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An Improved Study of the Excited Radiative Decay $\Upsilon(2S) \rightarrow \eta_b(1S)\gamma$ Using Lattice NRQCD

C. Hughes, R. Dowdall, G. Von Hippel,

R. Horgan, M. Wingate

Motivation : $\Upsilon(2S) \rightarrow \eta_b(1S)\gamma$

- Spin singlets: η_b

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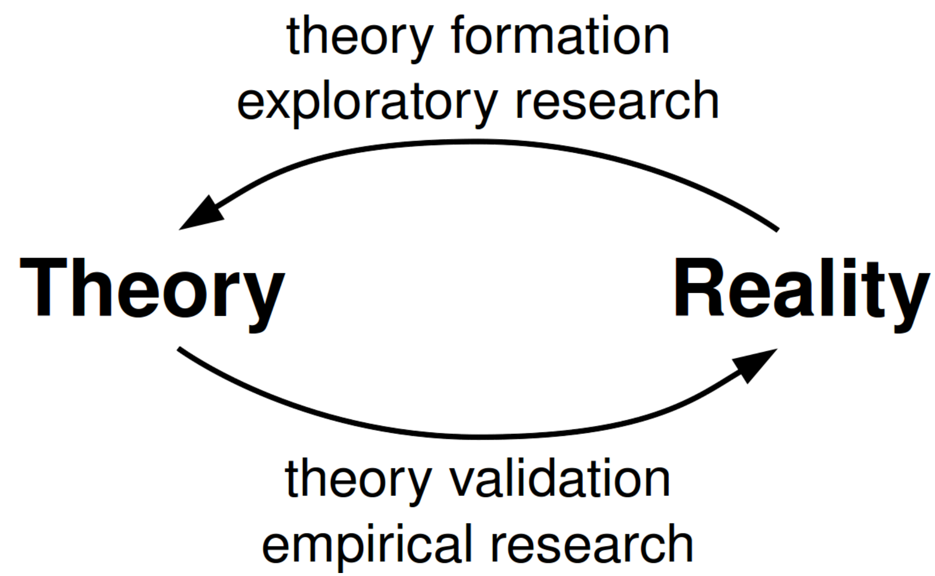
- Insight into heavy quark bound states in QCD
- Exclusion of parity odd Higgs
- Laboratory to test relativistic effects: NRQCD =? Experiment

Decay Rate: $\Upsilon(2S) \to \eta_b(1S)\gamma$

$$v^2 \sim 0.1,\ |\mathbf{q}_\gamma| \sim 0.6\text{GeV}$$

$$\mathcal{B}(\Upsilon(2S) \to \eta_b(1S)\gamma) = (3.9 \pm 1.5) \times 10^{-4}$$



Decay Rate: $\Upsilon(2S) \to \eta_b(1S)\gamma$

$$\langle \eta_b(p_i)|J^\mu(0)|\Upsilon(p_f, s_\Upsilon)\rangle = \frac{2\mathbf{V}(\mathbf{Q^2})}{M_\Upsilon + M_{\eta_b}} \varepsilon^{\mu\alpha\beta\tau} p_{i,\alpha} p_{f,\beta} \epsilon_{\Upsilon,\tau}(p_f, s_\Upsilon)$$

$$\Gamma_{\Upsilon\to\eta_b\gamma} = \alpha_{QED} e_q^2 \frac{16}{3} \frac{|\mathbf{q}|^3}{(M_\Upsilon + M_{\eta_b})^2} \left|\mathbf{V}^{\mathrm{lat}}(\mathbf{Q^2 = 0})\right|^2$$

Currents and Power Counting from NRQCD + NRQED

$$|\mathbf{q}_\gamma| \sim mv^2, v^2 \sim 0.1$$

$$M4 : \frac{\omega_4}{2M}\psi_b^\dagger \sigma \cdot \mathbf{B}^{\mathbf{QED}} \psi_b \sim |\mathbf{q}_\gamma|^2 \sim v^4$$

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Potential Model Predictions: S. Godfrey, J. L. Rosner, arXiv:hep-ph/0104253

Table III: Predictions for overlap integrals and branching ratios in hindered M1 transitions between n^3S_1 and n'^1S_0 $b\bar{b}$ levels, taking into account relativistic corrections.

