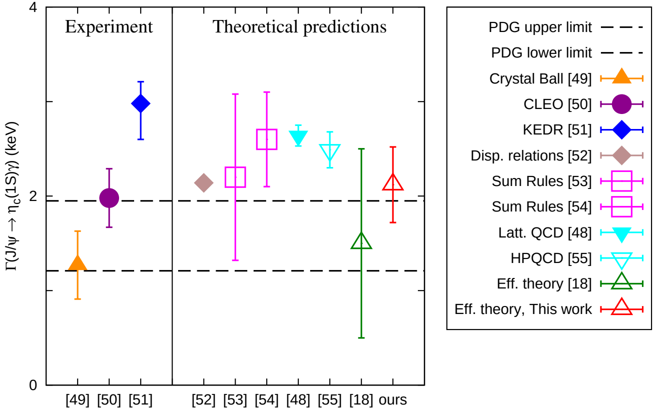


Figure: *Comparison of different theoretical and experimental predictions for* $\Gamma_{J/\psi\to\eta_c\gamma}$.

$$\Gamma(n^3S_1 \to n^1S_0\gamma) = \frac{4}{3}\alpha e_Q^2\frac{k_\gamma^3}{m^2}\left[(1+\kappa)^2 - \frac{5}{3}\frac{\langle p^2\rangle_n}{m^2}\right],$$

$$\kappa = \kappa^{(1)}\alpha_s(m) + \kappa^{(2)}\alpha_s^2(m)$$

Hindered transitions. Strict weak coupling: Brambilla, Jia, Vairo

$$\Gamma(n^3S_1 \to n'^1S_0\gamma) \overset{n\neq n'}{=} \frac{4}{3}\alpha e_Q^2 \frac{k_\gamma^3}{m^2}\left[\frac{k_\gamma^2}{24}\,{}_{n'}\langle r^2\rangle_n^C + \frac{5}{6}\frac{{}_{n'}\langle p^2\rangle_n^C}{m^2} - \frac{2}{m^2}\frac{{}_{n'}\langle V_{S^2}^C(\vec{r})\rangle_n}{E_n^C - E_{n'}^C}\right]^2,$$

$$\Gamma(n^1S_0 \to n'^3S_1\gamma) \overset{n\neq n'}{=} 4\alpha e_Q^2 \frac{k_\gamma^3}{m^2}\left[\frac{k_\gamma^2}{24}\,{}_{n'}\langle r^2\rangle_n^C + \frac{5}{6}\frac{{}_{n'}\langle p^2\rangle_n^C}{m^2} + \frac{2}{m^2}\frac{{}_{n'}\langle V_{S^2}(\vec{r})\rangle_n^C}{E_n^C - E_{n'}^C}\right]^2,$$

$$V_{S^2}(\vec{r}) = \frac{4}{3}\pi C_f D^{(2)}_{S^2,s}(\nu)\delta^{(3)}(\vec{r})$$

$$D^{(2)}_{S^2,s}(\nu) = \alpha_s(\nu)$$

Hindered transitions. Improved weak coupling: Pineda, Segovia

$$\Gamma(n^3S_1 \to n'^1S_0\gamma) \overset{n\neq n'}{=} \frac{4}{3}\alpha e_Q^2 \frac{k_\gamma^3}{m^2}\left[\frac{k_\gamma^2}{24}{}_{n'}\langle r^2\rangle_n + \frac{5}{6}\frac{{}_{n'}\langle p^2\rangle_n}{m^2} - \frac{2}{m^2}\frac{{}_{n'}\langle V_{S^2}(\vec{r})\rangle_n}{E_n - E_{n'}}\right]^2,$$

$$\Gamma(n^1S_0 \to n'^3S_1\gamma) \overset{n\neq n'}{=} 4\alpha e_Q^2 \frac{k_\gamma^3}{m^2}\left[\frac{k_\gamma^2}{24}{}_{n'}\langle r^2\rangle_n + \frac{5}{6}\frac{{}_{n'}\langle p^2\rangle_n}{m^2} + \frac{2}{m^2}\frac{{}_{n'}\langle V_{S^2}(\vec{r})\rangle_n}{E_n - E_{n'}}\right]^2,$$

$$V_{S^2}(\vec{r}) = \frac{4}{3}\pi C_f D^{(2)}_{S^2,s}(\nu)\delta^{(3)}(\vec{r})$$

$$D^{(2)}_{S^2,s}(\nu) = \alpha_s(\nu)c_F^2(\nu) - \frac{3}{2\pi C_f}\left(d_{sv}(\nu) + C_f d_{vv}(\nu)\right)$$

