Phenomenology of the heavy quarkonium electric dipole transitions

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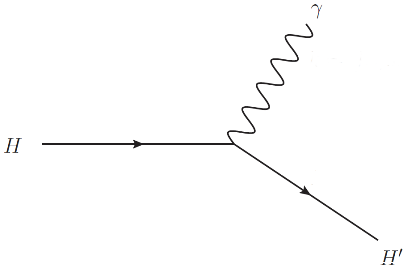
Outline

Motivation

Electric dipole transitions (E1) of heavy quarkonium:

- $|\Delta L| = 1$
- $|\Delta S| = 0$
- Example transition: $1^3P_J \to 1^3S_1\gamma$:
    - in charmonium: $\chi_c \to J/\psi\gamma$
    - in bottomonium: $\chi_b \to \Upsilon\gamma$



Experimental results available:

- $h_c \to \eta_c\gamma$ first measured at BESIII in 2010
- $h_b \to \eta_b\gamma$ first measured in 2011 at BaBar and BELLE
- First measurement of the EM branching ratios of the χ_b states by CLEO and BaBar in 2011

Branching fraction of E1 transition significant for some states (from PDG):

- $\mathcal{BR}(\chi_{b1}(1P) \to \Upsilon(1S)\gamma) \approx 34 \pm 2\%$,
- $\mathcal{BR}(h_b(1P) \to \eta_b(1S)\gamma) \approx 49 \pm 8\%$

The formula for the E1 transition rates including NLO relativistic corrections has the following structure

$$\Gamma^{\rm E1}_{H\to H'} = \Gamma^{(0)}_{H\to H'}(1 + R_{H\to H'} + \delta\Gamma_{H\to H'})$$

Brambilla, Pietrulewicz and Vairo, *PRD 85, 094005 (2012)*

where the $R_{H\to H'} + \delta\Gamma_{H\to H'}$ are v^2 suppressed.
For instance, in the case of the $n^3P_J \to n'^3S_1\gamma$ the terms in this formula read:

$$\begin{aligned}\Gamma^{(0)}_{n^3P_J\to n'^3S_1} &= \frac{4}{9}\alpha_{em}e_Q^2k_\gamma^3I_3^2,\\ \delta\Gamma_{n^3P_J\to n'^3S_1} &= -\frac{k_\gamma^2}{60}\frac{I_5}{I_3} - \frac{k_\gamma}{6m} + \left(\frac{J(J+1)}{2} - 2\right)\left(\frac{1}{m^2}\frac{I_2^{(')} + 2I_1}{I_3} - \frac{k_\gamma}{2m}\right),\end{aligned}$$

k_γ is the energy of the emitted photon. $R_{H\to H'}$ accounts for the relativistic corrections to the quarkonium wavefunction, it has the general structure

$$R_{H\to H'} = \frac{1}{A^{(0)}_{H\to H'}}\left(-^{(0)}\langle H'\gamma|\int d^3R\mathcal{L}^{(0)}_{E_1}|H\rangle^{(1)} -^{(1)}\langle H'\gamma|\int d^3R\mathcal{L}^{(0)}_{E_1}|H\rangle^{(0)} + \ldots\right)$$

where

$$I_N^{(k)} = \int_0^\infty dr r^N R_{n'0}(r) \frac{d^k}{dr^k} R_{n1}(r),$$

$$\left(-\frac{\nabla_r^2}{m} + V^{(0)}\right)\phi_H^{(0)} = E_H^{(0)}\phi_H^{(0)},$$

with the quarkonium wavefunctions given by

$$|H(P)\rangle^{(0)} = \int d^3R \int d^3r\, e^{iP\cdot R}\mathrm{Tr}\left\{\phi_H^{(0)} S^\dagger(r,R)|US\rangle\right\},$$

$$|H(P)\rangle^{(1)} = \sum_{H'\neq H} {}^{(0)}\langle H'(P)| \int d^3R \int d^3r \mathrm{Tr}\{S^\dagger \delta h S\}|H(P)\rangle^{(0)} \frac{|H'(P)\rangle^{(0)}}{E_{H'}^{(0)} - E_H^{(0)}}.$$

δh accounts for the relativistic corrections to the quark-antiquark potential

$$\delta h = -\frac{p^4}{4m^3} + \delta V.$$

In order to evaluate the formula of the rates we need to include the relativistic corrections to the potential $\delta V \sim \mathcal{O}(1/m^2)$

- To evaluate transitions between lower quarkonium states (weakly-coupled) we can use the perturbative expressions for these corrections.
- For transitions that involve higher states (strongly-coupled) we need to include non-perturbative terms in the potential.