	$n=2,\ n'=1$ $k=$ [illegible] MeV		$n=3,\ n'=1$ $k=908$ MeV		$n=3,\ n'=2$ $k=352$ MeV	
Reference	$\|I\|$	$\mathcal{B}$ (10^{-4})	$\|I\|$	$\mathcal{B}$ (10^{-4})	$\|I\|$	$\mathcal{B}$ (10^{-4})
ZB83 [8]	0.08	15	0.041	22	0.095	7.0
GOS84 [9] (a)	(b)	7.9	(b)	(b)	(b)	(b)
GOS84 [9] (c)	(b)	5.4	(b)	(b)	(b)	(b)
GI85 [10] (d)	0.05	7.4	0.029	11	0.054	2.2
GI85 [22] (e)	0.08	13	0.043	25	0.078	4.7
ZSG91 [12] (f)	0.02	1.4	(b)	(b)	(b)	(b)
ZSG91 [12] (g)	0.00	~ 0	(b)	(b)	(b)	(b)
LNR99 [14] (h)	(b)	0.46	(b)	1.4	(b)	0.13
LNR99 [14] (i)	(b)	0.05	(b)	0.05	(b)	0.40

[a] Scalar confining potential. [b] Not quoted.
[c] Vector confining potential.
[d] Based on quoted transition moments.
[e] Based on matrix elements between $^3S_1$ and $^1S_0$ wave functions.
[f] Scalar-vector confining potential of Ref. [21].
[g] Scalar confining potential of Ref. [21].
[h] Without exchange current. [i] With exchange current.

VS.

Experimental Result (Babar): K.A. Olive *et. al.* (Particle Data Group)

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$$M6 : \frac{\omega_7}{2M^3}\psi_b^\dagger \{\mathbf{D}^2, \sigma \cdot \mathbf{B}^{\mathbf{QED}}\}\psi_b \sim v^2|\mathbf{q}_\gamma|^2 \sim v^6$$

$$E4 : \frac{i\omega_3}{8M^2}\psi_b^\dagger \sigma \cdot [\mathbf{D}\times, \mathbf{E}^{\mathbf{QED}}]\psi_b \sim |\mathbf{q}_\gamma|^3 \sim v^6$$

$$E6 : \frac{3i\omega_8}{64M^4}\psi_b^\dagger \sigma \cdot \{\mathbf{D}^2, [\mathbf{D}\times, \mathbf{E}^{\mathbf{QED}}]\}\psi_b \sim v^2|\mathbf{q}_\gamma|^3 \sim v^8$$

Currents and Power Counting from NRQCD + NRQED

Need matching coefficient of L.O. Current Operator:

$$M4 : \frac{\omega_4}{2M}\psi_b^\dagger \sigma \cdot \mathbf{B}^{\mathbf{QED}} \psi_b \sim |\mathbf{q}_\gamma|^2 \sim v^4 \Longrightarrow \sim v^5$$

This study finds (including mixing from higher order currents):

$$\omega_4 \approx 1.03 + \mathcal{O}(\alpha_s^2)$$

NRQCD Action

$$
\begin{aligned}
aH &= aH_0 + a\delta H_{v^4} + a\delta H_{v^6} \\
aH_0 &= -\frac{\Delta^{(2)}}{2am_b}, \\
a\delta H_{v^4} &= -c_1 \frac{(\Delta^{(2)})^2}{8(am_b)^3} + c_2 \frac{i}{8(am_b)^2} \left(\nabla \cdot \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \cdot \nabla \right) \\
&\quad - c_3 \frac{1}{8(am_b)^2} \sigma \cdot \left(\tilde{\nabla} \times \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \times \tilde{\nabla} \right) \\
&\quad - c_4 \frac{1}{2am_b} \sigma \cdot \tilde{\mathbf{B}} + c_5 \frac{\Delta^{(4)}}{24am_b} - c_6 \frac{(\Delta^{(2)})^2}{16n(am_b)^2}, \\
a\delta H_{v^6} &= -c_7 \frac{1}{8(am_b)^3} \left\{ \Delta^{(2)}, \sigma \cdot \tilde{\mathbf{B}} \right\} \\
&\quad - c_8 \frac{3i}{64(am_b)^4} \left\{ \Delta^{(2)}, \sigma \cdot \left(\tilde{\nabla} \times \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \times \tilde{\nabla} \right) \right\} \\
&\quad + c_9 \frac{1}{8(am_b)^3} \sigma \cdot \tilde{\mathbf{E}} \times \tilde{\mathbf{E}}
\end{aligned}
$$

Lattice Methodology

Two Point Calculation

1. Build interpolating operators $\mathcal{O}(n, t_0)$,

   which overlap with states having specific quantum numbers J^{PC}, e.g., Υ, η_b

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NB: Get on-shell photon data numerically by using twisted boundary conditions with twist:

$$\theta_{nm} = \left(\left(\frac{m^2_{\Upsilon(nS)} + m^2_{\eta_b(mS)}}{2m_{\Upsilon(nS)}} \right)^2 - m^2_{\eta_b(mS)} \right)^{\frac{1}{2}}$$

Three Point Calculation

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NB: Details swept under the rug, ask if interested!

