bottomonium spectroscopy
with lattice NRQCD

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1. spectrum results
2. free-form smearing

tadpole-improved lattice NRQCD

$$H = \frac{-\Delta^{(2)}}{2M_0} - c_1\frac{(\Delta^{(2)})^2}{8{M_0}^3} - \frac{c_3}{{U_0}^4}\frac{g}{8{M_0}^2}\boldsymbol{\sigma}\cdot(\Delta\times\mathbf{E}-\mathbf{E}\times\Delta) + c_5\frac{a^2\Delta^{(4)}}{24M_0}$$
$$+\frac{c_2}{{U_0}^4}\frac{ig}{8{M_0}^2}(\Delta\cdot\mathbf{E}-\mathbf{E}\cdot\Delta) - \frac{c_4}{{U_0}^4}\frac{g}{2M_0}\boldsymbol{\sigma}\cdot\mathbf{B} - c_6\frac{a(\Delta^{(2)})^2}{16n{M_0}^2} + O(v^6)$$

The stability parameter n is algorithmic, not physical; we use $n = 4$.

The tadpole factor U_0 is the mean value of a gauge link.

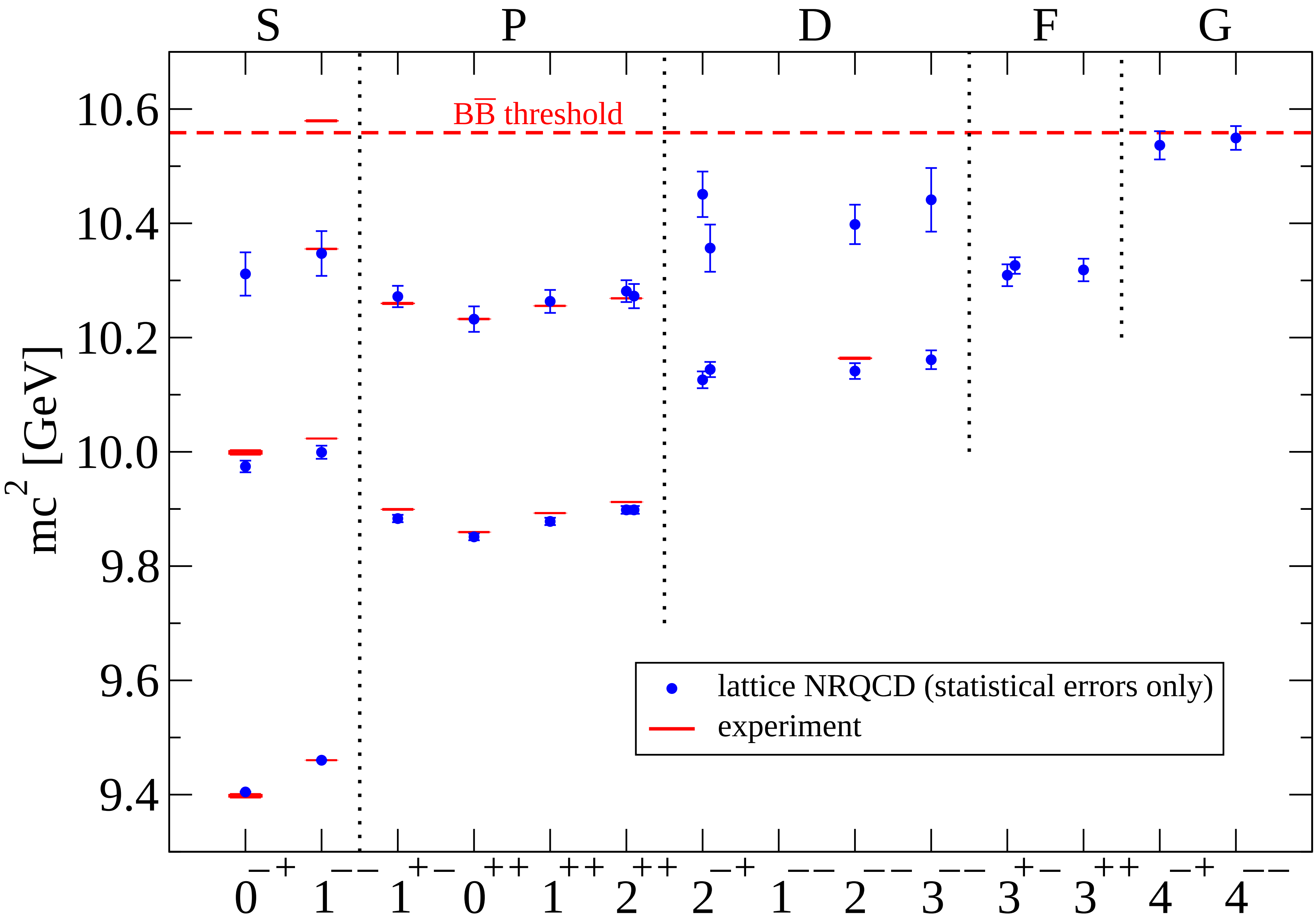
The coefficients $c_i = 1 + O(\alpha_s)$ are short-distance (perturbative) physics.

The bottom quark bare mass M_0 is set by fitting the experimental $1S$ mass.

- Keeping only $O(v^2)$ and $O(v^4)$ and using $c_i = 1$ is a good start.
  Systematic errors are reduced by going beyond this starting point.