The $Q\bar{Q}$ potential

In the equal mass case the relativistic corrections to the quark-antiquark potential can be organized in powers of $1/m$:

$$V = V^{(0)} + \frac{V^{(1)}}{m} + \frac{V^{(2)}}{m^2} + \cdots$$

where

$$\begin{aligned} V^{(2)} &= V^{(2)}_{SD} + V^{(2)}_{SI}, \\ V^{(2)}_{SI} &= \frac{1}{2}\left\{\mathbf{p}^2, V^{(2)}_{p^2}(r)\right\} + \frac{V^{(2)}_{L^2}(r)}{r^2}\mathbf{L}^2 + V^{(2)}_r(r), \\ V^{(2)}_{SD} &= V^{(2)}_{LS}(r)\,\mathbf{L}\cdot\mathbf{S} + V^{(2)}_{S^2}(r)\left(\frac{\mathbf{S}^2}{2} - \frac{3}{4}\right) + V^{(2)}_{S_{12}}(r)\mathbf{S}_{12}, \end{aligned}$$

The potentials $V^{(i)}_{\hat{O}}$ were obtained in terms of operator insertions in the expectation value of the rectangular Wilson loop. For instance for the $1/m$ correction we have

$$V^{(1)}(r) = -\lim_{T\to\infty}\int_0^T dt\, t\, \langle\langle g\mathbf{E}_1(t)\cdot g\mathbf{E}_1(0)\rangle\rangle_c,$$

where $\langle\langle \ldots \rangle\rangle \equiv \langle \ldots W_\square \rangle / \langle W_\square \rangle$ and

$$\langle\langle O_1(t_1) O_2(t_2) \rangle\rangle_c = \langle\langle O_1(t_1) O_2(t_2) \rangle\rangle - \langle\langle O_1(t_1) \rangle\rangle \langle\langle O_2(t_2) \rangle\rangle$$

Brambilla, Pineda, Soto and Vairo, *PRD 63, 014023 (2001)*

The other potentials follow the same way

Pineda and Vairo, *PRD 63, 054007 (2001)*

- In the short-distance regime, $r\Lambda_{QCD} \ll 1$, these correlators can be computed in perturbation theory.
- In the long-distance regime, $r\Lambda_{QCD} \sim 1$, these correlators should be computed in Lattice QCD.

Not all these correlators have been calculated in the Lattice. To evaluate the rates we take the intermediate approach of computing the correlators in the **effective string theory (EST)**:
The static quark-antiquark potential is given by

$$V^{(0)}(r) = \lim_{T, T_W \to \infty} \frac{i}{T} \ln \langle W_\square \rangle$$

where

$$W_{\Box} \equiv \mathrm{P}\exp\left\{-ig\oint_{r\times T_W} dz^{\mu}A_{\mu}(z)\right\},$$

is the rectangular Wilson loop. In the long distance limit this leads to a static potential with a linear, string-like, dependence in r:

$$V^{(0)} = \sigma r$$

where σ can be identified as the string tension.
The EST states that at long distances ($r\Lambda_{QCD} >> 1$)

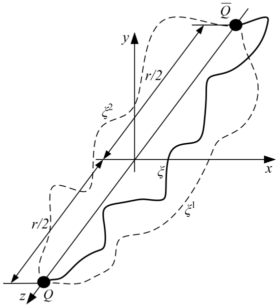
$$\lim_{T_W\to\infty}\langle 0|W_{\Box}(T_W,r)|0\rangle = Z\int \mathcal{D}\xi^1\mathcal{D}\xi^2\, e^{iS_{string}(\xi^1,\xi^2)}$$

where

$$S_{string} = \int dtdz\,\mathcal{L}(\partial^{\mu}\xi^l) = -\sigma\int dtdz\,\left(1-\frac{1}{2}\partial_{\mu}\xi^l\partial^{\mu}\xi^l\right),$$

Since both are expected to be valid at long distances, one could expect to have a mapping relating the operator insertions in the Wilson loop and degrees of freedom of the EST.

