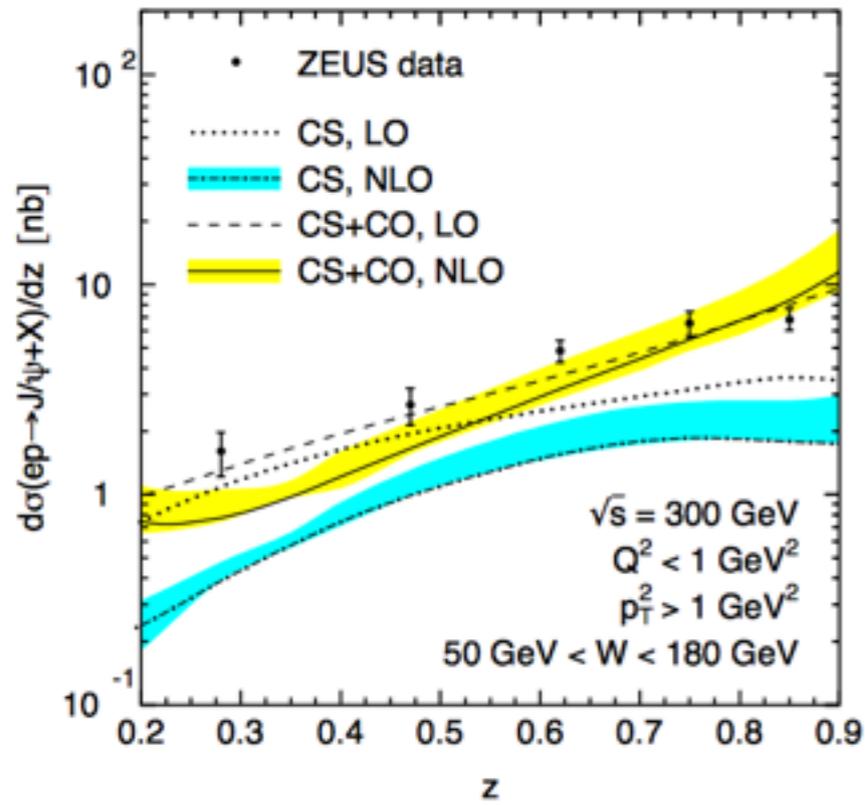
# Global Fits with NLO CSM + COM



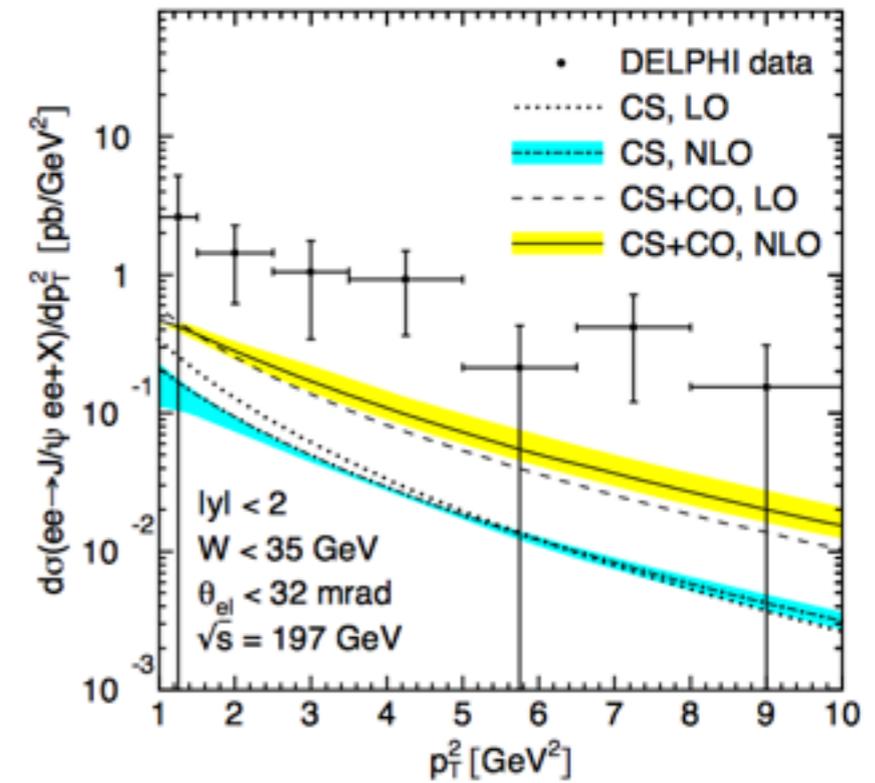
$$e^+e^-, \gamma\gamma, \gamma p, p\bar{p}, pp \rightarrow J/\psi + X$$

fit to 194 data points, 26 data sets,  
Butenschoen and Kniehl, PRD 84 (2011) 051501

# NLO: CSM + COM Required to Fit Data



$$ep \rightarrow J/\psi + X$$



$$\gamma^* \gamma^* \rightarrow J/\psi + X$$

# Status of NRQCD approach to $J/\psi$ Production

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NLO: COM + CSM required for most processes

**extracted LDME satisfy NRQCD v-scaling**

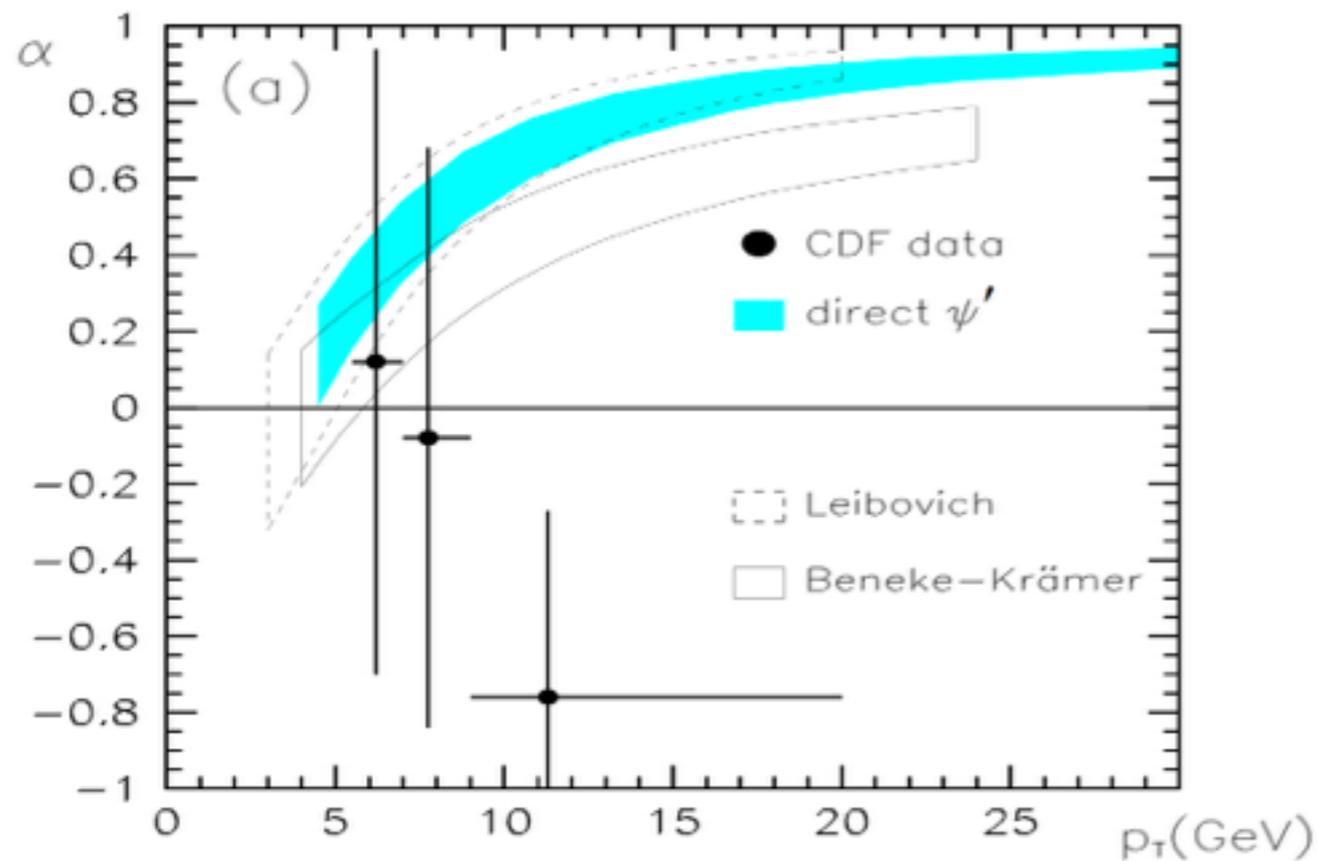
$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle = 1.32 \text{ GeV}^3 :$$

$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle$	$(4.97 \pm 0.44) \times 10^{-2} \text{ GeV}^3$
$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$	$(2.24 \pm 0.59) \times 10^{-3} \text{ GeV}^3$
$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle$	$(-1.61 \pm 0.20) \times 10^{-2} \text{ GeV}^5$

