Update on the determination of α_s from the QCD static energy

Xavier Garcia i Tormo
Universität Bern

Based on:

A. Bazavov, N. Brambilla, XGT, P. Petreczky, J. Soto and A. Vairo, Phys. Rev. D **86**, 114031 (2012) [arXiv:1205.6155 [hep-ph]];

Phys. Rev. D **90**, 074038 (2014) [arXiv:1407.8437 [hep-ph]]

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Does the uncertainty in the world average really reflects our current understanding of the value of α_s?
Increasing corroboration of α_s value, by extracting it from independent quantities, is crucial; exhaustively analyze theoretical errors entering in each determination.

α_s from the QCD static energy

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$$E_0(r) \sim -C_F \frac{\alpha_s(1/r)}{r} \left(1 + O(\alpha_s) + O(\alpha_s^2) + O(\alpha_s^3, \alpha_s^3 \ln \alpha_s) + \cdots\right)$$

Fischler'77; Billoire'80; Peter'96'97; Schröder'98; Kniehl Penin Steinhauser Smirnov'01; Smirnov Smirnov Steinhauser'08'10; Anzai Kiyo Sumino'10; Brambilla Pineda Soto Vairo'99; Kniehl Penin'99; Brambilla X.G.T. Soto Vairo'06

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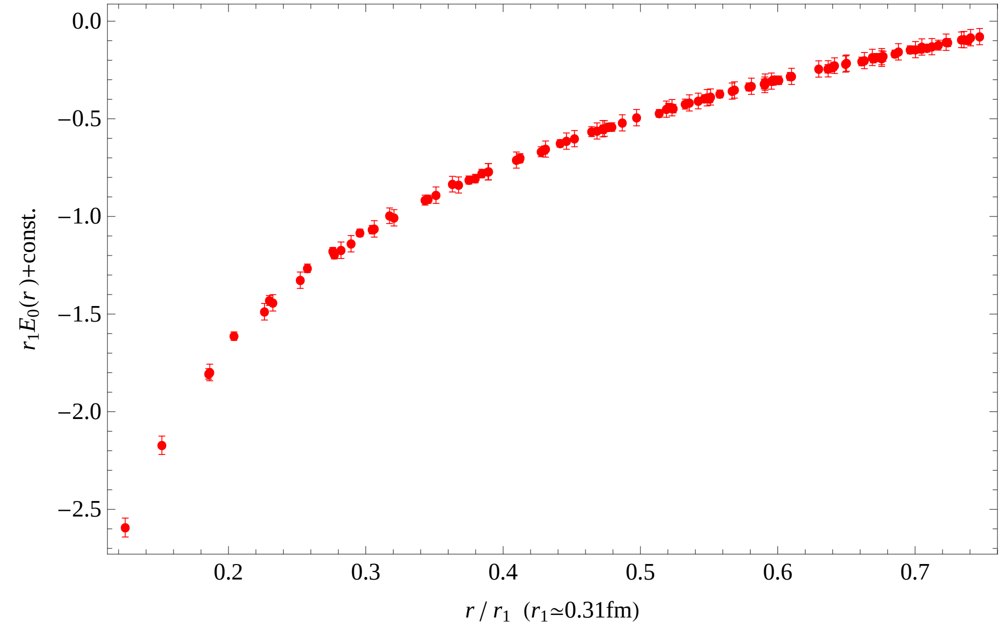