Using symmetry considerations the following mapping between the transverse string coordinates and the operator insertions in the correlators appearing in the relativistic corrections to the quark-antiquark potential has been obtained (**Perez-Nadal and Soto,** *PRD79, 114002 (2009)* & **Kogut and Parisi** *PRL47, 1089 (1981)*)



$$\psi^\dagger(t)\mathbf{E}^l(t,\frac{\mathbf{r}}{2})\psi(t) \mapsto \Lambda^2\partial_z\xi^l(t,\frac{r}{2})$$
$$\chi^\dagger(t)\mathbf{E}^l(t,-\frac{\mathbf{r}}{2})\chi(t) \mapsto -\Lambda^2\partial_z\xi^l(t,-\frac{r}{2})$$
$$\psi^\dagger(t)\mathbf{B}^l(t,\frac{\mathbf{r}}{2})\psi(t) \mapsto \Lambda'\epsilon^{lm}\partial_t\partial_z\xi^m(t,\frac{r}{2})$$
$$\chi^\dagger(t)\mathbf{B}^l(t,-\frac{\mathbf{r}}{2})\chi(t) \mapsto \Lambda'\epsilon^{lm}\partial_t\partial_z\xi^l(t,-\frac{r}{2})$$
$$\psi^\dagger(t)\mathbf{E}^3(t,\frac{\mathbf{r}}{2})\psi(t) \mapsto \Lambda''^2$$
$$\chi^\dagger(t)\mathbf{E}^3(t,-\frac{\mathbf{r}}{2})\chi(t) \mapsto -\Lambda''^2$$
$$\psi^\dagger(t)\mathbf{B}^3(t,\frac{\mathbf{r}}{2})\psi(t) \mapsto \Lambda'''\epsilon^{lm}\partial_t\partial_z\xi^l(t,\frac{r}{2})\partial_z\xi^m(t,\frac{r}{2})$$
$$\psi^\dagger(t)\mathbf{B}^3(t,-\frac{\mathbf{r}}{2})\psi(t) \mapsto \Lambda'''\epsilon^{lm}\partial_t\partial_z\xi^l(t,-\frac{r}{2})\partial_z\xi^m(t,-\frac{r}{2}).$$

Using this mapping for the $1/m$ correction to the potential we get

$$\langle\langle \mathbf{E}^l(t,\frac{\mathbf{r}}{2})\mathbf{E}^m(0,\frac{\mathbf{r}}{2})\rangle\rangle_c \mapsto \Lambda^4\partial_z\partial_{z'}\langle\xi^{\mathbf{l}}(\mathbf{t},\mathbf{r}/2)\xi^{\mathbf{m}}(\mathbf{0},\mathbf{r}/2)\rangle = \Lambda^4\partial_z\partial_{z'}\mathbf{G}_{\mathbf{F}}^{\mathbf{lm}}(\mathbf{t},\mathbf{r}/2;\mathbf{0},\mathbf{r}/2)$$

where the mapping into the EST is expressed in terms of the two point correlator of the string degrees of freedom:

$$G_F^{lm}(it,z;it',z') = \frac{\delta_{lm}}{4\pi\sigma}\ \ln\left(\frac{\cosh(\frac{\pi}{r}(t-t'))+\cos(\frac{\pi}{r}(z+z'))}{\cosh(\frac{\pi}{r}(t-t'))-\cos(\frac{\pi}{r}(z-z'))}\right)$$

then the potential is given by

$$\begin{aligned} V^{(1)}(r) \quad &\mapsto \quad -g^{2\Lambda^4}\lim_{\epsilon\to 0+}\int_{\epsilon}^{\infty} dt\, t\,\partial_z\partial_{z'}G_F^{ll}(t,r/2;0,r/2) \\ &= \quad \frac{2g^2\Lambda^4}{\sigma\pi}\lim_{\epsilon\to 0+}\int_{\frac{\pi\epsilon}{2r}}^{\infty} dt\,\frac{t}{\sinh^2(t)} \end{aligned}$$

$$V^{(1)} = \frac{2g^2\Lambda^4}{\sigma\pi}\ln(\sqrt{\sigma}r) + \text{Infinite constant}$$