Bayesian Fitting

- Simultaneously fit two point correlator for Υ, η_b data to

$$C_{2pt}(n_{src}, n_{snk}) = \sum_{i}^{m} a_i(n_{src}) a_i(n_{snk}) \exp(-E_i t)$$

and three point correlator data in order to

$$C_{3pt}(n_{src}, n_{snk}) = \sum_{i,f}^{m} a_i(n_{src}) V_{i,f} b_f(n_{snk}) \exp(-E_i t) \exp(-E_f(T-t))$$

and extract what we need: $V_{i,f}$

Coulomb Gauge Fixed Ensembles

MILC Configurations ($n_f = 2+1+1$ HISQ)

Set	β	a_Υ (fm)	am_l	am_s	am_c	$L \times T$	n_{cfg}
1	5.8	0.1474(5)(14)(2)	0.013	0.065	0.838	16×48	1020
2	6.0	0.1219(2)(9)(2)	0.0102	0.0509	0.635	24×64	1052
3	6.3	0.0884(3)(5)(1)	0.0074	0.037	0.440	32×96	1008

$$m_\pi^{lat} \approx 300\text{MeV}$$

Previous Lattice Calculation

R. Lewis, R. Woloshyn, arXiv:1207.3825 first study of the decay includes:

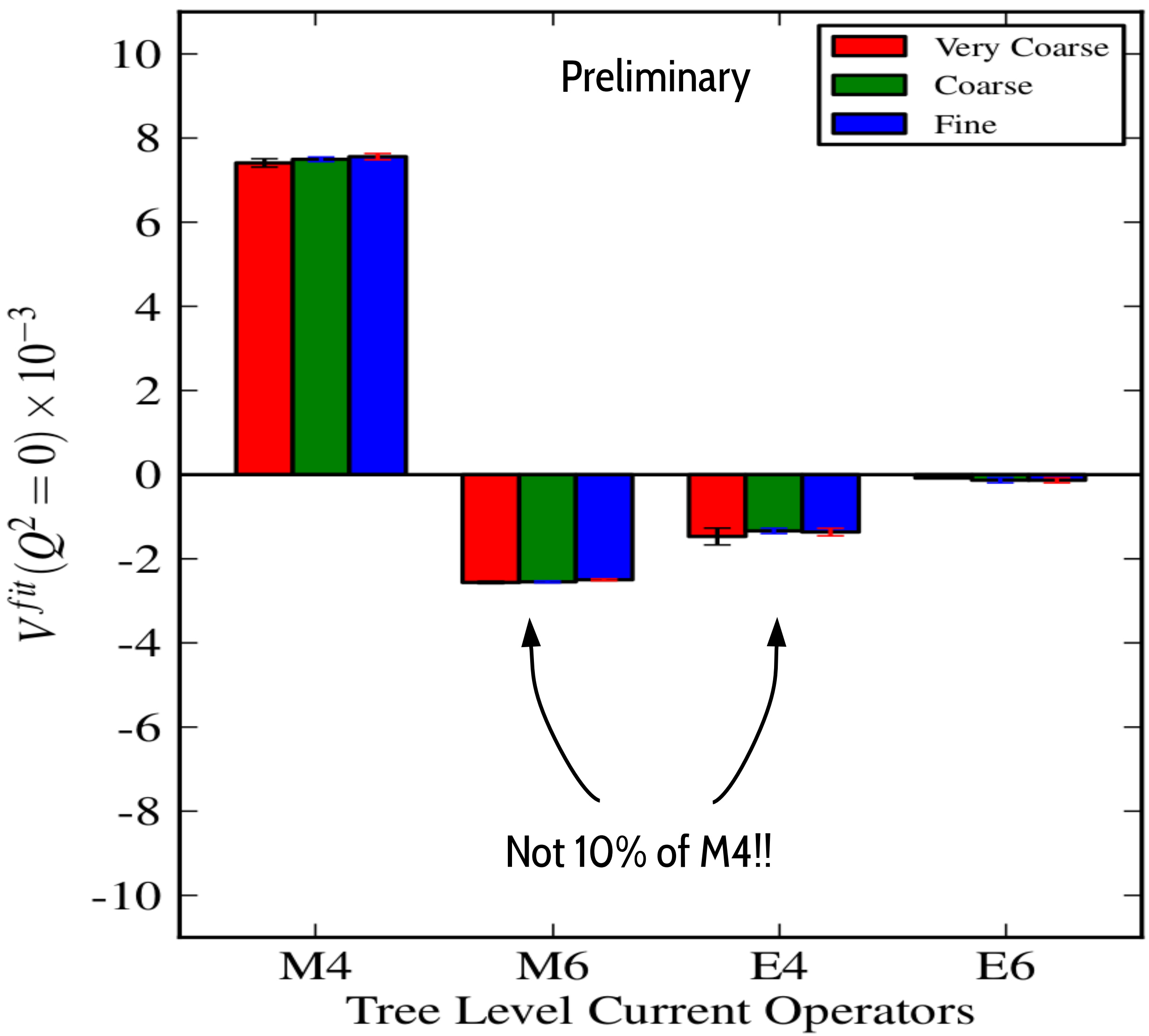
- One gluon ensemble
- 192 gauge fields and 16 time sources
- No radiative corrections to action or currents
- Extrapolate to on-shell photon matrix element using off-shell data