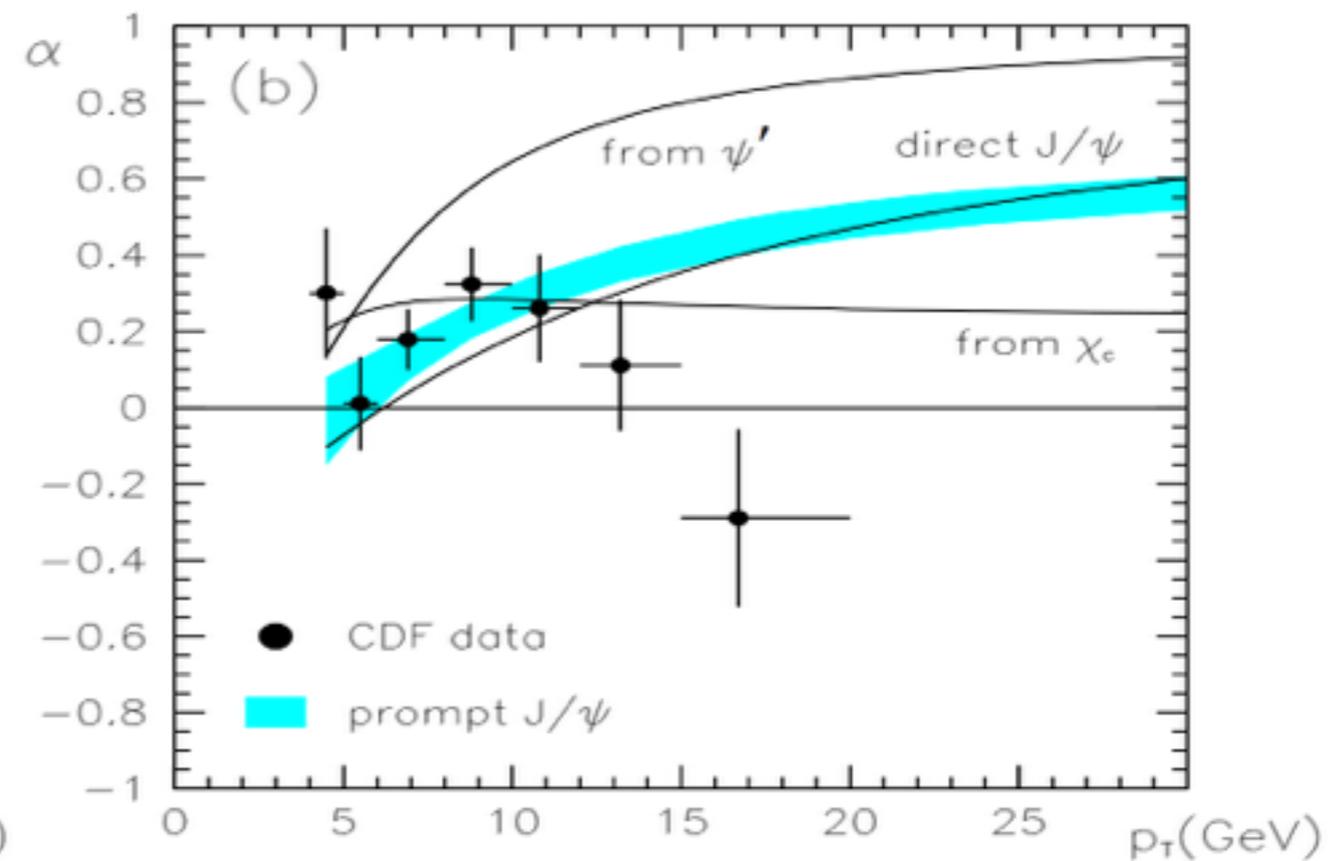
$$\chi_{\text{d.o.f.}}^2 = 857/194 = 4.42$$

# Polarization Puzzle

$^3S_1^{[8]}$  fragmentation at large  $p_T$  predicts transversely polarized  $J/\psi$ ,  $\psi'$