bottomonium spectrum from lattice NRQCD

Wurtz,Lewis,Woloshyn arXiv:1409:7103



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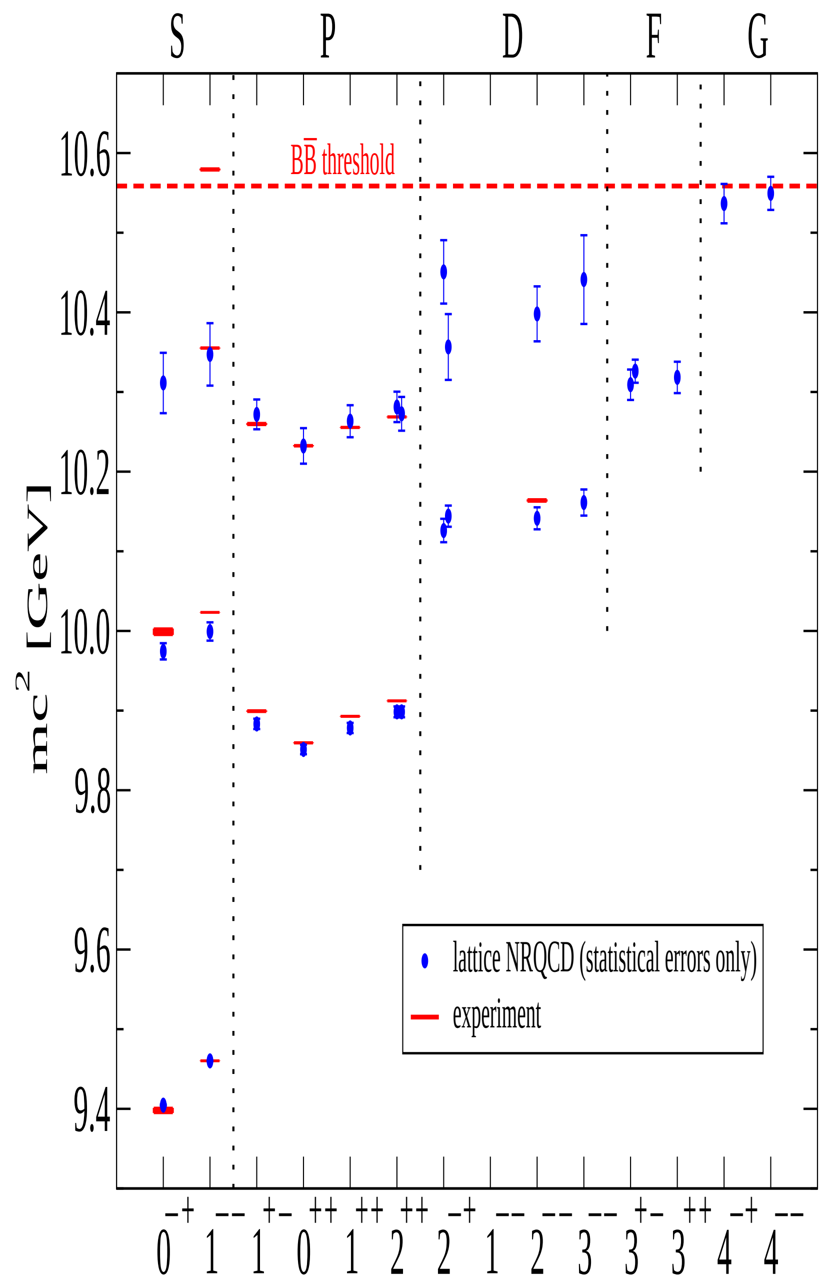
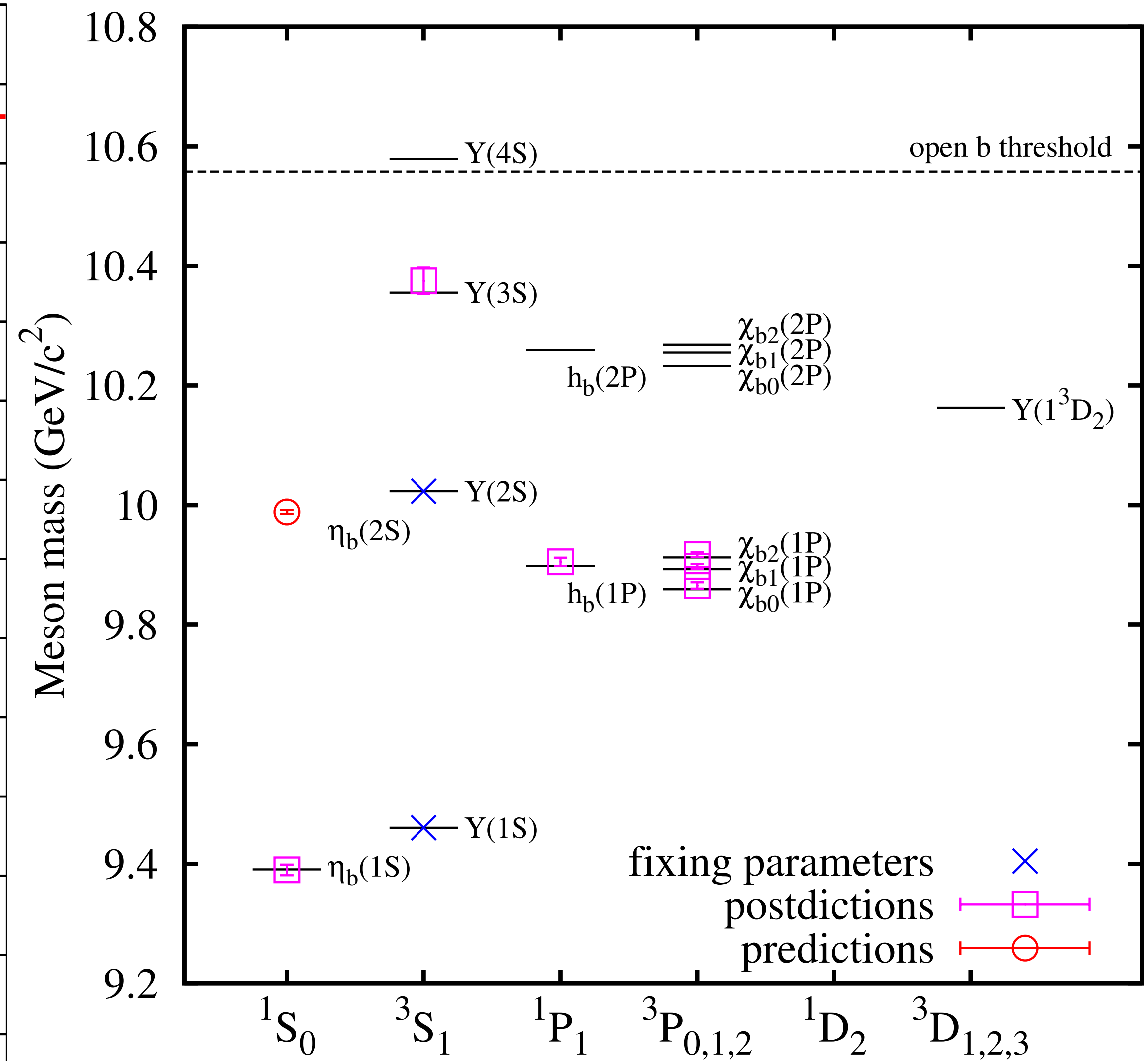