Our Improved Lattice Calculation

This study of the decay includes:

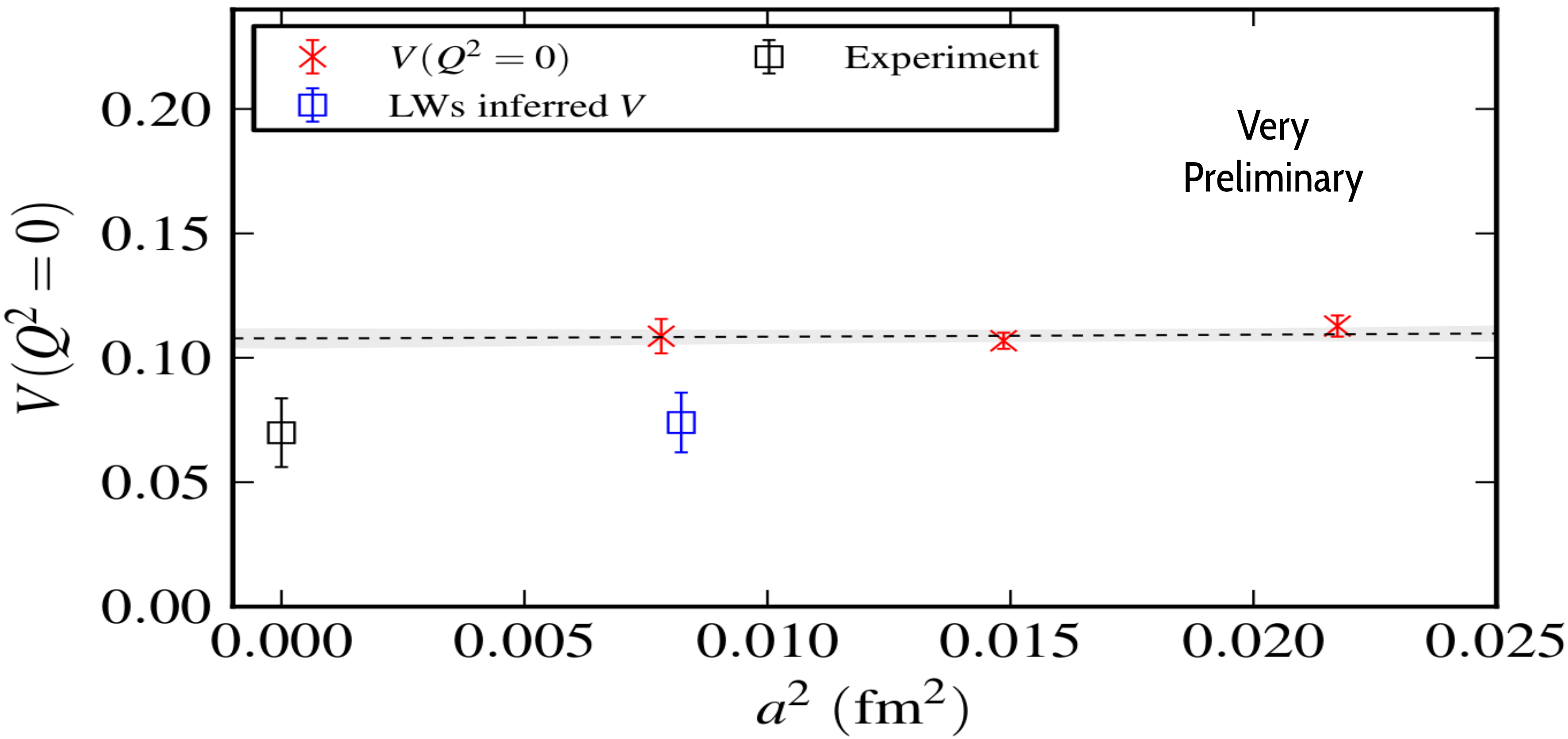
- Three gluon ensembles
- ~1000 gauge fields and 16 time sources, including physical pion mass.
- Order alpha in v^4 Action and Order alpha in L.O. Current radiative corrections – N.B!!
- On-shell photon data