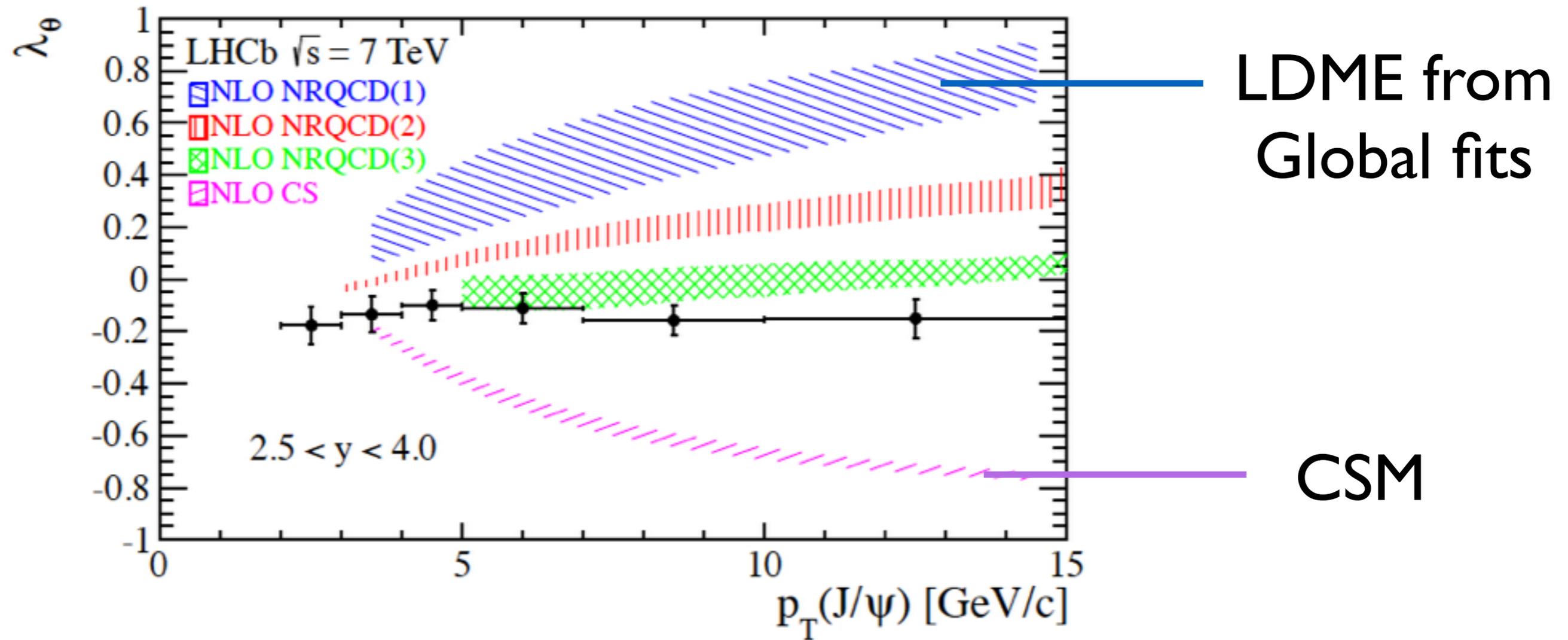


$\psi'$

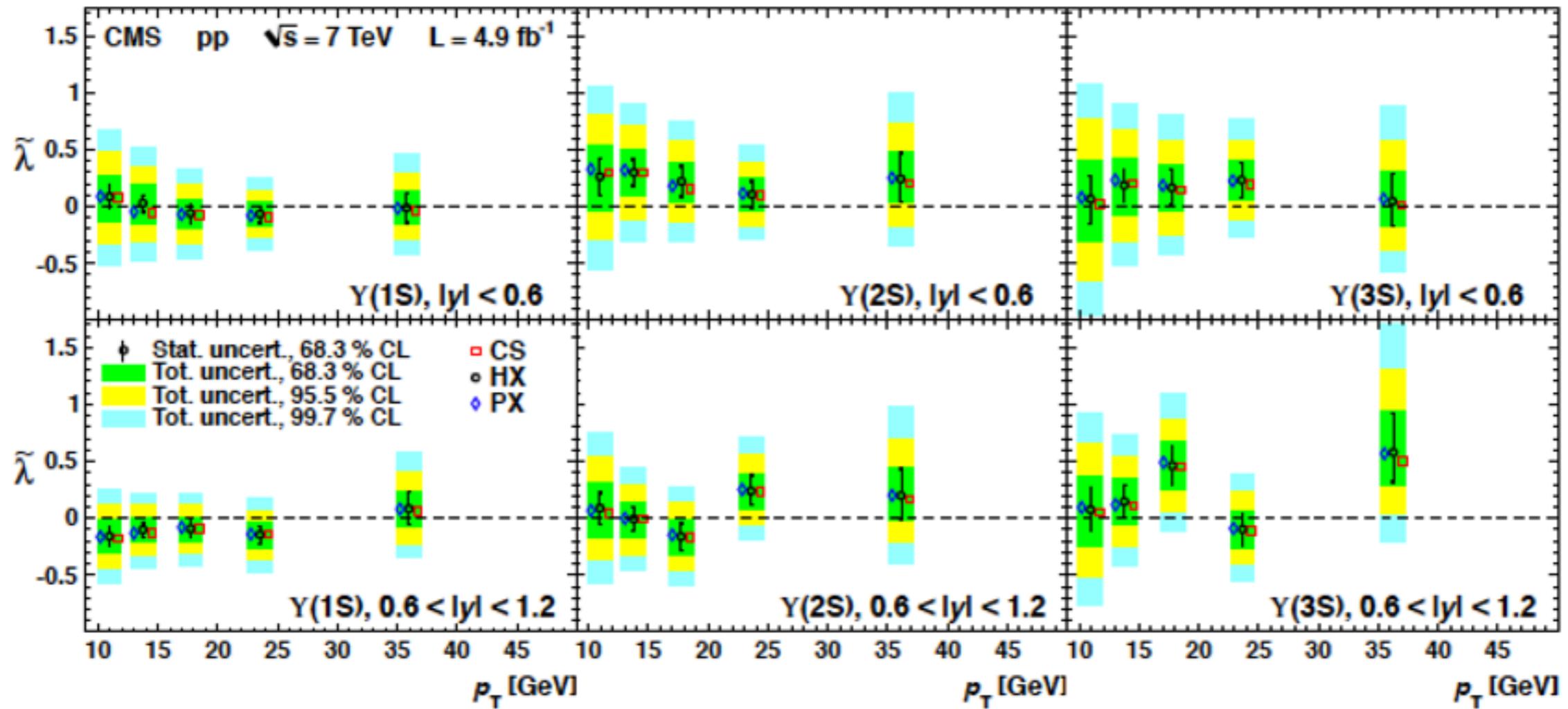


$J/\psi$

# Polarization of $J/\psi$ at LHCb



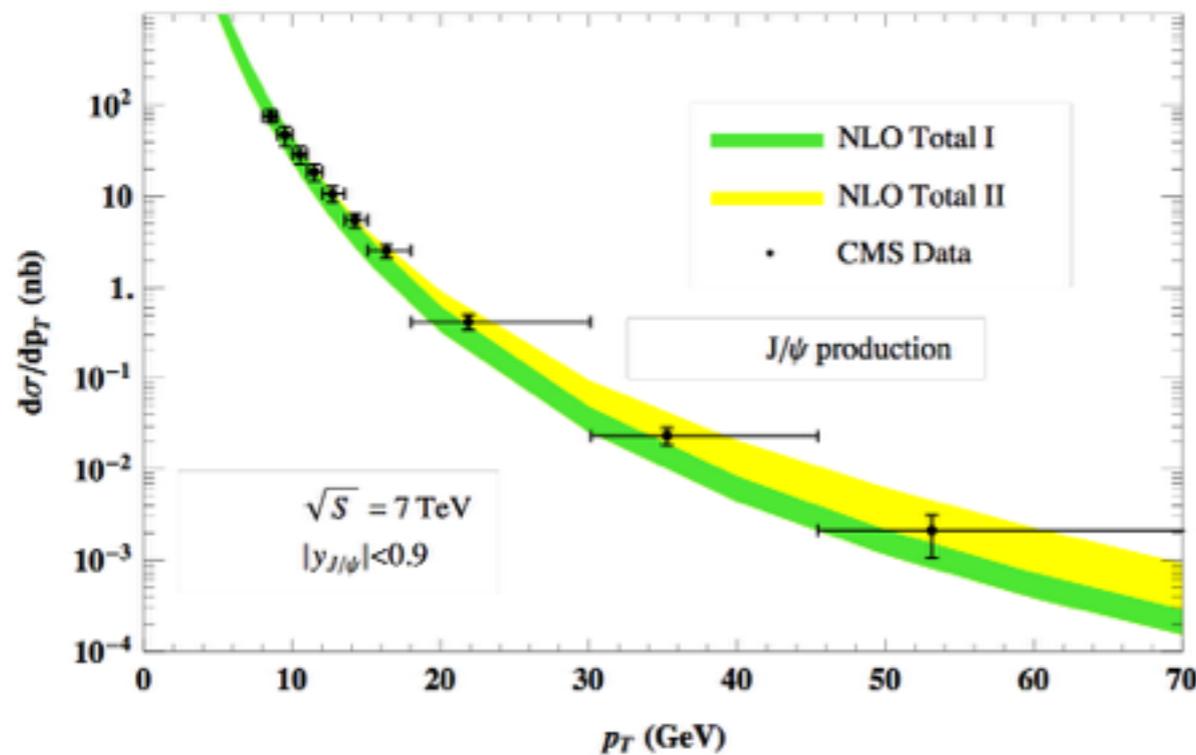
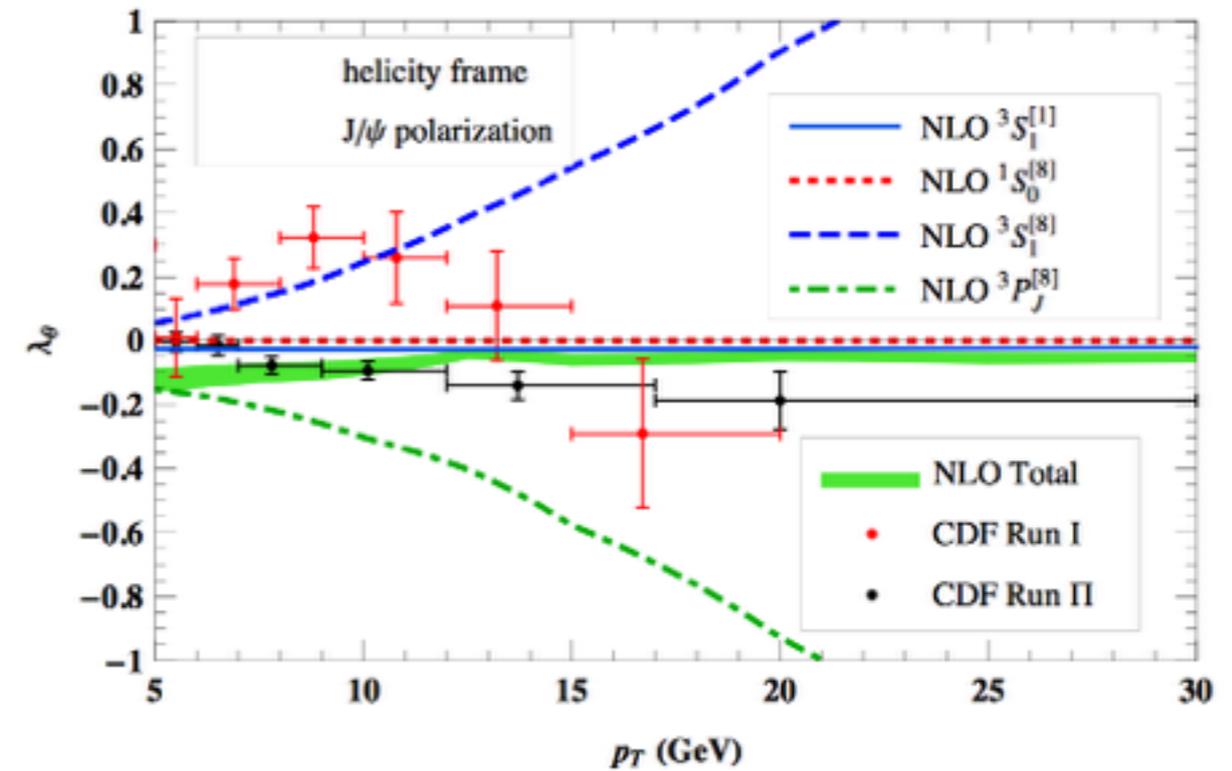
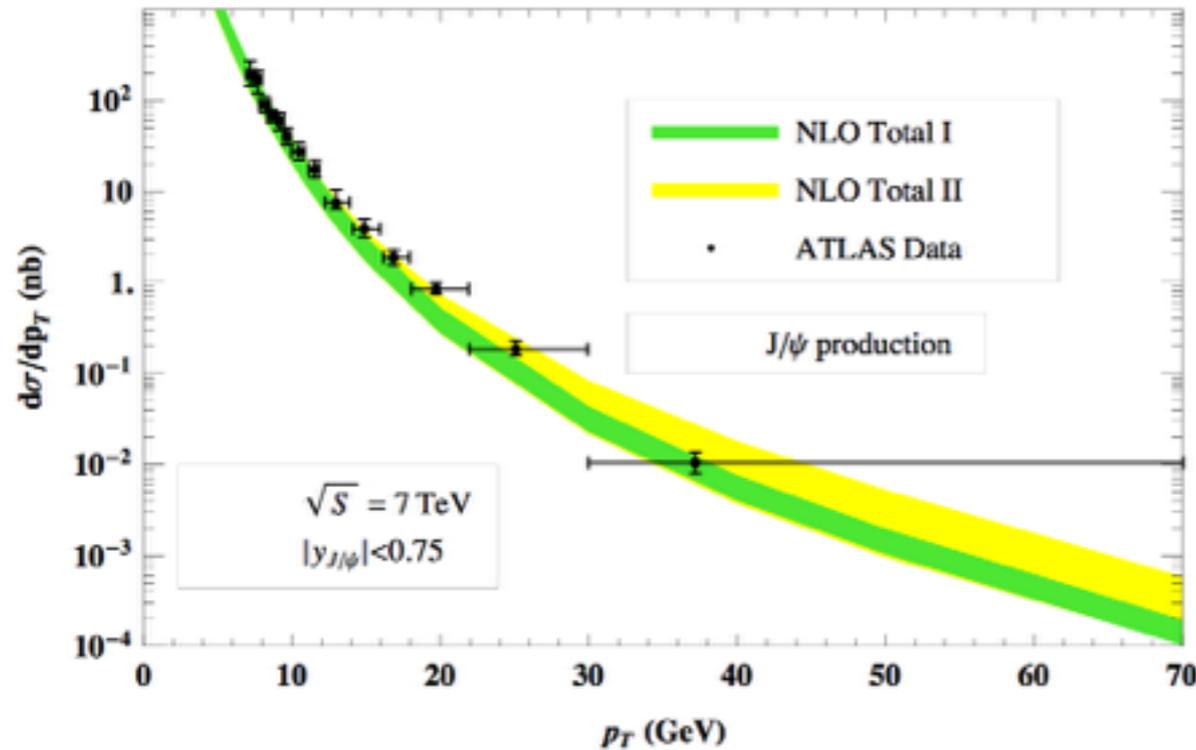
# Polarization of $\Upsilon(nS)$ at CMS



# Recent Attempts to Resolve J/ψ Polarization Puzzle

simultaneous NLO fit to CMS, ATLAS high  $p_T$  production, polarization

Chao, et. al. PRL 108, 242004 (2012)



$\langle \mathcal{O}(^3S_1^{[1]}) \rangle$ GeV <sup>3</sup>	$\langle \mathcal{O}(^1S_0^{[8]}) \rangle$ 10 <sup>-2</sup> GeV <sup>3</sup>	$\langle \mathcal{O}(^3S_1^{[8]}) \rangle$ 10 <sup>-2</sup> GeV <sup>3</sup>	$\langle \mathcal{O}(^3P_0^{[8]}) \rangle / m_c^2$ 10 <sup>-2</sup> GeV <sup>3</sup>
1.16	8.9 ± 0.98	0.30 ± 0.12	0.56 ± 0.21
1.16	0	1.4	2.4
1.16	11	0	0