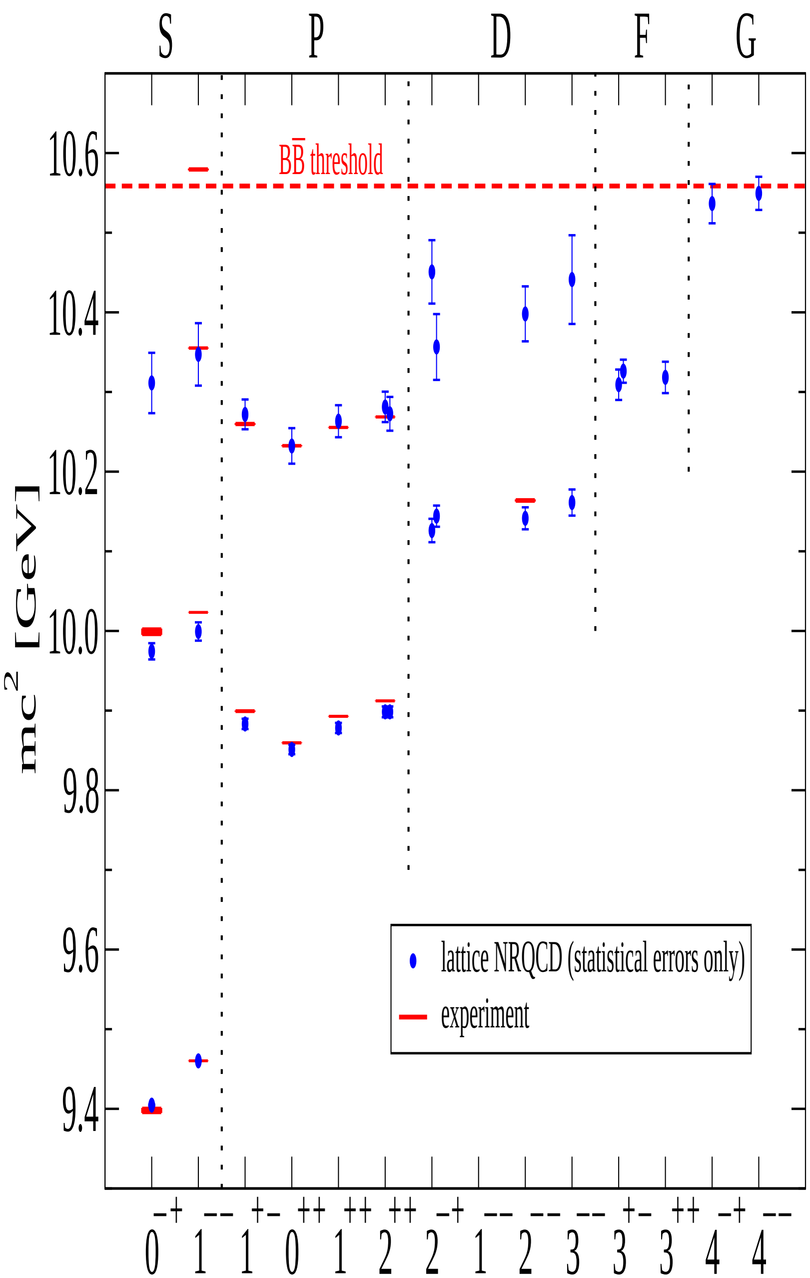
HPQCD, PRD85, 054509 (2012)



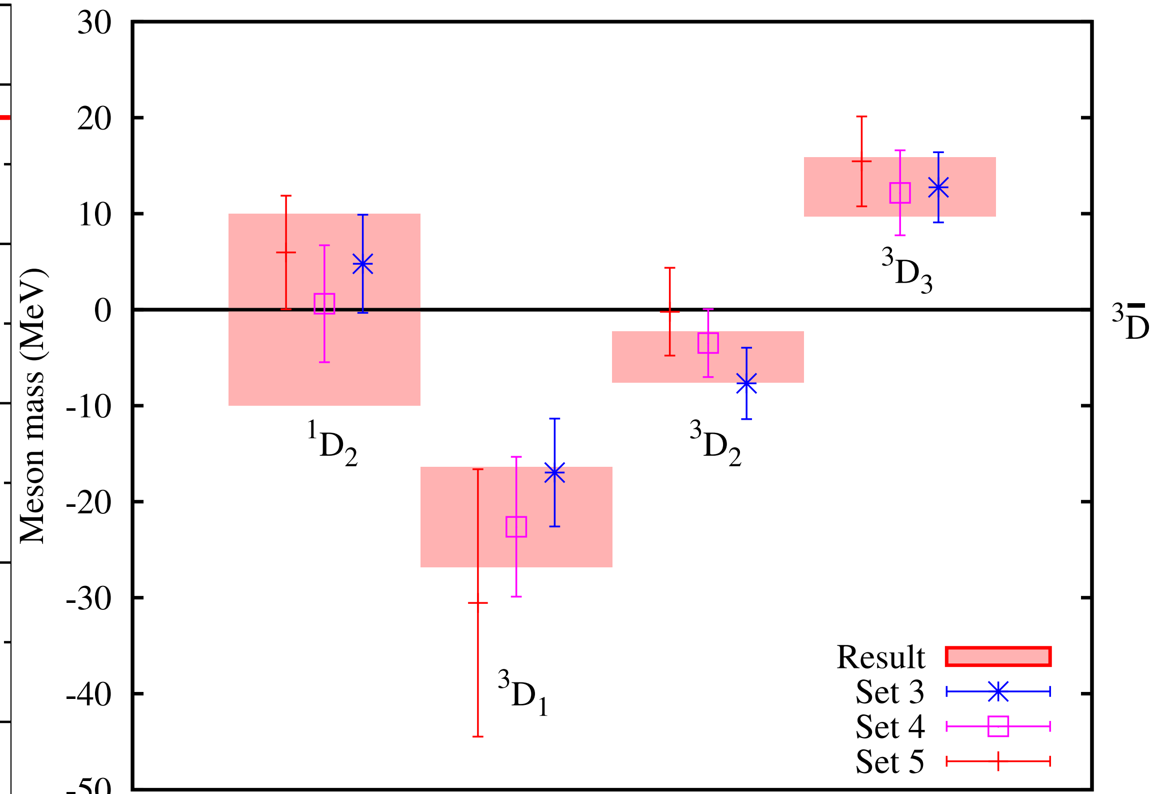
A complete error budget is obtained.

bottomonium spectrum from lattice NRQCD

Wurtz, Lewis, Woloshyn arXiv:1409:7103



HPQCD, PRL108, 102003 (2012)



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bottomonium spectrum from lattice NRQCD