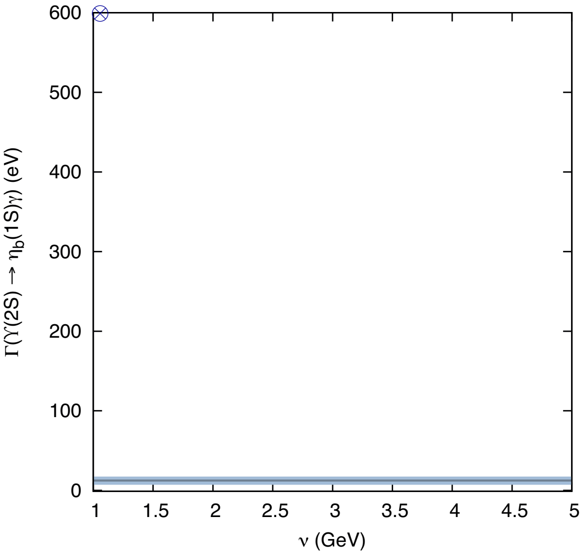
depends on the NRQCD Wilson coefficients. With LL accuracy they read

$$c_F(\nu) = z^{-C_A}, \qquad d_{sv}(\nu) = d_{sv}(m),$$

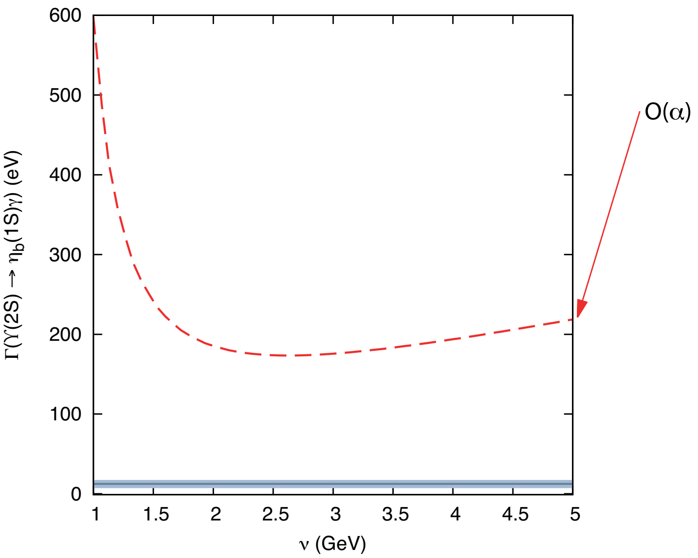
$$d_{vv}(\nu) = d_{vv}(m) + \frac{C_A}{\beta_0 - 2C_A}\pi\alpha_s(m)(z^{\beta_0 - 2C_A} - 1),$$

$$z = \left[\frac{\alpha_s(\nu)}{\alpha_s(m)}\right]^{\frac{1}{\beta_0}} \simeq 1 - \frac{1}{2\pi}\alpha_s(\nu)\ln\left(\frac{\nu}{m}\right),$$

$$d_{sv}(m) = C_f\left(C_f - \frac{C_A}{2}\right)\pi\alpha_s(m), d_{vv}(m) = -\left(C_f - \frac{C_A}{2}\right)\pi\alpha_s(m).$$