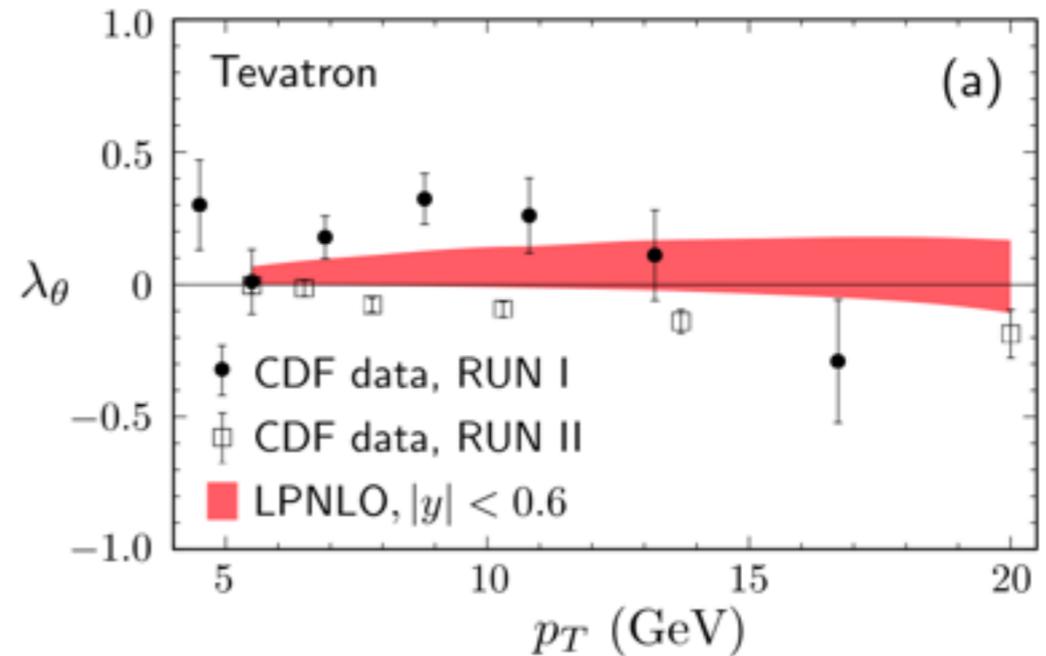
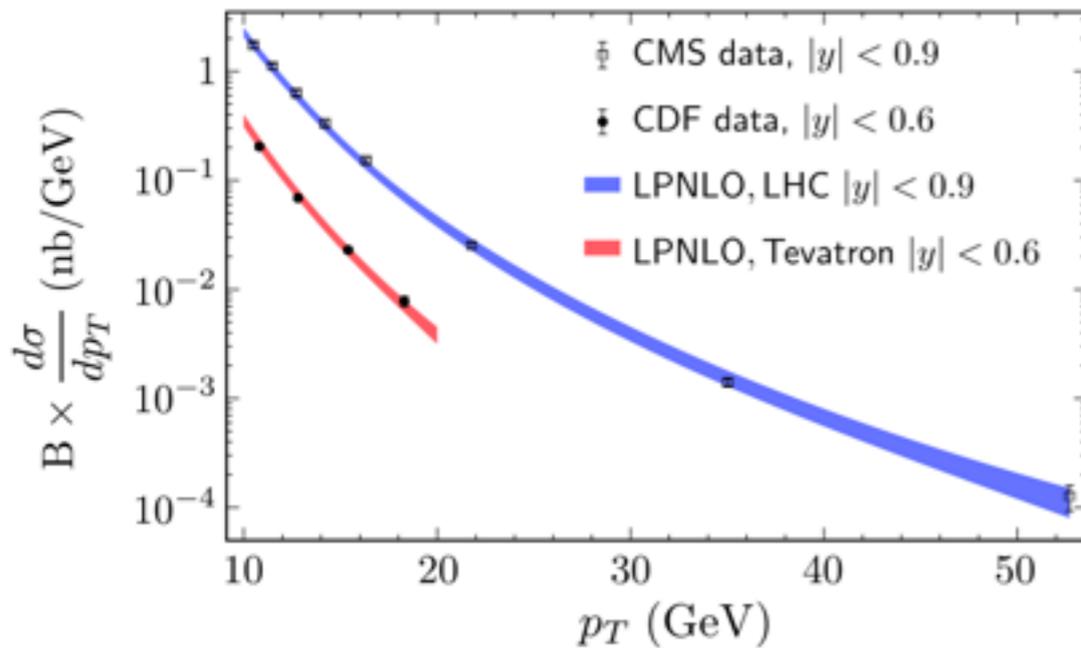
# Recent Attempts to Resolve $J/\psi$ Polarization Puzzle

i) large  $p_t$  production at CDF

Bodwin, et. al., PRL 113, 022001 (2014)

ii) resum logs of  $p_t/m_c$  using AP evolution

iii) fit COME to  $p_t$  spectrum, predict basically no polarization



**Extracted COME inconsistent with global fits**

$$\langle \mathcal{O}^{J/\psi} (^1S_0^{(8)}) \rangle = 0.099 \pm 0.022 \text{ GeV}^3$$

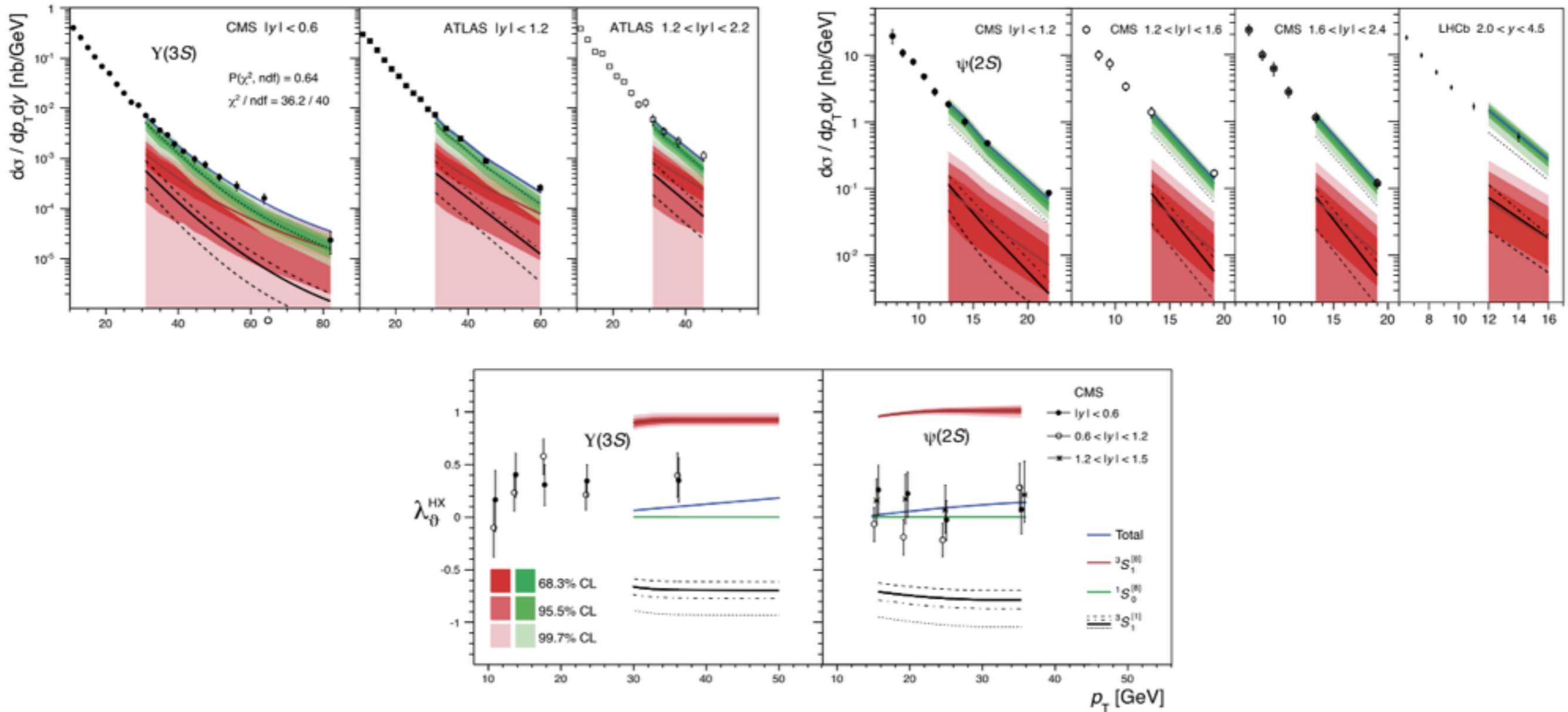
$$\langle \mathcal{O}^{J/\psi} (^3S_1^{(8)}) \rangle = 0.011 \pm 0.010 \text{ GeV}^3$$

$$\langle \mathcal{O}^{J/\psi} (^3P_0^{(8)}) \rangle = 0.011 \pm 0.010 \text{ GeV}^5$$

# Recent Attempts to Resolve J/ψ Polarization Puzzle

Faccioli, et. al. PLB736 (2014) 98

Lourenco, et. al., NPA, in press



argue for  $^1S_0^{(8)}$  dominance in both  $\psi(2S)$  &  $Y(3S)$  production

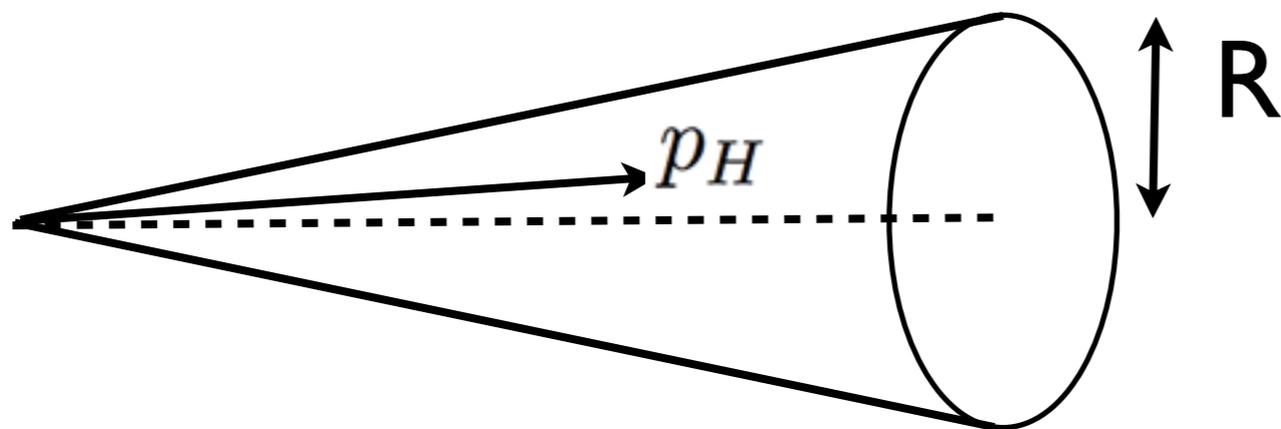
# Fragmenting Jet Functions

Procura, Stewart, arXiv:0911.4980

Jain, Procura, Waalewijn, arXiv:1101.4953

Procura, Waalewijn, arXiv:1111.6605

jets with identified hadrons



Jet Energy:  $E$

$$p_H^+ = z p_{\text{jet}}^+$$

cross sections determined by **fragmenting jet function (FJF)**:

$$\mathcal{G}_g^h(E, R, \mu, z)$$

inclusive hadron production: fragmentation functions

$$\frac{1}{\sigma_0} \frac{d\sigma^h}{dz} (e^+e^- \rightarrow h X) = \sum_i \int_z^1 \frac{dx}{x} C_i(E_{\text{cm}}, x, \mu) D_i^h(z/x, \mu)$$