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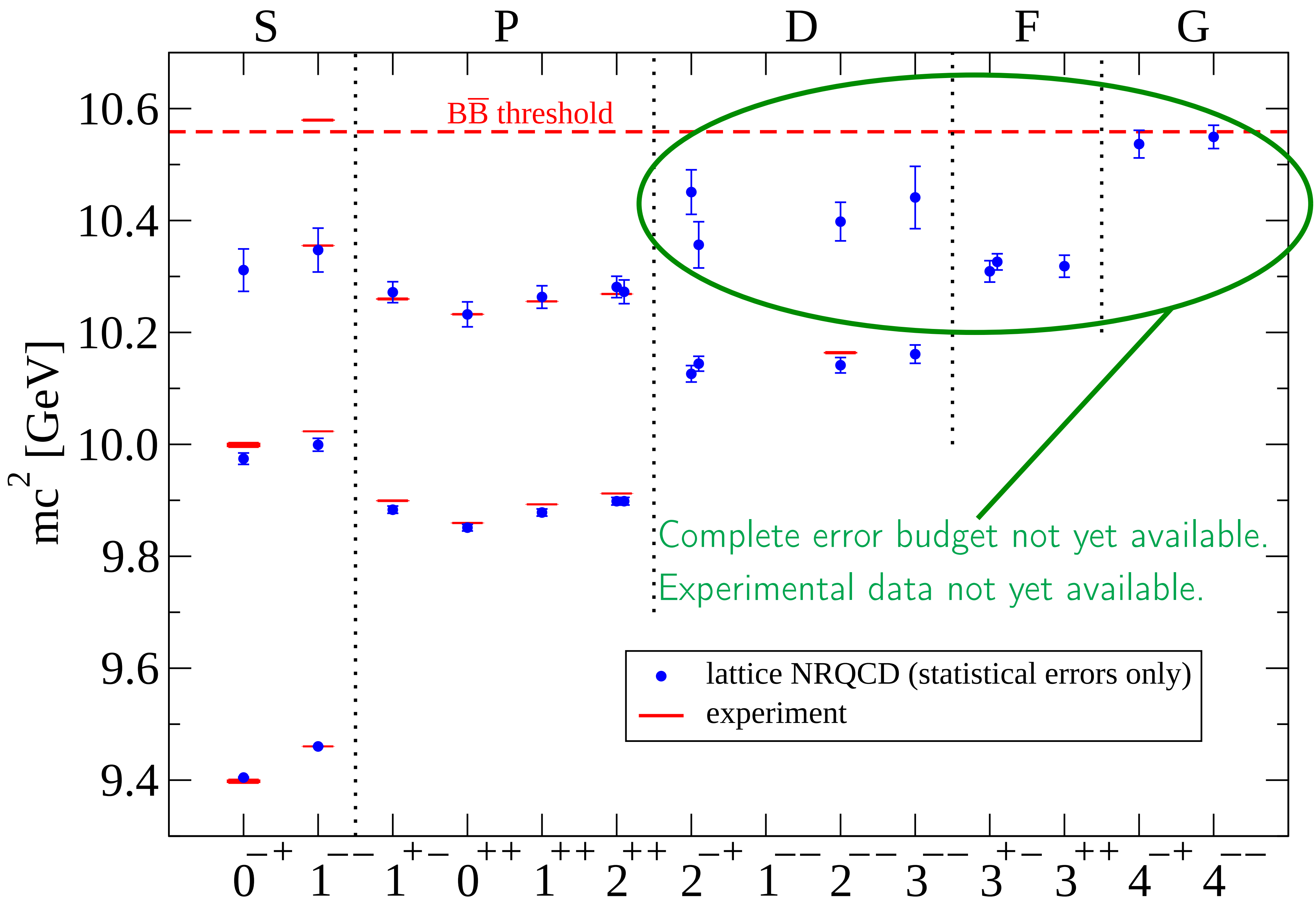
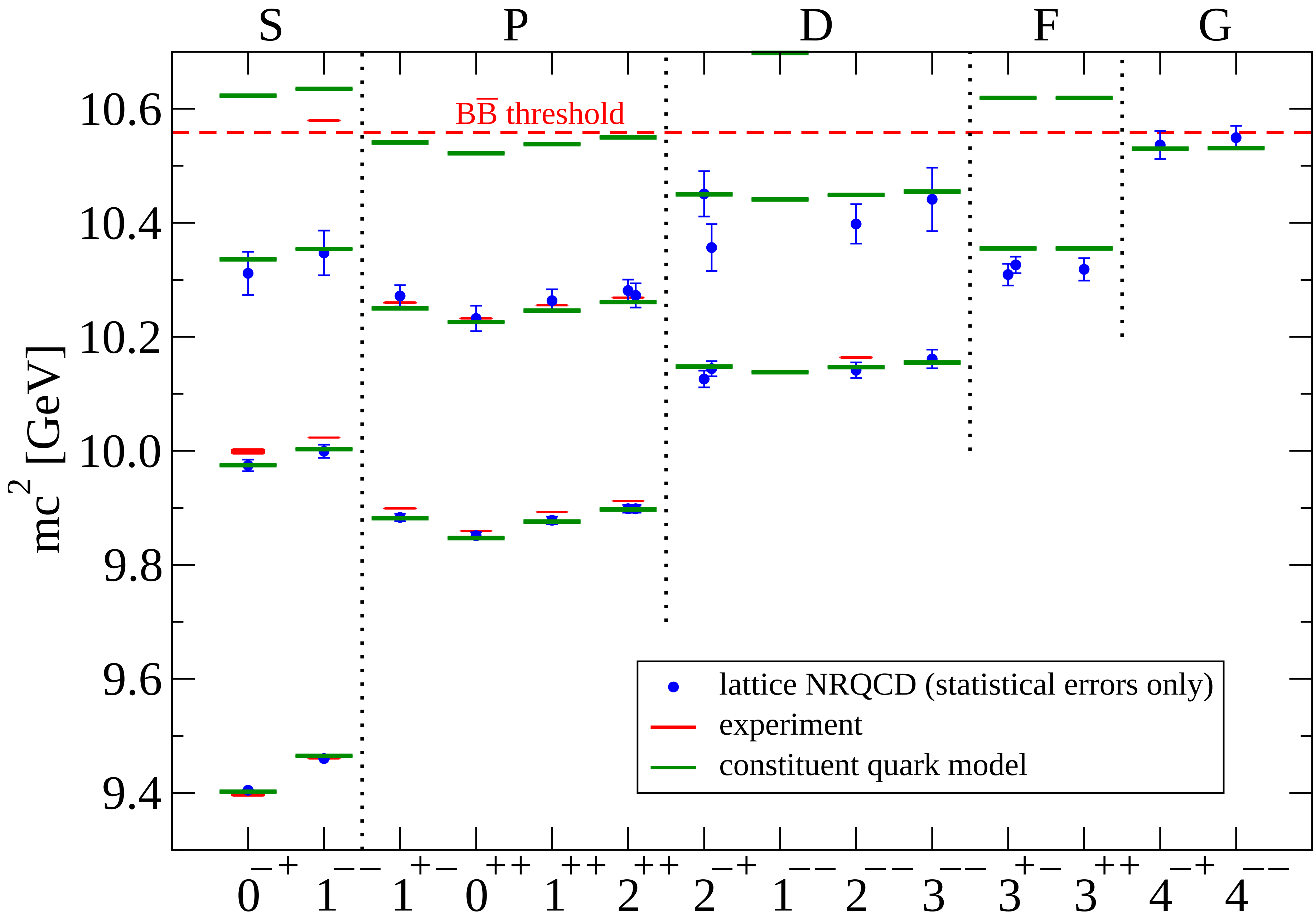