Results for Radiatively Improved $\mathcal{O}(v^4)$ Action with $\mathcal{O}(v^6)$ Corrections



Results for Radiatively Improved $\mathcal{O}(v^4)$ Action with $\mathcal{O}(v^6)$ Corrections

$$V(Q^2) = \sum_i^{currents} V^i(Q^2)$$ with statistical errors only



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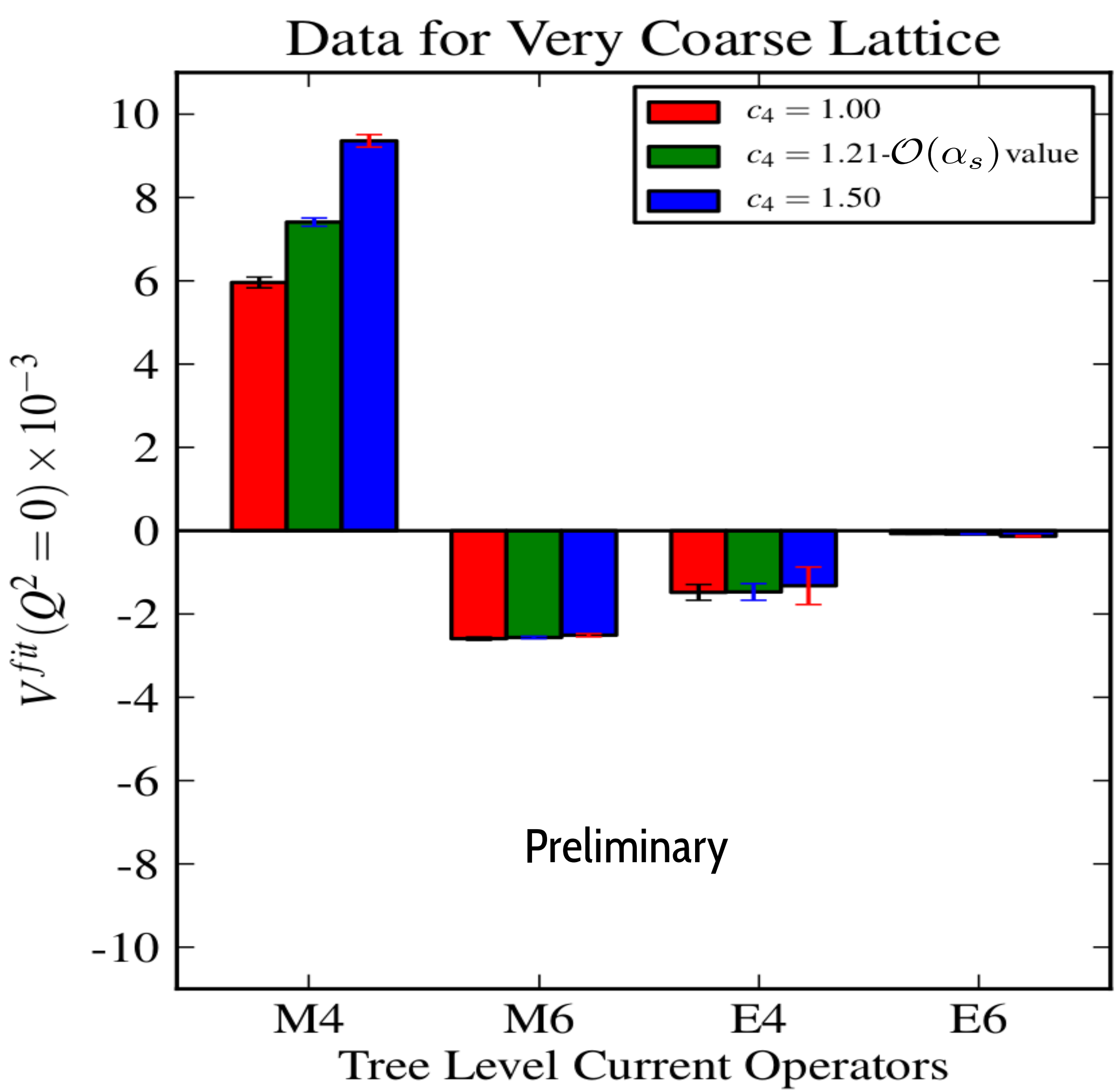

