Threshold production of top-quark pairs at NNNLO

Jan Piclum

UNIVERSITÄT BERN

based on work in collaboration with

M. Beneke, Y. Kiyo, P. Marquard,

A. Penin, T. Rauh, D. Seidel, M. Steinhauser

Threshold Production

relative velocity of quark-antiquark pair is small:

$$\sqrt{s} = E + 2m_Q \approx 2m_Q \quad \Rightarrow \quad v = \sqrt{\frac{E}{m_Q}} \ll 1\,; \quad v \sim \alpha_s(m_Q v)$$

- multi-scale problem: mass m_Q, momentum $m_Q v$, energy $m_Q v^2$
- perturbation theory breaks down due to terms proportional to $\frac{\alpha_s}{v}$
  $\rightsquigarrow$ Coulomb resummation
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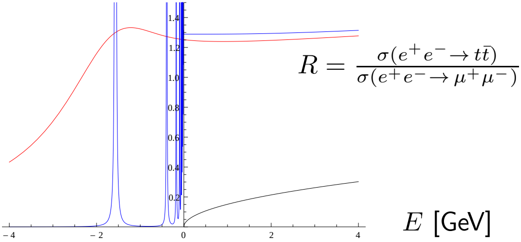
- formation of bound states below threshold
- $b\bar{b}$: bound-state resonances
- $t\bar{t}$: large width prevents existence of bound states



Effective Theory Setup

[Thacker, Lepage; Lepage, Magnea, Nakhleh, Magnea, Hornbostel; Bodwin, Braaten, Lepage]
[Pineda, Soto; Beneke, Signer, Smirnov; Brambilla, Pineda, Soto, Vairo]

scale hierarchy: $m_t \gg m_t v \gg m_t v^2 \gg \Lambda_{\rm QCD}$

QCD	full theory

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↓ integrate out hard modes: $k^0 \sim k^i \sim m_t$
hard subgraphs become point-like vertices
$\rightsquigarrow$ hard matching coefficients

NRQCD

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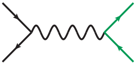
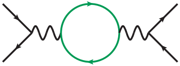
NRQCD

↓ integrate out soft modes: $k^0 \sim k^i \sim m_t v$
soft subgraphs become instantaneous, non-local vertices
$\rightsquigarrow$ potentials

pNRQCD — contains potential quarks and ultrasoft gluons use optical theorem:

$\int \mathrm{d}\Phi_{\mathrm{PS}} \Big|$  $\Big|^2 \sim \mathrm{Im}$  $\sim \mathrm{Im}\Pi(q)$

compute polarisation function in pNRQCD: $\Pi(q) \sim c_v^2\, G(\vec{0},\vec{0};E)$

- compute hard matching coefficients
- compute potential
- solve Schrödinger equation
- compute ultrasoft corrections

power counting:

$$\sum_n \left(\frac{\alpha_s}{v}\right)^n \times \Big\{1;\ \underbrace{\alpha_s, v}_{\text{NLO}};\ \underbrace{\alpha_s^2, \alpha_s v, v^2}_{\text{NNLO}};\ \underbrace{\alpha_s^3, \alpha_s^2 v, \alpha_s v^2, v^3}_{\text{NNNLO}};\ \dots\Big\}$$

Theory Status

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  - potential contributions [Beneke, Kiyo, Schuller; Kniehl, Penin, Smirnov, Steinhauser]
  - 3-loop static potential [Anzai, Kiyo, Sumino; Smirnov, Smirnov, Steinhauser]
  - ultrasoft corrections [Beneke, Kiyo, Penin]
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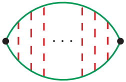
  - pNRQCD [Pineda, Signer]
- electroweak corrections [Guth, Kühn; Hoang, Reißer; Eiras, Steinhauser; Kiyo, Seidel, Steinhauser]
- finite-width effects [Fadin, Khoze; Hoang, Reißer, Ruiz-Femenía; Beneke, Jantzen, Ruiz-Femenía; Penin, JP]
- Higgs contribution [Harlander, Jeżabek, Kühn; Eiras, Steinhauser; Beneke, JP, Rauh]

Computing the Green's Function

Coulomb Green's function at the origin:

$$G_C(E) = \langle 0| \frac{1}{H_0 - E} |0\rangle = \langle 0| G^{(0)} |0\rangle$$



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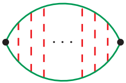
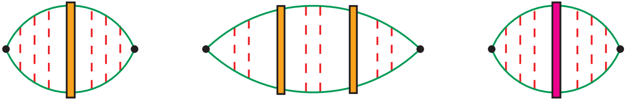


corrections due to higher order potential can be computed perturbatively:

$$\begin{aligned} \delta G(E) &= \langle 0 | \frac{1}{H_0 + \delta V - E} | 0 \rangle - G_C(E) \\ &= -\langle 0 | G^{(0)} \delta V^{(1)} G^{(0)} | 0 \rangle \\ &\quad + \langle 0 | G^{(0)} \delta V^{(1)} G^{(0)} \delta V^{(1)} G^{(0)} | 0 \rangle - \langle 0 | G^{(0)} \delta V^{(2)} G^{(0)} | 0 \rangle + \ldots \end{aligned}$$



Extracting Divergences

- divergent diagrams have to be calculated in $D = 4 - 2\epsilon$
- Coulomb Green function only known in $D = 4$
- adding Coulomb exchanges decreases degree of divergence