comparing a constituent quark model

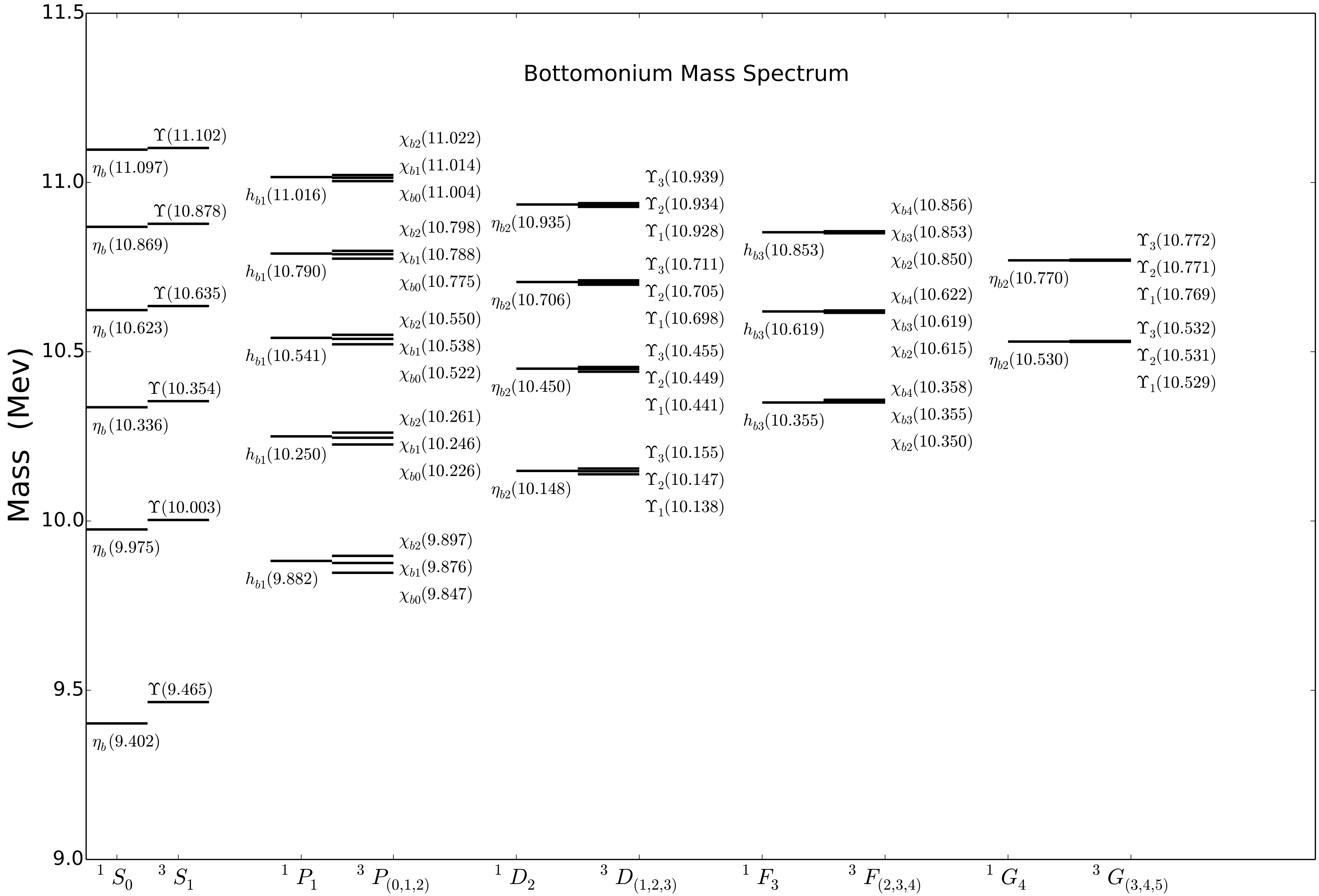
Wurtz,Lewis,Woloshyn arXiv:1409:7103

Godfrey,Moats (preliminary)



results from a constituent quark model

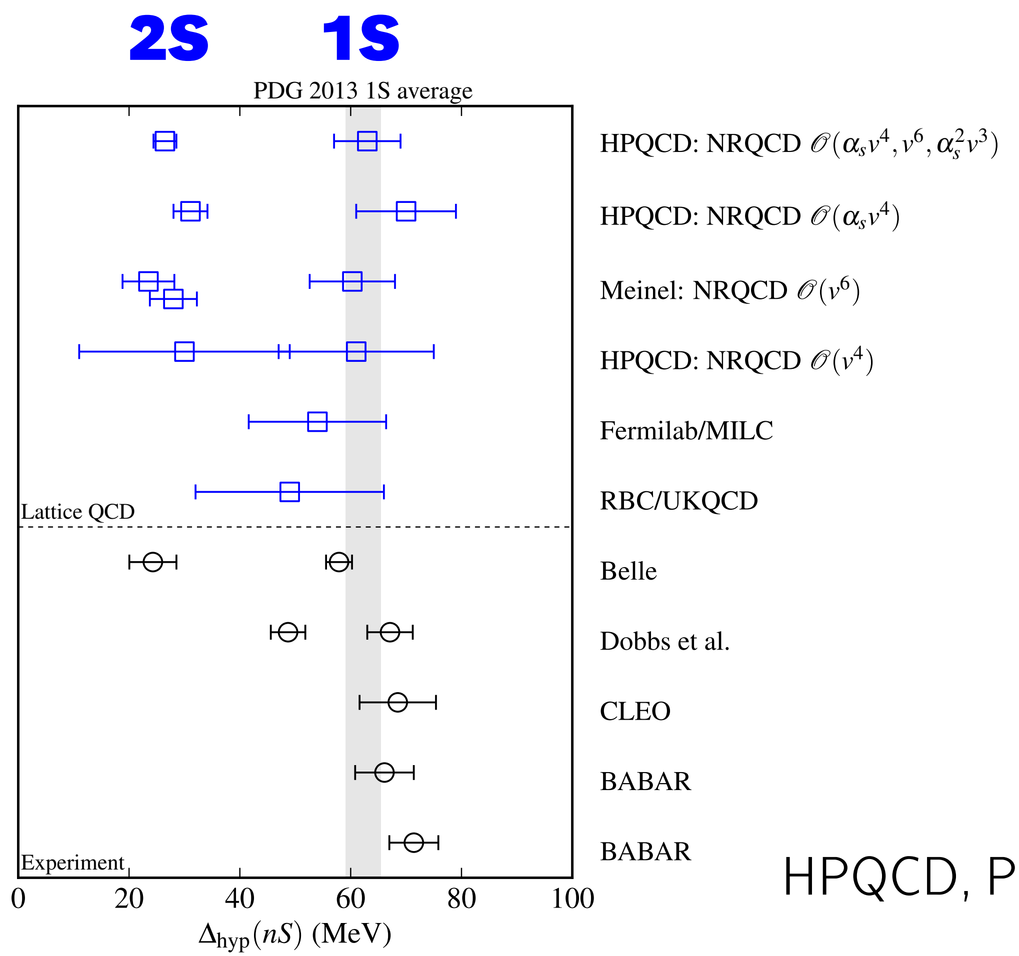
Godfrey and Moats (preliminary)