Data ($n_f = 2+1$): Bazavov *et al.* (HotQCD Coll.)'14

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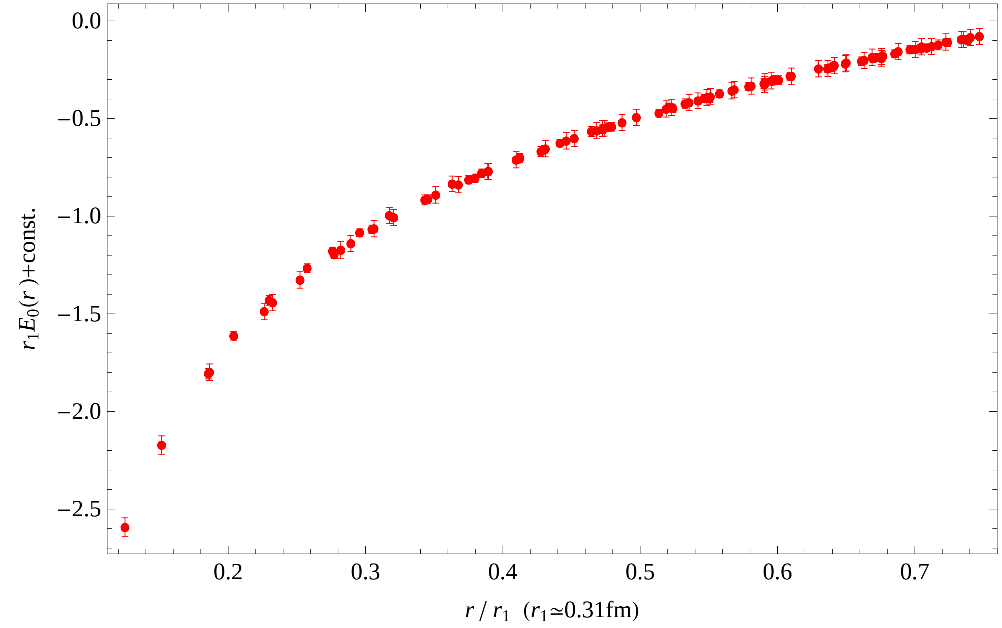

Update: New data and at shorter distances.
Allow us to perform more detailed analyses

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Perturbative expression best suited for the comparison. Use pert. expression for the force

$$E_0 \sim -\frac{C_F}{r}\alpha_E(r,\nu) + RS(\rho)$$

Beneke'98; Hoang *el al.*'99; Pineda'01

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$\alpha_E(r,\nu)$: series in $\alpha_s(\nu)$, contain $\ln(r\nu)$ terms
$RS(\rho)$: series in $\alpha_s(\rho)$, affected by uncertainties in computation of renormalon

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$$E_0 \sim -\frac{C_F}{r}\alpha_E(r,\nu) + RS(\rho) \rightarrow$$

$$\rightarrow F(r,\nu) = \frac{dE_0}{dr} = \frac{C_F}{r^2}\left(\alpha_E(r,\nu) - r\alpha'_E(r,\nu)\right)$$

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No logs, not affected by renormalon uncertainties

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- Integrate numerically, compare with data for energy, $E_0(r) + const.$

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Additionally one can

- Put all lattice data together *normalization errors*
- Analysis for each value of lattice spacing (β), then take average *less data each set*