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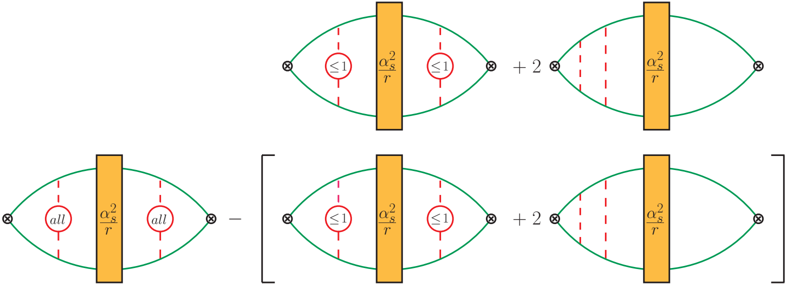
example: NLO P-wave contribution [Beneke, JP, Rauh, 2013]

divergent :

finite :



Finite-Width Divergences

- $E \to E + i\Gamma_t$ at LO $\rightsquigarrow$ Γ_t/ϵ at NNLO
- finite-width divergences lead to scale dependence
- dependence does not decrease in higher orders
- divergences are cancelled by non-resonant contributions

example: LO P-wave Green function [Beneke, JP, Rauh, 2013]

$$G_0^P(E) = \frac{m_t^4}{4\pi}\left\{\frac{\alpha_s C_F}{2}\left[\frac{1}{2\epsilon} - \ln\left(\frac{-4m_t E}{\mu_w^2}\right) + 4\right]\frac{E}{m_t} + \dots\right\}$$

$$\rightsquigarrow \mathrm{Im} G_0^P(E + i\Gamma_t) = \frac{m_t^4}{4\pi}\left\{\frac{\alpha_s C_F}{2}\left[\frac{1}{2\epsilon} - \ln\left(\frac{4m_t\sqrt{E^2 + \Gamma_t^2}}{\mu_w^2}\right) + 4\right]\frac{\Gamma_t}{m_t} + \dots\right\}$$

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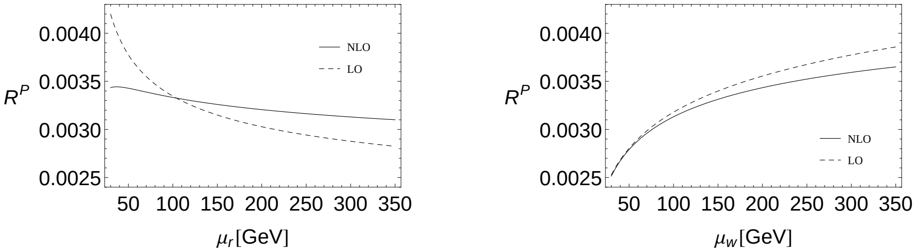
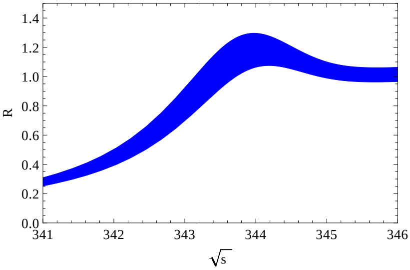
example: P-wave contribution close to threshold [Beneke, JP, Rauh, 2013]

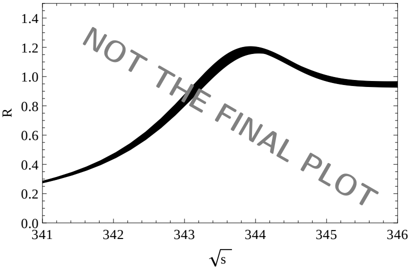


(effect is less pronounced for S-wave)

NNLO:

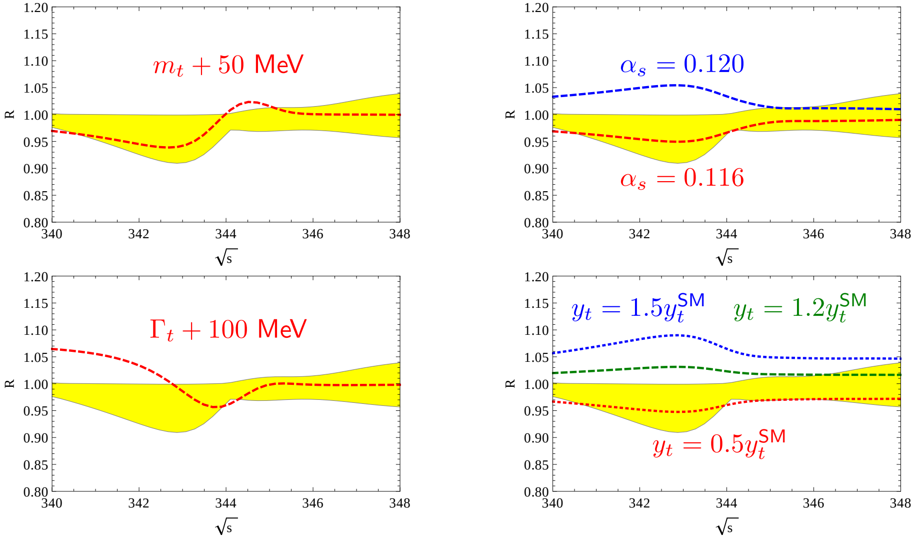


NNNLO:



PS scheme: $m_t^{\mathrm{PS}} = 171.5$ GeV, $\Gamma_t = 1.33$ GeV, $\alpha_s(M_Z) = 0.118$
$\mu = 50 - 350$ GeV

Parameter Dependence (preliminary)



Summary

- threshold production allows for very precise and theoretically clean determination of top-quark mass
- very high theoretical precision is required
- NNNLO pNRQCD calculation is almost done

lots of work still to be done, e.g.:

- electroweak/finite-width corrections
- combination of fixed-order and resummed calculations