2S hyperfine splitting

In chronological order…		[MeV]
lattice NRQCD	Meinel, PRD82, 114502 (2010)	$23.5 \pm 4.1_{\rm stat} \pm 2.1_{\rm syst} \pm 0.8_{\rm exp}$
lattice NRQCD	HPQCD, PRD85, 054509 (2012)	35 ± 3
experiment	Dobbs et al, PRL109 082001 (2012)	$48.7 \pm 2.3 \pm 2.1$
lattice NRQCD	Lewis,Woloshyn, PRD85, 114509 (2012)	24 ± 3 (statistical only)
experiment	Belle, PRL109, 232002 (2012)	$24.3 \pm 3.5^{+2.8}_{-1.9}$

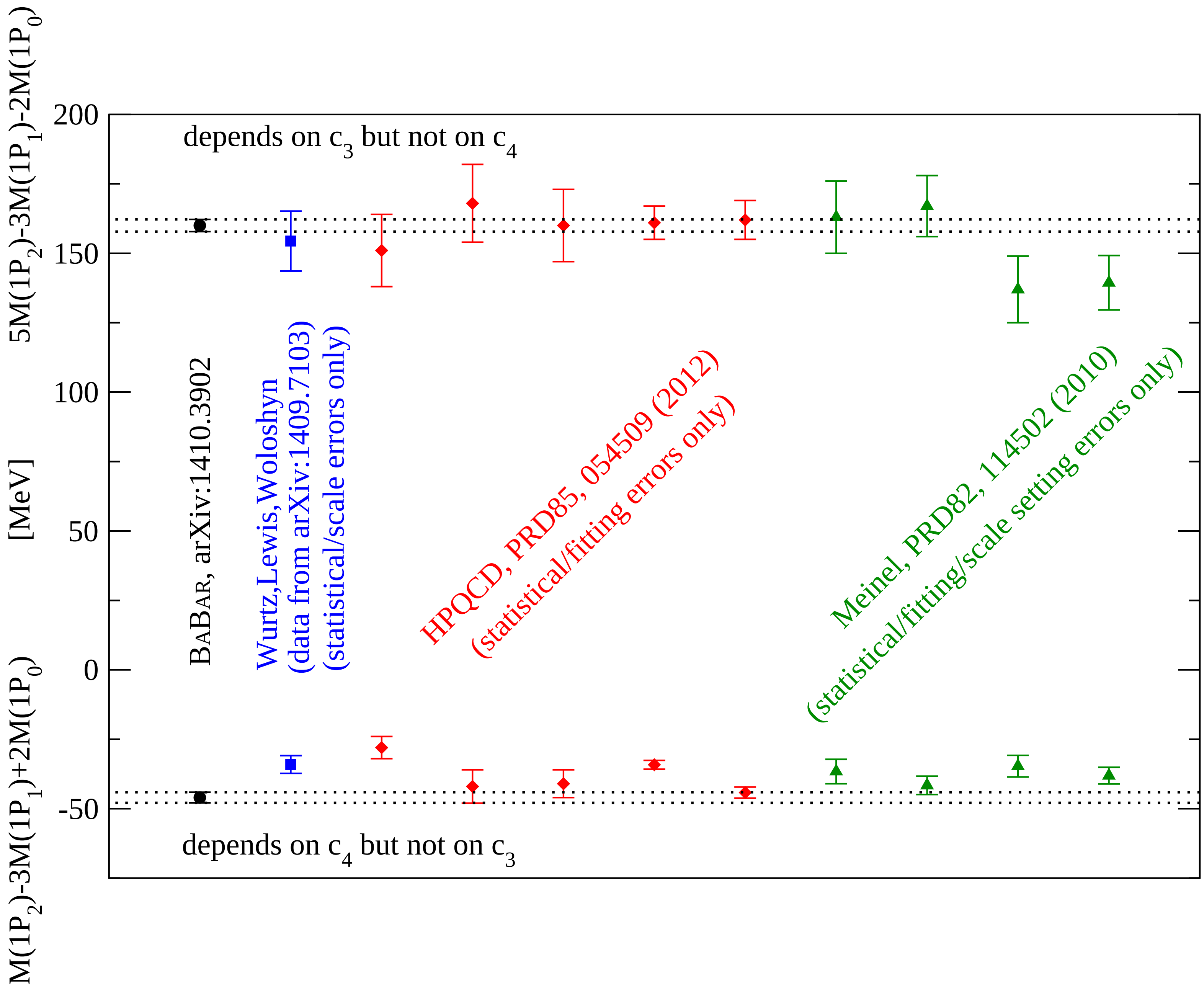
considering systematic errors:



HPQCD, PRD89, 031502 (2014)

mass combinations for specific NRQCD coefficients

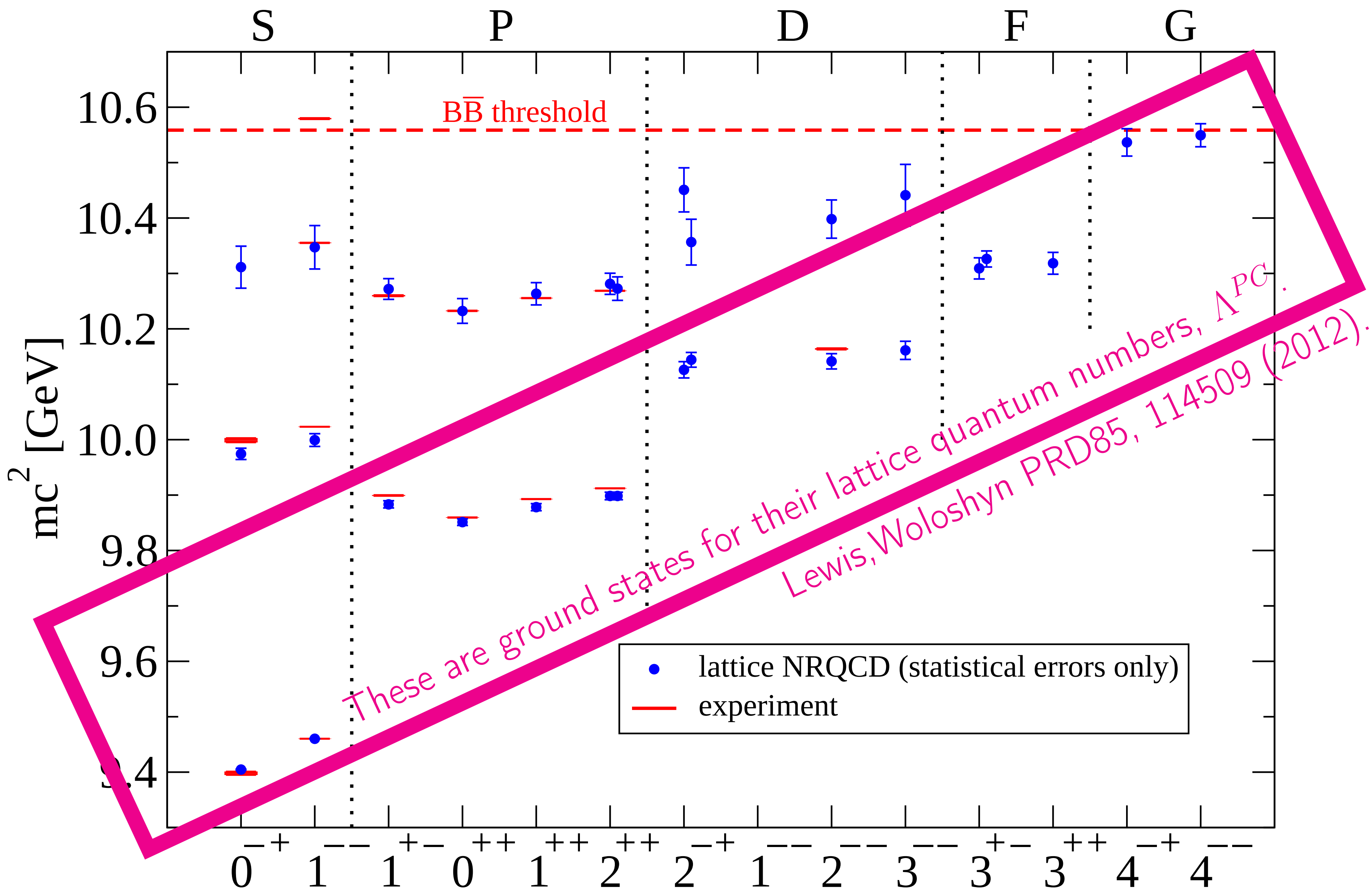
BaBar, arXiv:1410:3902 includes Section VI.C for this topic.