no RG , $V_s^{(0)} = -C_f \frac{\alpha_s(\mu)}{r}$, $\mu = 1$ GeV



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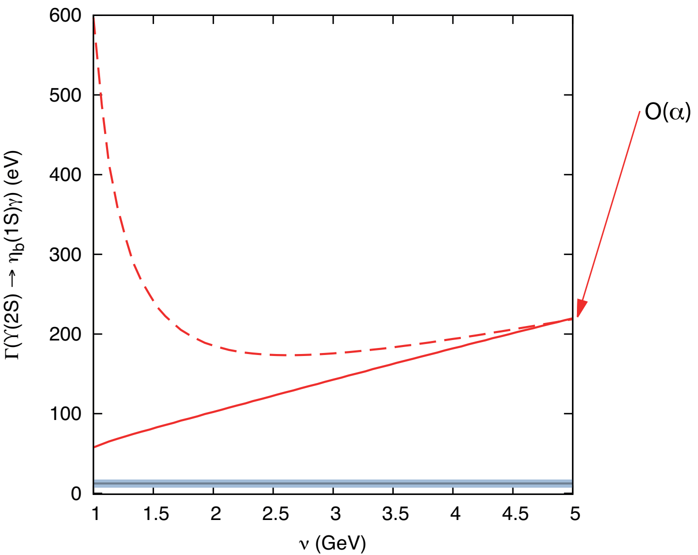

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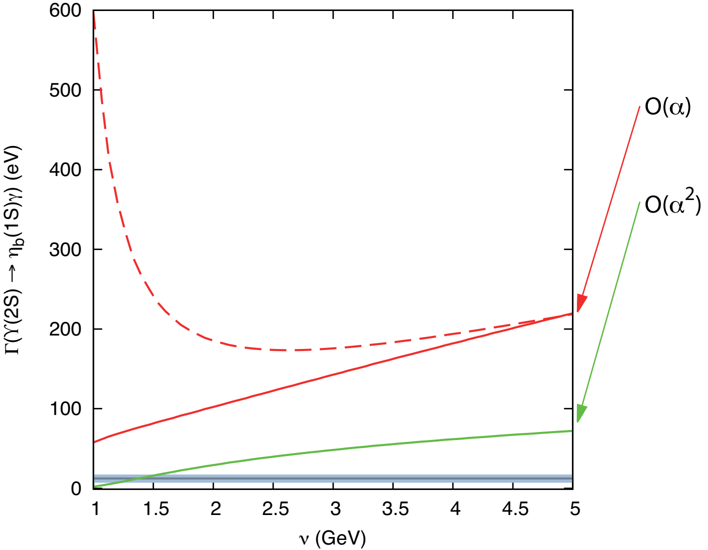
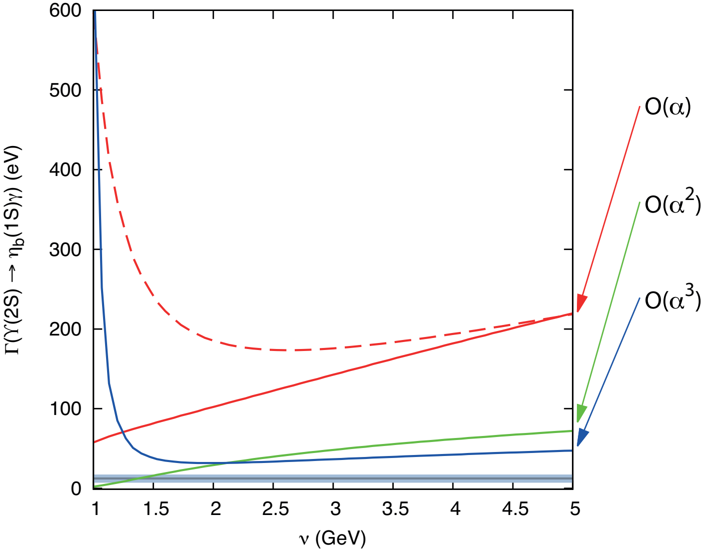
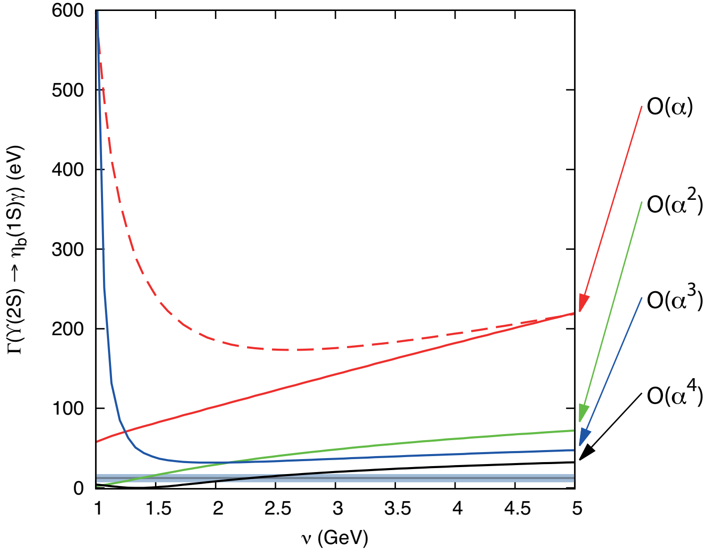