The potentials in the EST read

$$V^{(0)} = \sigma r,$$

$$V^{(1)}(r) = \frac{2g^2\Lambda^4}{\pi\sigma}\ln(\sqrt{\sigma}r) + \mu_1\,,$$

$$V^{(2)}_{p^2}(r) = 0\,,$$

$$V^{(2)}_{L^2}(r) = -\frac{g^2\Lambda^4 r}{6\sigma}\,,$$

$$V^{(2)}_{LS}(r) = -\frac{\mu_2}{r} - \frac{2c_F^{(1)}g^2\Lambda^2\Lambda'}{\sigma\, r^2}\,,$$

$$V^{(2)}_{S^2}(r) = \frac{2\pi^3 c_F^{(1)}c_F^{(2)}g^2\Lambda'''^{\,2}}{45\sigma^2 r^5}$$

$$V^{(2)}_{\mathbf{S}_{12}}(r) = \frac{\pi^3 c_F^{(1)}c_F^{(2)}g^2\Lambda'''^{\,2}}{90\sigma^2 r^5}\,,$$

$$V^{(2)}_{r}(r) = -\frac{9\,\zeta_3\, g^4\Lambda^8 r}{2\pi^3\sigma^2} + \mu_3 + \frac{\mu_4}{r^2} + \frac{\mu_5}{r^4} + \frac{\pi^3 c_F^{(1)\,2}g^2\Lambda'''^{\,2}}{30\sigma^2 r^5}.$$

Keeping the LO terms in r, dropping the renormalization constants and applying further known constraints to the potentials (i.e. Poincare invariance) the equal mass long-range $Q\bar{Q}$ potential can be reduced to

$$V(r)^{\mathrm{long-range}} \approx \sigma r + \frac{1}{m}\left[\frac{2\sigma}{\pi}\ln(\sqrt{\sigma}r)\right] + \frac{1}{m^2}\left(-\frac{\sigma}{6r}\mathbf{L}^2 - \frac{\sigma}{2r}\mathbf{L}\cdot\mathbf{S} - \frac{9\,\zeta_3\,\sigma^2 r}{2\pi^3}\right)$$

N. Brambilla, M. Groher, H.M. and A. Vairo, *arXiv:1407.7761*

There is some freedom in the way one can construct the potential for the full r range, for instance, one could introduce a cut-off, r_c, to divide between the short and long distance regimes:

$$V(r) = \begin{cases} V^{\text{pert.}}(r) & \text{if } r < r_c \\ V^{\text{long-range}}(r) & \text{if } r > r_c \end{cases}$$

In a first attempt to evaluate the rates we took the simpler approach of just sum both contributions:

$$V(r) = V^{\text{pert.}}(r) + V^{\text{long-range}}(r)$$

The perturbative potentials appearing in the relativistic corrections are given by

$$V^{(0)} = -C_F \frac{\alpha_{V_s}(r)}{r}$$

$$V^{(1)} = -\frac{C_F C_A \alpha_s(r)^2}{2r^2}$$

$$V^{(2)}_{p^2} = -\frac{C_F \alpha_s(r)}{r}$$

$$V^{(2)}_{L^2} = \frac{C_F \alpha_s(r)}{2r}$$

$$V^{(2)}_{r} = \pi C_F \alpha_s(r) \delta^{(3)}(\mathbf{r})$$

$$V^{(2)}_{S^2} = \frac{4\pi C_F \alpha_s(r)}{3} \delta^{(3)}(\mathbf{r})$$

$$V^{(2)}_{LS} = \frac{3 C_F \alpha_s(r)}{2r^3}$$

$$V^{(2)}_{S_{12}} = \frac{C_F \alpha_s(r)}{4r^3}$$

Buchmüller et al., *PRD 24, 3003 (1981)*
Gupata and Radford , *PRD 24, 2307 (1981)*
Gupata and Radford , *PRD 25, 3430 (1982)*
Pantaleone et al. , *PRD 33, 777 (1986)*
Titard and Yndurain, *PRD 49, 6007 (1994)*

We take the further simplification of freezing $\alpha_{V_s}, \alpha_s(r) \mapsto a$ where a will be a parameter that we will fix later.
In this approach the quark-antiquark potential is given by