BaBar also provides 2P results as a helpful challenge for lattice studies.

hypercubic symmetry allows some J values to mix.

$J = 0, 1, 2, 3, \ldots$ collapses to just 5 irreps: $\Lambda = A_1, A_2, E, T_1, T_2$



operator smearing improves the signal

Three smearing options:

Gaussian smearing

$$\tilde{\psi}_y(x) = \left(1 + \frac{\alpha}{n}\Delta\right)^n \psi(y)$$

- enhance ground state
- only Gaussian shape

Coulomb gauge smearing

$$\tilde{\psi}(x) = \sum_y f(x-y)\psi(y)$$

- enhance any state
- must fix a gauge

free-form smearing

$$\tilde{\tilde{\psi}}_y(x) = \frac{f(x-y)\tilde{\psi}_y(x)}{\langle||\tilde{\psi}_y(x)||\rangle}$$

- enhance any state
- new method*

*von Hippel, Jäger, Rae and Wittig, JHEP 1309, 014 (2013)

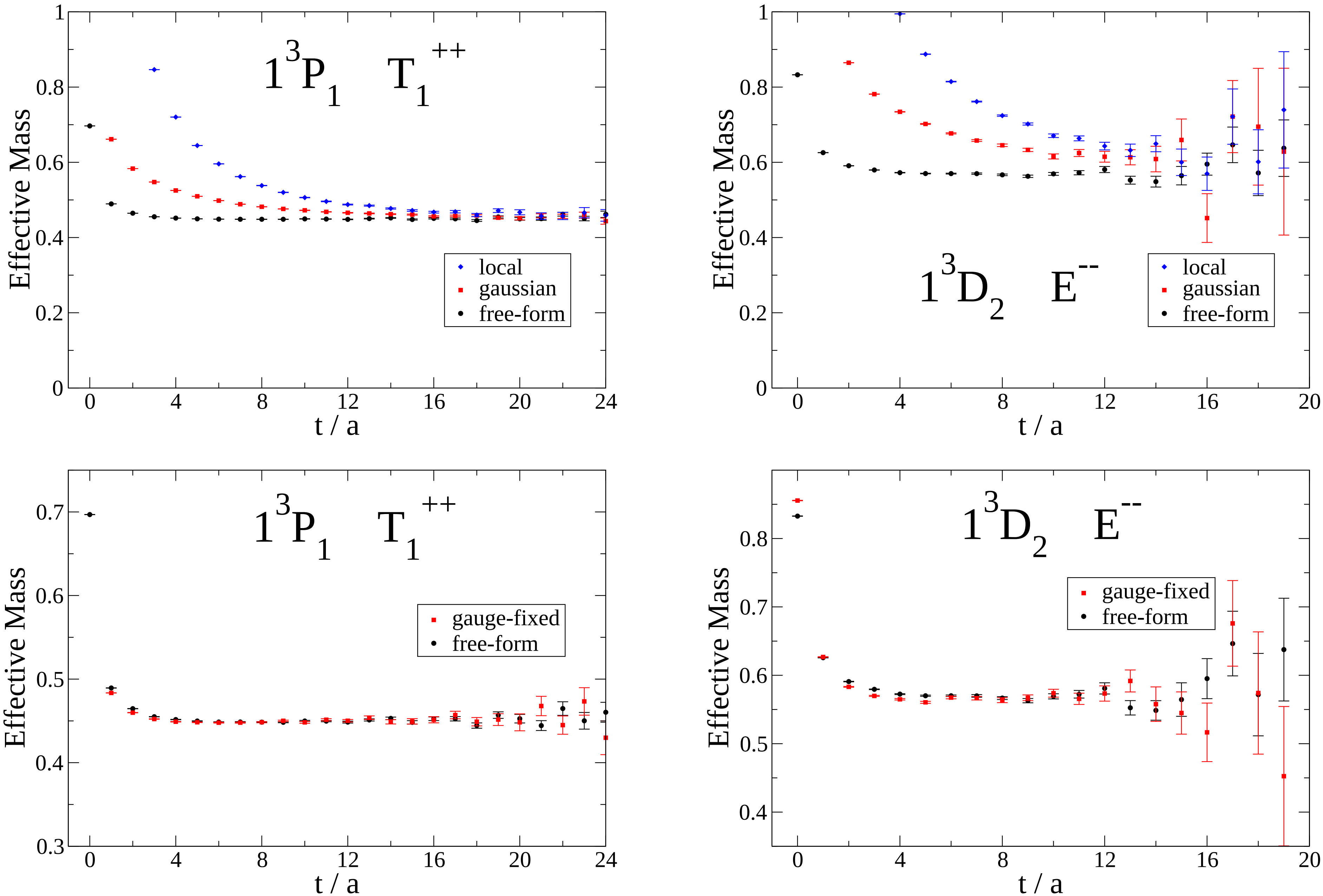
We are the first to apply free-form smearing to NRQCD.