jet cross sections: jet functions

$$\frac{d\sigma^h}{dz}(E, R) = \int d\Phi_N \text{tr}[H_N S_N] \prod_{\ell} J_{\ell}$$

$$\mathcal{G}_g^h(E, R, \mu, z) \longrightarrow D_i^h(z/x, \mu), J_{\ell}$$

relationship to jet function:

$$\sum_h \int_0^1 dz z D_j^h(z, \mu) = 1$$

$$\longrightarrow J_i(E, R, z, \mu) = \frac{1}{2} \sum_h \int \frac{dz}{(2\pi)^3} z \mathcal{G}_i^h(E, R, z, \mu)$$

cross section for jet w/ identified hadron from jet cross section

$$\frac{d\sigma^h}{dz}(E, R) = \int d\Phi_N \text{tr}[H_N S_N] \prod_{\ell} J_{\ell} J_i(E, R, z, \mu)$$

$$\longrightarrow \frac{d\sigma^h}{dz}(E, R) = \int d\Phi_N \text{tr}[H_N S_N] \mathcal{G}_i^h(E, R, z, \mu) \prod_{\ell} J_{\ell} .$$

relationship to fragmentation functions

$$\mathcal{G}_i^h(E, R, z, \mu) = \sum_i \int_z^1 \frac{dz'}{z'} \mathcal{J}_{ij}(E, R, z', \mu) D_j^h\left(\frac{z}{z'}, \mu\right) \left[1 + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{4E^2 \tan^2(R/2)}\right)\right]$$

**matching coefficients calculable in perturbation theory**

$$\frac{\mathcal{J}_{gg}(E, R, z, \mu)}{2(2\pi)^3} = \delta(1-z) + \frac{\alpha_s(\mu)C_A}{\pi} \left[ \left(L^2 - \frac{\pi^2}{24}\right) \delta(1-z) + \hat{P}_{gg}(z)L + \hat{\mathcal{J}}_{gg}(z) \right]$$

$$\hat{\mathcal{J}}_{gg}(z) = \begin{cases} \hat{P}_{gg}(z) \ln z & z \leq 1/2 \\ \frac{2(1-z+z^2)^2}{z} \left(\frac{\ln(1-z)}{1-z}\right)_+ & z \geq 1/2. \end{cases}$$

$$L = \ln[2E \tan(R/2)/\mu].$$

scale for  $\mathcal{J}_{ij}(E, R, z, \mu)$

sum rule for matching coefficients

$$\sum_j \int_0^1 dz z \mathcal{J}_{ij}(R, z, \mu) = 2(2\pi)^3 J_i(R, \mu)$$

# NRQCD fragmentation functions

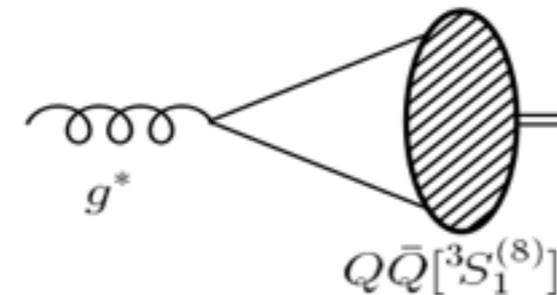
Braaten, Yuan, hep-ph/9302307

Braaten, Chen, hep-ph/9604237

Braaten, Fleming, hep-ph/9411365

Perturbatively calculable at the scale  $2m_c$

$$D_g^{\psi(8)}(z, 2m_c) = \frac{\pi\alpha_s(2m_c)}{3M_\psi^3} \langle O^\psi(^3S_1^{(8)}) \rangle \delta(1-z)$$



$$D_g^{\psi(1)}(z, 2m_c) = \frac{5\alpha_s^3(2m_c)}{648\pi^2} \frac{\langle O^\psi(^3S_1^{(1)}) \rangle}{M_\psi^3} \int_0^z dr \int_{(r+z^2)/2z}^{(1+r)/2} dy \frac{1}{(1-y)^2(y-r)^2(y^2-r)^2} \sum_{i=0}^2 z^i \left( f_i(r, y) + g_i(r, y) \frac{1+r-2y}{2(y-r)\sqrt{y^2-r}} \ln \frac{y-r+\sqrt{y^2-r}}{y-r-\sqrt{y^2-r}} \right),$$

**Altarelli-Parisi evolution:  $2m_c$  to  $2E \tan(R/2)$**

# FJF in terms of fragmentation function

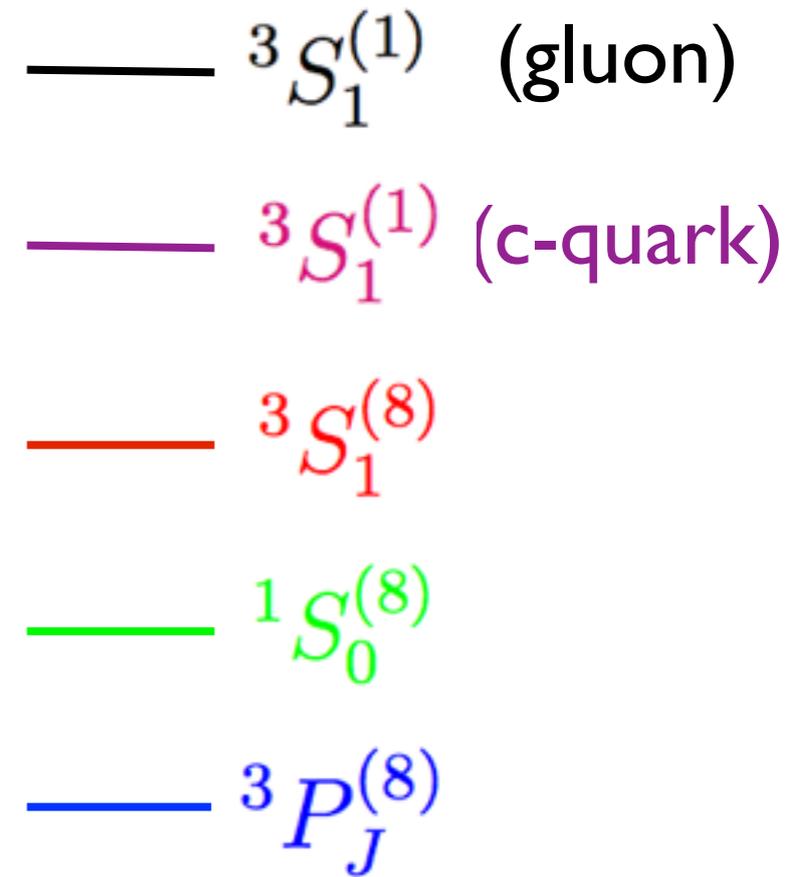
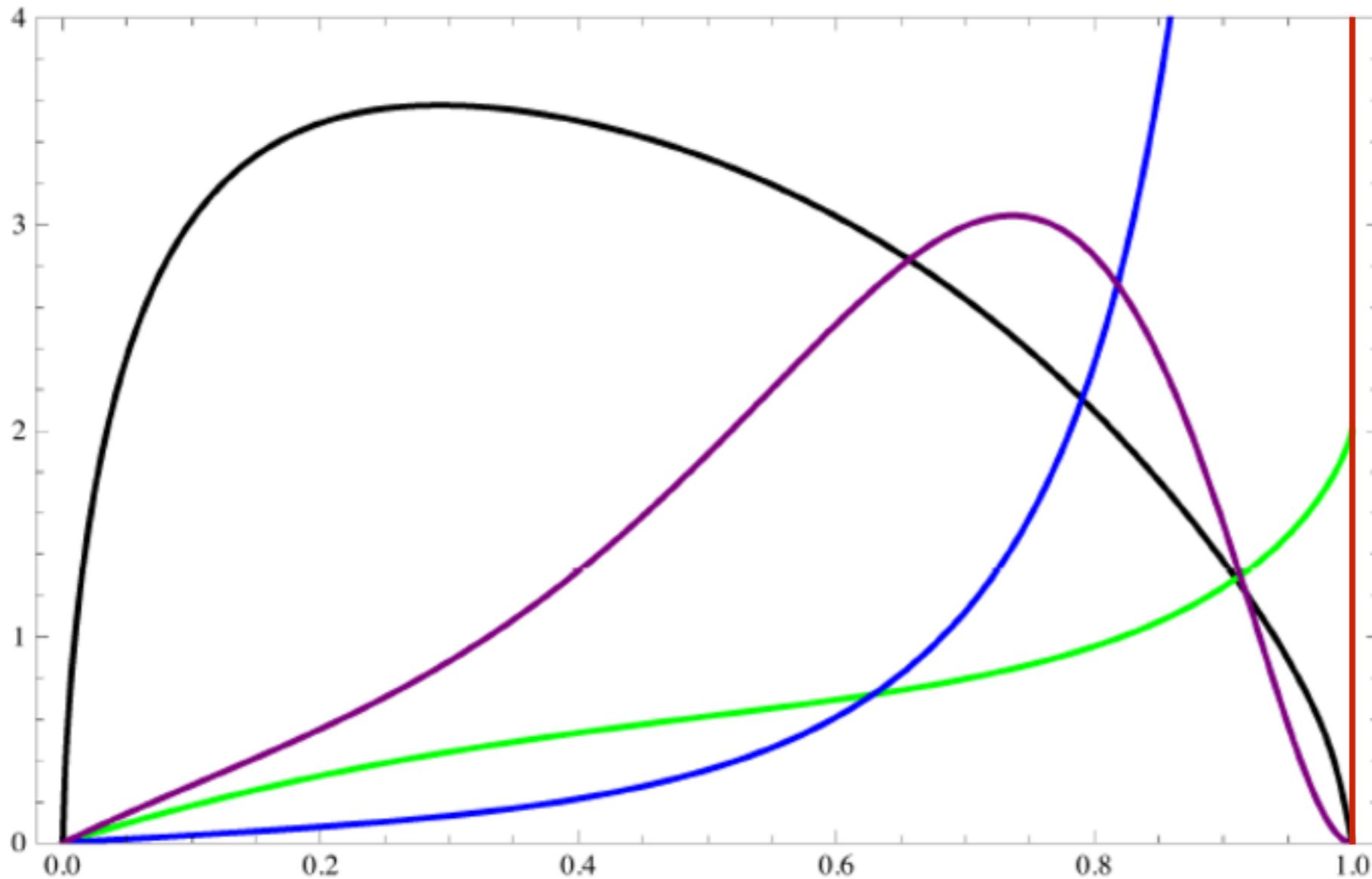
$$\begin{aligned}
 \mathcal{G}_g^\psi(E, R, z, \mu) = & D_{g \rightarrow \psi}(z, \mu) \left( 1 + \frac{C_A \alpha_s}{\pi} \left( L_{1-z}^2 - \frac{\pi^2}{24} \right) \right) \\
 & + \frac{C_A \alpha_s}{\pi} \left[ \int_z^1 \frac{dy}{y} \tilde{P}_{gg}(y) L_{1-y} D_{g \rightarrow \psi} \left( \frac{z}{y}, \mu \right) \right. \\
 & + 2 \int_z^1 dy \frac{D_{g \rightarrow \psi}(z/y, \mu) - D_{g \rightarrow \psi}(z, \mu)}{1-y} L_{1-y} \\
 & \left. + \theta \left( \frac{1}{2} - z \right) \int_z^{1/2} \frac{dy}{y} \hat{P}_{gg}(y) \ln \left( \frac{y}{1-y} \right) D_{g \rightarrow \psi} \left( \frac{z}{y}, \mu \right) \right]
 \end{aligned}$$