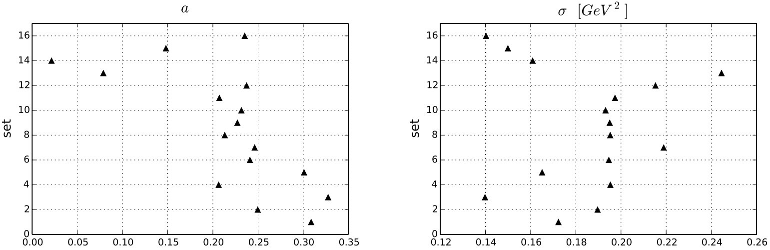
$$
\begin{aligned}
V(a,\sigma,m,r) \quad = \quad & \overbrace{-\frac{C_F a}{r} + \sigma r}^{\equiv V^{(0)}} + \frac{1}{m} \overbrace{\left\{ -\frac{C_F C_A a^2}{2r^2} + \frac{2\sigma}{\pi} \log(\sqrt{\sigma} r) \right\}}^{\equiv V^{(1)}} \\
+ \quad & \underbrace{\frac{1}{m^2} \left\{ \frac{1}{2} \left\{ \mathbf{p}^2, -\frac{C_F a}{r} \right\} + \left(\frac{C_F a}{2r} - \frac{\sigma}{6r} \right) \mathbf{L}^2 + \left(\frac{3 C_F a}{2r^3} - \frac{\sigma}{2r} \right) \mathbf{L}\cdot\mathbf{S} \right.
+ \left. \frac{4\pi C_F a}{3} \delta^{(3)}(\mathbf{r}) \mathbf{S}^2 + \frac{C_F a}{4r^3} \mathbf{S}_{12}(\hat{\mathbf{r}}) + \pi C_F a \delta^{(3)}(\mathbf{r}) - \frac{9\,\zeta_3\,\sigma^2 r}{2\pi^3} \right\}}_{\equiv V^{(2)}}
\end{aligned}
$$

Evaluation of the transition rates

Parameter fixing:

We fix the parameters of the potential requiring them to reproduce the masses of quarkonium states:

$$\underbrace{2m_{c,b} + E_{nl}^{(0)}}_{LO} + \underbrace{\frac{\langle nl|V^{(1)}(r)|nl\rangle}{m_{c,b}}}_{NLO} + \underbrace{\frac{1}{m_{c,b}^2}\sum_{m\neq n}^{\infty}\frac{|\langle nl|V^{(1)}(r)|ml\rangle|^2}{E_{nl}^{(0)} - E_{ml}^{(0)}} + \frac{\langle nljs|V^{(2)}(r)|nljs\rangle}{m_{c,b}^2}}_{NNLO} = M(n^{2s+1}L_J)$$



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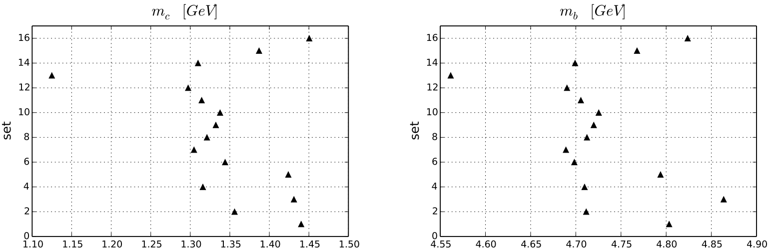
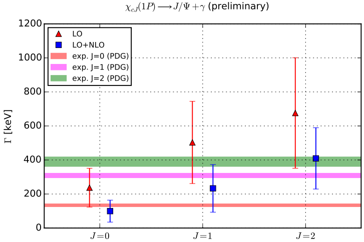
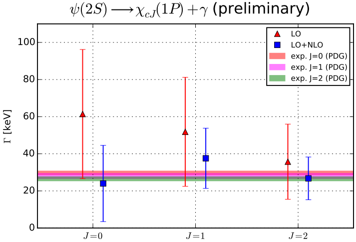
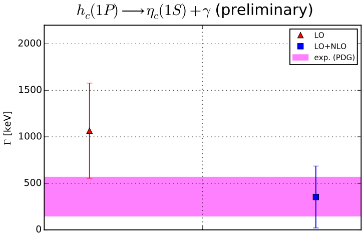


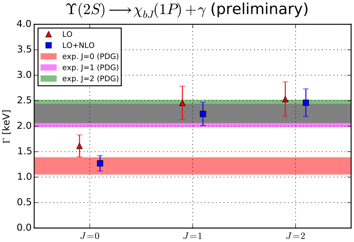
Results: Charmonium

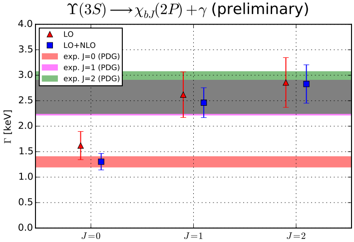






Results: Bottomonium





Summary and Outlook

- The NLO relativistic corrections get us closer to the experimental values.
- Next step: elaborate more on the perturbative potential.
- A pure perturbative potential can be suitable to describe the $1P \to 1S\gamma$ process (work in progress).

THANKS FOR YOUR ATTENTION!

Backup

Comparison with the recent BaBar results (arXiv:1410.3902)