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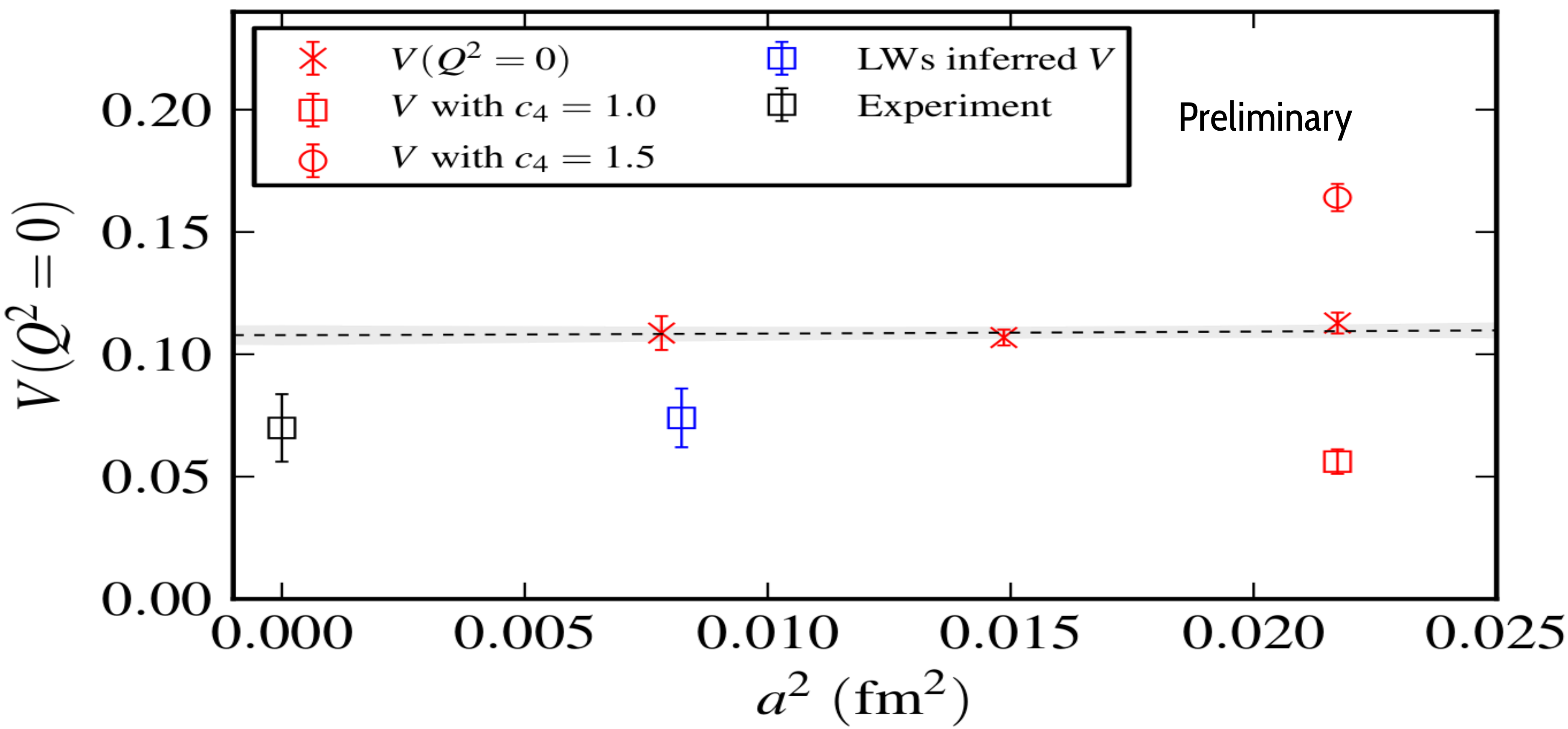