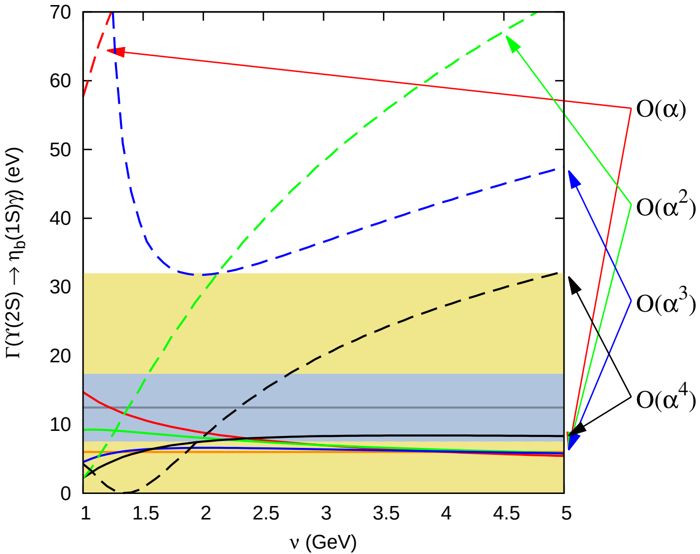
RG , $V_s^{(0)} = -C_f \frac{\alpha_s(\mu)}{r}\left(1 + a_1 \frac{\alpha_s(\mu)}{4\pi}\right)$



$$\text{RG} \; , \; V_s^{(0)} = -C_f \frac{\alpha_s(\mu)}{r} \left(1 + a_1 \frac{\alpha_s(\mu)}{4\pi} + a_2 \frac{\alpha_s^2(\mu)}{(4\pi)^2} \right)$$



$$\text{RG}\ ,\ V_s^{(0)} = -C_f \frac{\alpha_s(\mu)}{r} \left(1 + a_1 \frac{\alpha_s(\mu)}{4\pi} + a_2 \frac{\alpha_s^2(\mu)}{(4\pi)^2} + a_3 \frac{\alpha_s^3(\mu)}{(4\pi)^3} \right)$$



$$\text{RG}\ ,\ V_s^{(0)} = -C_f \frac{\alpha_s(1/r)}{r}\left(1 + a_1\frac{\alpha_s(1/r)}{4\pi} + a_2\frac{\alpha_s^2(1/r)}{(4\pi)^2} + a_3\frac{\alpha_s^3(1/r)}{(4\pi)^3}\right)$$

$$\Gamma^{(\rm th)}_{\Upsilon(2S)\to\eta_b(1S)\gamma} = 0.006 \pm 0.006(\mathcal{O}(v^5))^{+0.026}_{-0.006}(N_m)^{-0.001}_{+0.001}(\alpha_s)^{-0.000}_{+0.000}(m_{\overline{\rm MS}})\,{\rm keV}.$$

$$\Gamma^{(\rm th)}_{\Upsilon(2S)\to\eta_b(1S)\gamma} = 6^{+26}_{-06}\,{\rm eV}.$$

Matrix element.

Experimental number: 0.035(7)

Ours: $0.025^{+0.031}_{-0.025}$

Lewis, Woloshyn: $\mathcal{O}(v^4) = 0.080(5)$; $\mathcal{O}(v^6) = 0.032(5)$

Both agreement but different physics. To be clarified.

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