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Analyses α_s extraction

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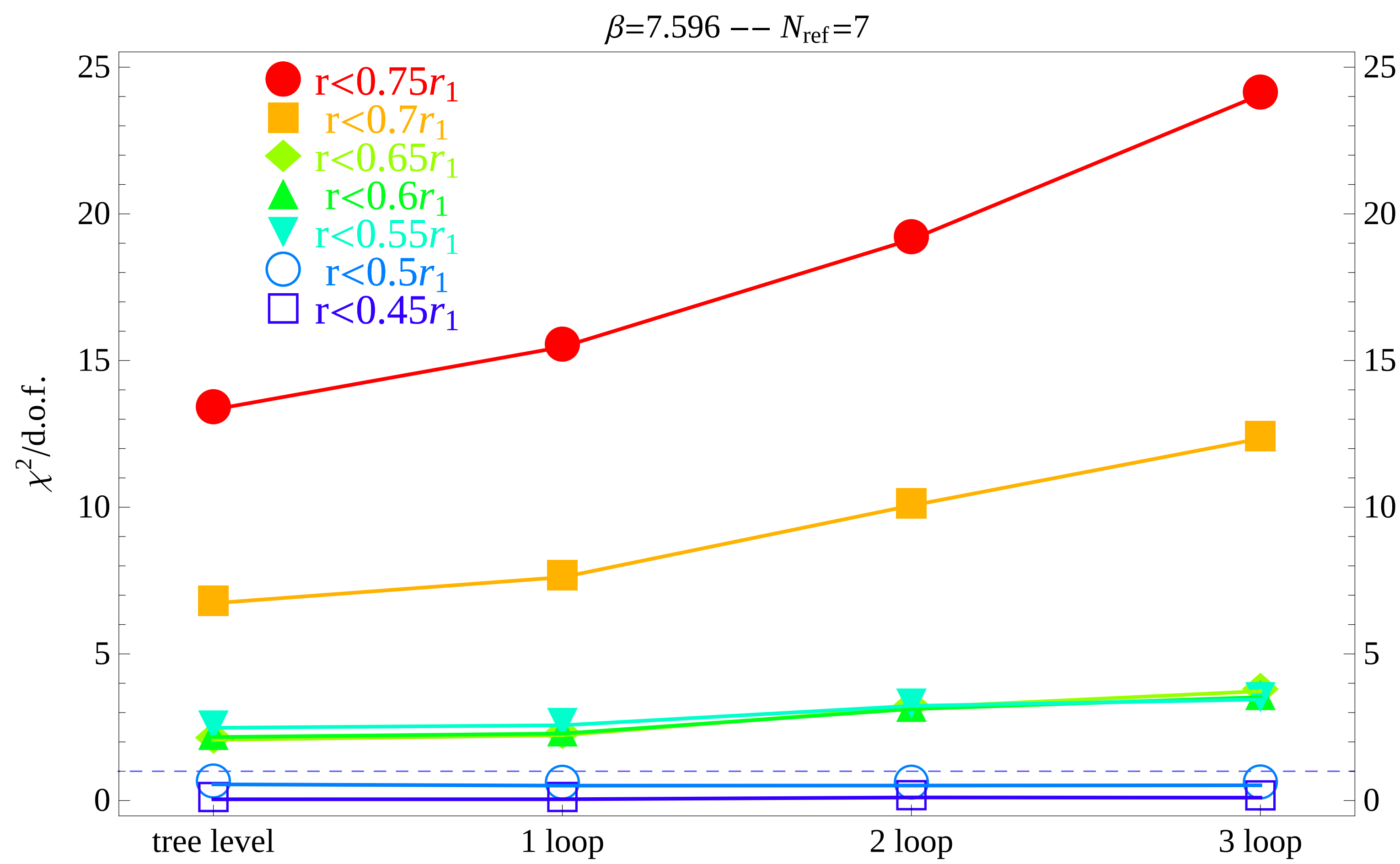
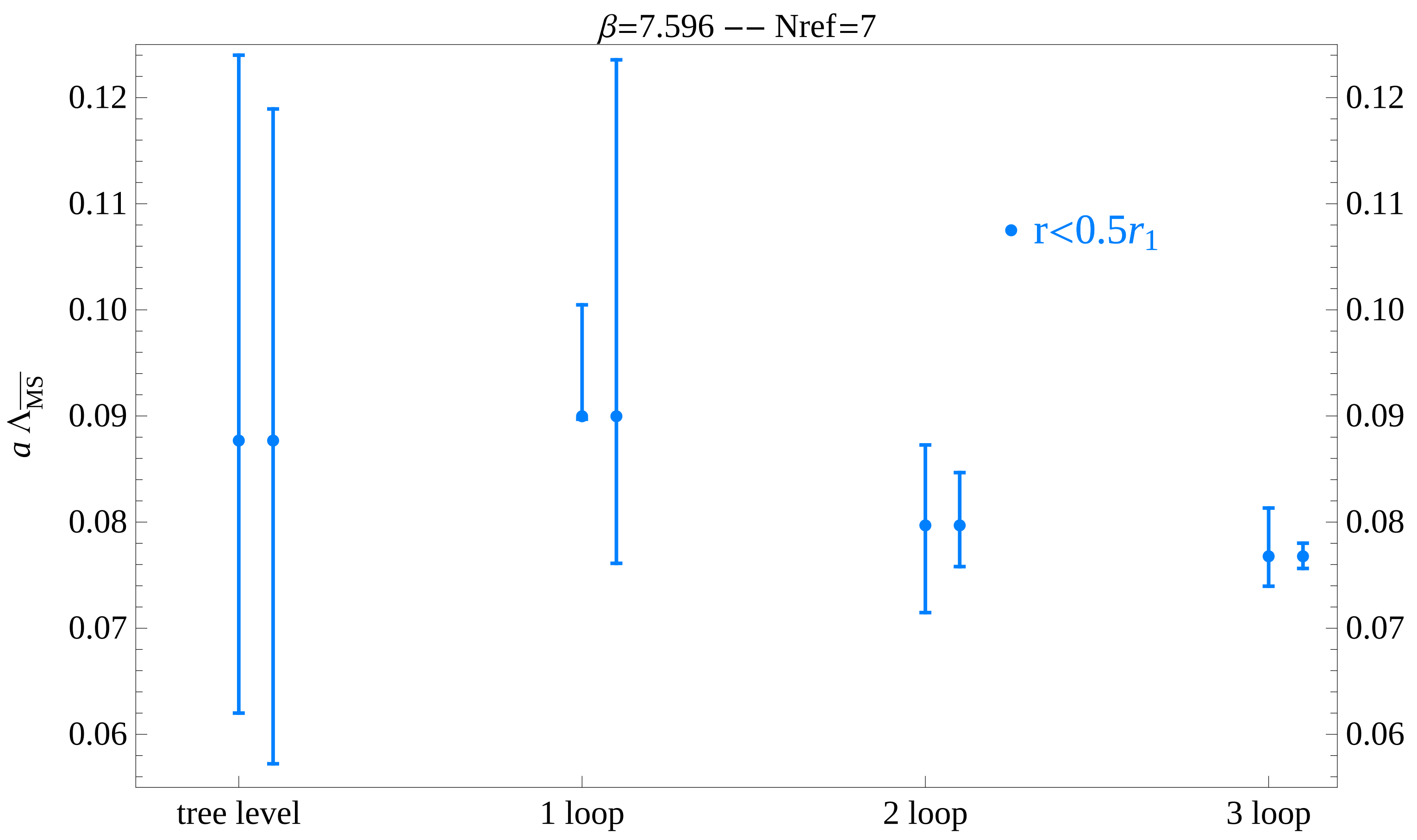
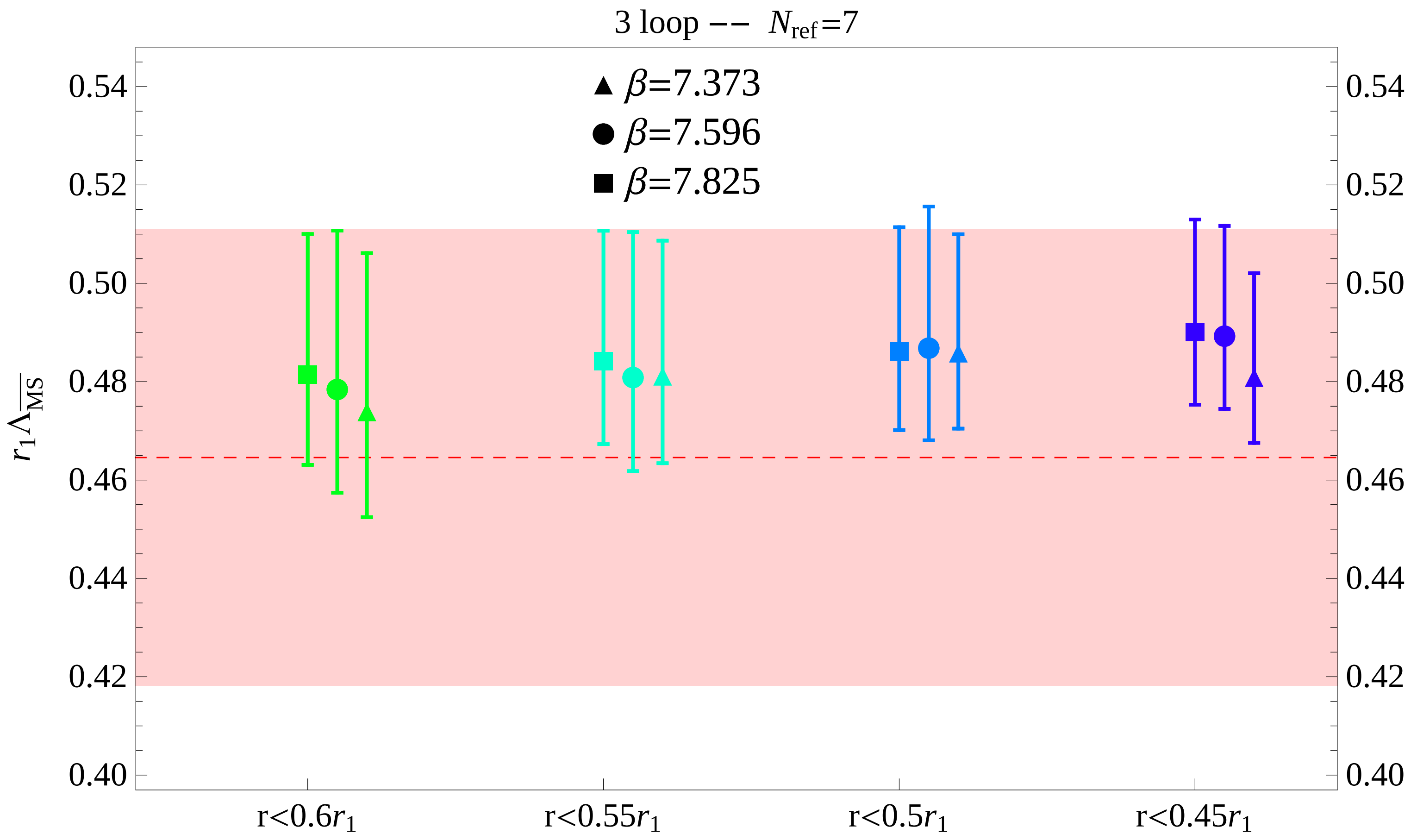
- Estimate pert. uncertainty: Repeat fits with scale variation, and adding $\pm(C_F/r^2)\alpha_{\rm s}^{n+2}$