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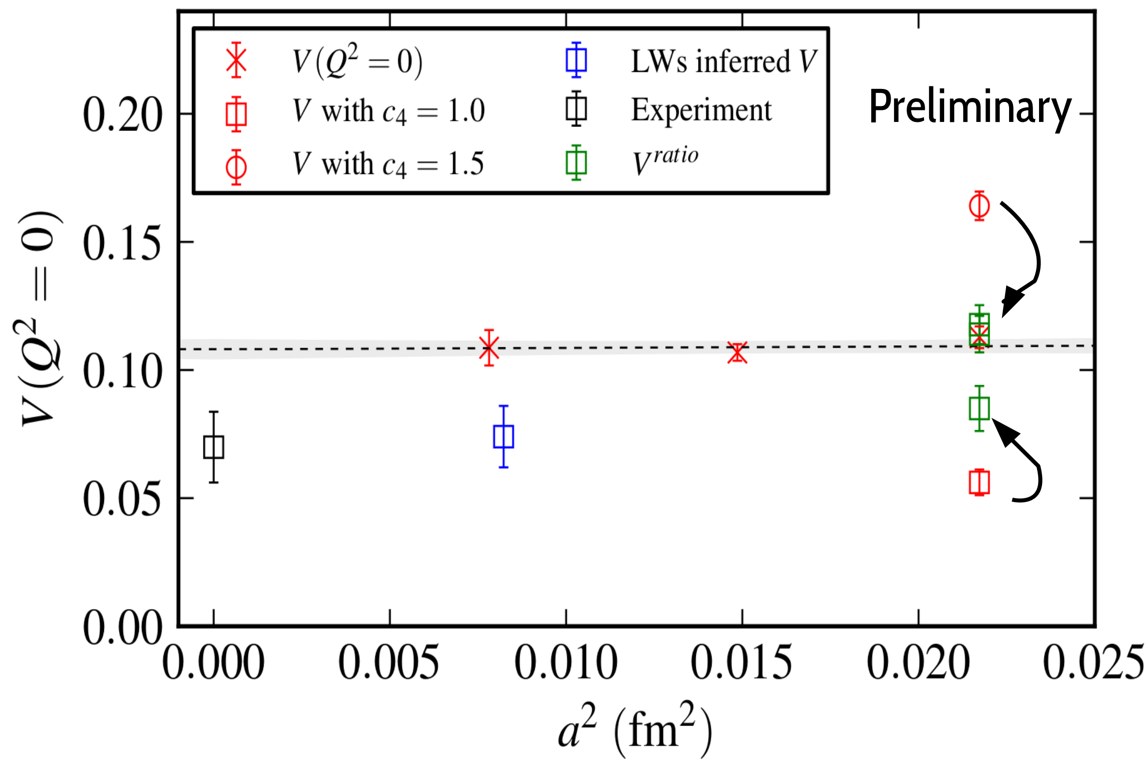
Remove sensitivity to c_4

$$V^{ratio} = \frac{(M_{\Upsilon(1S)} - M_{\eta_b(1S)})^{exp}}{(M_{\Upsilon(1S)} - M_{\eta_b(1S)})^{lat}} V$$



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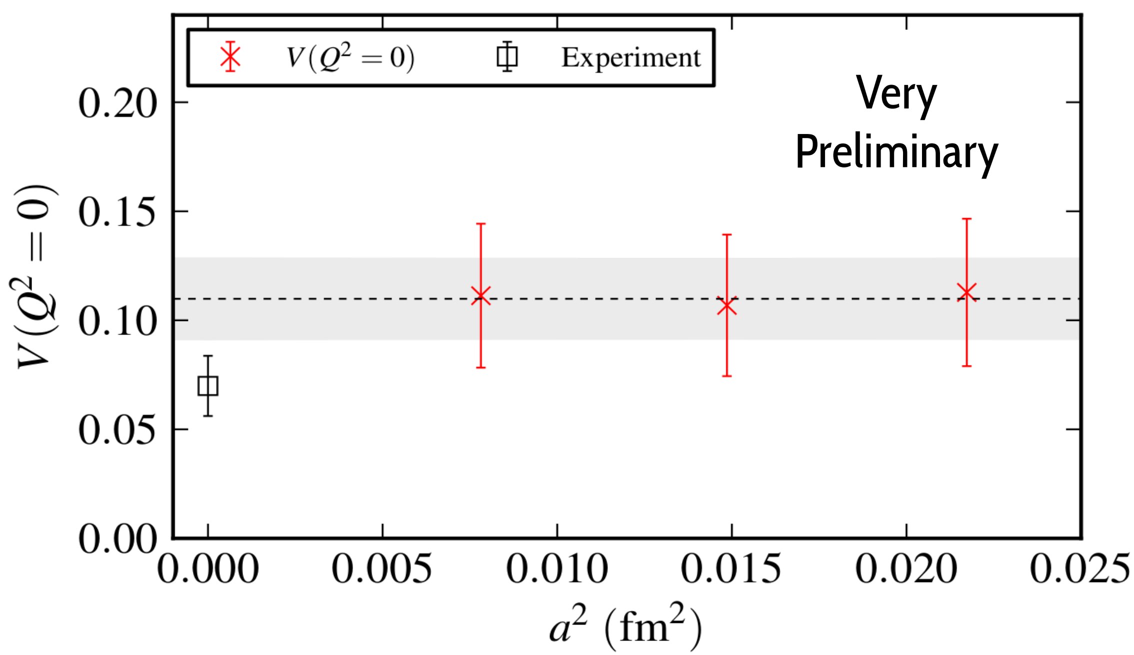
Introduce a 10% systematic error on L.O. current from neglecting $\mathcal{O}(\alpha_s^2)$ terms in matching, and 30% in all other currents from neglecting $\mathcal{O}(\alpha_s)$ terms.