Our tests use wall sources and hydrogen-like wave functions

(radius and node positions are parameters)

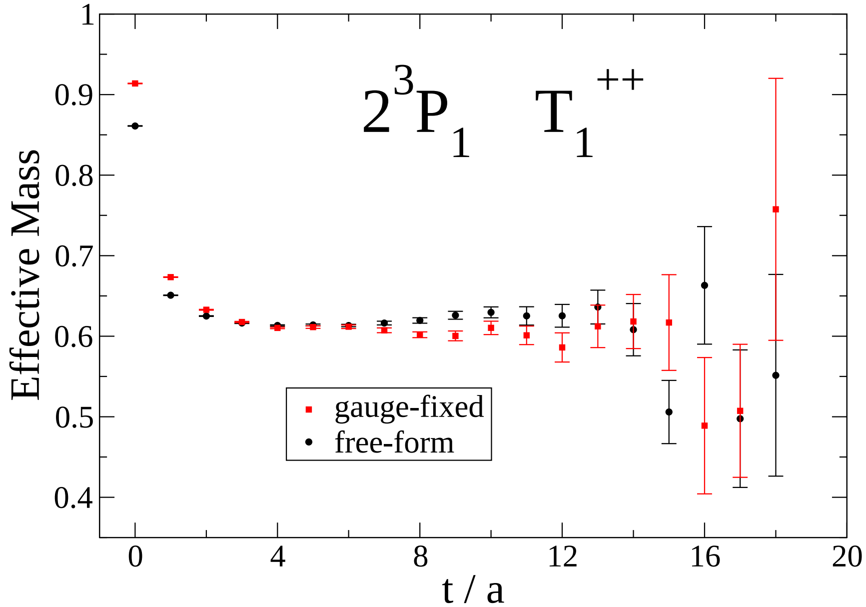
comparing ground state smearings

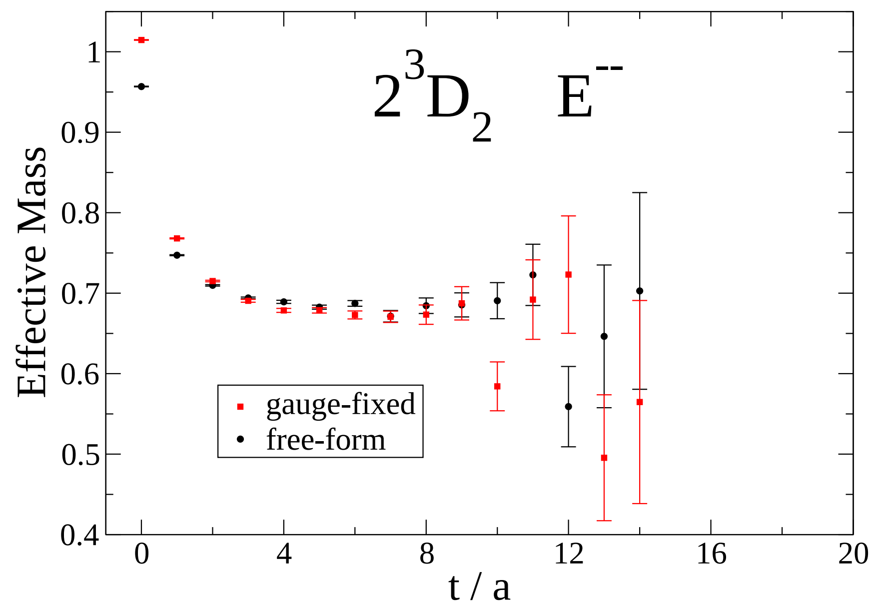
Wurtz,Lewis,Woloshyn arXiv:1409:7103

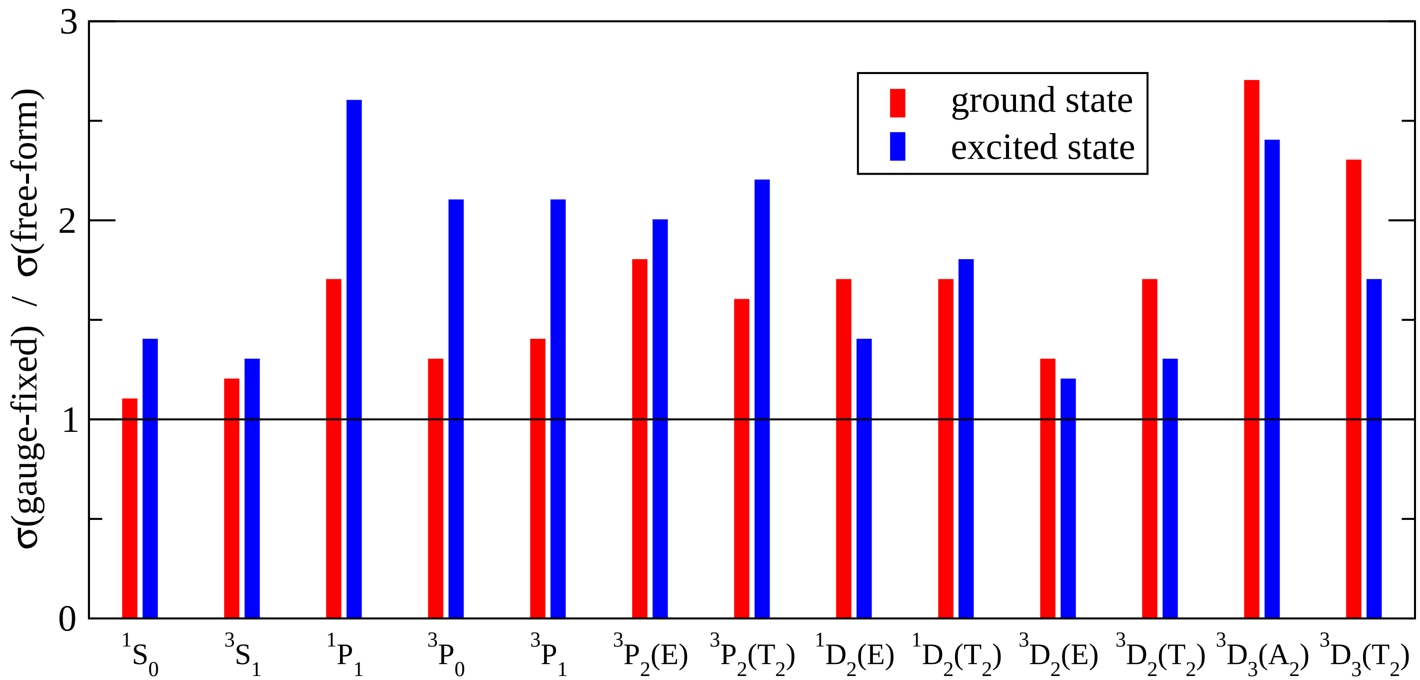


more smearing comparisons

Wurtz,Lewis,Woloshyn arXiv:1409:7103







conclusions

- Lattice studies have observed many bottomonium states.
- Lattice NRQCD agrees with experiment where available.
- Free-form smearing is a valuable new tool.