$$L_{1-z} = \ln \left( \frac{2E \tan(R/2)(1-z)}{\mu} \right)$$

**For large E, FJF  $\sim$  NRQCD frag. function (at scale  $2E \tan(R/2)$ )**

$$\mathcal{G}_g^h(E, R, \mu = 2E \tan(R/2), z) \rightarrow D_g^\psi(z, 2E \tan(R/2)) + O(\alpha_s)$$

# NRQCD FF's (at scale $2m_c$ )

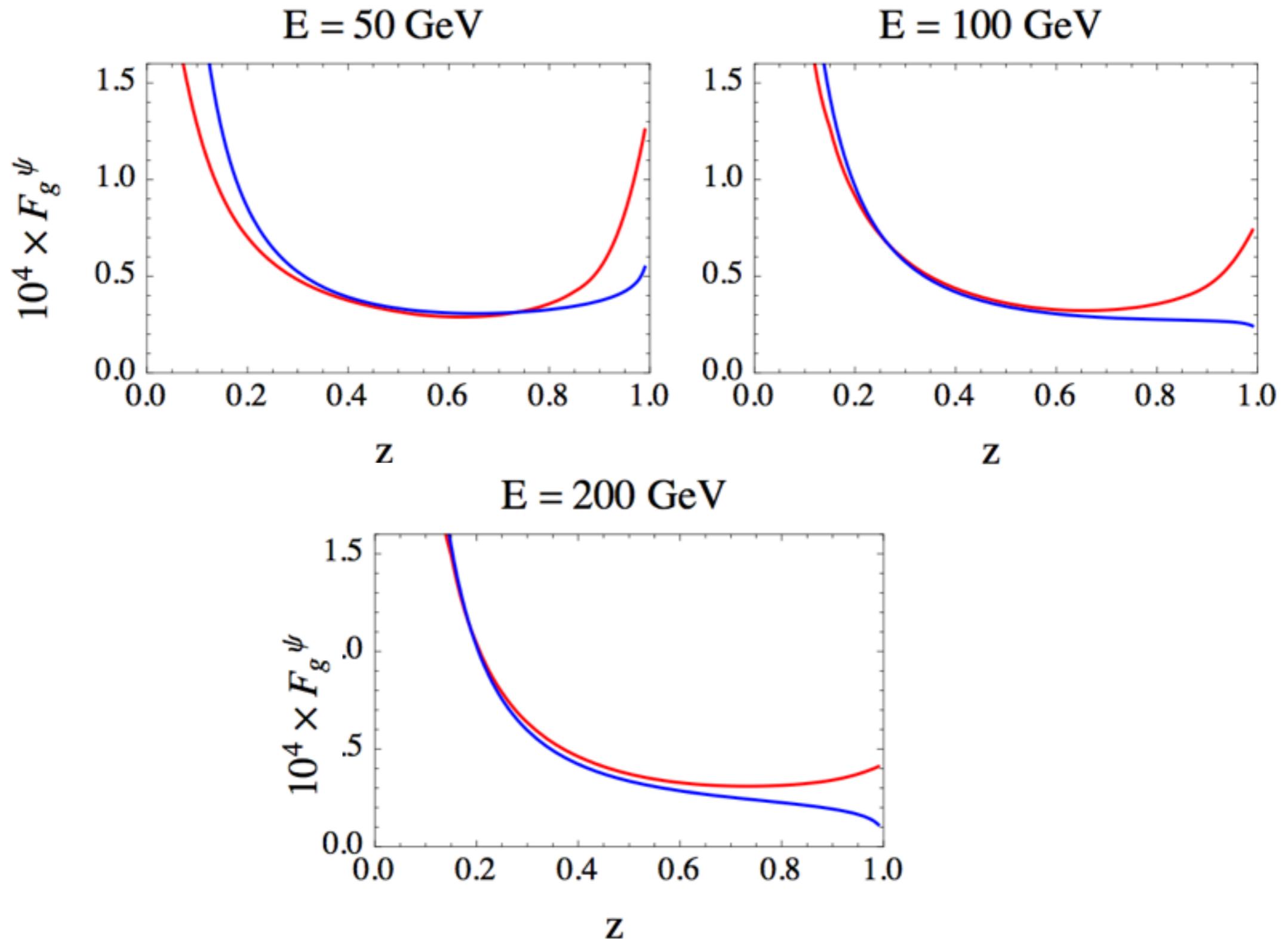


(normalization arbitrary)