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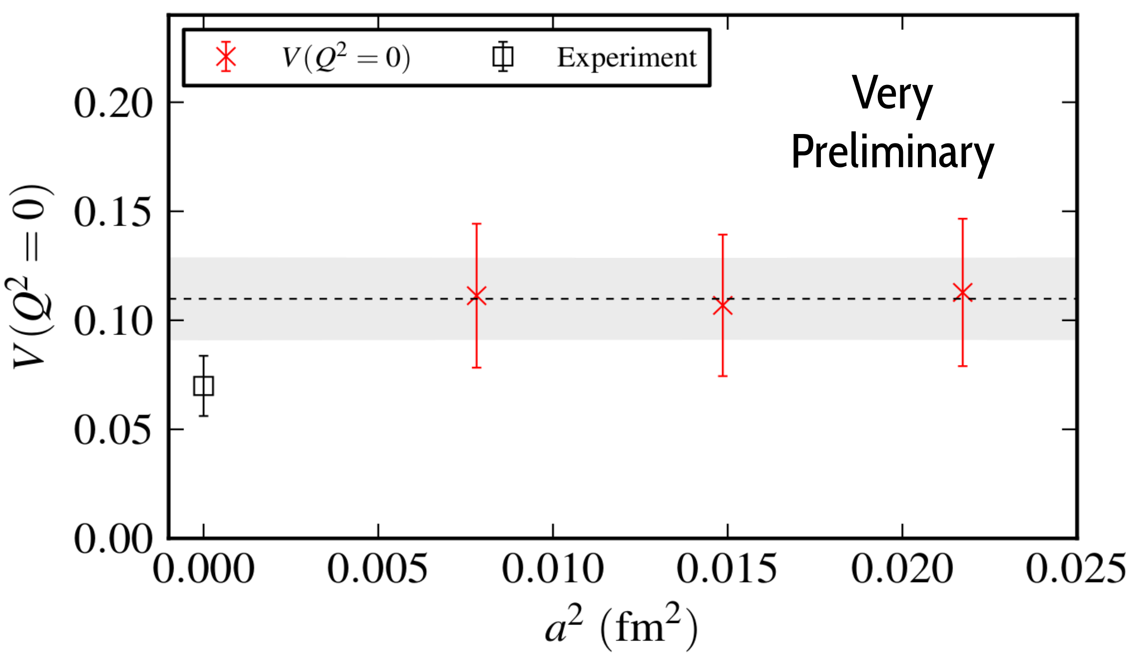

NB: Progress Report. Take with pinch of salt, further analysis of sensitivity and full error budget still needs to be done.

Summary

- L.O. current suppressed due to orthogonality of radial wavefunctions

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  - Relativistic corrections in current (Need multiple current corrections)
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- Lattice NRQCD is a first principals tool that has been systematically improved.
- Can aid in reliably pinning down this difficult to predict decay.

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Questions

Back Up Slides

Decay Rate: $\Upsilon(2S) \to \eta_b(1S)\gamma$

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Non-Relativistic QCD

- Is an Effective Field Theory to describe b-quarks
- Has expansion parameter : $v^2 \sim 0.1$
- Shows up in two places: the propagator and interaction Lagrangian (which also has NRQED)

Potential Model for L.O. Current

$$\Gamma_{\Upsilon\to\eta_b\gamma} = \alpha_{QED} e_q^2 \frac{4}{3m_b^2} |\mathbf{q}_\gamma|^3 \left| \int r^2 dr \phi^*_{\eta_b}(1S) j_0(\frac{|\mathbf{q}_\gamma| r}{2}) \phi_\Upsilon(2S) \right|^2$$

$$V(Q^2)_{nm} \propto \int r^2 dr \phi^*_{\eta_b}(mS) j_0(\frac{|\mathbf{q}_\gamma| r}{2}) \phi_\Upsilon(nS)$$

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- $V(Q^2)^{\mathrm{Hyd}}_{21} \propto \underbrace{\frac{a_0^2|\mathbf{q}|^2}{16}}_{\sim v^2} \left(1 + \frac{a_0^2|\mathbf{q}|^2}{16}\right)^{-3} \quad \xrightarrow{|\mathbf{q}| \to 0} 0$

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$\Longrightarrow$ Suppressed more than naively expected!!
Difficult to predict in potential model.

Fitting

$$V(a^2, am_b) = V_{\text{phys}} \times \left[1 + \sum_{j=1,2} k_j (a\Lambda)^{2j} (1 + k_{jb}\delta x_m + k_{jbb}(\delta x_m)^2)\right]. \quad (4)$$

The lattice spacing dependence is set by a scale $\Lambda = 500$ MeV, and $\delta x_m = (am_b - 2.7)/1.5$ allows for mild dependence on the effective theory cutoff am_b. We take priors of 0(1) on all the coefficients except k_1 which is 0.0(3) since the action includes radiatively improved a^2 lattice spacing corrections. We have tested that our results are not sensitive to the fit form or the priors.