β=7.596 -- N_ref=7
r<0.75r_1
r<0.7r_1
r<0.65r_1
r<0.6r_1
r<0.55r_1
r<0.5r_1
r<0.45r_1
χ²/d.o.f.
0
5
10
15
20
25
tree level
1 loop
2 loop
3 loop



β=7.596 –– Nref=7
a Λ_MS
r<0.5r_1
0.12
0.11
0.10
0.09
0.08
0.07
0.06
tree level
1 loop
2 loop
3 loop


3 loop −− N_{ref}=7
▲ β=7.373
● β=7.596
■ β=7.825
$r_1\Lambda_{\overline{MS}}$
0.54
0.52
0.50
0.48
0.46
0.44
0.42
0.40
r<0.6r_1
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All the results perfectly compatible with each other. It shows that the extraction is robust

Result for α_s

$$r_1\Lambda_{\overline{\rm MS}} = 0.495^{+0.028}_{-0.018} \to \Lambda_{\overline{\rm MS}} = 315^{+18}_{-12}\ \mathrm{MeV}$$

$$\alpha_s(1.5\ \mathrm{GeV}, n_f = 3) = 0.336^{+0.012}_{-0.008}$$

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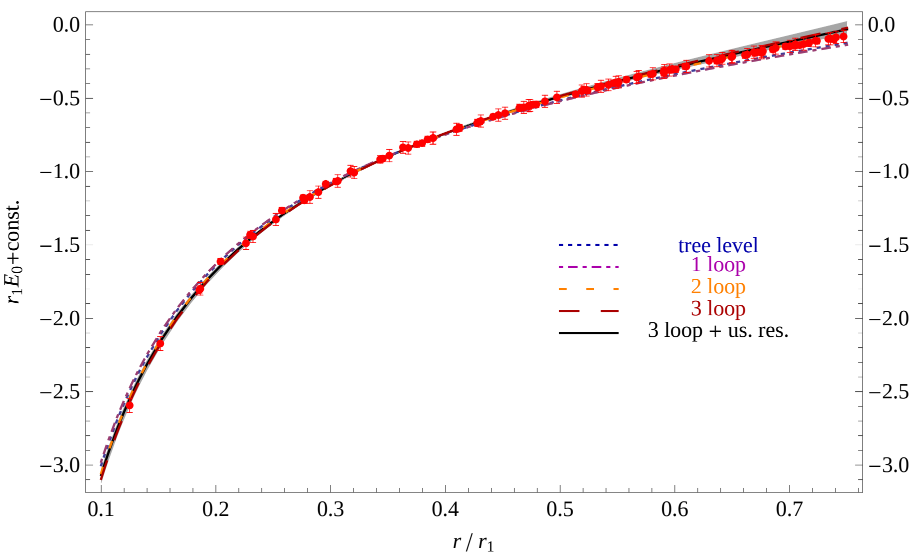
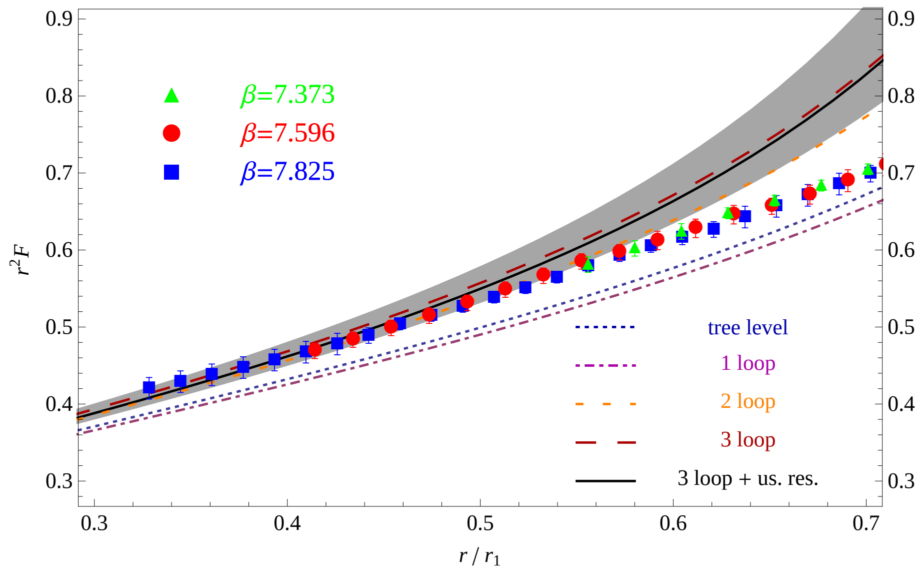
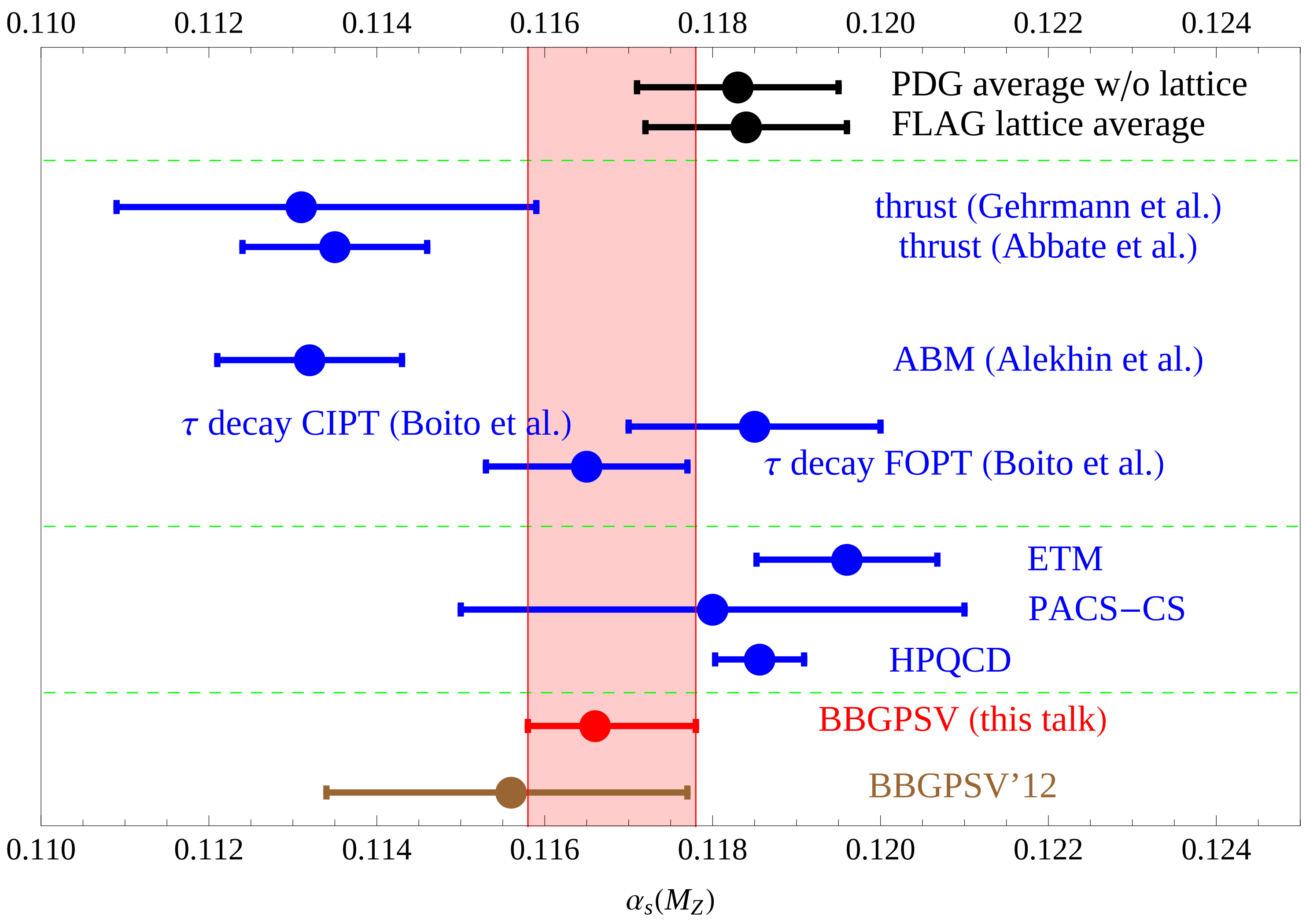




Comparison with other results



Conclusions

Determination of α_s by comparing lattice data for the short-distance part of the QCD static energy with perturbation theory (3 loop + resummation of ultrasoft logs accuracy)

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