Evolution to  $2E \tan(R/2)$  will soften discrepancies

# Color-Octet $^3S_1$ fragmentation function, FJF

M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, arXiv:1406.2295



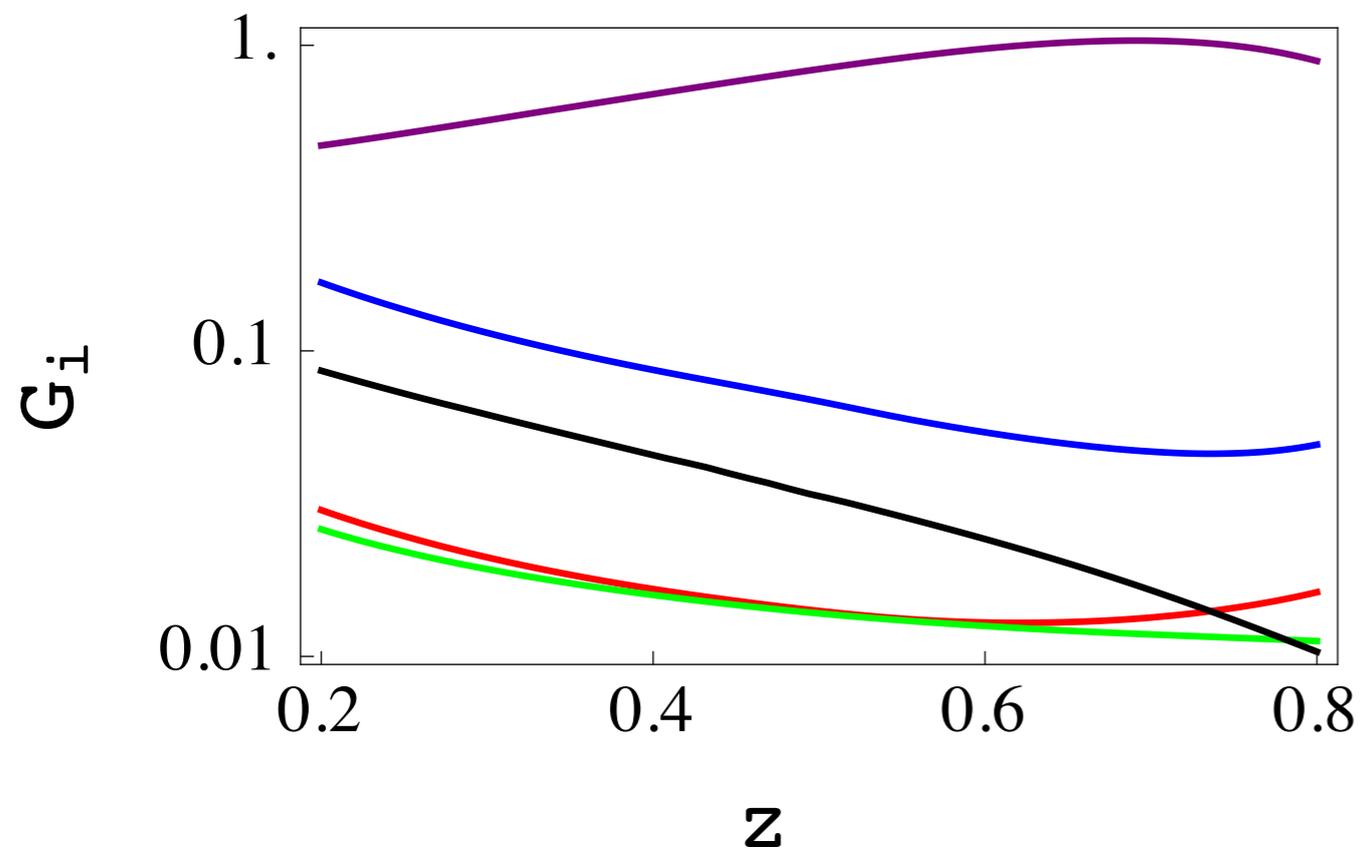
— fragmentation function

— fragmenting jet function

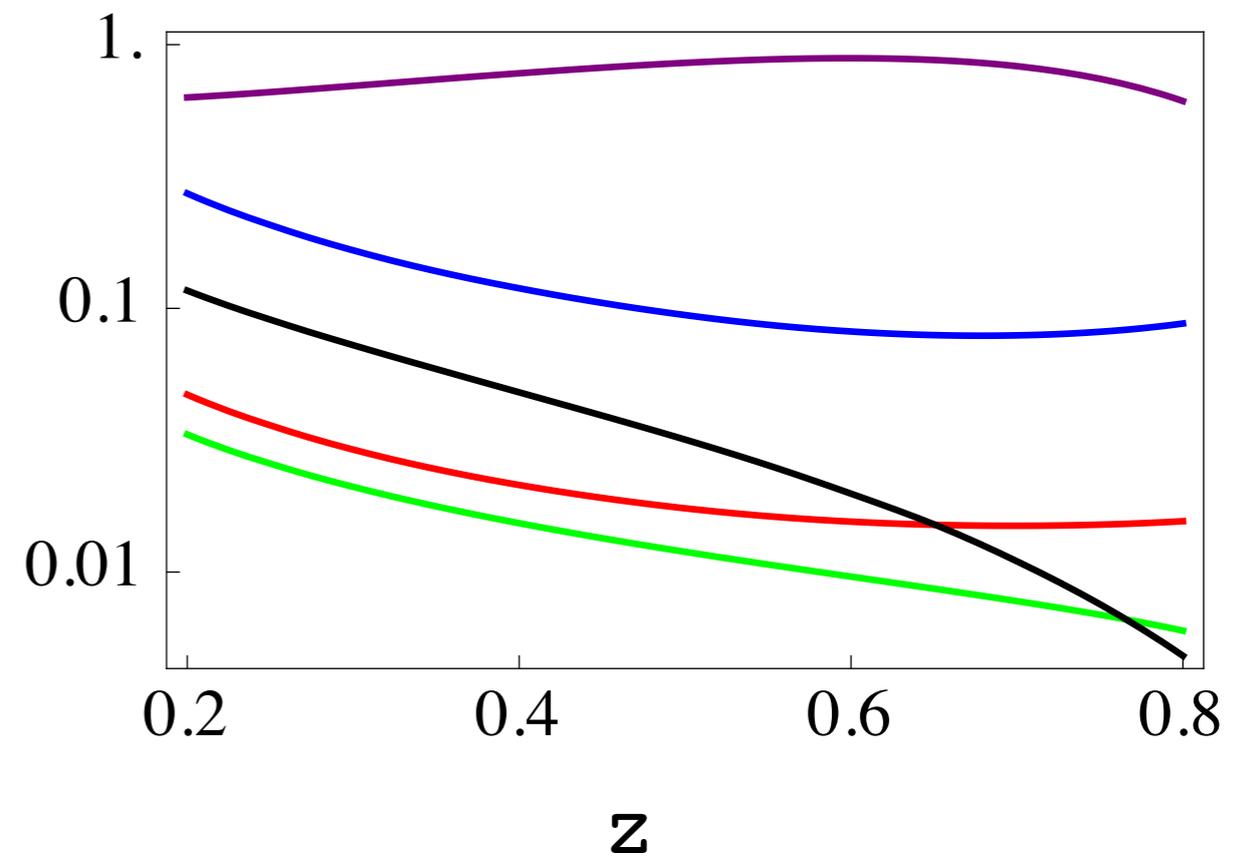
# FJF's at Fixed Energy vs. $z$

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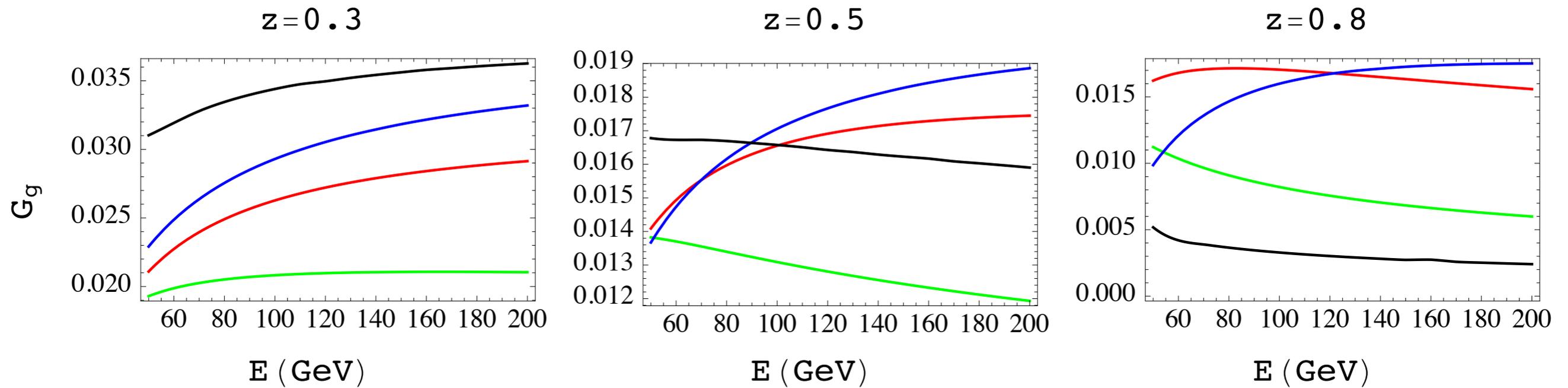
$E = 50 \text{ GeV}$



$E = 200 \text{ GeV}$



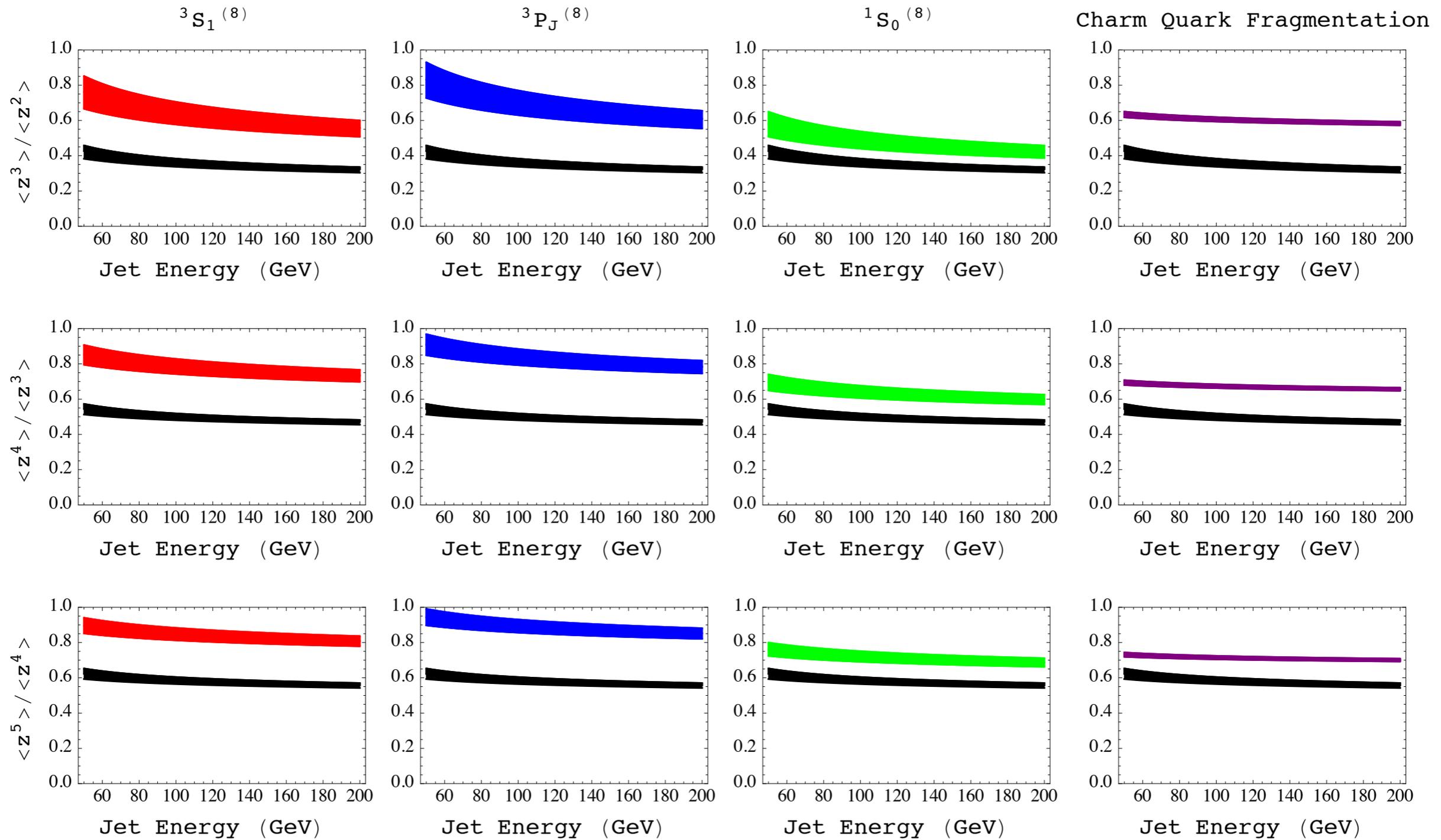
# FJF's at Fixed z vs. Energy



$^1S_0^{(8)}$  dominance predicts negative slope for z vs. E if  $z > 0.5$

# Ratios of Moments

$$E \tan(R/2) < \mu < 4E \tan(R/2)$$

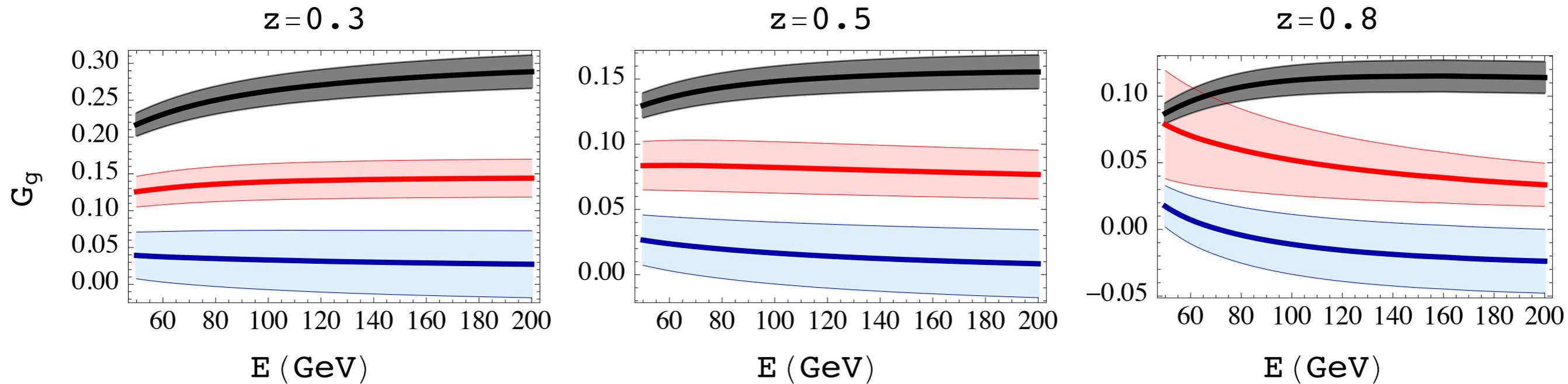


## Ratios of Moments

$$\frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{3P_J^{(8)}} \approx \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{3S_1^{(8)}} > \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{1S_0^{(8)}} \approx \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{\text{c-quark}} > \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{3S_1^{(1)}}$$

# Gluon FJF for different extractions of LDME

fix  $z$ , vary energy



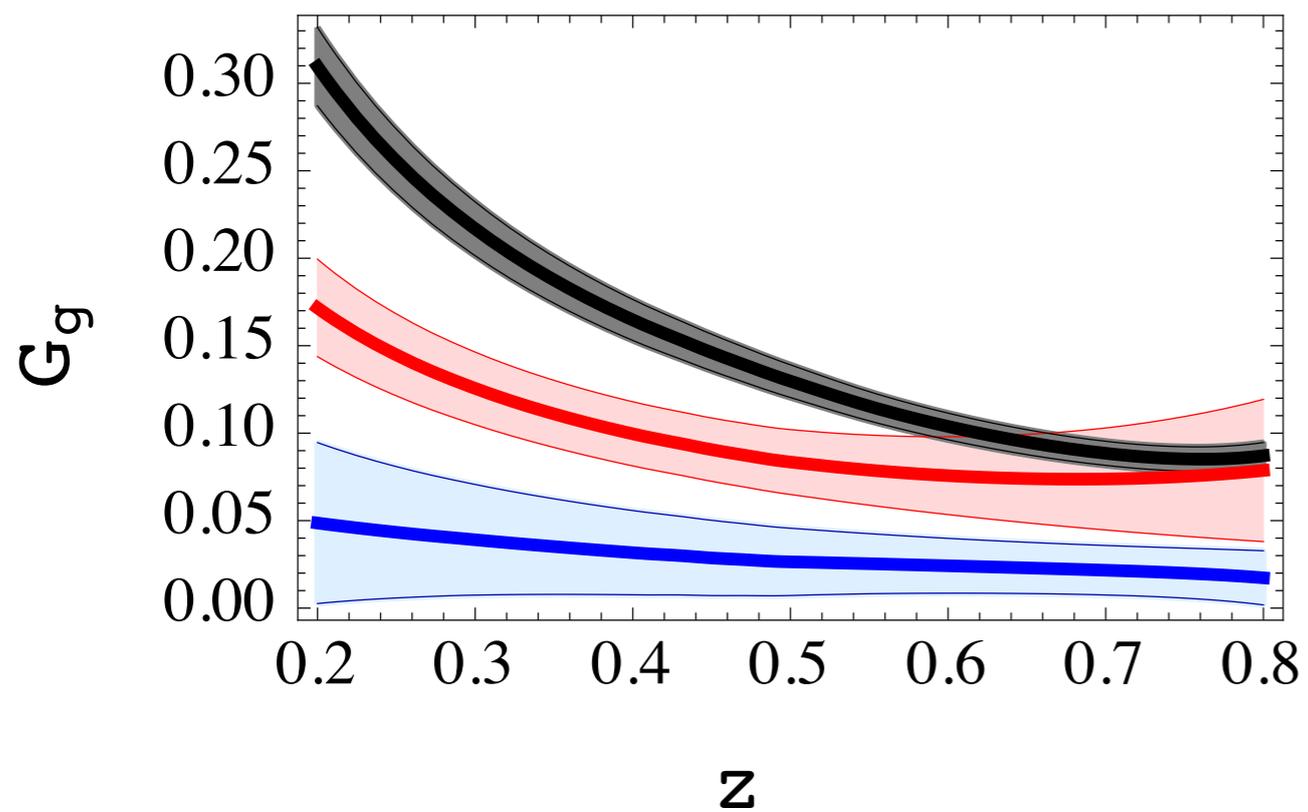
- Butenschoen and Kniehl, PRD 84 (2011) 051501, arXiv:1105.0822
- Bodwin, et. al. arXiv:1403.3612
- Chao, et. al. PRL 108, 242004 (2012)

# Gluon FJF for different extractions of LDME

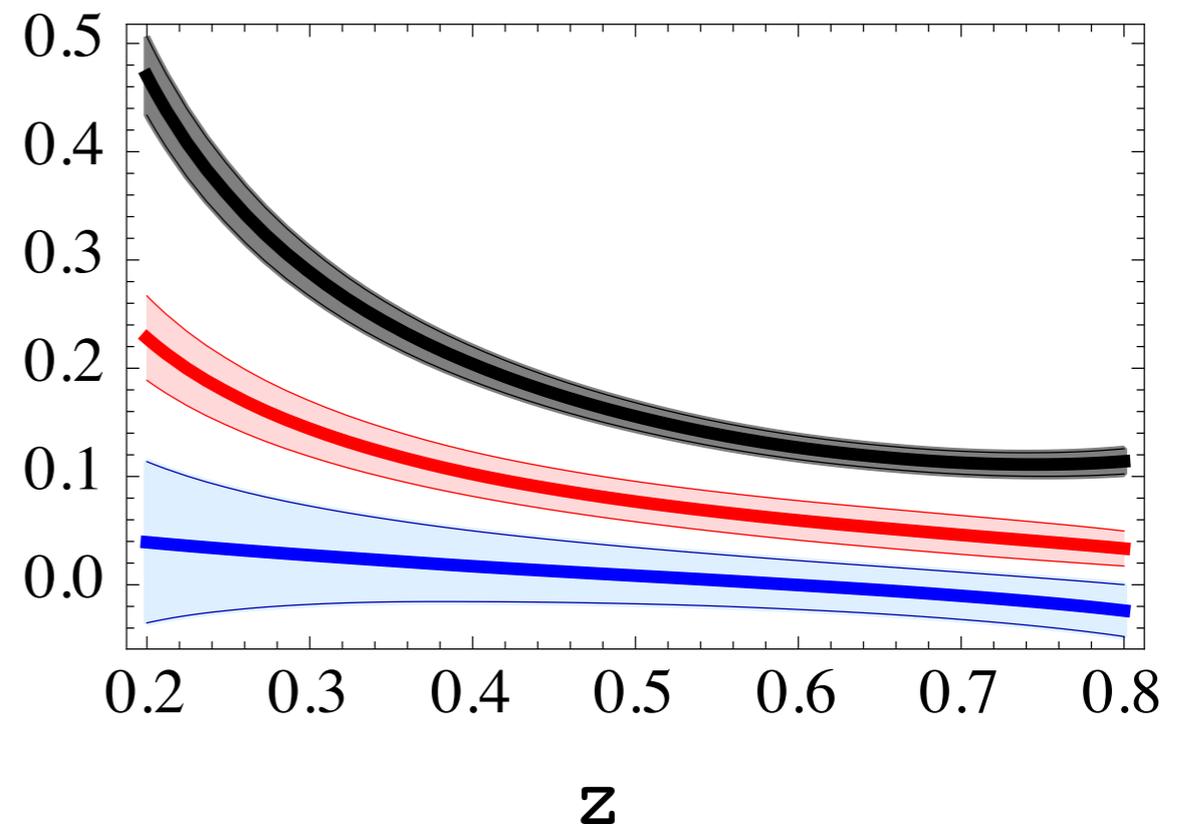
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fix energy, vary  $z$

$E = 50 \text{ GeV}$



$E = 200 \text{ GeV}$



# Conclusions

NRQCD describes much world data on quarkonium data but puzzles, esp. polarization, remain

existing analyses focus on inclusive  $p_t$  spectra, polarization can we find other observables distinguish various production mechanisms at high  $p_T$ ?

**measuring  $Q\bar{Q}$  within jets, and using jet observables should provide insights into  $Q\bar{Q}$  production**

quarkonium fragmenting jet functions (FJFs)

**If  $^1S_0^{(8)}$  mechanism dominates high  $p_T$  production FJF should have negative slope for  $z(E)$ , for  $z > 0.5$**

**Backup**

fragmentation function (QCD)

$$D_q^h(z) = z \int \frac{dx^+}{4\pi} e^{ik^-x^+/2} \frac{1}{4N_c} \text{Tr} \sum_X \langle 0 | \vec{\eta} \Psi(x^+, 0, 0_\perp) | Xh \rangle \langle Xh | \bar{\Psi}(0) | 0 \rangle \Big|_{p_h^\perp=0}$$

fragmentation function (SCET)

$$D_q^h\left(\frac{p_h^-}{\omega}, \mu\right) = \pi\omega \int dp_h^+ \frac{1}{4N_c} \text{Tr} \sum_X \vec{\eta} \langle 0 | [\delta_{\omega, \bar{p}} \delta_{0, p_\perp} \chi_n(0)] | Xh \rangle \langle Xh | \bar{\chi}_n(0) | 0 \rangle$$

Jet function (SCET)

$$J_u(k^+\omega) = -\frac{1}{\pi\omega} \text{Im} \int d^4x e^{ik \cdot x} i \langle 0 | \text{T} \bar{\chi}_{n, \omega, 0_\perp}(0) \frac{\vec{\eta}}{4N_c} \chi_n(x) | 0 \rangle$$

fragmentation jet function (SCET)

$$\mathcal{G}_{q, \text{bare}}^h(s, z) = \int d^4y e^{ik^+y^-/2} \int dp_h^+ \sum_X \frac{1}{4N_c} \text{tr} \left[ \frac{\vec{\eta}}{2} \langle 0 | [\delta_{\omega, \bar{p}} \delta_{0, p_\perp} \chi_n(y)] | Xh \rangle \langle Xh | \bar{\chi}_n(0) | 0 \rangle \right]$$

$$\delta(p^+/z - P_H^+) \rightarrow \delta(p^+/z - P_H^+) \delta(p^- - s/p^+)$$

**FF**

**FJF**